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Research Article

Study of (*p*, *d*) reaction on ^{209}Bi at 42 Mev

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ABSTRACT

The ^{209}Bi(p,d)^{208}Bi reaction has been studied with 42 MeV protons. The laboratory angles are 25°, 30°, and 35°. Energy spectra have been calculated theoretically as an incoherent sum of many shell-orbits constituents based on the Distorted-Wave Born Approximation (DWBA). An asymmetric Lorentzian form strength response function having energy-dependent spreading width is adopted in this analysis. The calculated deuteron energy spectrum is reasonably well reproduced in the direct reaction region. The agreement between the experiment and the calculated double differentials cross-sections up to all experimental points is reasonable and satisfactory, thereby yielding confidence in the theoretical models used.

Introduction

One nucleon transfer is fundamental to applications; however, this is the fundamental research for any application. Removing or adding one neutron from a nucleus using a (d,p) or (p,d) transfer reaction has been used popularly for nuclear structure studies (Timofeyuk and Johnson, 2020). All studies concerning nuclear technology depend on nuclear data. In order to study the operation and performance of fission power plants, fusion devices and accelerators, simulation codes must have a wide range of nuclear data such as cross-sections and decay properties for all the materials of interest in the device (Forrest, 2011). Therefore, there is a continuous demand for high-quality updates of the main nuclear physics databases of the experimental and theoretical results for nucleons induced interactions at various incident energies and for all kinds of emitted particles all over the energy and the emission angles.

Generally, the spectrum of the emitted particles from one nucleon transfer reaction can be divided into three parts, i.e., direct, pre-equilibrium, and evaporation processes. The evaporation and pre-equilibrium processes are dealt with statistical models, and in present days some computer codes are available to calculate double differential cross sections. However, the continuum spectrum in the direct reaction region is too scanty to analyze easily. Research programmes had been carried out to build theoretical models by Lewis (Lewis, 1975) and Mataba et al. (Matoba et al., 1995 and 1996).

Therefore, these authors proposed predictive models for analyzing continuum spectrum in the direct reaction region, using an asymmetric Lorentzian shaped strength function having energy-dependent spreading widths multiplied by Distorted-Wave Born Approximation (DWBA) predicted cross section for the reproduction of double differential cross-sections. These models were successfully applied for the (p,d) reactions (Syafarudin et al., 2002; Sultana et al., 2004) and then applied for the (n,d) reactions (Sultana et al., 2003; Sultana, 2006; Sultana et al., 2009; Sultana et al., 2010; Sultana, 2016) with a slight modification, which demonstrated reasonable ability to reproduce the data. Proton-induced neutron

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pick-up reaction on ^{209}Bi for the direct reaction region has been studied in the present work at Ep = 42 MeV. Here, the laboratory angles are 25°, 30°, and 35° for the said incident energy.

The theoretically calculated spectra are compared with the experimental ones, and it is observed that the measured spectra are well reproduced.

Theoretical Approach

Direct Reaction Calculations

The experimental data for the analysis are taken from (Harada et al., 2002). The differential cross-section $\frac{d\sigma}{d\Omega}$, is the cross-section per unit solid angle and is a function of reaction angle and energy. DWUCK-4 calculates the differential cross-section at an angle for incident energy given as an input. $\frac{d^2\sigma}{d\Omega dE}$, the double differential cross-section, at the reaction angle is the cross-section per unit solid angle and unit energy interval of the ejectile. The theoretical calculations of the double differential cross-sections have been performed by considering a direct reaction model as an incoherent sum of the DWBA-predicted direct reaction components over an energy interval and multiplied by spin-weighted spectroscopic factors of the predicted states expressed as:

$$\frac{d^2\sigma}{d\Omega dE} = 2.30 \sum_{l,j} \left[\frac{C^2 S_{l,j}(E)}{2j+1} \times \left(\frac{d\sigma}{d\Omega}\bigg|_{l,J}^{DW} (E) \right) \right] \qquad (1)$$

where $d\sigma/d\Omega|_{l,j}^{DW}(E)$ is the cross-section calculated by the DWBA code, DWUCK-4 (Kunz, unpublished). $C^2 S_{l,j}(E)$ are the spectroscopic factors, expressed as:

$$C^2 S_{l,j}(E) = \left(\sum C^2 S_{l,j} \right) \times f_{l,j}(E) \qquad (2)$$

where C^2 is Clebsch-Gordan coefficients for coupling angular momenta (l,j). $S_{l,j}(E)$ is the spectroscopic factor. $f_{l,j}(E)$ in eqn. (2) refers to the distribution of the strength function, describe the deuteron spectra, and is expressed as an asymmetric Lorentzian function (Matoba et al., 1995 and 1996; Mahaux and Sartor, 1989 and 1991):

$$f_{l,j}(E) = \frac{n_0}{2\pi} \frac{\Gamma(E)}{\left(|E-E_F|-E_{l,j}\right)^2 + \Gamma^2(E)/4} \qquad (3)$$

The strength function $f_{l,j}(E)$ integrates over the energy integrand dE to unity starting from certain initial energy to a final energy of the ejectile, the condition

$$\int_i^f f_{l.j}(E) dE = 1 \qquad (4)$$

has to be satisfied.

The deuteron is ejected at various energies starting from ~ 16 to 37 MeV (Figs. 1, 2, and 3). The incident proton energy is 42 MeV for all the angles $\theta_{\text{Lab}} = 25°$, 30°, and 35°.

n_0 in eqn. (3) is the renormalization constant, and E_F is the Fermi energy. The Fermi energy can be calculated using an empirical formula given by Hisamochi et al. (Hisamochi et al.,1993). The sum rule of the spectroscopic factor and the centroid energy ($E_{l,j}$) calculations for $j = l \pm \frac{1}{2}$ shell orbits have been carried out by using a Bardeen, Cooper, and Schrieffer (BCS) calculation (Bardeen et al., 1957). The concept is taken from the behaviour of electrons in the superconducting phenomenon to estimate a nucleon occupancy rate in spin-orbit space. In this calculation, the required single- particle energies are calculated by the prescription of Bohr and Mottelson (Bohr and Mottelson, 1996). Spreading width (Γ) is expressed by a function proposed by Brown and Rho (Brown and Rho, 1981) as well as by Mahaux and Sartor (Mahaux and Sartor, 1989 and 1991) as:

$$\Gamma(E) = \frac{\epsilon_0 (E - E_F)^2}{(E - E_F)^2 + E_0^2} + \frac{\epsilon_1 (E - E_F)^2}{(E - E_F)^2 + E_1^2} \qquad (5)$$

where ϵ_0, ϵ_1, $E_{0,}$ and E_1 are constants and express the effects of nuclear dumping in the nucleus (Matoba et al., 1995). The estimated values of the constants (Matoba *et al.*, 1995) are

ϵ_0=19.4 MeV, E_0 =18.4 MeV,

ϵ_1= 1.40 MeV, E_1 =1.60 MeV (6)

The sum rule of the spectroscopic factors of nucleon orbits for $T \pm \frac{1}{2}$ isospin states are calculated with shell model prescription (French and Macfarlane, 1961)

$$C^2 S_{l,j} = \frac{n_{p(l,j)}}{2T+1} \qquad (7)$$

Here, $n_{p(l,j)}$ and $n_{n(l,j)}$ are the numbers of protons and neutrons respectively for each (l, j) orbit, and T is the target isospin.

Calculation of spectroscopic factor in Seniority Scheme

Seniority is defined as a number of nucleons that appear from the breakdown of nucleon pairing.

The spectroscopic factor for one nucleon separation from n particles in a shell for even and odd particle systems are, respectively,

$C^2S = n$ and $C^2S = \frac{2j+2-n}{2j+1}$ (8)

We calculate C^2S from the BCS formalism (Bardeen *et al.,* 1957). It is appropriate to multiply the strength function by a constant as follows:

The C^2S for the ground state and for low-lying states resulting from the n particle system can be estimated, multiplying by a constant χ, where χ is:

$$\chi = \frac{\frac{2j+2-n}{2j+1}}{n} \qquad (9)$$

Results and Discussion

Double differential cross-sections (DDXs) are calculated for the ^{209}Bi(p,d)^{208}Bi reaction at 42 MeV. The laboratory angles are 25°, 30°, and 35°, as shown in Figs. 1, 2, and 3. Experimental and theoretical results are given by the circles and lines, respectively. Three global potentials (Becchetti and Greenlees, 1969; Koning and Delaroche, 2003; Menet et al., 1971) are used here for protons, while for deuteron, an adiabatic potential (Becchetti and Greenlees 1969; Koning and Delaroche, 2003; Menet et al., 1971) based on the proton and neutron potentials are constructed for the DWUCK-4 calculations as shown in Table 1.

The solid, dotted, and short-long-dashed lines represent the DDXs for Beccheti and Greenlees (Becchetti and Greenlees, 1969), Koning and Delaroche (Koning and Delaroche, 2003), and Menet et al. (Menet et al., 1971) potentials, respectively. As ^{209}Bi is an odd mass target nucleus, so seniority scheme has been applied to extract the spectroscopic factor of the ground state. But for ^{209}Bi, the multiplication factor of the seniority scheme is 1. Therefore, no modification is required in the seniority scheme in the case of ^{209}Bi.

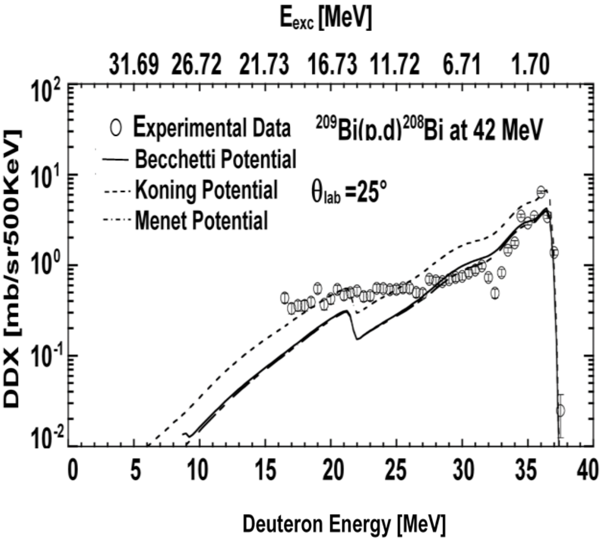

Fig. 1. Double differential cross sections for the ^{209}Bi(p,d)^{208}Bi reaction at 42 MeV for 25° angle.

For Figs. 1, 2, and 3, at 25°, 30°, and 35° laboratory angles, the predicted DDXs are in good agreement with experimental ones for the Beccheti and Greenlees (1967) and Menet et al. (1971) potentials in the lower excitation region. Still, their description isS somewhat underestimated above 12–13 MeV excitation energy. The satisfactory prediction is possible with Koning and Delaroche's (Koning and Delaroches, 2003) potential throughout the measured excitation energies except for a short span of excitation energy ∼ 3-4 MeV in the very forward angles, where the theoretical description is a little bit overestimated.

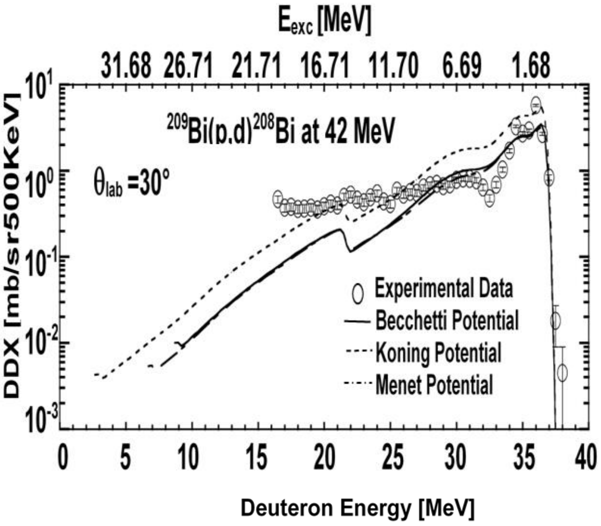

Fig. 2. Double differential cross sections for the ^{209}Bi(p,d)^{208}Bi reaction at 42 MeV for 30° angle.

It is evident that the theoretical predictions generally agree with the experimental ones. There is still some scope for improving predicted data by using other optical model potentials.

It should be noted that the calculated double differential cross sections agree with experimental data up to about ∼ 18 MeV excitation energy region and not to speak of the direct reaction region where the agreement between the experiment and theory is equally good by three different sets of optical model potentials (Becchetti and Greenlees, 1969; Koning and Delaroche, 2003; Menet et al., 1971).

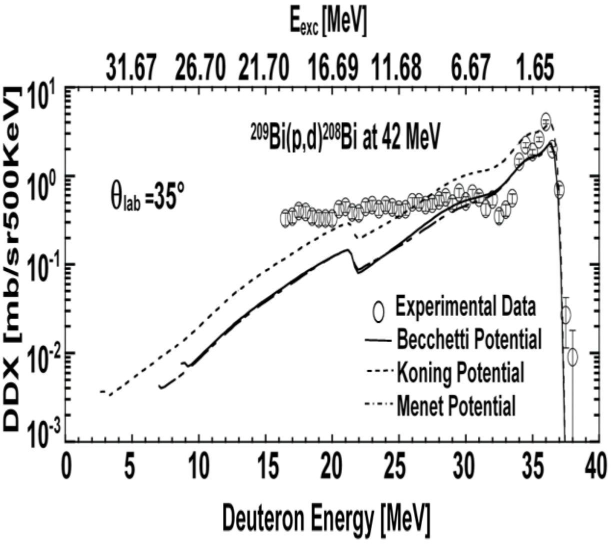

Fig. 3. Double differential cross sections for the ^{209}Bi(p,d)^{208}Bi reaction at 42 MeV for 35° angle.

Conclusions

The ^{209}Bi(p,d)^{208}Bi reaction has been analyzed with a direct reaction model. Here, the incident energy is 42 MeV. The calculated DDXs agree with the experimental data both in magnitude and shape. Three global potentials are used for proton and deuteron channels in the theoretical model for the DWUCK-4 calculation for being confident about the theoretical method of reproducing DDXs. Nuclear data for the charged particles production through reactions induced either by proton or neutron are rather scarce. More so with the double-differential cross-section data by proton and neutron, where the data is limited, the quality of data is relatively poor because of experimental difficulties. That is why theoretical models with strong predictive ability are required to simulate the experimental data. Our effort was to reproduce the DDXs in the direct region. We believe that the different optical model parameters may improve the quality of agreement between the experiment and theory.

It is be noted that nuclear reaction data, especially proton and neutron-induced reactions though not directly related to practical application, but are the prerequisite for the fundamental research works for any practical application.

Table 1. Optical model Table parameters used in the DWBA calculations for the ^{209}Bi(p,d)^{208}Bi reaction at 42 MeV.

Becchetti and Greenlees potential in ref. (Becchetti and Greenlees, 1969)

Particle	V	r	a	r_c	W_v	W_s	r′	a′	V_{so}	r_{so}	a_{so}
	(MeV)	(fm)	(fm)	(fm)	(MeV)	(MeV)	(fm)	(fm)	(MeV)	(fm)	(fm)
Proton	51.09	1.17	0.75	1.25	6.54	3.77	1.32	0.65	6.20	1.01	0.75
Deuteron	a	1.17	0.78	1.25	b	b	1.29	0.65	6.20	1.06	0.75
Neutron	c	1.25	0.65								

[a] $V = 110.3 - 0.64(E_d/2) + 0.4Z/A^{1/3}$ (MeV)
[b] $W_v = 0.44(E_d/2) - 4.26$ (MeV), $Ws = 24.8 - 0.50(E_d/2)$ (MeV), E_d is the deuteron kinetic energy.
[c] Well depth adjusted to fit the separation energy.

Koning and Delaroche potential in ref. (Koning and Delaroche, 2003)

Particle	V	r	a	r_c	W_v	W_s	r′	a′	V_{so}	r_{so}	a_{so}
	(MeV)	(fm)	(fm)	(fm)	(MeV)	(MeV)	(fm)	(fm)	(MeV)	(fm)	(fm)
Proton	39.05	1.24	0.65	1.22	3.62	7.75	1.25	0.51	5.42	1.08	0.59
Deuteron	a	1.24	0.65	1.22	a	a	1.25	0.57	a	1.08	0.59
Neutron	b	1.25	0.65								

[a] Adiabatic potentials with those of ref. (Koning and Delaroche, 2003)
[b] Well depth adjusted to fit the separation energy.

Menet et al. potential in ref. (Menet et al., 1971)

Particle	V	r	a	r_c	W_v	W_s	r′	a′	V_{so}	r_{so}	a_{so}
	(MeV)	(fm)	(fm)	(fm)	(MeV)	(MeV)	(fm)	(fm)	(MeV)	(fm)	(fm)
Proton	51.69	1.16	0.75	1.25	4.98	5.29	1.37	0.61	6.04	1.06	0.75
Deuteron	a	1.16	0.75	1.25	b	b	1.37	c	6.04	1.06	0.75
Neutron	d	1.25	0.65								

[a] $V = 99.8 - 0.44(E_d/2) + 0.4Z/A^{1/3}$ (MeV)
[b] $W_v = 2.4 + 0.18(E_d/2)$ (MeV), $Ws = 8.40 - 0.10(E_d/2)$ (MeV), E_d is the deuteron kinetic energy.
[c] $a' = 0.74 - 0.008(E_d/2) + 1.0(N\text{-}Z)/2A$, [d] Well depth adjusted to fit the separation energy.

For all potentials nonlocality, finite-range and spin orbit term are shown below:

Particle	Nonlocality parameters (fm)	Finite-range parameter (fm)	Thomas-Fermi spin orbit term
Proton	0.85	0.621	λ= 25
Neutron	0.85	0.621	
Deuteron	0.54		

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Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this article.

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