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Research Article

Goal programming approach for multi-objective optimization to the transportation problem in uncertain environment using fuzzy non-linear membership functions

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ABSTRACT

The ultimate goal of the decision maker (DM) is to take right decisions to optimize the profit or loss of the organization when the parameters of the transportation problem are ambiguous because of some uncontrollable effects. In this paper, mathematical models are proposed using fuzzy non-linear membership functions and the inverse uncertain normal distribution has been used to eliminate the uncertainty in the parameters which will help the DM to find a compromise solution of the uncertain multi-objective transportation problem (UMOTP) and to achieve the desired goals for a chosen level of confidence for the uncertain parameters. The compromise solutions of the uncertain multi-objective transportation problem are presented to obtain the DM satisfaction if the problem becomes achievable for this preferred confidence level of the parameters. Numerical illustration is given where Linear Programming Problems (LPPs) are resolved with LINGO and the graphs are designed with the help of MATLAB 18.00.

Introduction

When the aspiration level to each of the objectives in a Multiobjective Transportation Problem (MOTP) is identified, the fuzzy objectives turned as fuzzy goal programming. These fuzzy goals are then can be characterized by the fuzzy membership functions. In the present paper, goals of the DM for the specific objectives are considered as the goal to achieve. MOTP is a very distinctive variety of linear programming problem (LPP) where the restraints are equality or inequality form and the purposes are varying from each other. The primal simplex method in transportation problem was used by (Dantzing, 1963). All the proposed methods to solve MOTP breed a set of compromise solution. The Goal programming technique with the priority along with various situations such as environmental constraints, organizational goal and bureaucratic decision structures and many more has a vast use to solve different problems involving multiple objectives. (Zadeh, 1965), (Bellman and Zadeh, 1970) gave a brief description about a new technique for decision making in a fuzzy situation. (Lee and Moore, 1973) optimize the TP with numerous objectives applying the goal programming concept. (Zimmermann, 1978) applied the fuzzy programming and linear programming concept with numerous objective functions with some fuzzy membership function to solve MOTP. The converted ordinary values of the uncertain parameters were calculated using uncertain normal distribution proposed by (Liu, 2008), (Liu, 2010), (Liu, 2009(b)). (Wahed and Lee 2006), together with the concept of fuzzy membership function, formulate a LPP that develop the uncertain measure theorem. (Hasan 2017) uses fuzzy TOPSIS for a perfect choice that can reduces the cost and sufferings of the DM. (Hasan 2015) developed an algorithm to ensure the quickest flow of material at the lowest cost.

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Table 1. Related Research and their Contributions

References	Objective Functions			Uncertain Supply	Uncertain Demand	Using Fuzzy Logic	Goal Programming
	Uncertain TP Cost	Uncertai n Profit	Uncertain Damage cost				
Anukokila et al (2017)	–	–	–	–	–	√	√
Bit et al. (1993)	√	√	–	–	–	√	–
Bit et al. (1992)	–	–	–	–	–	–	√
Cadenas and Verdegay (2000)	–	–	–	–	–	√	-
Liu (2008)	√	–	√	–	–	–	–
Das et al. (1999)	√	√	√	–	–	–	–
Wahed (2001)	–	–	–	–	–	√	√
Maity and Roy (2015)	–	–	–	√	√	–	–
Roy and Midya (1988)	√	–	–	–	–	√	–
Surapati and Roy (2008)	–	–	–	–	–	√	√
Shenh and Yao (2012)	√	–	√	–	–	–	–
Zangiabadi and Maleki (2007)	–	–	–	–	–	–	√
Jagtap and Kawale (2017)	–	–	–	–	–	–	√
Wahed and Lee (2006)	–	–	–	–	–	√	√
Wahed and Abo-Sinna (2001)	–	–	–	–	–	√	√
Gupta and Kumar (2012)	–	–	–	–	–	√	–
Delgado et al. (1989)	–	–	–	–	–	√	–
Kaur and Kumar (2011)	–	–	–	–	–	–	√
This study	√	√	√	√	√	√	√

From the literature review and the interpretations provided in Table 1, it can be seen that some gap attained in model development for MOTP in uncertain environment using fuzzy goal programming. In the present research, we have work hard to remove this gap by extending the work of (Wahed and Lee, 2006) combining the uncertainty not only in the objective functions but also in all parameters which will be very helpful for the DM regarding decision making in very uncertain situation. The notable executions of the designed study are shortened as follows:

- Fuzzy goal programming is implemented to an uncertain parameter problem.
- Uncertain MOTP is changed applying the uncertain normal distribution concept.
- DM’s confidence levels are in consideration
- Non-linear membership functions of fuzzy programming approach are used to model the algorithm.

Nomenclature

Notations	Descriptions
x_{ij}	Transporting amount from i^{th} origin to j^{th} destination
C_{ij}	Cost parameter in unit price for i^{th} origin to j^{th} destination
a_i	Supply parameters
b_j	Demand parameters
$\hbar$	Uncertain variable
$\Re$	Uncertain distribution
ϕ	Normal uncertain distribution
Ω	Uncertain measure function
e	Expected value of the parameter
σ	Standard deviation
ω	Confidence level
η	Satisfaction level of the DM
d_i^+	Positive deviation or over achievement from i^{th} goal
d_i^-	Negative deviation or under achievement from i^{th} goal
$\mu_k(Z^k)$	Linear membership function for the k^{th} objective
β_{ij}	Uncertain distribution for cost
θ_i	Uncertain distribution for demand
ψ_j	Uncertain distribution for supply
G^k	Desired goal of k^{th} objective
Z^k	Objective of the k^{th} goal
L_k	Lower bound of the k^{th} objective
U_k	Upper bound of the k^{th} objective

Materials and Methods

If C_{ij} is the per unit transportation cost from i^{th} origin to j^{th} destination and x_{ij} is the unknown quantity to be shifted from the i^{th} origin to j^{th} destination then the original transportation model is written as:

$$\left.\begin{array}{l} \text{Minimize} \quad : Z(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij}. \\ \text{Subject to :} \\ \sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots, m \\ \sum_{i=1}^{m} x_{ij} \ge b_j, \quad j = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots, n \\ \forall\ x_{ij} \ge 0, \quad i = 1,2,\ldots m; \quad j = 1,2,\ldots, n \end{array}\right\} \ldots(1)$$

And the feasibility condition is $\sum_{i=1}^{m} a_i \ge \sum_{j=i}^{n} b_j$.

Where $a_1, a_2, \ldots, a_m$ are the m sources (origins) and $b_1, b_2, \ldots, b_n$ are the n destinations (demands) Taking the multiple objective idea, the original model can be written as follows:

$$\left.\begin{array}{l} \text{Minimize} \quad : Z(x) = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}^{k}x_{ij}. \\ \text{Subject to :} \\ \sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots, m \\ \sum_{i=1}^{m} x_{ij} \ge b_j, \quad j = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots, n \\ \forall\ x_{ij} \ge 0, \quad i = 1,2,\ldots, m; \quad j = 1,2,.., n \end{array}\right\} \ldots(2)$$

feasibility condition $\sum_{i=1}^{m} a_i \ge \sum_{j=i}^{n} b_j$, where C_{ij}^{k} is the unit cost for transporting from i^{th} origin to j^{th} destination , $a_i\ (i = 1,2,\ldots\ldots, m)$ is the supply and $b_j\ (j = 1,2,\ldots\ldots, n)$ is the demand parameter for the $k^{th}\ (k = 1,2,\ldots\ldots, K)$ objective function of the MOTP.

Introducing inverse measure theorem, equation (2) can be written as follows:

$$\left.\begin{array}{l}\text{Max/Min } : Z^k(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\Omega\left(C_{ij}^k\right)\geq \alpha_{ij}\right]x_{ij}.\\ \text{Subject to }:\\ \Omega\left(\sum_{j=1}^{n}x_{ij}\leq a_i\right)\geq \gamma_i \quad i=1,2,3,\dots\dots\dots,m\\ \Omega\left(\sum_{i=1}^{m}x_{ij}\geq b_j\right)\geq \delta_j, \quad j=1,2,3,\dots\dots,n\\ \forall\ x_{ij}\geq 0,\quad i=1,2,\dots\dots m;\quad j=1,2\dots\dots,n\end{array}\right\}\dots(3)$$

The inverse normal uncertainty distribution of Normal uncertain variable $N(e,\sigma)$ is defined as $\Re^{-1}(x)=e+\frac{\sqrt{3}\sigma}{\pi}\ln\left(\frac{\omega}{1-\omega}\right)$,where' $\ln$ 'denotes natural logarithm and ω is the confidence level of the DM. Considering the uncertainty distributions β_{ij}, θ_i and ψ_j for cost C_{ij} $(i=1,2,\dots\dots,m;\ j=1,2,\dots\dots,n)$, demand $a_i(i=1,2,\dots\dots,m)$ and supply parameters $b_j(j=1,2,\dots\dots,n)$ respectively, the inverse measure shows the following results:

$\Omega\left(C_{ij}^k\right)\geq \alpha_{ij}\approx C_{ij}^k\geq \beta_{ij}^{-1}\left(1-\alpha_{ij}\right)$;

$\Omega\left(\sum_{j=1}^{n}x_{ij}\leq a_i\right)\geq \gamma_i \Rightarrow \sum_{j=1}^{n}x_{ij}\leq \Re_i^{-1}(1-\gamma_i)$ and

$\Omega\left(\sum_{i=1}^{m}x_{ij}\geq b_j\right)\geq \delta_j$ is reduced to $\sum_{i=1}^{m}x_{ij}\leq \psi_j^{-1}\delta_j$.

Then equation (3) is reduced as follows:

$$\left.\begin{array}{l}\text{Max/Min } : Z^k(x)=\sum_{i=1}^{m}\sum_{j=1}^{n}\left[\beta_{ij}^{-1}\left(1-\alpha_{ij}\right)\right]x_{ij}.\\ \text{Subject to }:\\ \sum_{j=1}^{n}x_{ij}\leq \theta_i^{-1}(1-\gamma_i),\quad i=1,2,3,\dots\dots\dots,m\\ \sum_{i=1}^{m}x_{ij}\leq \psi_j^{-1}\delta_j, \qquad j=1,2,3,\dots\dots\dots,n\\ \forall\ x_{ij}\geq 0,\quad i=1,2,\dots m;\quad j=1,2,\dots\dots,n\end{array}\right\}\dots(4)$$

Model for Non-linear Membership Function

Step 1: Obtain the crisp values of the parameters using normal uncertain distribution.

Step 2: Construct the given MOTP for uncertain parameters to an ordinary TP using the crisp number obtained from step 1.

Step 3: Single objective TP are solved ignoring all others objectives.

Step 4: Define non-linear membership function $\mu(Z^k)$ for the k^{th} objective function.

Step 5: Change the fuzzy model obtained in **Step** 4, as follows:

$$\left.\begin{array}{l}\text{Maximize } \quad \xi\\ \text{Subject to}\\ \xi\leq \mu\left(Z^k\right)\\ \sum_{j=1}^{n}X_{ij}\leq a_i, i=1,2,3,\dots\dots m\\ \sum_{i=1}^{m}X_{ij}\geq b_j, j=1,2,3,\dots\dots,n\\ \xi\geq 0,\quad X_{ij}\geq 0, i=1,2,\dots,m;\\ j=1,2,\dots,n\end{array}\right\}\dots(5)$$

Step 6: Advance Step 5 as a goal programming problem as follows:

$$\left.\begin{array}{l}\text{Maximize } \quad \xi\\ \text{Subject to}\\ \xi\leq \mu\left(Z^k\right)\\ \sum_{j=1}^{n}X_{ij}\leq a_i,\quad i=1,2,3,\dots,m\\ \sum_{i=1}^{m}X_{ij}\geq b_j,\quad j=1,2,3,\dots,n\\ Z^k(x)-d_k^{+}+d_k^{-}=G^k,\\ k=1,2,3,\dots\dots\dots,K\\ \xi\geq 0,\quad X_{ij}\geq 0,\\ i=1,2,\dots\dots m; j=1,2,\dots\dots,n\end{array}\right\}\dots(6)$$

Step 7: Solve the model in **Step** 6.

Step 8: Reveal the feasible solution to the DM. If DM satisfies, go to **Step** 9; otherwise, go through **Step** 1 to **Step** 7.

Step 9: Stop.

For the computational algorithm, a flow chart is presented in Fig.1.

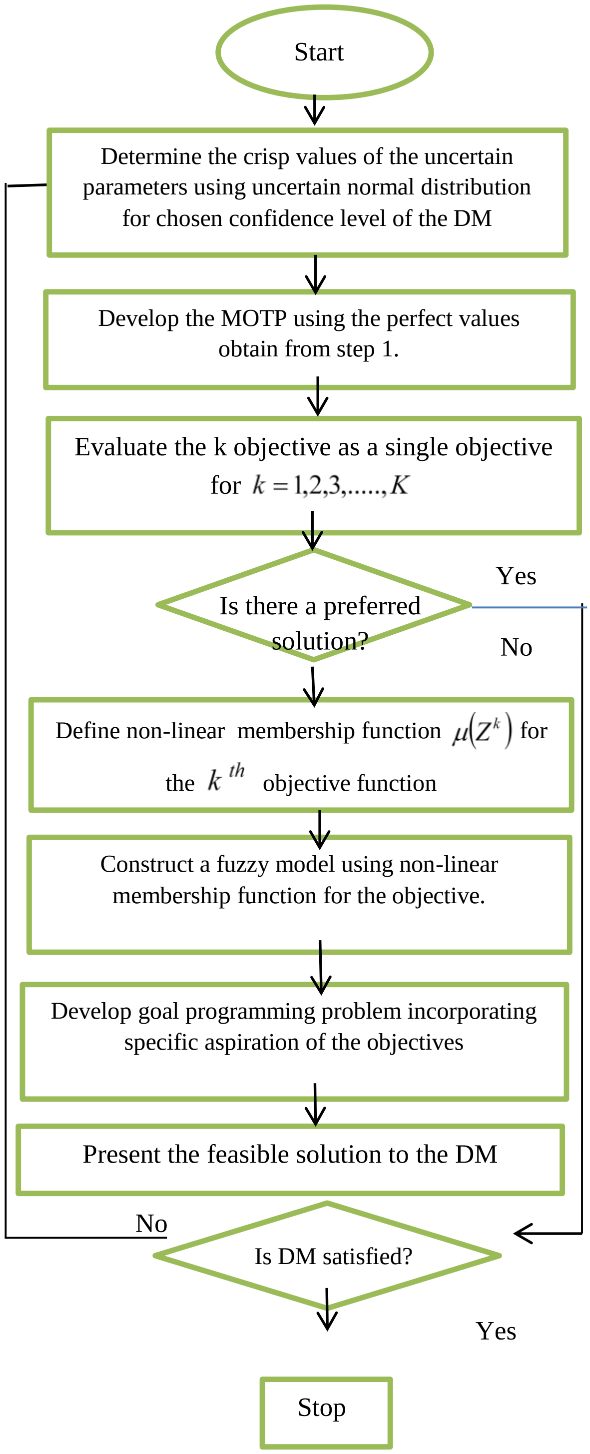

Fig. 1. Flow chart of the proposed model for goal programming using non-linear membership functions

Results and Discussions

To testify the feasibility of the proposed model, consider a MOTP with the uncertainty in transportation cost, profit, damage cost due to delay or early supply. Demand and supply in the market are also considered as uncertain. The DM wants to deliver the goods from three sources S_1, S_2, S_3 to four destination points D_1, D_2, D_3, D_4 and optimize the objectives as:

(i) The transportation cost must be from \$3000 to \$3500 (Z^1)

(ii) Profit will be not more \$1200 (Z^2) and not less than \$900

(iii) Damage cost have to be in between \$700 to \$1000 (Z^3)

The data tables of this example are taken from Uddin et al., 2021) and (Maity et al., 2016):

Table 2. Data for uncertain transportation cost C_{ij}^1

	D_1	D_2	D_3	D_4
S_1	(20, 2)	(18, 2)	(22, 3)	(24, 3)
S_2	(10, 1)	(12, 2)	(15, 3)	(13, 1)
S_3	(22, 3)	(20, 3)	(24, 2)	(23, 2)

Table 3. Data for uncertain profit C_{ij}^2

	D_1	D_2	D_3	D_4
S_1	(5,1)	(6, 1.5)	(4, 1)	(3, 0.5)
S_2	(6,1)	(5,1.5)	(5, 0.5)	(4,1)
S_3	(9,1)	(8,1.5)	(8, 2)	(10, 2)

Table 4. Data for uncertain damage cost C_{ij}^3

	D_1	D_2	D_3	D_4
S_1	(4,1)	(4,1)	(3,1)	(5, 2)
S_2	(3,1)	(6,1)	(4,1)	(4,1)
S_3	(4, 1.5)	(3,1)	(4,1)	(5, 1.5)

Table 5. Data for uncertain demand

b_1	b_2	b_3	b_4
(40, 3)	(36, 4)	(35, 5)	(40, 3)

Table 6. Data for uncertain supply

a_1	a_2	a_3
(55, 4)	(60, 5)	(70, 4)

Mathematical Illustration using Exponential Membership Function (EMF)

Using the inverse uncertain distribution with confidence level $\omega = 0.78$ Tables 2, 3, 4, 5 and 6 reduced to Tables 7, 8, 9, 10 and 11 respectively.

Table 7. Crisp value for transportation cost C_{ij}^1 for confidence level $\omega = 0.78$

	D_1	D_2	D_3	D_4
S_2	10.70	13.40	17.10	13.70
S_3	24.10	22.10	25.40	24.40

Table 8. Crisp value for profit C_{ij}^2 for confidence level

	D_1	D_2	D_3	D_4
S_1	5.70	7.05	4.70	3.35
S_2	6.70	6.05	5.35	4.70
S_3	9.70	9.05	9.40	11.40

Table 9. Crisp value for damage cost C_{ij}^3 for confidence level $\omega = 0.78$

	D_1	D_2	D_3	D_4
S_1	4.70	4.70	3.87	6.40
S_2	3.70	6.70	4.70	4.70
S_3	5.05	3.70	4.70	6.05

Table 10. Crisp value for demand for confidence level $\omega = 0.78$

b_1	b_2	b_3	b_4
42.1	38.8	38.5	42.1

Table 11. Crisp value for supply for confidence level $\omega = 0.78$

a_1	a_2	a_3
52.2	56.5	67.2

In this present problem, the goal of the DM is the transportation cost must be from \$3000 to \$3500, profit will be no more \$1200 and no less than \$900 and the damage cost have to be in between \$700 to \$1000 (Z^3).

Therefore, the membership functions are:

$$\mu_1(Z^1(x)) = \frac{3500 - Z^1(x)}{3500 - 3000},$$

$$\mu_2(Z^2(x)) = \frac{1200 - Z^2(x)}{1200 - 900}, \quad \text{and}$$

$$\mu_3(Z^3(x)) = \frac{1000 - Z^3(x)}{1000 - 700}.$$

Let η be the satisfaction level of the DM, Then from model (6), we have the following LPPs: Max Subject to:

$$Z^1 = 21.4x_{11} + 19.4x_{12} + 24.1x_{13} + 26.1x_{14} + 10.7x_{21} + 13.4x_{22} + 17.1x_{23} + 13.7x_{24} + 24.1x_{31} + 22.1x_{32} + 25.4x_{33} + 24.4x_{34}$$

$$Z^2 = 5.7x_{11} + 7.05x_{12} + 4.7x_{13} + 3.35x_{14} + 6.7x_{21} + 6.05x_{22} + 5.35x_{23} + 4.7x_{24} + 9.7x_{31} + 9.05x_{32} + 9.4x_{33} + 11.4x_{34}$$

$$Z^3 = 4.7x_{11} + 4.7x_{12} + 3.7x_{13} + 6.4x_{14} + 3.7x_{21} + 6.7x_{22} + 4.7x_{23} + 4.7x_{24} + 5.05x_{31} + 3.7x_{32} + 4.7x_{33} + 6.05x_{34}$$

$$\exp\left(\frac{-Z^1 + 3500}{500}\right) \geq 0.63\xi + 0.37$$

$$\exp\left(\frac{-Z^2 + 1200}{200}\right) \geq 0.63\xi + 0.37$$

$$\exp\left(\frac{-Z^3 + 1000}{200}\right) \geq 0.63\xi + 0.37$$

$$21.4x_{11} + 19.4x_{12} + 24.1x_{13} + 26.1x_{14} + 10.7x_{21} + 13.4x_{22} + 17.1x_{23} + 13.7x_{24} + 24.1x_{31} + 22.1x_{32} + 25.4x_{33} + 24.4x_{34} + d_1^- - d_1^+ = 3000$$

$$5.7x_{11} + 7.05x_{12} + 4.7x_{13} + 3.35x_{14} + 6.7x_{21} + 6.05x_{22} + 5.35x_{23} + 4.7x_{24} + 9.7x_{31} + 9.05x_{32} + 9.4x_{33} + 11.4x_{34} + d_1^- - d_1^+ = 900$$

$$4.7x_{11} + 4.7x_{12} + 3.7x_{13} + 6.4x_{14} + 3.7x_{21} + 6.7x_{22} + 4.7x_{23} + 4.7x_{24} + 5.05x_{31} + 3.7x_{32} + 4.7x_{33} + 6.05x_{34} + d_1^- - d_1^+ = 700$$

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 52.2$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 56.5$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 67.2$$
$$x_{11} + x_{21} + x_{31} \geq 42.1$$
$$x_{12} + x_{22} + x_{32} \geq 38.8$$
$$x_{13} + x_{23} + x_{33} \geq 38.5$$
$$x_{14} + x_{24} + x_{34} \geq 42.1$$

$x_{ij} \geq 0$ for all intger i,j. Using LINGO software, the optimal compromise solution is obtained as follows:

$x_{11} = 5.93,\quad x_{12} = 38.80,\quad x_{13} = 7.469,\quad x_{14} = 0,$
$x_{21} = 36.170,\quad x_{22} = 0,\quad x_{23} = 0,\quad x_{24} = 20.330,$
$x_{31} = 0.00,\quad x_{32} = 0,\quad x_{33} = 31.03,\quad x_{34} = 30,$
$d_1^- = 470.926,\ d_2^+ = 7.787,\ d_3^- = 255.20,$
$\eta = 0.7021,\ Z^1 = 3030.30,\quad Z^2 = 1207.80,$
$Z^3 = 744.80$

The overall satisfaction of the DM for confidence level $\omega = 0.78$ is $\xi = 0.9394$,which indicates 93.94%. Table 12, represents the values of the satisfaction level of the DM, values of the objectives and deviation from the desired goal corresponding to several confidence levels.

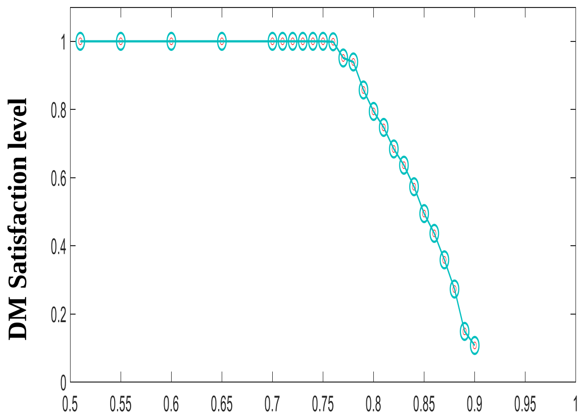

Fig. 2. Satisfaction level versus Confidence level of goal programming using exponential membership function.

From Fig. 2, it observed that, using exponential membership function, the DM's level of satisfaction is at pick level 0.51 to 0.76 because the entire desired DM goal is being met in that region. Table 12, for confidence level point from the confidence 0.77, objective for maximizing profit have shown insignificant over achievement from the goal and that is why DM satisfaction level undertakes from that point and incessantly declines until arriving at the poorest satisfaction level 10.76% for the confidence level 0.90. The infeasibility of the problem occurs for the confidence level 0.00 to 0.50 and 0.91 to onwards.

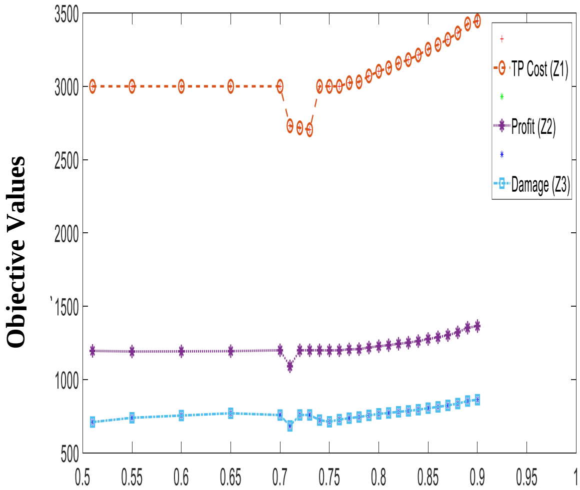

Fig. 3. Objective values versus Confidence level of goal programming using exponential membership function.

Fig. 3 shows that the profit and cost due to damage are rise when the confidence level surges only exception at 0.71 whereas the TP cost deceases from the confidence level 0.70 to 0.75 after unveiling a stability from the confidence level 0.50 to 0.70 and then it gradually increases until the confidence level 0.90. Moreover, from Table 12, we can declare that profit becomes under achievement from the confidence level 0.76 and that is why the DM's satisfaction level decreasing from its height desired level 100%.

Table 12. Computational results using exponential membership function in accordance to different confidence levels

ω (DM Confidence Level)	η (DM Satisfaction Level)	Objective Values			Deviations from Goal (Positive or Negative)						Satisfaction (%)	Feasible Solution (FS)/No-feasible Solution (No-FS)
		Z^1 Not More 3500 (TP Cost)	Z^2 Not More 1200 (Profit)	Z^3 Not more 1000 (Damage Cost)	d_1^+	d_1^-	d_2^+	d_2^-	d_3^+	d_3^-		
0.00-0.50	--	--	--	--	--	--	--	--	--	--	--	No-FS
0.51	1.000	3000.00	1195.91	710.52	--	500.00	--	4.09	--	289.47	100%	FS
0.55	1.000	3000.00	1191.63	739.72	--	500.00	--	8.37	--	260.28	100%	FS
0.60	1.000	3000.00	1192.85	755.08	--	500.00	--	7.15	--	244.92	100%	FS
0.65	1.000	3000.00	1193.94	770.43	--	500.00	--	6.06	--	229.56	100%	FS
0.70	1.000	3000.00	1200.00	758.34	--	500.00	--	0.00	--	241.66	100%	FS
0.71	1.000	2730.9	1091.00	684.21	--	500.00	--	109.0	--	315.8	100%	FS
0.72	1.000	2716.94	1200.00	758.81	--	783.06	--	0.00	--	241.19	100%	FS
0.73	1.000	2704.00	1200.00	759.00	--	796.00	--	0.00	--	241.00	100%	FS
0.74	1.000	3000.00	1200.00	724.00	--	500.00	--	0.00	--	275.98	100%	FS
0.75	1.000	3000.00	1199.33	712.45	--	500.00	--	0.67	--	287.54	100%	FS
0.76	0.9987	3000.00	1200.00	727.27	--	500.00	--	0.00	--	272.72	99.87%	FS
0.77	0.9511	3024.41	1206.24	737.16	--	475.58	6.25	--	--	262.84	95.11%	FS
0.78	0.9394	3030.30	1207.80	744.80	--	470.	7.80	--	--	255.2	93.94%	FS
0.79	0.8572	3071.00	1218.85	756.00	--	429.00	17.1	--	--	244.00	85.72%	FS
0.80	0.7943	3102.83	1227.75	766.58	--	397.17	27.7	--	--	233.41	79.43%	FS
0.81	0.7474	3126.29	1234.66	772.53	--	373.70	34.6	--	--	227.47	74.74%	FS
0.82	0.6841	3157.92	1244.37	780.51	--	342.07	44.3	--	--	219.48	68.41%	FS
0.83	0.6365	3181.73	1252.00	786.60	--	318.26	52.0	--	--	313.39	63.65%	FS
0.84	0.5733	3213.34	1262.61	794.82	--	286.65	62.6	--	--	205.17	57.33%	FS
0.85	0.4945	3252.71	1276.97	805.31	--	248.52	76.6	--	--	194.67	49.45%	FS
0.86	0.4365	3284.17	1288.65	813.91	--	215.82	88.6	--	--	186.08	43.65%	FS
0.87	0.3588	3320.58	1303.48	824.32	--	179.41	103.4	--	--	175.67	35.88%	FS
0.88	0.2731	3363.43	1322.47	836.61	--	136.56	122.4	--	--	163.38	27.31%	FS
0.89	0.1488	3425.58	1353.67	854.00	--	74.42	153.6	--	--	146.00	14.88%	FS
0.90	0.1076	3446.17	1365.19	862.91	--	55.05	165.1	--	--	137.08	10.76%	FS
0.91-1.00	---	---	---	---	--	---	---	---	--	---	---	No-FS

'---'indicates not applicable

Mathematical Illustration for Hyperbolic Membership Function (HMF)

Using the inverse uncertain distribution with confidence level $\omega = 0.75$ Table 2, 3, 4, 5 and 6 changes to Table 13, 14, 15, 16 and 17 respectively as follows:

Table 13. Crisp value for transportation cost C_{ij}^{1} for confidence level

	D_1	D_2	D_3	D_4
S_1	21.22	19.22	23.83	25.83
S_2	10.61	13.22	16.83	13.61
S_3	23.83	21.83	25.22	24.22

Table 14. Crisp value for profit C_{ij}^{2} for confidence level $\omega = 0.75$

	D_1	D_2	D_3	D_4
S_1	5.61	6.92	4.61	3.31
S_2	6.61	5.92	5.31	4.61
S_3	9.61	8.92	9.92	11.22

Table 15. Crisp value for damage cost C_{ij}^{3} for confidence level $\omega = 0.75$

	D_1	D_2	D_3	D_4
S_1	4.61	4.61	3.61	6.22
S_2	3.61	6.61	4.61	4.61
S_3	4.92	3.61	4.61	5.92

Table 16. Crisp value for demand for confidence level $\omega = 0.75$

b_1	b_2	b_3	b_4
41.8	38.4	38.05	41.83

Table 17. Crisp value for supply for confidence level $\omega = 0.75$

a_1	a_2	a_3
52.6	57	67.56

Here we set the goal that the transportation cost must be in between \$3000 and \$3500, profit will be no more \$1200 and no less than \$900 and the damagecost have to be from \$700 to \$1000 (Z^3). Using this goals, the membership functions have the following from:

$$\mu_1(Z^1(x)) = \frac{3500 - Z^1(x)}{3500 - 3000},$$

$$\mu_2(Z^2(x)) = \frac{1200 - Z^2(x)}{1200 - 900}, \text{and}$$

$$\mu_3(Z^3(x)) = \frac{1000 - Z^3(x)}{1000 - 700}.$$

Let η be the satisfaction level of the DM, then from equation (6), using hyperbolic membership function, we have the following LPPs: Max ξ Subject to:

$$Z^1 = 21.22x_{11} + 19.22x_{12} + 23.83x_{13} + 25.83x_{14} + 10.61x_{21} + 13.22x_{22} + 16.83x_{23} + 13.61x_{24} + 23.83x_{31} + 21.83x_{32} + 25.22x_{33} + 24.22x_{34}$$

$$Z^2 = 5.616x_{11} + 6.92x_{12} + 4.61x_{13} + 3.31x_{14} + 6.61x_{21} + 5.92x_{22} + 5.31x_{23} + 4.61x_{24} + 9.61x_{31} + 8.92x_{32} + 9.22x_{33} + 11.22x_{34}$$

$$Z^3 = 4.61x_{11} + 4.61x_{12} + 3.61x_{13} + 6.22x_{14} + 3.61x_{21} + 6.61x_{22} + 4.61x_{23} + 4.61x_{24} + 4.92x_{31} + 3.61x_{32} + 4.61x_{33} + 5.92x_{34}$$

$$Z^1\left(\frac{6}{500}\right) + \xi \le \left(\frac{3}{500}\right)(3500 + 3000)$$

$$Z^2\left(\frac{6}{200}\right) + \xi \le \left(\frac{3}{200}\right)(1200 + 1000)$$

$$Z^3\left(\frac{6}{200}\right) + \xi \le \left(\frac{3}{200}\right)(1000 + 800)$$

$$Z^1 + d_1^- - d_1^+ = 3000 \quad Z^2 + d_1^- - d_1^+ = 900$$

$$Z^3 + d_1^- - d_1^+ = 700$$

$$x_{11} + x_{12} + x_{13} + x_{14} \le 52.56$$

$$x_{21} + x_{22} + x_{23} + x_{24} \le 56.95$$

$$x_{31} + x_{32} + x_{33} + x_{34} \le 67.56$$

$$x_{11} + x_{21} + x_{31} \ge 41.83$$

$$x_{12} + x_{22} + x_{32} \ge 38.44$$

$$x_{13} + x_{23} + x_{33} \ge 38.05$$

$$x_{14} + x_{24} + x_{34} \ge 41.83$$

$$x_{ij} \ge 0 \text{ for all intger } i,j.$$

Using LINGO software, the optimal compromise solution can be decorated as follows:

$x_{11} = 26.71$, $x_{12} = 19.865$, $x_{13} = 5.985$,
$x_{14} = 0$, $x_{21} = 15.12$, $x_{22} = 0.00$, $x_{23} = 0.00$,
$x_{24} = 41.83$, $x_{31} = 0.00$, $x_{32} = 18.575$,
$x_{33} = 32.065$, $x_{34} = 0.00$, $d_1^- = 464.885$,
$d_2^- = 130.992$, $d_3^- = 301.388$, $\eta = 0.9297$,
$Z^1 = 3035.115$, $Z^2 = 1069.008$, $Z^3 = 698.611$

The overall satisfaction of the DM for confidence level $\omega = 0.75$ is $\xi = 0.9297$, which indicates 92.97%. Table 18, represents the satisfaction level of the DM, the targeted values and the deviation from the intended goal.

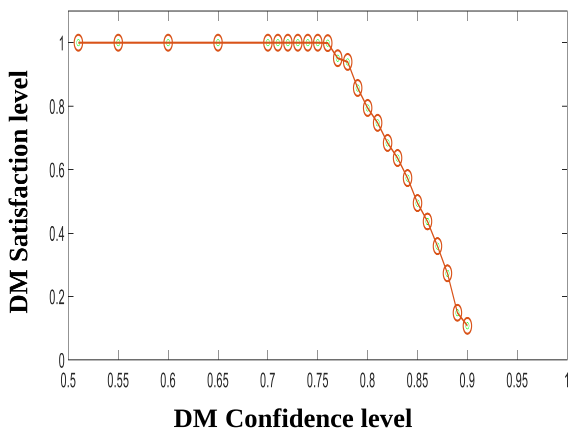

Fig. 4. Satisfaction level versus Confidence level of goal programming using hyperbolic membership function.

Fig. 4 present the satisfaction level of the DM using HMF configuring 100% from the confidence level 0.51 to 0.74 because all the desired goal of the DM is satisfied within this region and then continuously decreases from the confidence level 0.75 until arriving at the worst satisfaction level 64.18% 0.80. The infeasibility of the problem occurs for the confidence level 0.00 to 0.50 and 0.80 to onwards.

Fig. 5 revels that the objective values of Z^2 and Z^3 increase when the confidence level increases only exception at 0.73 where Z^1 have a very tiny deceases

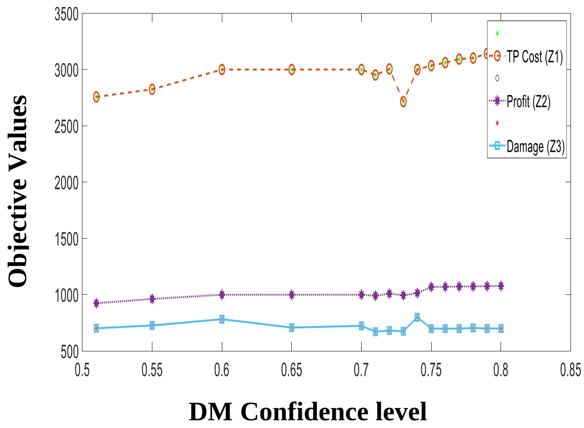

Fig. 5. Objective values versus Confidence level of goal programming using hyperbolic membership function.

and immediate gradually increases until the confidence level 0.80. Moreover, from table 18, it observed that the objective values become under achievement from the confidence level 0.75 and that is why the DM's satisfaction level decreasing from its height desired level 100%.

Comparative Results Obtain from Different Non-linear Membership Functions.

From the previous discussions, it is clear that there are slight differences in the objective values and the satisfaction level of the decision maker for the reporting membership functions corresponding to the DM choices. In the Table 19 below, we will have an explicit overview of the information gather from the previous calculation.

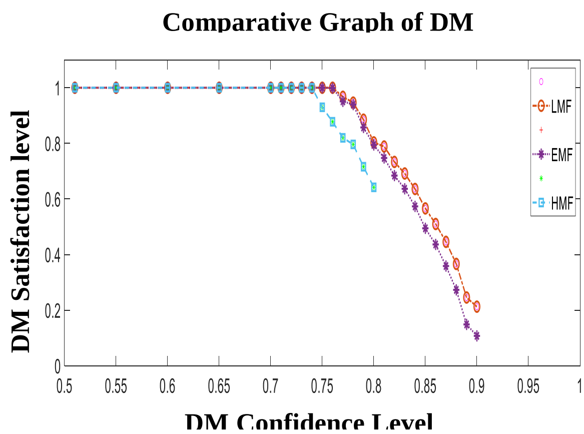

Fig. 6. Comparative graph of DM confidence level versus DM Satisfaction level of goal programming.

Table 18. Computational results using hyperbolic membership function in accordance to different confidence levels.

ω (DM Confidence Level)	ξ (DM Satisfaction Level)	Objective Values			Deviations from Goal (Positive or Negative)						Satisfaction (%)	Feasible Solution (FS)/No feasible Solution (No-FS)
		Z^1 Not More 3500 (TP Cost)	Z^2 Not More 1200 (Profit)	Z^3 NotMore 1000 (Damage Cost)	d_1^+	d_1^-	d_2^+	d_2^-	d_3^+	d_3^-		
0.00-0.50	---	---	---	---	---	---	---	---	--	---	---	No-FS
0.51	1.000	2756.94	924.37	702.00	---	743.05	---	275.62	---	298.10	100%	FS
0.55	1.000	2824.73	962.69	727.00	---	675.26	---	237.30	---	273.00	100%	FS
0.60	1.000	3000.00	1000.00	782.55	---	500.00	---	200.00	---	217.45	100%	FS
0.65	1.000	3000.00	1000.00	708.04	---	500.00	---	200.00	---	291.95	100%	FS
0.70	1.000	3000.00	1000.00	723.57	---	500.00	---	200.00	---	276.42	100%	FS
0.71	1.000	2951.74	991.50	671.80	---	548.25	---	208.50	---	328.20	100%	FS
0.72	1.000	3005.49	1009.61	681.82	---	494.51	---	190.39	---	318.17	100%	FS
0.73	1.000	2715.27	994.64	675.71	---	784.73	---	205.36	---	324.28	100%	FS
0.74	1.000	3000.00	1014.71	800.00	---	500.00	---	185.28	---	200.00	100%	FS
0.75	0.9297	3035.11	1069.00	698.61	---	464.88	---	131.00	---	301.38	92.97%	FS
0.76	0.8776	3061.16	1070.74	698.76	---	438.83	---	129.25	---	301.24	87.76%	FS
0.77	0.8198	3090.07	1072.67	698.90	---	409.93	---	127.32	---	301.09	81.98%	FS
0.78	0.7962	3101.87	1073.45	704.65	---	398.12	---	126.54	---	295.34	79.62%	FS
0.79	0.7159	3142.02	1076.13	700.00	---	357.97	---	123.86	---	300.00	71.59%	FS
0.80	0.6418	3179.10	1078.60	700.37	---	320.89	---	121.39		299.63	64.18%	FS
0.81-1.00	---	---	---	---	---	---	---	---	--	---	---	No-FS

'---'indicates not applicable

Table 19. Comparative results of the membership functions for different confidence level of the decision maker.

Hyperbolic Membership Function (HMF)					Exponential Membership Function (EMF)					DM Confidence Level
Z^1	Z^2	Z^3	Satisfaction level (100%)	Feasible / No-feasible Solution	Z^1	Z^2	Z^3	Satisfaction level (100%)	Feasible / No-feasible Sol.	
3000.00	1000.00	723.57	100%	Feasible Sol.	3000.00	1200.00	758.34	100%	FS.	0.70
2951.74	991.50	671.80	100%	Feasible Sol.	2730.9	1091.00	684.21	100%	FS.	0.71
3005.49	1009.61	681.82	100%	Feasible Sol.	2716.94	1200.00	758.81	100%	FS.	0.72
2715.27	994.64	675.71	100%	Feasible Sol.	2704.00	1200.00	759.00	100%	FS.	0.73
3000.00	1014.71	800.00	100%	Feasible Sol.	3000.00	1200.00	724.00	100%	FS.	0.74
3035.11	1069.00	698.61	92.97%	Feasible Sol.	3000.00	1199.33	712.45	100%	FS.	0.75
3061.16	1070.74	698.76	87.76%	Feasible Sol.	3000.00	1200.00	727.27	99.87%	FS.	0.76
3090.07	1072.67	698.90	81.98%	Feasible Sol.	3024.41	1206.24	737.16	95.11%	FS.	0.77
3101.87	1073.45	704.65	79.62%	Feasible Sol.	3030.30	1207.80	744.80	93.94%	FS.	0.78
3142.02	1076.13	700.00	71.59%	Feasible Sol.	3071.00	1218.85	756.00	85.72%	FS.	0.79
3179.10	1078.600	700.37	64.18%	Feasible Sol.	3102.83	1227.75	766.58	79.43%	FS.	0.80
---	---	---	---	No Feasible Sol.	3126.29	1234.66	772.53	74.74%	No FS.	0.81
---	---	---	---	No Feasible Sol.	3157.92	1244.37	780.51	68.41%	No FS.	0.82
---	---	---	---	No Feasible Sol.	3181.73	1252.00	786.60	63.65%	No FS.	0.83
---	---	---	---	No Feasible Sol.	3213.34	1262.61	794.82	57.33%	No FS.	0.84
---	---	---	---	No Feasible Sol.	3252.71	1276.97	805.31	49.45%	No FS.	0.85

'---'indicates not applicable

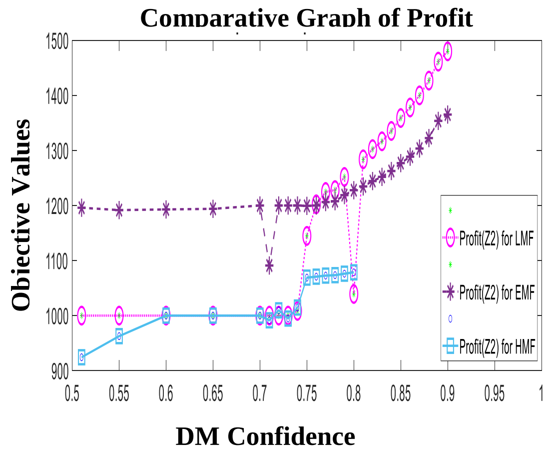

Fig. 7. Comparative graph of Objective values versus DM confidence level of goal programming for TP cost.

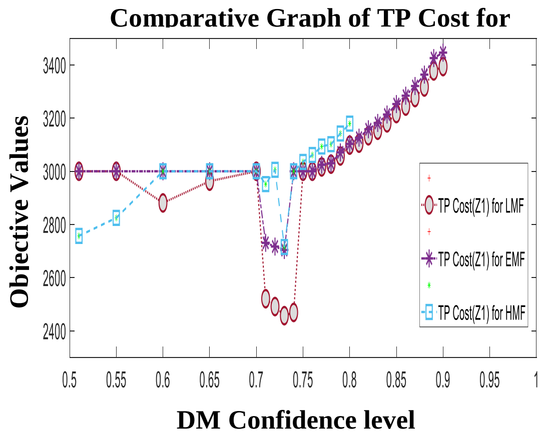

Fig. 8. Comparative graph of Objective values versus DM confidence level of goal programming for profit.

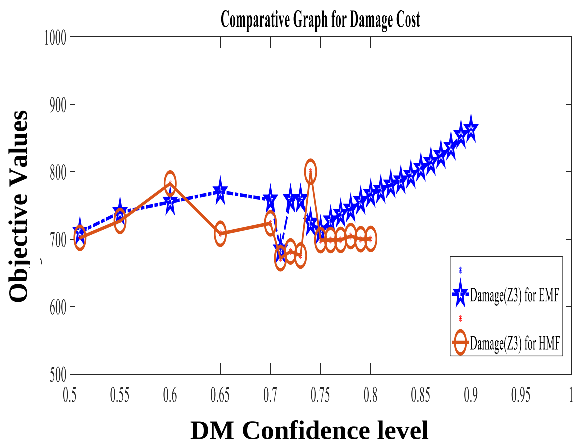

Fig. 9. Comparative graph of Objective values versus DM confidence level of goal programming for damage cost.

From the graphs presented in Figs. 6 to 9, we have a very specific observations of various parameters of the goal programming transportation problem in uncertain environment using fuzzy membership functions. Fig. 6 reveals the satisfaction level of the decision maker for various confidence level using the fuzzy membership functions, linear, exponential and hyperbolic. It is clear from the graph that the satisfaction of DM in all three cases, is height for the confidence level 0.51 to 0.75 and then continuously decreases from the confidence level 0.75 until arriving at the worst satisfaction level 10.16% at 0.90.

Fig. 7 unwrapped the transportation cost against the confidence level for the fuzzy membership functions. All the three function have shown almost same pattern throughout the region except 0.70 to 0.75. The linear membership function has shown more fluctuation regarding the objective value other than two in the area 0.72 to 0.74 whereas the others have same oscillation on that region. All the three membership function have shown the increasing behavior of TP cost from 0.75 to onwards.

Fig. 8 and Fig. 9 have shown some anomaly regarding the profit and damage cost for the chosen confidence level of the DM but have similar increasing pattern visible from 0.80 to onwards.

Conclusion

In this research, uncertain MOTP has been investigated based on the method presented in this paper. The uncertain parameters were resolved by uncertain normal distribution. MOTP in uncertain parameters using the fuzzy non-linear membership functions with their mathematical algorithms have shown with applicability of these algorithms by a heuristic example of same data table with a variety of confidence level of the DM for each case. Sometime the problem becomes infeasible for a chosen confidence level due to the violation of the feasibility condition of the transportation problem. The satisfactions in percent of the decision maker are obtained for a chosen confidence level that is accumulated in listed tables. From the comparative results, we see that the satisfactions level of the

DM using hyperbolic membership function has shown multiple time 100% in a region but solution is not feasible for a large scale of confidence level. On the other hand, the objective values using exponential membership functions are more considerable than that hyperbolic.

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Declarations

The contents of the paper were not published before or submitted for publication in any other journal and all the co-authors have given their consent for the article to be considered by the Editorial Board for publication in the Journal of Bangladesh Academy of Sciences.

Conflicting interests

The authors declare that there are no conflicts of interest.

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