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## Research Article

Optimization of interval cost multi-objective transportation problem in uncertain parameters:

> A modified least-cost method
Md. Tariqul Islam and Md. Musa Miah*

Department of Mathematics, Mawlana Bhashani Science and Technology University, Santosh, Tangail, Bangladeshs

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#### Abstract

This research aims to focus on multi-objective transportation problems, to reduce the cost of transporting goods from various sources or origins to a range of destinations while adhering to a predetermined mathematical framework. By using interval values, the policy maker indicated the source and destination parameters. This work presents a novel approach for finding initial basic feasible solutions (IBFS) that are extremely near to the best ones for a variety of transportation problems is presented. The proposed method can become a milestone in resolving the constraints to solve the transport problem, making the decision-makers regarding logistics and supply chains quite profitable. The prediction model is supported by an illustration drawn from a mathematical view. All the LPPs are evaluated with the help of LINGO.


## Introduction

The Transportation Problem (TP) can be treated as a unique mathematical programming issue that seeks to reduce the cost of conveying an item from numerous sources or origins to different destinations. A certain mathematical structure underlies this issue. The standard, straightforward technique is inappropriate for dealing with transportation challenges due to its unique structure. While the supply criteria $a_{i}$ could be manufactural plants, services, major shipping containers, etc. the destination factors $b_{j}$ may comprise minor stores, selling stores, etc., Transportation costs, the typical time it takes for goods to be carried, unmet requirements, and other things can all be represented by penalty factors $C_{i j}$ or components of decision variables. The primary goal of the vehicle routing problem is to establish the variety of quantities of a service that should be conveyed from numerous supply sites to diverse transfer stations, while decreasing the overall cost of transportation and lowering the price per unit of merchandise for users.

The factors of the transportation problem are the unit costs, or the cost of moving a single entity from one supply location to another, the numbers possible at the supply sites, and the volumes needed at the active nodes.

The following sequences are typically included in the solution process for the transportation issue:
Sequence 1: The transportation problem is expressed mathematically.
Sequence 2: Represent the problem in the form of a matrix.
Sequence 3: Obtaining an outcome of an initial basic feasible solution.

For this study, we have concentrated on sequence 3 to identify effective initial basic workable solutions to the transportation problem. Several researchers have thoroughly studied multi-objective transportation issues. A study entitled "The Delivery of a Material from Numerous Producers to Multiple Communities" (Hitchcock, 1941) initially identified the core

[^0]transportation issue. The fuzzy convex programming technique for multi-criteria decision making with setinclusive constraints was later refined by (Soyster, 1973; Chanas and Kuchta, 1996; Bit et al., 1992) with applications to inexact linear programming. Goal programming has been established (Miah et al., 2022; (Uddin et al., 2021) for multi-objective enhancement of the transportation issue in an unpredictable environment utilizing ambiguous non-linear transfer functions. An interactive explanation of the transportation issue with many objectives is shown by Ringuest and Rinks (1987) and Pandian and Natarajan (2010). Using interval cost, source, and destination factors, Das et al. (1999) and Hong (2009) established a model for optimizing a multi-objective transportation issue. In a different piece of work, Akilbasha et al. (2018) developed an original, accurate approach for resolving the complete interval integer transportation problem. Yu et al. (2015) and Ishibuchi and Tanaka (1990) generalized the existing idea of the interactive approach's solution to the multi-objective issue of traffic congestion using interval factors. The collection of all prehistoric solutions was enumerated using Isermann (1979) and Ahmed et al. (2016) approaches for solving a linear multi-objective transportation issue.

The fuzzy time - series network model has a new, ideal solution Murugesan and Kumar, 2013 that have put forth. Heuristic-based modifications (Shimshak et al., 1981; Kirca and Satir, 1990) to Vogel's approximation method that have undergone experimental study and are also part of the modern study's extension by Balakrishnan (1990), Shore (1970), Mathirajan and Meenakshi (2004), Korukoğlu and Balli (2011) and so on.
In this study, we postulate a model for dealing with the multi-objective shipping problem everywhere the origin and location criteria, as well as the decision variables co-efficient, take the shape of intervals. To create an appropriate mathematical model for the multi-objective issue of traffic congestion using periodic coefficients, we made use of various principles. To recognize an initial basic workable
solution to a vehicle routing problem, we have utilized certain approaches such as Northwest Corner Method (NWCM), Least Cost Method (LCM), Vogel's Approximation Method (VAM), Row Minimum Method (RMM) and Column Minimum Method (CMM). In order to illustrate the proposed solution method, numerical examples are provided.

## Nomenclature

$\mathrm{m}=$ Number of sources.
$\mathrm{n}=$ Number of destinations.
$\mathrm{z}=$ Total transportation cost.
$a_{i}=$ Supply quantities for source i .
$b_{j}=$ Demand quantities for destination j .
$x_{i j}=$ Allocation units from origin i to location j.
$c_{i j}=\mathrm{TP}$ cost for source i to location j .

## Materials and Methods

## Interval-based multi-objective transportation issues

The multi-objective transportation problem is when the optimization function coefficient takes the form of closed intervals, namely, $c_{i j}=\left[c_{L i j}, c_{U i j}\right]$ and the constraints or parameters are in the deterministic, closed and open intervals.

Minimize, $Z=\sum_{i=1}^{m} \sum_{j=1}^{n}\left[c_{L i j}, c_{R i j}\right] \quad \mathbf{x}_{\mathrm{ij}}$

## Case-I: Deterministic parameters

Subject to

$$
\begin{aligned}
& \sum_{i=1}^{m} x_{i j}=a_{i}, i=1,2,3 \ldots m \\
& \sum_{j=1}^{n} x_{i j}=b_{j}, j=1,2,3 \ldots n \\
& x_{i j} \geq 0, \quad i=1,2 \ldots m \text { and } j=1,2 \ldots n \\
& \text { With } \sum_{i=1}^{m} a_{i}=\sum_{j=1}^{n} \mathbf{b}_{j}
\end{aligned}
$$

## Case-I1: Closed interval parameters

Subject to
$\sum_{i=1}^{m} x_{i j}=\left[a_{L i}, b_{U j}\right], \quad i=1,2,3 \ldots m$
$\sum_{j=1}^{n} x_{i j}=\left[a_{L i}, b_{U j}\right], \quad j=1,2,3 \ldots . n$
$x_{i j} \geq 0, i=1,2 \ldots m$ and $j=1,2 \ldots n$

With $\sum_{i=1}^{m} a_{L i}=\sum_{j=1}^{n} b_{L j}$ and $\sum_{i=1}^{m} a_{U i}=\sum_{j=1}^{n} b_{U j}$

## Case-III: Open interval parameter

Subject to
$\sum_{i=1}^{m} x_{i j}=\left(a_{L i}, b_{U j}\right), \quad i=1,2,3 \ldots m$
$\sum_{j=1}^{n} x_{i j}=\left(a_{L i}, b_{U j}\right), \quad j=1,2,3 \ldots n$
$x_{i j} \geq 0, \quad i=1,2 \ldots m \quad$ and $\quad j=1,2 \ldots n$

With $\sum_{i=1}^{m} a_{L i}=\sum_{j=1}^{n} b_{L j}$ and $\quad \sum_{i=1}^{m} a_{U i}=\sum_{j=1}^{n} b_{U j}$
To derive the comparable conventional multiobjective transportation issue:

Minimize. $Z_{L}(x)=\sum_{i=1}^{m} \sum_{j=1}^{n} c_{L i j} x_{i j}$
Minimize. $Z_{U}(x)=\sum_{i=1}^{m} \sum_{j=1}^{n} c_{U i j} x_{i j}$
Subject to

$$
\begin{array}{ll}
\sum_{j=1}^{n} x_{i j} \leq a_{R i}, & \sum_{j=1}^{n} x_{i j} \geq a_{L i} \\
\sum_{i=1}^{m} x_{i j} \leq b_{R j}, & \sum_{i=1}^{m} x_{i j} \geq b_{L j}
\end{array}
$$

## Procedure for Defining the Transportation Problem

The steps listed below are included in the procedurefor solving a transpiration problem:

## 1. Create a matrix for the problem and formulate it.

The structure of the vehicle routing problem is correlated to that of LP issues. The price and quantity conditions for each transmitter and the receiver, respectively, serve as the constraints in this model, with the transportation expense acting as the objective function.

## 2. Find an initial basic feasible solution

To reach this initial basic solution, you can use one of the bellowing approaches:
i. North West Corner Method (NWCM)
ii. Least Cost Method (LCM)
iii. Vogel Approximation Method (VAM)
iv. Row Minima Method (RMM)
v. Column Minima Method (CMM)

Any of the current strategies must produce a solution that satisfies the requirements listed below:
i. A feasible solution is one that satisfies all supply and demand restrictions. The rim condition refers to this.

If m is the number of rows and n is the number of columns, the number of positive allocations must satisfy $m+n-1$. A non-degenerate basic viable solution is one that satisfies the requirements above.

## Construct an Initial Basic Feasible Solution: The Proposed Method

Step 1: Represent the problem as an original table, a matrix of the problem.

Step 2: Transportation problem must be balanced. If not balanced, have to do.

Step 3: The minimum odd cost is to be determined from all the costs in the original table. Nothing will happen if the original table's cost column does not include an odd cost until the cost cell contains an odd value. All costs must be divided by 2 .

Step 4: Create a new table that stores the minimum odd cost as it was in the corresponding cost cell in the table, which will be called the allocation table, and from each cell containing an odd value in the original table, only deduct the chosen lowest odd cost.

Step 5: Using the smallest supply and demand at the beginning of the allocation procedure, place this lowest of price and quantity in the initial allocation cells of the table created in Step 4 instead of the values of the allocation cells with odd values. The column should be removed if the demand is fulfilled, and when it supplied remove the row.
Step 6: Assign the lowest supply or demand at the location that was selected in the allocation cost values table by determining the minimal allocation cost values. To make the smallest allocation, a different allocation cost value should be selected rather than the same allocation cost values and deleted column again, if the demand is satisfied. Supply can end the row.
Step 7: Step 6 should be repeated until supply or demand is no longer.
Step 8: Every allocation should be returned to the original table.
Step 9: Therefore, the total of the original table's cost and matching assigned value may be determined.

## 3. Check the original answer for optimality

Numerical Illustrations

## Example-1 (Closed Interval Parameters)

A firm has three manufacturing plants (origins) with a capacity of $[5,7],[0,2]$, and $[8,12]$ units each. These facilities are A1, A2, and A3. There are a requirement of $[6,8],[4,6],[2,4]$, and $[1,3]$ units for each of these units to be delivered to the four storerooms B1, B2, B3, and B4. Following are the shipping costs and times from firms to storerooms:
$\mathbf{C}=\left[\begin{array}{cccc}{[2,4]} & {[3,5]} & {[11,15]} & {[7,9]} \\ {[1,5]} & {[0,2]} & {[6,8]} & {[1,3]} \\ {[5,9]} & {[8,12]} & {[15,17]} & {[9,13]}\end{array}\right]$

## Solution

Mathematical example 1 has been used to demonstrate the solution process.

Minimize, $\quad Z=\sum_{i=1}^{3} \sum_{j=1}^{4}\left[c_{L i j}, c_{R i j}\right] \mathbf{x}_{\mathrm{ij}}$
Subject to
$\sum_{j=1}^{4} x_{1 j}=[5,7], \quad \sum_{j=1}^{4} x_{2 j}=[0,2], \quad \sum_{j=1}^{4} x_{3 j}=[8,12]$,
$\sum_{i=1}^{3} x_{i 1}=[6,8], \quad \sum_{i=1}^{3} x_{i 2}=[4,6], \quad \sum_{i=1}^{3} x_{i 3}=[2,4]$,
$\sum_{i=1}^{3} x_{i 4}=[1,3], \quad x_{i j} \geq 0, \quad i=1,2,3$ and $j=1,2,3,4$.
With the equivalent cases
Subject to

$$
\begin{aligned}
& \sum_{j}^{4} x_{1 j} \leq 7, \quad \sum_{j}^{4} x_{1 j} \geq 5, \quad \sum_{j}^{4} x_{1 j} \leq 2, \quad \sum_{j}^{4} x_{1 j} \geq 0, \\
& \sum_{j}^{4} x_{3 j} \leq 12, \quad \sum_{j}^{4} x_{3 j} \geq 8, \sum_{i}^{3} x_{i 1} \leq 8, \quad \sum_{i}^{3} x_{i 1} \geq 6 \\
& \sum_{i}^{3} x_{i 2} \leq 6, \quad \sum_{i}^{3} x_{i 2} \geq 4, \quad \sum_{i}^{3} x_{i 3} \leq 4, \\
& \sum_{i}^{3} x_{3 i} \geq 2, \quad \sum_{i}^{4} x_{i 4} \leq 3, \quad \sum_{i}^{4} x_{i 4} \geq 1 .
\end{aligned}
$$

We consider the lower cost of intervals of the matrices as one metric, i.e., $C_{L}$, and similarly consider the matrices for the upper intervals i.e.; $C_{U}$.

$$
C_{L}=\left[\begin{array}{cccc}
2 & 3 & 11 & 7 \\
1 & 0 & 6 & 1 \\
5 & 8 & 15 & 9
\end{array}\right]
$$

And

$$
C_{U}=\left[\begin{array}{cccc}
4 & 5 & 15 & 9 \\
5 & 2 & 8 & 3 \\
9 & 12 & 17 & 13
\end{array}\right]
$$

Firstly, we find the lower cost of an interval of the transportation problem.
Utilizing the algorithm of the suggested approach:
The provided problem is discovered to be balanced. Since the total supplies are equal to the total demands. All price cells in transportation (Table 1) have the lowest odd cost of 1 , shown in cost cells $(2,1)$ and $(2,4)$.

The transportation's least odd cost is still in cells $(2,1)$ and $(2,4)$, but all other odd-valued price cells are removed from this odd expense in table 2. For example, in Transportation Table 1's cost cell (1, 2), the value is 3 , but in Table 2, it is $2=(3-1)$.

Table 1. Data of the lower cost of interval

| Facilities | Warehouses |  |  |  | Capacity |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{B}_{1}$ | $\mathbf{B}_{\mathbf{2}}$ | $\mathbf{B}_{3}$ | $\mathbf{B}_{4}$ |  |
| $\mathbf{A}_{\mathbf{1}}$ | 2 | 3 | 11 | 7 | 5 |
| $\mathbf{A}_{\mathbf{2}}$ | 1 | 0 | 6 | 1 | 0 |
| $\mathbf{A}_{3}$ | 5 | 8 | 15 | 9 | 8 |
| Requirement | 6 | 4 | 2 | 1 | 13 |

Table 2. All the cost cells are subtracted by the minimum odd cost

| Facilities | Warehouses |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Capacity |  |  |  |  |
| $\mathbf{B}_{\mathbf{1}}$ | $\mathbf{B}_{\mathbf{2}}$ | $\mathbf{B}_{\mathbf{3}}$ | $\mathbf{B}_{\mathbf{4}}$ |  |  |
| $\mathbf{A}_{\mathbf{2}}$ | 2 | 2 | 10 | 6 | 5 |
| $\mathbf{A}_{\mathbf{3}}$ | 4 | 0 | 6 | 1 | 0 |
| Requirement | 6 | 4 | 2 | 1 | 13 |

Table 3. Several cells in the Allocation table are assigned

| Facilities | Warehouses |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{B}_{\mathbf{1}}$ | $\mathbf{B}_{\mathbf{2}}$ | $\mathbf{B}_{3}$ | $\mathbf{B}_{4}$ |  |
| $\mathbf{A}_{\mathbf{1}}$ | 1 | 4 | 10 | 6 | $5 / 1 / 0$ |
|  | 2 | 2 |  |  |  |
| $\mathbf{A}_{\mathbf{2}}$ |  |  |  | 0 | $0 / 0$ |
|  | 1 | 0 | 6 | 1 |  |
| $\mathbf{A}_{3}$ | 5 |  | 2 | 1 |  |
|  | 4 | 8 | 14 | 8 | $8 / 3 / 2 / 0$ |
| Requirement | $6 / 5 / 0$ | $4 / 0$ | $2 / 0$ | $1 / 1 / 0$ | 13 |

According to step 5 of the proposed method, the smallest amount of either supply or demand that has been provided in cells $(2,4)$ is 0 . This value is assigned, and it is established that the supply is fulfilled. Whatever row in $\mathrm{A}_{2}$ needs to be used up.
The only cells to be considered are those in rows $\mathrm{A}_{1}$ and $A_{3}$. If cell $(1,1)$ and $(1,2)$ has the weakest cell
value 2 . However, of these two cells, 4 can only receive the least allocation $(1,2)$. After assigning this sum, column $B_{2}$ has now been destroyed.
Once more, it is discovered that cell value 2 is the lowermost cost in the follicular cells that occur in the cells $(1,1)$. Therefore, there is a minimum allocation of 1 created in cell $(1,1)$. Therefore, no additional calculations should take row $\mathrm{A}_{1}$ cells into account.

One cell from row A3 alone needs to be taken into account. The least expensive options are $4,8,14$, respectively, in Table 3's cells ( 3,1 ), ( 3,4 ), and ( 3,3 ). Finally finish the allocation by distributing 5,1 , and 2 to cells in table 3 marked $(3,1),(3,4)$, and ( 3,3 ), respectively, in Table 3.
All these transfers are made to transportation (Table 1), which is included in the final allocation Table 4. The initial basic feasible solution in line with the provided approach is shown by the discovery that there are 6 fundamental cells in this table, indicating the number of fundamental cells.

Table 4. Lower Initial basic feasible solution according to proposed method

| Facilities | Warehouses |  |  |  | Capacity |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{B}_{1}$ | $\mathbf{B}_{\mathbf{2}}$ | $\mathbf{B}_{\mathbf{3}}$ | $\mathbf{B}_{\mathbf{4}}$ |  |
| $\mathbf{A}_{\mathbf{1}}$ | 1 | 4 | 11 | 7 | 5 |
|  | 2 | 3 |  |  |  |
| $\mathbf{A}_{\mathbf{2}}$ | 1 | 0 | 6 | 0 | 0 |
|  |  |  |  | 1 |  |
| $\mathbf{A}_{3}$ | 5 | 8 | 2 | 1 | 8 |
|  | 5 |  | 15 | 9 |  |
| Requirement | 6 | 4 | 2 | 1 | 13 |

The solution to a given problem is

$$
\begin{array}{lll}
x_{11}=1, & x_{12}=4, & x_{24}=0 \\
x_{31}=5, & x_{33}=2, & x_{34}=1 .
\end{array}
$$

The total lower cost, $Z_{L}$

$$
\begin{aligned}
& =1 \times 2+4 \times 3+0 \times 1+5 \times 5+2 \times 15+1 \times 9 \\
& =2+12+25+30+9=78
\end{aligned}
$$

Secondly, we find to the upper cost of interval of the transportation problem.

Table 5. Data of the upper cost of interval

| Facilities | Warehouses |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{B}_{1}$ | $\mathbf{B}_{\mathbf{2}}$ | $\mathbf{B}_{3}$ | $\mathbf{B}_{4}$ |  |
| $\mathbf{A}_{1}$ | 4 | 5 | 15 | 9 | 7 |
| $\mathbf{A}_{2}$ | 5 | 2 | 8 | 3 | 2 |
| $\mathbf{A}_{3}$ | 9 | 12 | 17 | 13 | 12 |
| Requirement | 8 | 6 | 4 | 3 | 21 |

Utilizing the algorithm of the suggested approach:
The provided problem is discovered to be balanced. Due to the fact that the total supplies are equal to the total demands.
All price cells in transportation (Table 5) have the lowest odd cost of 3 , shown in cost cells $(2,4)$.
The transportation's least odd cost is still in cell $(2,4)$, but all other odd-valued price cells are removed from this odd expense in Table 6. For example, in Transportation Table 5's cost cell $(1,2)$, the value is 5, but in Table 6, it is $2=(5-3)$.

Table 6. All the cost cells are subtracted by the minimum odd cost

| Facilities | Warehouses |  |  |  | Capacity |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{B}_{1}$ | $\mathrm{B}_{2}$ | B3 | $\mathrm{B}_{4}$ |  |
| A1 | 4 | 2 | 12 | 6 | 7 |
| $\mathrm{A}_{2}$ | 2 | 2 | 8 | 3 | 2 |
| A3 | 6 | 12 | 14 | 10 | 12 |
| Requirement | 8 | 6 | 4 | 3 | 21 |
| Table 7. Several cells in the Allocation table are assigned |  |  |  |  |  |
| Facilities | Warehouses |  |  |  | pa |
|  | B1 | $\mathbf{B}_{2}$ | $\mathbf{B}_{3}$ | B4 | , |
| A1 | $\begin{aligned} & 1 \\ & 4 \end{aligned}$ | $\begin{aligned} & \hline 6 \\ & 2 \end{aligned}$ | 12 | 6 | 7/1/0 |
| A2 | 2 | 2 | 8 | $\begin{aligned} & 2 \\ & 3 \end{aligned}$ | 2/0 |
| A3 | $\begin{aligned} & 7 \\ & 6 \end{aligned}$ | 12 | $\begin{gathered} 4 \\ 14 \end{gathered}$ | $\begin{gathered} 1 \\ 10 \end{gathered}$ | 12/5/4/0 |
| Requirement | $\begin{gathered} 8 / 7 / \\ 0 \end{gathered}$ | 6/0 | $\begin{gathered} 4 / \\ 0 \end{gathered}$ | $\begin{gathered} 3 / 1 / \\ 0 \end{gathered}$ | 21 |

According to step 5 of the proposed method, the smallest amount of either supply or demand that has been provided in cells $(2,4)$ is 1 . This value is assigned, and it is established that the supply is fulfilled. Whatever row in $\mathrm{A}_{2}$ needs to be used up.
The only cells to be considered are those in rows A1 and A3. If the cell $(1,1)$ and $(1,2)$ has the weakest cell value 2. However, of these two cells, 5 can only receive the least allocation (1, 2). After assigning this sum, column $\mathrm{B}_{2}$ has now been destroyed.
Once more, it is discovered that cell value 2 is the lowermost cost in the follicular cells that occur in the cells $(1,1)$. Therefore, there is a minimum allocation of 1 created in cell $(1,1)$. Therefore, no additional calculations should take row $\mathrm{A}_{1}$ cells into account.
One cell from row $\mathrm{A}_{3}$ alone needs to be taken into account. The least expensive options are 6,10 , and 14 , respectively, in Table 7 's cells ( 3,1 ), $(3,4)$, and ( 3,3 ). Finally, finish the allocation by distributing 6,1 , and 3 to the cells $(3,1),(3,4)$, and (3, 3), respectively, in Table 7.
All these transfers are made to transportation (Table 5), which is included in the final allocation Table 8. The initial basic feasible solution in line with the provided approach is shown by the discovery that there are 6 fundamental cells in this table, indicating the number of fundamental cells.

Table 8. Upper Initial basic feasible solution according to the proposed method

| Facilities | Warehouses |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{B}_{\mathbf{1}}$ | $\mathbf{B}_{\mathbf{2}}$ | $\mathbf{B}_{3}$ | $\mathbf{B}_{\mathbf{4}}$ |  |
| $\mathbf{A}_{\mathbf{1}}$ | 1 | 6 | 15 | 9 | 7 |
| $\mathbf{A}_{\mathbf{2}}$ | 4 | 5 |  | 2 |  |
|  | 5 | 2 | 8 | 3 | 2 |
| $\mathbf{A}_{\mathbf{3}}$ | 7 |  | 4 | 1 | 12 |
| Requirement | 9 | 12 | 17 | 13 | 12 |

The solution to a given problem is

$$
\begin{array}{lll}
x_{11}=1, & x_{12}=6, & x_{24}=2, \\
x_{31}=7, & x_{33}=4, & x_{34}=1 .
\end{array}
$$

The total upper cost, ${ }_{Z}$

$$
\begin{aligned}
& =4 \times 1+5 \times 6+3 \times 2+9 \times 7+4 \times 17+1 \times 13 \\
& =4+30+6+63+68+13 \\
& =184
\end{aligned}
$$

Finally, the total transportation cost is $\mathrm{Z}=[78,184]$.

## Example-2 (Deterministic Parameters)

A firm has three manufacturing plants (origins) with a capacity of $\mathbf{6}, \mathbf{1}$, and $\mathbf{1 0}$ units each. These facilities are $\mathrm{A} 1, \mathrm{~A} 2$, and A 3 . There is a requirement of $7,5,3$, and 2 units to be delivered to the four storerooms B1, B2, B3, and B 4 . Following are the shipping costs and times from firms to storerooms:
$\mathbf{C}=\left[\begin{array}{cccc}{[2,4]} & {[3,5]} & {[11,15]} & {[7,9]} \\ {[1,5]} & {[0,2]} & {[6,8]} & {[1,3]} \\ {[5,9]} & {[8,12]} & {[15,17]} & {[9,13]}\end{array}\right]$

## Example-3 (Open Interval Parameters)

A firm has three manufacturing plants (origins) with a capacity of $(6,8),(8,10)$, and $(17,19)$ units each. These facilities are A1, A2, and A3. There are a requirement of $(4,6),(7,9),(6,8)$, and $(13,15)$ units for each of these units to be delivered to the four storerooms B1, B2, B3, and B4. Following are the shipping costs and times from firms to storerooms

$$
\mathbf{C}=\left[\begin{array}{cccc}
{[10,12]} & {[30,32]} & {[50,54]} & {[10,12]} \\
{[70,74]} & {[30,32]} & {[40,42]} & {[60,62]} \\
{[40,42]} & {[8,10]} & {[70,74]} & {[20,24]}
\end{array}\right]
$$

## Result Comparison and Graphical Interpretation

The entire transportation costs for the suggested strategy, the existing approaches, and the optimal solution are compared in Table 9. The optimal algorithm that provides a solution at the lowest cost is used in the comparison to determine which option has the lowest cost.
Table 9 shows that the new method provides the best IBFS when compared to other ways now in use since, on average, it provides the best price for the transportation issue. The achievement of IBFS is often displayed in Table 10. This data demonstrates the proposed method's superior performance. Our suggested approach has been used in several examples, and the solution results were superior to those of the other ways already used to address the transportation issue.

Table 9. Comparative Analysis of Initial Basic Feasible Solution

| Methods | Total cost, Z |  |  |
| :---: | :---: | :---: | :---: |
|  | Ex-1 | Ex-2 | Ex-3 |
| North-West <br> Corner | $[86,202]$ | $[116,166]$ | $[970,1074]$ |
| Least Cost | $[86,196]$ | $[112,160]$ | $[734,822]$ |
| Vogel's | $[78,186]$ | $[102,151]$ | $[734,822]$ |
| Approximation |  |  |  |
| Row Minima | $[86,196]$ | $[112,160]$ | $[1010,1114]$ |
| Column <br> Minima | $[86,202]$ | $[116,166]$ | $[734,822]$ |
| Proposed | $[78,184]$ | $[102,150]$ | $[734,822]$ |
| Approach |  |  |  |
| Optimal <br> Solution | $[77,184]$ | $[100,150]$ | $[698,790]$ |

Table 10. Difference between IBFS and optimal solution

| Methods | Difference between IBFS and <br> optimal solution |  |  |
| :---: | :---: | :---: | :---: |
|  | Ex-1 | Ex-2 | Ex-3 |
| NWCM | $[9,18]$ | $[16,16]$ | $[92,284]$ |
| LCM | $[9,12]$ | $[12,10]$ | $[36,32]$ |
| VAM | $[1,2]$ | $[2,1]$ | $[36,32]$ |
| RMM | $[9,12]$ | $[12,16]$ | $[312,324]$ |
| CMM | $[9,20]$ | $[16,16]$ | $[36,32]$ |
| Proposed | $[\mathbf{1 , 0}]$ | $[\mathbf{2 , 0}]$ | $[\mathbf{3 6 , 3 2}]$ |

Figures 1, 2, and 3 shows that the initial basic feasible solutions superior to the results obtained by conventional algorithms, which are either optimal or near to optimal. Another time, the performance of the solution changes in different ways, which is also possible in the case of the proposed method.


Fig. 1. Graphical Illustration of Various Solutions of Example-1


Fig. 2. Graphical Illustration of Various Solutions of Example-2


Fig. 3. Graphical Illustration of Various Solutions of Example-3

This research is conducted to support the method's typical performance. During such an approach, it was discovered that the suggested model performed better than some of the other procedures in the chart, as indicated in Figures 4 and 5, correspondingly.


Fig. 4. Graphical Represented of Performance of IBFS


Fig. 5. Graphical Representation of Mean of POD
Table 11. Percentage of Disparity (POD) and Mean of POD

| 0Methods | Percentage of Disparity (POD) |  |  | Mean of <br> POD |
| :---: | :---: | :---: | :---: | :---: |
|  | $[11.69,9.79]$ | $[16,10.67]$ | $[13.19,35.95]$ |  |
| LCM | $[11.69,6.53]$ | $[12,6.67]$ | $[5.16,4.05]$ | $[9.62,5.75]$ |
| VAM | $[1.3,1.09]$ | $[2,0.67]$ | $[5.16,4.05]$ | $[2.82,1.93]$ |
| RMM | $[11.69,6.53]$ | $[12,10.67]$ | $[44.7,41.02]$ | $[22.8,19.5]$ |
| CMM | $[11.69,10.87]$ | $[16,10.67]$ | $[5.16,4.05]$ | $[10.95,8.53]$ |
| Proposed | $[1.3,0]$ | $[2,0]$ | $[5.16,4.05]$ | $[2.8,1.35]$ |

The performance of Percentage of Disparity (POD) and Mean of POD is frequently shown in Table 11.

## Conclusions

The carrying cost is an essential component of the overall the cost configuration for any firm. The transportation issue was expressed as a linear programming problem, which was then resolved using the usual LP solvers to get the initial basic feasible solutions (IBFS) and the best option. In this research, we use mathematical illustration to describe the approach and effectiveness of our proposed method. 'LINGO' software is used to create the ideal solution Comparative analysis of the solutions using the suggested method and the other ways already in use, is illustrated using a sample problem. We already have some basic, workable solutions that are closer to the ideal ones, but the suggested method still produces predictable results.

The analysis of transportation issues is made more accessible by the suggested method's reducing the odd cost coefficients to even numbers, providing a solution far superior to other approaches. It affects lowering the cost of transportation in the objective function, which will assist in achieving the goal of maximizing profit. Finally, validating transportation costs utilizing the suggested strategy may produce a unique initial basic achievable solution. It may mark a turning point in removing obstacles that stand in the way of resolving the transport issue.

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## Conflicting interests

The authors acknowledge no potential conflicts of concern.

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[^0]:    *Corresponding author: [mmusa@mbstu.ac.bd](mailto:mmusa@mbstu.ac.bd)

