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AN EXTENSION OF FUHRBOTER'S STUDIES FOR WAVE HEIGHT: A THEORETICAL STUDY

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ABSTRACT

This theoretical study extends Fuhrboter's work and includes the effects of air bubble entrainment. Result shows that there is a correlation between the sudden reductions of wave height and entrained of air bubbles into the water. Moreover, the study reveals that air entrainment occurs very short way in plunging breakers comparatively to spilling breakers.

Key words : Fuhrboter's studies, Waive height, Theoretical study

INTRODUCTION

The breaking of waves in the near shore area is an important and fascinating phenomenon of wave hydrodynamics. When waves propagate to the near shore zone, wave profiles become steeper and eventually waves break. Once the waves start to break, a part of wave energy is transformed to turbulence and heat, resulting in wave height decrease towards the shore. During wave breaking, a large amount of air bubble is entrained into water, causing a large-scale disturbance in flow. Horikawa and Kao (1966) suggested that this entrained air bubbles are responsible for dissipating the wave energy, specially at initial stages. The problem is how to measure the dissipation of wave energy in the surf zone. Although many models have been developed based on different assumptions, but none of them is precise. After mixing the air with water, the flow field becomes complicated. However, in order to get knowledge of wave energy dissipation by the aeration effect, the distribution of void fraction may have to know.

During the last few decades, a number of studies (Horikawa and Kuo (1966); Fuhrboter (1970), Hwung *et al* (1992), Hoque and Aoki (2008)) have been carried out to develop energy dissipation models. Energy dissipation model which include air bubble effects have been developed by Fuhrboter (1970) and Hoque (2002). Fuhrboter (1970) proposed that the sudden reduction of wave height and wave energy inside the surf zone could be explained by the entrained air bubbles into water. Horikawa and Kuo (1966) showed that the dissipation rate under breaking waves depends on depth and bottom slope.

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Usually there are four types of wave breaking. Among them, the representative types of breaking wave of air entrainment are spilling breakers and plunging breakers. For plunging breakers, the entrainment of air bubbles is caused by the top of the wave forming a water jet projecting ahead of the wave face and entraining air when it impacts the free surface in front of the wave (Chanson and Lee 1997). With spilling breakers, the air bubbles travel with the wave as a surface roller and finally entrain into water. In this study the authors have tried to find out the effects of air bubble entrainment on the most important parameter, wave evolution on the basis of Fuhrboter's approach.

THEORETICAL APPROACH

Basic assumptions: The height of breaking wave and the rate of its decaying have very important influences on the near shore wave-induced phenomena, such as the generation of near shore currents, the transportation of sediments and so on. According to the linear wave theory, the local total wave energy within the one wavelength can be expressed as

$$E = \frac{1}{8}\rho_w gbLH^2 \tag{1}$$

where H, L, b and ρ_w represent the wave height, wavelength, width and water density, respectively.

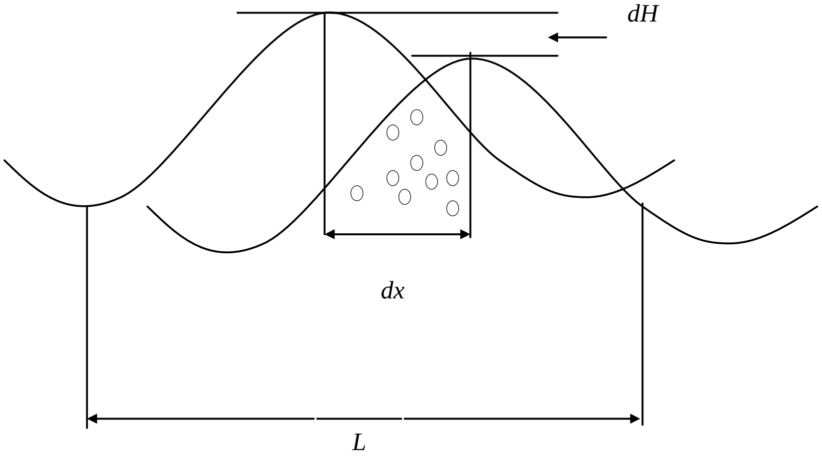

Fig. 1. Wave height reduction.

For a sudden reduction of wave height (dH) inside the surf zone which is shown in Fig. 1, the reduced wave energy (dE) is given by

$$dE = \frac{1}{4}\rho_w gbLHdH \tag{2}$$

This loss of energy must be transformed into other sorts of energy, so in turbulent motion and at last into heat by friction. By using the fact that the loss of wave energy is expressed in terms of that of turbulent energy. Before the energy is going into turbulence, there is a transfer mechanism by air entrainment and foam production.

In order to explain the effects of air entrainment in the surf zone, it is assumed that the distribution of air bubbles is exponential form proposed by Wu (1988) and Hoque and Aoki (2005):

$$C(z) = C_0 \exp(k_1 z) \tag{3}$$

where $C(z)$ is the part of the volume locally occupied by bubbles per unit width, k_1 is a decay parameter characterizing vertical distribution of air bubbles and C_0 denotes the reference concentration at the water surface $z = 0$.

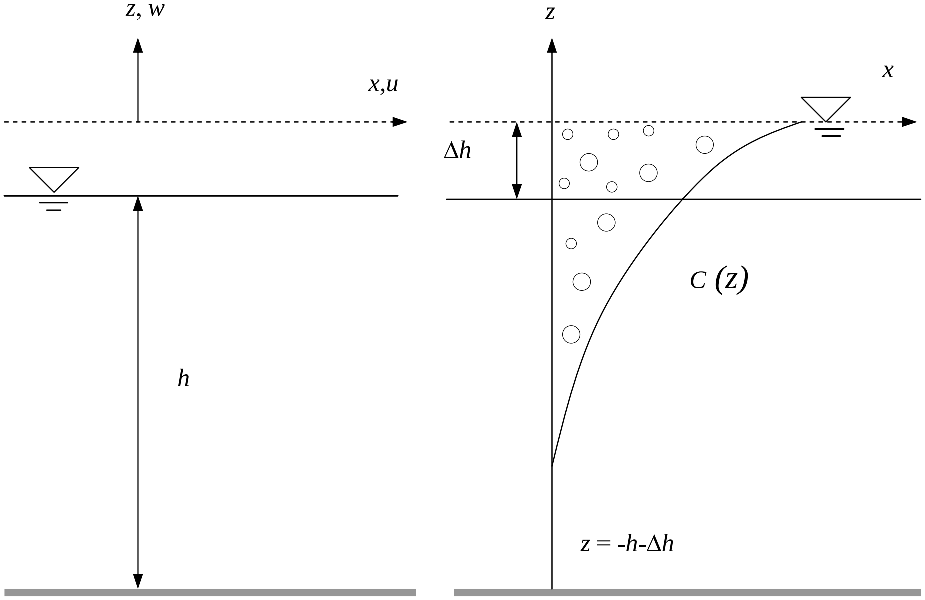

Fig. 2. Definition of the coordinate and static water level rise.

The rise of the free-surface level Δh is a function of the amount of entrained air and water depth (Fig. 2). The total volume of entrained air into water per unit width is defined as

$$\Delta h = \int_{-h-\Delta h}^{0} C(z)\, dz \tag{4}$$

where z is taken upward from the raised water surface.

Now Eq. (4) can be solved with the help of Eq. (3) in the following manner:

$$\Delta h = \frac{C_0}{k_1}\left[1 - e^{-k_1 h}\left\{1 - k_1 \Delta h + \frac{k_1 \Delta h}{2!} - \cdots\right\}\right]$$

Neglecting the higher order term of Δh, authors can write in explicit form of the above equation:

$$\Delta h = \frac{C_0}{k_1}\frac{(1 - e^{-k_1 h})}{(1 - C_0 e^{-k_1 h})} \tag{5}$$

Averaging procedure: It is very difficult to characterize the nature and behavior of every single bubble in the mixture. So in this respect, an averaging technique has been introduced by Biesheuvel and van Wijngaarden (1984) and by Hoque and Aoki (2006) for density ρ, where averaging is taken over the mixture can be written as

$$\rho = (1 - C)\rho_w + C\rho_a \tag{6}$$

where subscripts '*a*' and '*w*' denote the air and water. The air density is much smaller than the water density, so it can be neglected. So, we have

$$\rho = (1 - C_0 e^{-k_1 z})\rho_w \tag{7}$$

Static energy: When air bubbles are entered into the water, the water level increases by Δh. The excess static energy per unit horizontal area due to entrained air bubbles inside the surf zone can be expressed by

$$(dE)_{stat} = \int_{-h-\Delta h}^{0} \rho g z \, dz - \int_{-h-\Delta h}^{-\Delta h} \rho_w g z \, dz \tag{8}$$

Inserting the value of ρ, Eq. (8) becomes

$$(dE)_{stat} = \int_{-h-\Delta h}^{0} \rho_w (1 - C_0 e^{k_1 z}) g z \, dz - \int_{-h-\Delta h}^{-\Delta h} \rho_w g z \, dz \tag{9}$$

After simplification, the above equation can be written as

$$(dE)_{stat} = \rho_w g h^2 b C_0 \left[\frac{1 - e^{-k_1 h}}{(k_1 h)^2} - \frac{(1 - C_0)}{k_1 h}\frac{e^{-k_1 h}}{(1 - C_0 e^{-k_1 h})}\right] dx \tag{10}$$

where dx is the length of the water column.

Now combining Eqs. (2) and (10) authors get,

$$\frac{dH}{dx} = \frac{4h^2 C_0}{LH}\left[\frac{1 - e^{-k_1 h}}{(k_1 h)^2} - \frac{(1 - C_0)}{k_1 h}\frac{e^{-k_1 h}}{(1 - C_0 e^{-k_1 h})}\right] \tag{11}$$

This is the differential equation for wave height (H). Equation (11) can be solved for spilling and plunging breakers on the basis of empirical assumptions.

SOLUTIONS

Spilling breaker: In a spilling breaker, the energy which the wave has transported over many miles of sea is released gradually over a considerable distance. The wave peaks up until it is very steep but not vertical. Only the topmost portion of the wave curls over and descends on the forward slope of the wave, where it then slides down into the trough. Light foam may wash gently up the shore (Fig. 3a). Assuming that for a spilling breaker, the depth of aeration is in a linear relation to the reduction of wave height. So authors have

$$h \, \alpha \, H(x) \tag{12}$$

Authors need to compute $H(x)$ from Eq. (11). Integrating Eq. (11), while noting $H(x) = H_b$ at the breaking point $x = 0$, results in

$$H = H_b \exp\left\{ 4C_0 \left[\frac{1-e^{-k_1 H_b}}{(k_1 H_b)^2} - \frac{(1-C_0)e^{-k_1 H_b}}{k_1 H_b (1 - C_0 e^{-k_1 H_b})} \right] \frac{x}{L} \right\} \tag{13}$$

Fig. 3a. Spilling wave breakers.

In Fig. (3b), the variation of wave height as a function of x/L for different values of the C_0. Among these various exponential curves, curve for $C_0 = 0.15$ is the best choice, because Hoque (2002) measured this value near free surface in the spilling breakers.

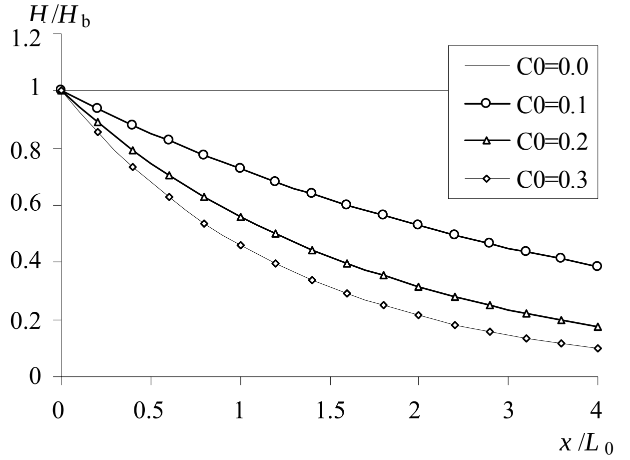

Fig. 3b. Wave height reduction.

Plunging breaker: In a plunging breaker, the energy is released suddenly into a downwardly directed mass of water. The crest advances faster than the base of the breaker, curls, and then descends violently into the wave trough. A considerable amount of air is trapped when this happens and this air escapes explosively behind the wave (Fig. 4a), throwing water high above the surface. The plunging breaker is characterized by a loud explosive sound.

For a plunging breaker, in first approximation the depth of aeration h can be considered to be of same order of magnitude as the breaker height H_b and to be constant during the process of breaking. So we have

$$h \,\alpha\, H_b = \text{constant} \tag{14}$$

Integrating Eq. (11) gives with $H(x) = H_b$ for $x = 0$

$$H = H_b \sqrt{8C_0 \left[\frac{1 - e^{-k_1 H_b}}{(k_1 H_b)^2} - \frac{(1 - C_0)e^{-k_1 H_b}}{k_1 H_b (1 - C_0 e^{-k_1 H_b})} \right] \frac{x}{L} + 1} \tag{15}$$

Fig. 4b compares the wave height curves for C_0 = 0.10, 0.20, 0.30 with the exact expression of linear theory. Thus, with the different specification of C_0, a part of wavelength is enough for the total destruction of the wave. The reference void fraction, C_0 was measured about 0.20 for plunging breaker (Hoque 2002).

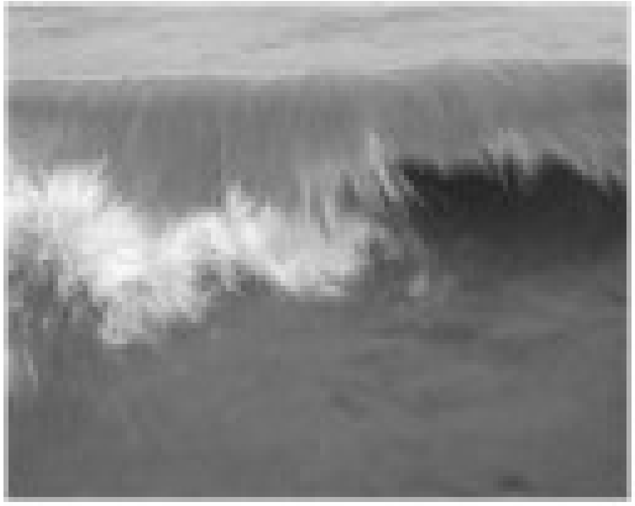

Fig. 4a. Plunging wave breakers.

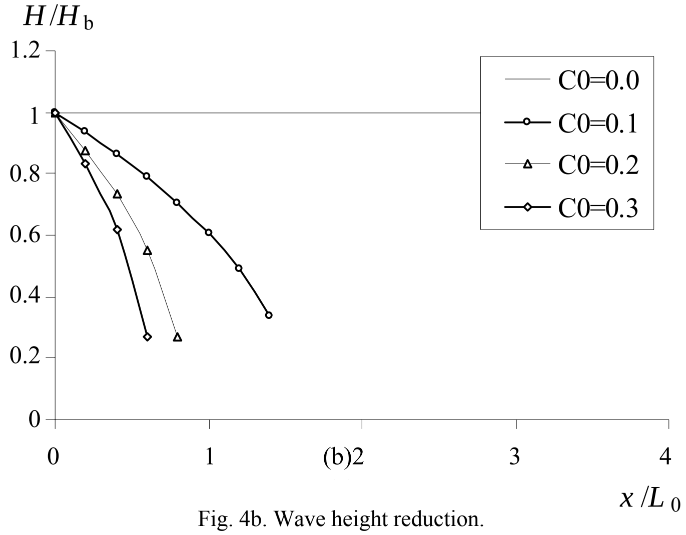

Fig. 4b. Wave height reduction.

RESULTS AND DISCUSSION

The dissipation of wave energy by entrained air under the breaking waves was studied theoretically both in spilling and plunging breakers. In the physical observation, most of wave energy is stored at first by the static energy of air bubbles which are driven into water (Fig. 1). Result shows that there is a correlation between the sudden reductions of wave height and entrained of air bubbles into the water (Figs 3b and 4b).

The theoretical study also provides that in a plunging breaker the wave energy is dissipated on a very short way whereas, in a spilling breaker this way is of the order of some wave lengths (Figs 3b and 4b).

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