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Research Article

Study on intutionistic fuzzy Hausdorff (T_2) bitopological spaces: theoretical insights

Saikh Shahjahan Miah* and Homayra Kabir

Department of Mathematics, Faculty of Science, Mawlana Bhashani Science and Technology University, Tangail, Bangladesh

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ABSTRACT

In this paper, we offer the concepts of some notions of Hausdorff (T_2) property in intuitionistic fuzzy Bitopological spaces. We demonstrate that each notion satisfies good extension property and establish that these notions satisfy hereditary properties. All the concepts of intuitionistic fuzzy Hausdorff topological spaces are preserved under one-one, onto fuzzy mappings.

Introduction

The root of fuzzy logic is the paper "Fuzzy Sets" by Zadeh (1965). Zadeh developed the idea of fuzzy sets in this research article 1965 as a mechanism to express and deal with vagueness and uncertainty in mathematical and computer systems. In contrast to classical set theory, which assumes that an element either belongs to a set or does not, fuzzy sets enable the representation of partial membership within a set. In the late 1960s, Chang (1968) made significant contributions to the study of fuzzy topological spaces. In his 1968 research, he formulated the notion of fuzzy topology. He pioneered the introduction of fuzzy topological spaces, a collection of fuzzy sets satisfied by the three conditions. Since Chang integrated fuzzy set theory with topology, many operations and properties were introduced within the fuzzy framework. The concept of separation axioms has significance in fuzzy topological spaces.

As a broader development of fuzzy topological space and as an extended generalization of bitopological spaces, which was initially introduced in the research article of Kelly (1963) named "Bitopological Spaces", Kandil and El-Shafee (1989) established various forms of separation axioms on fuzzy bitopological spaces in the research paper named "Biproximities and fuzzy bitopological spaces" in 1989. The theoretical foundation of fuzzy topological and bitopological spaces has undergone extensive research in several directions (Miah et al., 2017; Miah et al., 2018) in particular among those axioms, by the concepts of Hausdorff topological space, separations on bitopological spaces, fuzzy bitopological spaces, the concept of T_2-bitopological spaces has been established from the research articles of Sufiya et al. (1994), Nouh (1996), Miah and Amin (2017) and Amin et al. (2014). Again, as a generalized concept of fuzzy sets, in 1986, Atanosov (Atanassov and Stoeva, 1986; Atanassov, 1988) first put forward the idea of intuitionistic fuzzy sets, which expand the concept of the classical fuzzy set by introducing two membership functions, one that represents the degree of membership and the another that of non-membership. With the further advancement and development of this concept, Coker

*Corresponding author:<skhshahjahan@gmail.com>

(Coker, 1996a, 2000, 1997; Coker and Demirci 1995) and his colleagues pioneered the notions of intuitionistic fuzzy topological spaces in his research articles named "A note on intuitionistic sets and intuitionistic points," "On intuitionistic fuzzy points," "An introduction to intuitionistic fuzzy topological spaces," "An introduction to intuitionistic topological spaces ."According to the after approach, in 2014, Ahmed et al. (2014) introduced and conducted an exploration of the intuitionistic fuzzy T_2-spaces and established some relationships among them. The main focus of this paper is new developments and contributions to the expansion of intuitionistic fuzzy Hausdorff bitopological spaces. In this research paper's context, we define intuitionistic fuzzy Hausdorff bitopological spaces and illustrate their fundamental criteria, including the mappings, suitable extension property, and hereditary property.

Basic Notions and Preliminary Results

To attain our main result, defining specific terms and notions is necessary.

Definition 2.1 (Coker, 1996a, 2000; Bayhan and Coker, 2001)**:** Let us suppose that the object M has the form $M = (x, M_1, M_2)$ such that X has the subsets M_1 and M_2 that satisfy $M_1 \cap M_2 = \emptyset$. Then, we refer to M_1 as the set of M members and M_2 as the set of M non-members. Throughout this piece of writing, we will refer to the intuitionistic set as $M = (M_1, M_2)$.

Definition 2.2 (Coker 1996a; Coker 2000; Bayhan and Coker 2001) Let us Consider $M = (M_1, M_2)$ and $N = (N_1, N_2)$ to represent the intuitionistic sets on X. let $\{M_j : j \in J\}$ be an any family of intuitionistic sets in X where $M_j = ({M_j}^{(1)}, {M_j}^{(2)})$. Then

(a) $M \subseteq N$ if and only if $M_1 \subseteq N_1$ and $M_2 \supseteq N_2$.

(b) $M = N$ if and only if $M \subseteq N$ and $N \supseteq M$.

(c) $\bar{M} = (M_1,\ M_2)$, the complement of M.

(d) $\cap M_j = (\cap {M_j}^{(1)}, \cup {M_j}^{(2)})$.

(e) $\cup M_j = (\cup {M_j}^{(1)}, \cap {M_j}^{(2)})$.

(f) $\phi_{\sim} = (\phi, X)$ and $X_{\sim} = (X, \phi)$.

Definition 2.3 (Coker, 2000; Bayhan and Coker, 2001) An intuitionistic topology defined on a non-empty set X is a family τ of intuitionistic sets in X which satisfies the axioms listed following:

(1) $\phi_{\sim}, X_{\sim} \in \tau$.

(2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$. $\cup G_i \in \tau$ for any arbitrary family $G_i \in \tau$.

In this situation, the pair (X, τ) is referred to as an intuitionistic topological space, and every subset of X that possesses an intuitionistic set τ is called an intuitionistic open set in X.

Definition 2.4 (Atanassov, 1986; Coker, 1996a, 1997; Bayhan and Coker, 1996) Let us assume that X is a non-empty set and $I = [0,1]$. Let $M = \{(x, \mu_M(x), \vartheta_M(x))\}$ be the intuitionistic fuzzy set in X where x belongs to X and $\mu_M : X \to I$ represents the degree of membership and $\vartheta_M : X \to I$ represents the degree of non-membership; $\mu_M(x) + \vartheta_M(x) \leq 1$.

The set of all intuitionistic fuzzy sets in X is defined as $I(X)$. Notably, the simpler notation $(\mu_M, 1 - \mu_M)$ represents every fuzzy set μ_M in X as an intuitionistic fuzzy set.

In this work, instead of using $M = \{(x, \mu_M(x), \vartheta_M(x)), x \in X\}$,we shall use shorter notation $M = (\mu_M, \vartheta_M)$.

Definition 2.5 (Atanassov, 1986; Coker, 1996, 1997; Bayhan and Coker, 1996) Let us assume that the intuitionistic fuzzy sets in X are $M = (\mu_M, \vartheta_M)$ and $N = (\mu_N, \vartheta_N)$. Then

(1) $M \subseteq N$ if and only if $\mu_M \leq \mu_N$ and $\vartheta_M \geq \vartheta_N$.

(2) $M = N$ if and only if $M \subseteq N$ and $N \subseteq M$.

(3) $M^C = (\vartheta_M, \mu_M)$.

(4) $M \cap N = (\mu_M \cap \mu_N;\ \vartheta_M \cup \vartheta_N)$.

(5) $M \cup N = (\mu_M \cup \mu_N; \vartheta_M \cap \vartheta_N)$.

(6) $0_{\sim} = (0^{\sim}, 1^{\sim})$ and $1_{\sim} = (1^{\sim}, 0^{\sim})$.

Definition 2.6 (Coker, 1996b, 1997; Bayhan and Coker, 1996) An intuitionistic fuzzy topology on a non-empty set X is the collection t of intuitionistic fuzzy sets on X if it satisfies the following principles:

(1) $0_{\sim}, 1_{\sim} \in t$.

(2) If $M_1, M_2 \in t$, then$M_1 \cap M_2 \in t$.

(3) If $M_i \in t$ for each i, then $\cup M_i \in t$.

The pair (X, t) is an intuitionistic fuzzy topological space, and every intuitionistic fuzzy set in t possesses an intuitionistic fuzzy open set in X.

Definition 2.7 (Ahmed et al., 2014) An intuitionistic fuzzy topological space (X, t) is called

(a) IF-T_2(i) if for all $x_1, x_2 \in X$, $x_1 \neq x_2$ there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in t$ such that $\mu_M(x_1) = 1, \vartheta_M(x_1) = 0; \mu_N(x_2) = 1, \vartheta_N(x_2) = 0$ and $M \cap N = 0_{\sim}$.

(b) IF-T_2(ii) if for all $x_1, x_2 \in X, x_1 \neq x_2$ there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in t$ such that $\mu_M(x_1) = 1, \vartheta_M(x_1) = 0; \mu_N(x_2) > 1, \vartheta_N(x_2) = 0$ and $M \cap N = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0,1]$.

(c) IF-T_2(iii) if for all $x_1, x_2 \in X$, $x_1 \neq x_2$, there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in t$ such that $\mu_M(x_1) > 0, \vartheta_M(x_1) = 0; \mu_N(x_2) = 1, \vartheta_N(x_2) = 0$ and $M \cap N = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0,1]$.

(d) IF-T_2(iv) if for all $x_1, x_2 \in X, x_1 \neq x_2$ there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in t$ such that $\mu_M(x_1) > 1, \vartheta_M(x_1) = 0; \mu_N(x_2) > 0, \vartheta_N(x_2) = 0$ and $M \cap N = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0,1]$.

Definition 2.8 (Kelly, 1963)**:** Let us consider two general topologies, S and T, which are defined on any non-empty set X. The combination (X, S, T) is referred to as a bitopological space.

Definition 2.9 (Kandil and El-Shafee, 1989) The triple (X, s, t) is known as fuzzy bitopological space where s and t stand for two distinct fuzzy topologies defined on X.

3. Characteristics and Attributes of Intuitionistic Fuzzy Hausdorff (T_2) Bitopological Spaces

Definition: An intuitionistic fuzzy bitopological space (X, s, t) is called

(a) IFB-T_2(i) if for all $x_1, x_2 \in X$, $x_1 \neq x_2$ there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1) = 1, \vartheta_M(x_1) = 0; \mu_N(x_2) = 1, \vartheta_N(x_2) = 0$ and $M \cap N = 0_{\sim}$.

(b) IFB-T_2(ii) if for all $x_1, x_2 \in X, x_1 \neq x_2$ there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1) = 1, \vartheta_M(x_1) = 0; \mu_N(x_2) > 1, \vartheta_N(x_2) = 0$ and $M \cap N = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0,1]$.

(c) IFB-T_2(iii) if for all $x_1, x_2 \in X$, $x_1 \neq x_2$ there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1) > 0, \vartheta_M(x_1) = 0; \mu_N(x_2) = 1, \vartheta_N(x_2) = 0$ and $M \cap N = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0,1]$.

(d) IFB-T_2(iv) if for all $x_1, x_2 \in X$, $x_1 \neq x_2$ there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1) > 1, \vartheta_M(x_1) = 0; \mu_N(x_2) > 0, \vartheta_N(x_2) = 0$ and $M \cap N = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0,1]$.

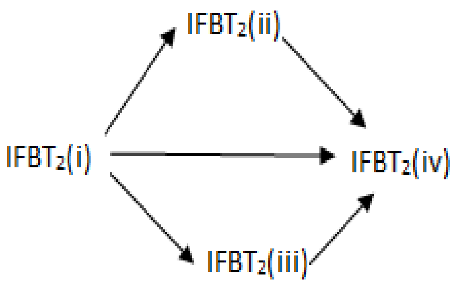
Definition: Let us assume that $\alpha \in (0,1)$. Then, an intuitionistic fuzzy bi-topological space (X, s, t) is referred to as

(a) α-IFB-T_2(i) if for all $x_1, x_2 \in X$, $x_1 \neq x_2$ there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1) = 1, \vartheta_M(x_1) = 0; \mu_N(x_2) \geq \alpha, \vartheta_N(x_2) = 0$ and $M \cap N = 0_{\sim}$.

(b) α-IFB-T_2(ii) if for all $x_1, x_2 \in X$, $x_1 \neq x_2$ there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1) \geq \alpha, \vartheta_M(x_1) = 0; \mu_N(x_2) \geq \alpha, \vartheta_N(x_2) = 0$ and $M \cap N = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0,1]$.

(c) α-IFB-T_2(iii) if for all $x_1, x_2 \in X$, $x_1 \neq x_2$ there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1) > 0, \vartheta_M(x_1) = 0; \mu_N(x_2) \geq \alpha, \vartheta_N(x_2) = 0$ and $M \cap N = (0^{\sim}, \gamma^{\sim})$ where $\gamma \in (0,1]$.

Theorem: Let the bitopological space $(X,)$ s, t be defined as an intuitionistic fuzzy bitopological space. Subsequently, the following consequences follow.



Proof: Given that (X,s,t) is IFB-T_2(i), then for all $x_1,x_2 \in X, x_1 \neq x_2$ there exist $M=(\mu_M,\vartheta_M), N=(\mu_N,\vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1)=1, \vartheta_M(x_1)=0$; $\mu_N(x_2)=1, \vartheta_N(x_2)=0$ and $M \cap N = 0_\sim$

$\Rightarrow \mu_M(x_1)>0, \vartheta_M(x_1)=0$; $\mu_N(x_2)>0, \vartheta_N(x_2)=0$ and $M \cap N = (0^\sim,\gamma^\sim)$ where $\gamma \in (0,1]$.

Which is IFBT$_2$ (iv).

Hence, IFB-T_2(i) $\Rightarrow$ IFBT$_2$ (iv).

Furthermore, let (X,s,t) is IFB-T$_2$ (ii), then for all $x_1,x_2 \in X, x_1 \neq x_2$ there exist $M=(\mu_M,\vartheta_M), N=(\mu_N,\vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1)=1, \vartheta_M(x_1)=0$;$\mu_N(x_2)>0,\ \vartheta_N(x_2)=0$ and $M \cap N=(0^\sim,\gamma^\sim)$ where $\gamma \in (0,1]$

$\Rightarrow \mu_M(x_1)>0,\ \vartheta_M(x_1)=0$;$\mu_N(x_2)>0,\ \vartheta_N(x_2)=0$ and $M \cap N=(0^\sim,\gamma^\sim)$ where $\gamma \in (0,1]$.

Which is also IFB-T$_2$(iv).

Hence, IFB-T$_2$ (ii) $\Rightarrow$ IFB-T$_2$ (iv).

Again, let (X,s,t) is IFB-T$_2$ (iii), then for all $x_1,x_2 \in X, x_1 \neq x_2$ there exist $M=(\mu_M,\vartheta_M)$, $N=(\mu_N,\vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1) \geq 0$, $\vartheta_M(x_1)=0$; $\mu_N(x_2)=1, \vartheta_N(x_2)=0$ and $M \cap N = (0^\sim,\gamma^\sim)$ where $\gamma \in (0,1]$

$\Rightarrow \mu_M(x_1)>0,\ \vartheta_M(x_1)=0$;$\mu_N(x_2)>0,\ \vartheta_N(x_2)=0$ and $M \cap N=(0^\sim,\gamma^\sim)$ where $\gamma \in (0,1]$.

Which is also IFB-T$_2$(iv).

Hence, IFB-T$_2$ (iii) $\Rightarrow$ IFB-T$_2$ (iv).

Furthermore, it can easily verify that IFB-T_2(i) $\Rightarrow$ IFB-T_2(ii), IFB-T_2(i) $\Rightarrow$ IFB-T_2(iii).

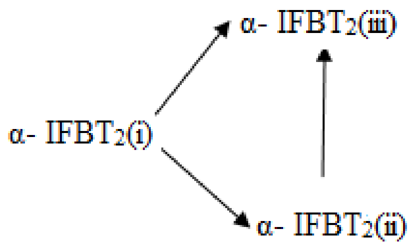
We give a counter example following to show that none of the reverse implications exist.

Counterexample: For IFB-T_2(iv) $\nRightarrow$ IFB-T$_2$ (i), let us assume that s and t are intuitionistic fuzzy topologies on $X=\{x_1,x_2\}$ generated by $M=\{x_1,0.3,0\}$ and $N=\{x_2,0.2,0\}$ respectively and the corresponding intuitionistic fuzzy bitopological space (X,s,t) which is IFB-T$_2$(iv) but not IFB-T_2(i).

For IFB-T_2(iv) $\nRightarrow$ IFB-T$_2$ (ii), let us assume that s and t are intuitionistic fuzzy topologies on $X=\{x_1,x_2\}$ generated by $M=\{x_1,0.4,0\}$ and $N=\{x_2,0.1,0\}$ respectively and the corresponding intuitionistic fuzzy bitopological space (X,s,t) which is IFB-T$_2$(iv) but not IFB-T_2(ii).

For IFB-T_2(iv) $\nRightarrow$ IFBT$_2$ (iii), let us assume that s and t are intuitionistic fuzzy topologies defined on $X=\{x_1,x_2\}$ generated by $M=\{x_1,0.9{,}0\}$ and $N=\{x_2,0.5{,}0\}$ respectively and the corresponding intuitionistic fuzzy bitopological space (X,s,t) which is IFB-T$_2$ (iv) but not IFB-T$_2$ (iii).

Theorem: The bitopological space (X,s,t) is an intuitionistic fuzzy bitopological space. Consider the following implications.



Proof: Let us assume that (X,s,t) is α-IFB-T_2(i). We shall prove that (X,s,t) is α-IFB-T_2(i). Since (X,s,t) is α-IFB-T_2(i) then for all $x_1,x_2 \in X, x_1 \neq x_2$ there exist $M=(\mu_M,\vartheta_M), N=(\mu_N,\vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1)=1,\ \vartheta_M(x_1)=0;\ \mu_N(x_2) \geq \alpha$, $\vartheta_N(x_2)=0$ and $M \cap N = 0_\sim$

$\Rightarrow \mu_M(x_1) \geq \alpha,\ \vartheta_M(x_1)=0$;$\mu_N(x_2) \geq \alpha,\ \vartheta_N(x_2)=0$ for any $\alpha \in (0,1)$ and $M \cap N=(0^\sim,\gamma^\sim)$.

Which is also α-IFB-T$_2$ (ii) space.

Hence α-IFB-T_2(i) $\Rightarrow$ α-IFB-T$_2$ (ii)

Again, suppose (X,s,t) is α-IFB-T$_2$ (ii). We will prove that (X,s,t) is α-IFB-T$_2$ (ii). Now, Since (X,s,t) is α-IFB-T$_2$ (ii), then for all $x_1,x_2 \in X, x_1 \neq x_2$ there exist $M=(\mu_M,\vartheta_M), N=(\mu_N,\vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1) \geq \alpha,\ \vartheta_M(x_1)=0$;$\mu_N(x_2) \geq \alpha$, $\vartheta_N(x_2)=0$ and $M \cap N=(0^\sim,\gamma^\sim)$ where $\gamma \in (0,1]$

$\Rightarrow \mu_M(x_1)>0,\ \vartheta_M(x_1)=0$;$\mu_N(x_2) \geq \alpha,\ \vartheta_N(x_2)=0$ for any $\alpha \in (0,1)$ and $M \cap N=(0^\sim,\gamma^\sim)$ where $\gamma \in (0,1]$.

Which is also α-IFB-T$_2$ (iii) space.

Hence α-IFB-T$_2$ (ii) $\Rightarrow$ α-IFB-T$_2$ (iii)

Hence it is verified that α-IFB-T_2(i) ⇒ α-IFB-T_2 (ii) ⇒ α-IFB-T_2 (iii)

Again, suppose that (X,s,t) is α-IFB-T_2(i) space. We will establish the proof that (X,s,t) is α-IFB-T_2 (iii). Since, (X,s,t) is α-IFB-T_2(i), then for all $x_1,x_2 \in X, x_1 \neq x_2$ there exist $M=(\mu_M,\vartheta_M), N=(\mu_N,\vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1)=1$, $\vartheta_M(x_1)=0$; $\mu_N(x_2) \geq \alpha$, $\vartheta_N(x_2)=0$ and $M \cap N = 0_\sim$

$\Rightarrow \mu_M(x_1)>0$, $\vartheta_M(x_1)=0$; $\mu_N(x_2) \geq \alpha$, $\vartheta_N(x_2)=0$ for any $\alpha \in (0,1)$ and $M \cap N = 0^\sim, \gamma^\sim)$ where $\gamma \in (0,1]$.

Which is also α-IFB-T_2 (iii) space.

Hence α-IFB-T_2(i) ⇒ α-IFB-T_2 (iii).

We give a counter example following to show that none of the reverse implications exist.

Counterexample: For α-IFB-T_2(iii) ⇏ α-IFB-T_2(i), let us assume that s and t are intuitionistic fuzzy topologies on $X=\{x_1,x_2\}$ generated by $M=\{x_1,0.5,0\}$ and $N=\{x_2,0.6,0\}$ respectively. The corresponding intuitionistic fuzzy bitopological space on X defined by $\{M,N\}$ is (X,s,t). For $\alpha=0.3$, the IFBTS (X,s,t) is α-IFB-T_2(iii) but not α-IFB-T_2(i).

For α-IFB-T_2(iii) ⇏ α-IFBT_2 (ii), let us assume that s and t are intuitionistic fuzzy topologies on $X=\{x_1,x_2\}$ generated by $M=\{x_1,0.1,0\}$ and $N=\{x_2,0.4,0\}$ respectively and the corresponding intuitionistic fuzzy bitopological space on X defined by $\{M,N\}$ is (X,s,t). For $\alpha=0.3$, the IFBTS (X,s,t) is α-IFB-T_2(iii) but not α-IFB-T_2(ii).

Theorem: Let us consider an intuitionistic fuzzy bitopological space defined by (X,s,t), and $0<\alpha \leq \beta<1$. Then

(a) β-IFB-T_2(i) ⇒ α-IFB-T_2(i)

(b) β-IFB-T_2(ii) ⇒ α-IFB-T_2(ii)

(c) β-IFB-T_2(iii) ⇒ α-IFB-T_2(iii)

Proof: **(b)** Let (X,s,t) be an intuitionistic fuzzy bitopological space and is defined as β-IFB-T_2(ii). Then for all $x_1,x_2 \in X, x_1 \neq x_2$ there exist $M=(\mu_M,\vartheta_M), N=(\mu_N,\vartheta_N) \in (s \cup t)$ such that $\mu_M(x_1) \geq \beta$, $\vartheta_M(x_1)=0$; $\mu_N(x_2) \geq \beta$, $\vartheta_N(x_2)=0$ and $M \cap N = (0^\sim,\gamma^\sim)$

$\Rightarrow \mu_M(x_1) \geq \alpha$, $\vartheta_M(x_1)=0$;$\mu_N(x_2) \geq \alpha$, $\vartheta_N(x_2)=0$ and $M \cap N = (0^\sim,\gamma^\sim)$ since $0<\alpha \leq \beta<1$.

Which is α-IFB-T_2 (ii) space.

Hence β-IFB-T_2 (ii) ⇒ α-IFB-T_2 (ii).

Similarly, we can prove that β-IFB-T_2(i) ⇒ α-IFB-T_2(i) and β-IFB-T_2(iii) ⇒ α-IFB-T_2(iii).

To show that none of the reverse implication exist, we give a counter example following

Counterexample: For β-IFB-T_2(i) ⇏ α-IFB-T_2(i), let us assume that s and t are intuitionistic fuzzy topologies defined on $X=\{x_1,x_2\}$ generated by $M=\{x_1,1,0\}$ and $N=\{x_2,0.5,0\}$ respectively and the corresponding intuitionistic fuzzy bitopological space on X is (X,s,t) defined by $\{M,\}$ N. For $\alpha=0.4$ and $\beta=0.6$, we can say that the IFBTS (X,s,t) is α-IFB-T_2(i) but not β-IFB-T_2(i).

For β-IFB-T_2(ii) ⇏ α-IFB-T_2(ii), let us assume that s and t are intuitionistic fuzzy topologies defined on $X=\{x_1,x_2\}$ generated by $M=\{x_1,0.6,0\}$ and $N=\{x_2,0.3,0\}$ respectively and the corresponding intuitionistic fuzzy bitopological space on X is (X,s,t) defined by $\{M,N\}$. For $\alpha=0.3$ and $\beta=0.7$, we can say that the IFBTS (X,s,t) is α-IFB-T_2(ii) but not β-IFB-T_2(ii).

For β-IFB-T_2(iii) ⇏ α-IFB-T_2(iii), let us assume that s and t are intuitionistic fuzzy topologies defined on $X=\{x_1,x_2\}$ generated by $M=\{x_1,0.2,0\}$ and $N=\{x_2,0.3,0\}$ respectively and the corresponding intuitionistic fuzzy bitopological space on X is (X,s,t) defined by $\{M,N\}$. For $\alpha=0.2$ and $\beta=0.4$, we can say that the IFBTS (X,s,t) is α-IFB-T_2(iii) but not β-IFB-T_2(iii).

Theorem: Consider an intuitionistic fuzzy bitopological space which is defined (X,s,t), characterized with $U \subseteq X$ and $s_U=\{M \mid U: M \in s\}$; $t_U=\{N \mid U: M \in t\}$; $\alpha \in (0,1)$.Then

(a) (X,s,t) is IFB-T_2(i) ⇒ (U,s_U,t_U) is IFB-T_2(i)

(b) (X,s,t) is IFB-T_2(ii) ⇒ (U,s_U,t_U) is IFB-T_2(ii)

(c) (X,s,t) is IFB-T_2(iii) ⇒ (U,s_U,t_U) is IFB-T_2(iii)

(d) (X,s,t) *is* IFB-T_2(iv) ⇒ (U,s_U,t_U) is IFB-T_2(iv)

(e) (X,s,t) *is* α-IFB-T_2(i) ⇒ (U,s_U,t_U) is α-IFB-T_2(i)

(f) (X,s,t) *is* α-IFB-T_2(ii) ⇒ (U,s_U,t_U) is α-IFB-T_2(ii)

(g) (X,s,t) *is* α-IFB-T_2(iii) ⇒ (U,s_U,t_U)) is α-IFB-T_2(iii)

Proof: (a) Consider an intuitionistic fuzzy bitopological space defined (X,s,t) as IFB-T_2(i). We will establish the proof that $(U,\ s_U,\ t_U)$ is IFB-T_2(i). Let $x_1, x_2 \in U,\ x_1 \neq x_2$, then $x_1, x_2 \in X,\ x_1 \neq x_2$ as $U \subseteq X$. Since (X,s,t) is IFB-T_2(i), then there exist $M_U = \left(\mu_{M_U}, \vartheta_{M_U}\right), N_U = \left(\mu_{N_U}, \vartheta_{M_U}\right) \in (s_U \cup t_U)$ such that $\mu_{M_U}(x_1) = 1,\ \vartheta_{M_U}(x_1) = 0;\ \mu_{N_U}(x_2) = 1,$ $\vartheta_{N_U}(x_2) = 0$ and $M_U \cap N_U = 0_\sim$

$\Rightarrow\ \mu_{M_U}|U(x_1) = 1,\quad \vartheta_{M_U}|U(x_1) = 0; \mu_{N_U}|U(x_2) = 1, \vartheta_{N_U}|U(x_2) = 0$ and $M_U \cap N_U = 0_\sim$.

Since $\{(\mu_{M_U}|U, \vartheta_{M_U}|U),\ \ (\mu_{N_U}|U, \vartheta_{N_U}|U)\} \in (s_U \cup t_U)$.

Therefore, the intuitionistic fuzzy bitopological space $(U,\quad sU, t_U)$ is IFB-T_2(i).

Similarly, we can prove (b), (c), (d), (e), (f), and (g).

Definition: An intuitionistic bitopological space (IBTS) $(X, \tau_{1,}\tau_{2,})$ is called IB-T_2 Space if for all $x, y \in X, x \neq y$ there exists $C = (C_{1,}\ C_2),\ \ D = \left(D_{1,}\ D_2\right) \in (\tau_{1,}\tau_{2,})$ such that $x_1 \in C_{1,} x_2, \in D_{1,}\ and\ C \cap D = \phi_\sim$.

Good Extension Property

Theorem: Let an intuitionistic bitopological space is defined by the triple $(X, \tau_{1,}\tau_{2,})$ and an intuitionistic fuzzy bitopological space is defined by the triple (X, t_1, t_2). Then,

(a) $(X, \tau_{1,}\tau_{2,})$ is IB-T_2⇔$(X, t_{1,}t_{2,})$ is IFB-T_2(i)

(b) $(X, \tau_{1,}\tau_{2,})$ is IB-T_2⇒$(X, t_{1,}t_{2,})$ is IFB -T_2(ii)

(c) $(X, \tau_{1,}\tau_{2,})$ is IB-T_2⇒$(X, t_{1,}t_{2,})$ is IFB -T_2(iii) $(X, \tau_{1,}\tau_{2,})$ is IB-T_2⇒$(X, t_{1,}t_{2,})$ is IFB -T_2(iv).

Proof: (b) suppose that $(X, \tau_{1,}\tau_{2,})$ is an IB-T_2 Space. We prove that $(X, t_{1,}t_2)$ is IFB-T_2(ii). Given that (X, t_1, τ_2) is IB-T_2. So, for all $x_1,\ x_2 \in X,\ x_1 \neq x_2$ there exist $C = (C_{1,} C_2),\ \ D = \left(D_{1,} D_2\right) \in (\tau_1 \cup \tau_{2,})$ such that $x_1 \in C_{1,},\ x_2 \in D_{1,}\ and\ C \cap D = \phi_\sim$

$\Rightarrow 1_{C_1}(x_1) = 1; 1_{D_1}(x_2) = 1$ and $C \cap D = \phi_\sim$

$\Rightarrow 1_{C_1}(x_1) = 1,\qquad 1_{C_2}(x_1) = 0;\qquad 1_{D_1}(x_2) > 0,$ $1_{D_2}(x_2) = 0$ and $C \cap D = \phi_\sim$.

Let $1_{C_1} = \mu_M,\ 1_{C_2} = \vartheta_M,\ 1_{D_1} = \mu_N,\ 1_{D_2} = \vartheta_N$, then $\mu_M(x_1) = 1,\ \ \vartheta_M(x_1) = 0;\ \ \mu_N(x_2) > 0,\ \vartheta_N(x_2) = 0$ and $M \cap N = (0^\sim, \gamma^\sim)$ where $\gamma \in (0,1]$.

Since $\{(\mu_M, \vartheta_M), (\mu_N, \vartheta_N)\}\ \in (t_{1,} \cup t_2)$

$\Rightarrow (X, t_1, t_2)$ is IFB-T_2 (ii).

Hence, IB-T_2 ⇒ IFB-T_2(ii)

(c) Suppose, $(X, \tau_{1,}\tau_{2,})$ is an IB-T_2 Space. We must prove that $(X, t_{1,}t_2)$ is IFB- T_2(iii). Given that $(X, \tau_{1,}\tau_{2,})$ is IB-T_2 then for all $x_1, x_2 \in X,\ x_1 \neq x_2$ there exist $C = (C_{1,} C_2),\ \ D = \left(D_{1,} D_2\right) \in (\tau_1 \cup \tau_{2,})$ such that $x_1 \in C_{1,}, x_2 \in D_{1,}$ and $C \cap D = \phi_\sim$

$\Rightarrow 1_{C_1}(x_1) = 1; 1_{D_1}(x_2) = 1$ and $C \cap D = \phi_\sim$

$\Rightarrow 1_{C_1}(x_1) > 0,\qquad 1_{C_2}(x_1) = 0;\qquad 1_{D_1}(x_2) = 1,$ $1_{D_2}(x_2) = 0$ and $C \cap D = \phi_\sim$.

Let $1_{C_1} = \mu_M,\ 1_{C_2} = \vartheta_M,\ 1_{D_1} = \mu_N,\ 1_{D_2} = \vartheta_N$, then $\mu_M(x_1) > 0,\ \ \vartheta_M(x_1) = 0;\ \ \mu_N(x_2) = 1,\ \vartheta_N(x_2) = 0$ and $M \cap N = (0^\sim, \gamma^\sim)$ where $\gamma \in (0,1]$.

Since $\{(\mu_M, \vartheta_M), (\mu_N, \vartheta_N)\}\ \in (t_{1,} \cup t_2)$

$\Rightarrow (X, t_1, t_2)$ is IFB-T_2 (iii).

Hence, IB-T_2 ⇒ IFB-T_2(iii)

Similarly, we can show IB-T_2 ⇔ IFB-T_2(i) and IB-T_2⇒IFB -T_2(iv).

We give a counterexample following to show that none of the reverse implications exists.

Counterexample: For IFB-T_2(ii) ⇏ IB-T_2, let us assume that t_1 and t_2 are intuitionistic fuzzy topologies on $X = \{x_1, x_2\}$ generated by $M = \{x_1, 1{,}0\}$ and $N = \{x_2, 0.2{,}0\}$ respectively. They form the intuitionistic fuzzy bitopological space (X, t_1, t_2) on X defined by $\{M, N\}$. So, the IFBTS (X, t_1, t_2) is IFB-T_2(ii) but not IB-T_2.

For IFB-T_2(iii) ⇏ IB-$T_{2,}$ let us assume that t_1 and t_2 are intuitionistic fuzzy topologies on $X =$

$\{x_1, x_2\}$ generated by $M = \{x_1, 0.2,0\}$ and $N = \{x_2, 1,0\}$ respectively. They form the intuitionistic fuzzy bitopological space (X, t_1, t_2) on X defined by $\{M,\}$ N. So, the IFBTS (X, t_1, t_2) is IFB-T_2(iii) but not IB-T_2.

For IFB-T_2(iv) $\not\Rightarrow$ IB-T_2, let us assume that t_1 and t_2 are intuitionistic fuzzy topologies on $X = \{x_1, x_2\}$ generated by $M = \{x_1, 0.3,0\}$ and $N = \{x_2, 0.2,0\}$ respectively. They form the intuitionistic fuzzy bitopological space (X, t_1, t_2) on X defined by $\{M,\}$ N. So, the IFBTS (X, t_1, t_2) is IFB-T_2(ii) but not IB-T_2.

Theorem: Let us consider that an intuitionistic bitopoloical space is defined by $(X, \tau_{1,} \tau_{2,})$ and an intuitionistic fuzzy bitopoloical space is defined by $(X, t_{1,} t_{2,})$. Then we consider the implications given below:

(a) IB-$T_2 \Rightarrow \alpha$-IFB-T_2(i)
(b) IB-$T_2 \Rightarrow \alpha$-IFB-T_2(ii)
(c) IB-$T_2 \Rightarrow \alpha$-IFB-T_2(iii)

Proof: (b) Suppose (X, t_1, τ_2) is an IB-T_2 Space. We shall prove that $(X, t_{1,} t_2)$ is α-IFB-T_2(ii). Since $(X, t_1, \tau_{2,})$ is IB-T_2, then for all $x_1, x_2 \in X, x_1 \neq x_2$ there exist $C = (C_{1,} C_2), D = (D_{1,} D_2) \in (\tau_1 \cup \tau_{2,})$ such that $x_1 \in C_1,\ x_2 \in D_1$ and $C \cap D = \phi_\sim$

$\Rightarrow\ 1_{C_1}(x_1) = 1; 1_{D_1}(x_2) = 1$ and $C \cap D = \phi_\sim$

$\Rightarrow\ 1_{C_1}(x_1) = 1; 1_{D_1}(x_2) \geq \alpha$ for any $\alpha \in (0,1)$ and $C \cap D = \phi_\sim$

$\Rightarrow 1_{C_1}(x_1) \geq \alpha, \quad 1_{C_2}(x_1) = 0; \quad 1_{D_1}(x_2) \geq \alpha, 1_{D_2}(x_2) = 0$ for any $\alpha \in (0,1)$ and $C \cap D = \phi_\sim$.

Let $1_{C_1} = \mu_M,\ 1_{C_2} = \vartheta_M,\ 1_{D_1} = \mu_N,\ 1_{D_2} = \vartheta_N$, then $\mu_M(x_1) \geq \alpha, \vartheta_M(x_1) = 0; \mu_N(x_2) \geq \alpha,\ \vartheta_N(x_2) = 0$ for any $\alpha \in (0,1)$ and $M \cap N = (0^\sim, \gamma^\sim)$ where $\gamma \in (0,1]$.

Since $\{(\mu_M, \vartheta_M), (\mu_N, \vartheta_N)\} \in (t_{1,} \cup t_2)$

$\Rightarrow (X, t_1, t_2)$ is α-IFB-T_2 (ii).

Hence, IB-$T_2 \Rightarrow \alpha$-IFB-T_2(ii).

Similarly, we can show that IB-$T_2 \Rightarrow \alpha$-IFB-T_2(ii) and IB-$T_2 \Rightarrow \alpha$-IFB-T_2(iii).

We give a counterexample following to show that none of the reverse implications exists.

Counterexample: For α-IFB-T_2(i)$\not\Rightarrow$IB-T_2, let us assume that t_1 and t_2 are intuitionistic fuzzy topologies defined on $X = \{x_1, x_2\}$ generated by $M = \{x_1, 1,0\}$ and $N = \{x_2, 0.4,0\}$ respectively and they form the intuitionistic fuzzy bitopological space (X, t_1, t_2) on X defined by $\{M, N\}$. For $\alpha = 0.3$, we can say that the IFBTS (X, t_1, t_2) is α-IFB-T_2(i) but not IB-T_2.

For α-IFB-T_2(ii)$\not\Rightarrow$IB-T_2**,** let us assume that t_1 and t_2 are intuitionistic fuzzy topologies defined on $X = \{x_1, x_2\}$ generated by $M = \{x_1, 0.4,0\}$ and $N = \{x_2, 0.6,0\}$ respectively and they form the intuitionistic fuzzy bitopological space (X, t_1, t_2) on X defined by $\{M, N\}$. For $\alpha = 0.4$, we can say that the IFBTS (X, t_1, t_2) is α-IFB-T_2(ii) but not IB-T_2.

For α-IFB-T_2(iii)$\not\Rightarrow$IB-T_2, let us assume that t_1 and t_2 are intuitionistic fuzzy topologies defined on $X = \{x_1, x_2\}$ generated by $M = \{x_1, 0.1,0\}$ and $N = \{x_2, 0.7,0\}$ respectively and they form the intuitionistic fuzzy bitopological space (X, t_1, t_2) on X defined by $\{M, N\}$. For $\alpha = 0.2$, we can say that the IFBTS (X, t_1, t_2) is α-IFB-T_2(iii) but not IB-T_2.

Theorem: Let us consider the two triples (X, t_1, t_2) and (Y, s_1, s_2) define two intuitionistic fuzzy bitopological spaces on X and Y respectively and $f: X \rightarrow Y$ be one- one, onto and continuous open mapping, then

(a) (X, t_1, t_2) is IFB-T_2(i) $\Leftrightarrow$ (Y, s_1, s_2) is IFB-T_2(i)

(b) (X, t_1, t_2) is IFB-T_2(ii) $\Leftrightarrow$ (Y, s_1, s_2) is IFB-T_2(ii)

(c) (X, t_1, t_2) is IFB-T_2(iii) $\Leftrightarrow$ (Y, s_1, s_2) is IFB-T_2(iii)

(d) (X, t_1, t_2) is IFB-T_2(iv) $\Leftrightarrow$ (Y, s_1, s_2) is IFB-T_2(iv)

(e) (X, t_1, t_2) is α-IFB-T_2(i) $\Leftrightarrow$ (Y, s_1, s_2) is α- IFB-T_2(i)

(f) (X, t_1, t_2) is α-IFB-T_2(ii) $\Leftrightarrow$ (Y, s_1, s_2) is α-IFB-T_2(ii)

(g) (X, t_1, t_2) is α-IFB-T_2(iii) $\Leftrightarrow$ (Y, s_1, s_2) is α-IFB-T_2(iii)

Proof: (a) Let (X, t_1, t_2)**,** which defines an intuitionistic fuzzy topological space on X, be IFB-T_2(i). We shall prove that (Y, s_1, s_2), which defines the intuitionistic fuzzy bitopological space on Y, is also IFB-T_2(i).

Let $y_1, y_2 \in Y$, $y_1 \neq y_2$ and $Z = (\mu_Z, \vartheta_Z) \in (s_1 \cup s_2)$ such that $Z(y_1) \neq Z(y_2)$.

Since f is onto, there exists $x_1, x_2 \in X$ such that $x_1 = f^{-1}(y_2)$ and $x_2 = f^{-1}(y_2)$. Again, since $y_1 \neq y_2$, $f^{-1}(y_1) \neq f^{-1}(y_2)$ as f is one-one and onto.

Hence $x_1 \neq x_2$.

We have, $W = (\mu_W, \vartheta_W) \in (t_1 \cup t_2)$ such that $W = f^{-1}(Z)$ i.e., $(\mu_W, \vartheta_W) = (f^{-1}(\mu_Z), f^{-1}(\vartheta_Z))$ as f is Intuitionistic Fuzzy-continuous. Now,

$W(x_1) = \{\mu_W(x_1) = (f^{-1}(\mu_Z))(x_1) = \mu_Z(f(x_1)) = \mu_Z(y_1), \vartheta_W(x_1) = (f^{-1}(\vartheta_Z))(x_1) = \vartheta_Z(f(x_1)) = \vartheta_Z(y_1)\}$ and $W(x_2) = \{\mu_W(x_2) = (f^{-1}(\mu_Z))(x_2) = \mu_Z(f(x_2)) = \mu_Z(y_2), \vartheta_W(x_2) = (f^{-1}(\vartheta_Z))(x_2) = \vartheta_Z(f(x_2)) = \vartheta_Z(y_2)\}$.

Which implies that $W(x_1) \neq W(x_2)$ since $Z(y_1) \neq Z(y_2)$.

Therefore, since (X, t_1, t_2) is IFB-T_2(i), then there exist $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in (t_1 \cup t_2)$ such that $\mu_M(x_1) = 1$, $\vartheta_M(x_1) = 0$; $\mu_N(x_2) = 1$, $\vartheta_N(x_2) = 0$ and $M \cap N = 0_\sim$.

Let us put $A = f(M)$ and $B = f(N)$ where $A = (\mu_A, \vartheta_A), B = (\mu_B, \vartheta_B) \in (s_1 \cup s_2)$ as f is Intuitionistic Fuzzy-continuous. Now we can write,

$\{\mu_A(y_1) = (f(\mu_M))(y_1) = \mu_M(f^{-1}(y_1)) = \mu_M(x_1) = 1, \vartheta_A(y_1) = (f(\vartheta_M))(y_1) = \vartheta_M(f^{-1}(y_1)) = \vartheta_M(x_1) = 0\}$;$\{\mu_B(y_2) = (f(\mu_N))(y_2) = \mu_N(f^{-1}(y_2)) = \mu_N(x_2) = 1, \vartheta_B(y_2) = (f(\vartheta_2))(y_2) = \vartheta_N(f^{-1}(y_2)) = \vartheta_N(x_2) = 0\}$ and $A \cap B = 0_\sim$.

Hence $(A, B) \in (s_1 \cup s_2)$.

Therefore, (Y, s_1, s_2) is IFB-T_2(i).

Conversely, let the intuitionistic fuzzy topological space (Y, s_1, s_2) be IFB-T_2(i). Let $x_1, x_2 \in X$, $x_1 \neq x_2$ and $W = (\mu_W, \vartheta_W) \in (t_1 \cup t_2)$ such that $W(x_1) \neq W(x_2)$.

Since f is one-one, then there exist $y_1, y_2 \in (s_1 \cup, and\ y2s_2)$ such that $y_1 = f(x_1)$ and $y_2 = f(x_2)$ and $f(x_1) \neq f(x_2)$, i.e., $y_1 \neq y_2$.

We have, $Z = (\mu_Z, \vartheta_Z) \in (s_1 \cup s_2)$ such that $Z = (\mu_Z, \vartheta_Z) \in (s_1 \cup s_2)$ such that $Z = f(W)$, i.e., $(\mu_Z, \vartheta_Z) = (f(\mu_W, \vartheta_W))$ as f is Intuitionistic Fuzzy-continuous. Now we write,

$Z(y_1) = \{(f(\mu_W))(y_1) = \mu_W(f^{-1}(y_1)) = \mu_W(x_1), (f(\vartheta_W))(y_1) = \vartheta_W(f^{-1}(y_1)) = \vartheta_W(x_1)\}$ and $\{Z(y_2) = \{(f(\mu_W))(y_2) = \mu_W(f^{-1}(y_2)) = \mu_W(x_2), (f(\vartheta_W))(y_2) = \vartheta_W(f^{-1}(y_2)) = \vartheta_W(x_2)\}$.

Therefore, $Z(y_1) \neq Z(y_2)$ since $W(x_1) \neq W(x_2)$.

Since (Y, s_1, s_2) is IFB-T_2(i), then there exist $A = (\mu_A, \vartheta_A), B = (\mu_B, \vartheta_B) \in (s_1 \cup s_2)$ such that $\mu_A(y_1) = 1$, $\vartheta_A(y_1) = 0$; $\mu_B(y_2) = 1$, $\vartheta_b(y_2) = 0$ and $A \cap B = 0_\sim$.

Now, let us put $M = f^{-1}(A)$ and $N = f^{-1}(B)$ where $M = (\mu_M, \vartheta_M), N = (\mu_N, \vartheta_N) \in (t_1 \cup t_2)$ as f is Intuitionistic Fuzzy-continuous. Now we write, $\{(f^{-1}(\mu_A))(x_1) = \mu_A(f(x_1)) = \mu_A(y_1) = 1, (f^{-1}(\vartheta_A))(x_1) = \vartheta_A(f(x_1)) = \vartheta_A(y_1) = 0\}$;$(f^{-1}(\mu_B))(x_2) = \mu_B(f(x_2)) = \mu_B(y_2) = 1, (f^{-1}(\vartheta_B))(x_2) = \vartheta_B(f(x_2)) = \vartheta_B(y_2) = 0\}$ and $M \cap N = 0_\sim$.

Hence $(M, N) \in (t_1 \cup t_2)$.

Therefore, (X, t_1, t_2) is IFB-T_2(i).

Hence**)**(X, t_1, t_2) is IFB-T_2(i) $\Leftrightarrow$ (Y, s_1, s_2) is IFB-T_2(i).

Similarly, we can easily prove (b), (c), (d), (e), (f), and (g).

Conclusion

The main contribution of this paper is the establishment of some new concepts of intuitionistic fuzzy Hausdorff bitopological spaces. We discuss some of these concepts and show that the mappings, hereditary, and good extension properties hold among them. All of our results have the purpose of helping the researchers to establish an extensive framework for the expansion of intuitionistic fuzzy Hausdorff bitopology, which is currently being researched by others and will help advance this aspect of modern mathematics.

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