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Research Article

Intuitionistic set and its relations

Nasrin Ifnath Nithun, Tahmina Tasnim Trisha*, Md. Sahadat Hossain, Shahriar Kobir and Israt Jahan Swarna

Department of Mathematics, University of Rajshahi, Rajshahi, Bangladesh

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ABSTRACT

We present a brief overview on Intuitionistic sets and its relations which cuts across some definitions, operations, relations and functions on intuitionistic set. In this article, we have shown various operations and their applications using intuitionistic set. We have also seen that relationships can be established using intuitionistic sets, which work similarly to usual relations. Reflexive, Symmetric, Antisymmetric and Transitive relations can be described using intuitionistic set theory, which demonstrated better results than the previous classical set.

Introduction

Since the introduction of intuitionistic sets, numerous researchers have made significant contributions to the field. By exploring various properties of classical sets, they have also introduced new concepts to broaden its generalization.

Intuitionistic set is the generalization of classical set and intuitionistic logic is the generalization of classical logic. Many researchers have worked on intuitionistic set. Atanassov (Atanassov 1986,1988,2001) is one of them who firstly gave the knowledge about it.

After that, many mathematicians also worked on it. Following Zadeh's (Zadeh 1965) introduction of fuzzy sets, classical topological spaces evolved into a new dimension known as "Fuzzy Topological Spaces," as defined by Chang (Chang 1968). Subsequently, Atanassov introduced intuitionistic fuzzy sets, which serve as a generalization of fuzzy sets. Later, Coker (Coker 1996) and colleagues defined intuitionistic fuzzy topological spaces, along with the concepts of intuitionistic sets and intuitionistic topological spaces.

In this paper, we investigate different types of intuitionistic set operations like De Morgan's law, Associative law, Commutative law, Distributive law and relation like Reflexive relation, Symmetric relation, Antisymmetric relation and Transitive relations.

Preliminaries

Brief introduction of intuitionistic sets and relations

Definition (Atanassov, 1986; Islam et. al., 2018): An intuitionistic set A defined on a non-empty set X can be expressed as follows:

An intuitionistic set A takes the form $= (A_1, A_2)$, where:A_1 is a subset of X, representing the members of the intuitionistic set $A. A_2$is a subset of X. representing the non-members of the intuitionistic set A. The defining condition is that.$A_1 \cap A_2 = \emptyset$. This ensures that an element cannot belong to both the members and non-members of A. For the purposes of this discussion, we simplify the notation by using $A = (A_1, A_2)$, omitting X from the expression. This means that the focus is on the classification of elements into members and non-members without explicitly referencing the underlying set X.

Remark (Atanassov, 1986; Prova and Hossain, 2022): Every subset A of a non-empty set X can be considered as an intuitionistic set in the following

*Corresponding author:<roufmollah1970@gmail.com>

way: We define A as an intuitionistic set with the form $A = (A, A^C)$, where: A is the subset itself, representing the members of the intuitionistic set. A^c (the complement of A in X) consists of all elements in X that are not in A, representing the non-members. This means that the intuitionistic set A effectively classifies elements of X into those that belong to A and those that do not. The notation emphasizes the dual nature of A as both a member set and its complement, encapsulating the full structure of the intuitionistic set.

Definition: Universal set is an intuitionistic set which contains all the elements or objects of other intuitionistic sets, including its own elements. It is usually denoted by the symbol "$U_\sim$"

$U_\sim = (U, \emptyset)$

Where, U is considered as interior point and $\emptyset$ is considered as exterior point.

Definition: Null set is a set which contains no value or element. It is denoted by $\emptyset = (\emptyset, U)$ .Where, $\emptyset$ is considered as interior point and U is considered as exterior point.

Definition: (Bayhan and Coker, 2001; Ahmed et. al., 2014): Null the intuitionistic sets M and N in P be of the forms

$M = (M_1, M_2)$ and $N = (N_1, N_2)$ respectively. Furthermore, let $\{M_j, j \in J\}$ be an arbitrary family of intuitionistic sets in P, where $M_j = \left(M_j^{(1)}, M_j^{(2)}\right)$

Then:

a. $M \subseteq N$ if and only if $M_1 \subseteq N_1$ and $M_2 \supseteq N_2$;

b. $M = N$ if and only if $M \subseteq N$ and $N \supseteq M$;

c. $\overline{M} = (M_1, M_2)$ denotes the complement of A;

d. $\cap M_j = \left(\cap M_j^{(1)}, \cup M_j^{(2)}\right)$;

e. $\cup M_j = \left(\cup M_j^{(1)}, \cap M_j^{(2)}\right)$;

f. $\emptyset_\sim = (\emptyset, P)$ and $X_\sim = (P, \emptyset)$.

Intuitionistic set function and its relation

Definition: A function f is an intuitionistic function if for every $p \in P$, there exists a unique $q \in Q$ such that $f(p) = q$. This means that each element of P is associated with exactly one element of , ensuring the well-defined nature of the function.

Mathematically, $f: P \longrightarrow Q$

If M is an intuitionistic set in P then,

$f(M) = \big(f(M_1), f(M_2) - f(M_1)\big)$

Example 1: $f: R \longrightarrow R$ be a function defined by

$f(x) = x^2 + 3x + 1$

If, M is an intuitionistic set

$M = (\{2,3,8\}, \{1,4,7\})$ then

$f(M_1) = \{11,19,89\}$

$f(M_2) = \{5,29,71\}$

$f(M_2) - f(M_1) = \{5,29,71\}$

$f(M) = (f(M_1), f(M_2) - f(M_1))$

$f(M) = (\{11,19,89\}, \{5,29,71\})$

$f : R \rightarrow R$ be a function defined by

$f(x) = x^2$

If N is an intuitionistic set then,

$N = (\{1,2,3\}, \{-1,-3,4\})$

$$f(N_1) = \{1,4,9\}$$

$$f(N_2) = \{1,9,16\}$$

$$f(N_2) - f(N_1) = \{16\}$$

$$f(N) = (f(N_1), f(N_2) - f(N_1))$$

$f(N) = (\{1,4,9\}, \{16\})$

Definition: Intuitionistic relations can represent data mappings where a value in one set is transformed into a value in another set based on defined functions. This is akin to functional programming, where the relationship is constructed through function definitions.

Visualizing the relation as a directed graph can help. Each element in the domain (P) connects to one or more elements in the range (Q), illustrating how relationships are formed based on constructive criteria.

R be an intuitionistic relation from intuitionistic set M to N then $R \subseteq M \times N$

$M \times N = \{(p, q) : p \in M, q \in N\}$

Example 2: Let, $M = (M_1, M_2)$ and $N = (N_1, N_2)$ be two intuitionistic set where,

$$M = (\{1,2\}, \{a, b\})$$

$$N = (\{3,4\}, \{c, d\})$$

Then a relation, $R = (\{(1,4), (2,3)\}, \{(a, d)\})$

Theorem: De-Morgan's Law for intuitionistic set:

$$(M \cap N)^c = M^c \cup N^c$$

Proof:

Let $M = (M_1\,, M_2)$ and $N = (N_1\,, N_2)$ be two intuitionistic set then,

$M \cap N = (M_1 \cap N_1\,, M_2 \cup N_2)$

$$(M \cap N)^C = (M_2 \cup N_2\,, M_1 \cap N_1)^c$$

Again,

$$M^c = (M_2\,, M_1)\; and\; N^c = (N_2\,, N_1)$$

$$M^c \cup N^C = (M_2 \cup N_2\,, M_1 \cap N_1)$$

$$= (M \cap N)^C \text{ (proved)}$$

Theorem: Commutative law for intuitionistic set:

For all intuitionistic sets M, N

$M = (M_1\,,\; M_2)$

$N = (N_1\,,\; N_2)$

$M \cup N = N \cup M$ and

$M \cap N = N \cap M$

Proof:

For interior points p_1 and q_1,

$Let, p_1 \in (M_1 \cup N_1)$

$\Rightarrow p_1 \in M_1$ or $p_1 \in N_1$

$\Rightarrow p_1 \in N_1$ or $p_1 \in M_1$

$\Rightarrow p_1 \in (N_1 \cup M_1)$

$\therefore (M_1 \cup N_1) \subseteq (N_1 \cup M_1)$

Again let,$q_1 \in (N_1 \cup M_1)$

$\Rightarrow q_1 \in N_1\; or\; q_1 \in M_1$

$\Rightarrow q_1 \in M_1\; or\; q_1 \in N_1$

$\Rightarrow q_1 \in (M_1 \cup N_1)$

$\therefore (N_1 \cup M_1) \subseteq (M_1 \cup N_1)$

$(M_1 \cup N_1) = (N_1 \cup M_1)$

For exterior points p_2 and q_2,

Let, $p_2 \in (M_2 \cap N_2)$

$\Rightarrow p_2 \in M_2\; and\; p_2 \in N_2$

$\Rightarrow p_2 \in N_2\; and\; p_2 \in M_2$

$\Rightarrow p_2 \in (N_2 \cap M_2)$

$\therefore (M_2 \cap N_2) \subseteq (N_2 \cap M_2)$

Again let,$q_2 \in (N_2 \cap M_2)$

$\Rightarrow q_2 \in N_2\; and\; q_2 \in M_2$

$\Rightarrow q_2 \in M_2\; and\; q_2 \in N_2$

$\Rightarrow q_2 \in (M_2 \cap N_2)$

$\therefore (N_2 \cap M_2) \subseteq (M_2 \cap N_2)$

$(M_2 \cap N_2) = (N_2 \cap M_2)$

Hence, $(M \cup N) = (N \cup M)$ (proved)

Similarly,$(M \cap N) = (N \cap M)$

Theorem: Associative Law for intuitionistic set:

For all intuitionistic sets M, N, O

$M = (M_1\,, M_2)$

$N = (N_1\,, N_2)$

$O = (O_1\,, O_2)$

$(M \cup N) \cup O = M \cup (N \cup O)$ and

$(M \cap N) \cap O = M \cap (N \cap O)$

Proof:

For interior points x_1 and y_1,

Let, $p_1 \in (M_1 \cup N_1) \cup O_1$

$\Rightarrow p_1 \in (M_1 \cup N_1)\; or\; p_1 \in O_1$

$\Rightarrow p_1 \in M_1\; or\; p_1 \in N_1\; or\; p_1 \in O_1$

$\Rightarrow p_1 \in M_1\; or\; p_1 \in (N_1 \cup O_1)$

$\Rightarrow p_1 \in M_1 \cup (N_1 \cup O_1)$

$\therefore (M_1 \cup N_1) \cup O_1 \subseteq M_1 \cup (N_1 \cup O_1)$

Again let,$q_1 \in M_1 \cup (N_1 \cup O_1)$

$\Rightarrow q_1 \in M_1\; or\; q_1 \in (N_1 \cup O_1)$

$\Rightarrow q_1 \in M_1\; or\; q_1 \in N_1\; or\; q_1 \in O_1$

$\Rightarrow q_1 \in (M_1 \cup N_1)\; or\; q_1 \in O_1$

$\Rightarrow q_1 \in (M_1 \cup N_1) \cup O_1$

$\therefore M_1 \cup (N_1 \cup O_1) \subseteq (M_1 \cup N_1) \cup O_1$

$(M_1 \cup N_1) \cup O_1 = M_1 \cup (N_1 \cup O_1)$

For exterior points p_2 and q_2,

Let,$p_2 \in (M_2 \cap N_2) \cap O_2$

$\Rightarrow p_2 \in (M_2 \cap N_2)$ and $P_2 \in O_2$

$\Rightarrow p_2 \in M_2$ and $p_2 \in N_2$ and $p_2 \in O_2$

$\Rightarrow p_2 \in M_2$ and $p_2 \in (N_2 \cap O_2)$

$\Rightarrow p_2 \in M_2 \cap (N_2 \cap O_2)$

$\therefore (M_2 \cap N_2) \cap O_2 \subseteq M_2 \cap (N_2 \cap O_2)$

Again let, $q_2 \in M_2 \cap (N_2 \cap O_2)$

$\Rightarrow q_2 \in M_2$ and $q_2 \in (N_2 \cap O_2)$

$\Rightarrow q_2 \in M_2$ and $q_2 \in N_2$ and $q_2 \in O_2$

$\Rightarrow q_2 \in (M_2 \cap N_2)$ and $q_2 \in O_2$

$\Rightarrow q_2 \in (M_2 \cap N_2) \cap O_2$

$\therefore M_2 \cap (N_2 \cap O_2) \subseteq (M_2 \cap N_2) \cap O_2$

$(M_2 \cap N_2) \cap O_2 = M_2 \cap (N_2 \cap O_2)$

Hence,$(M \cup N) \cup O = M \cup (N \cup O)$ (proved)

Similarly,$(M \cap N) \cap O = M \cap (N \cap O)$

Theorem: Distributive law for intuitionistic set:

For all intuitionistic sets M, N, O

$M = (M_1, M_2)$, $N = (N_1, N_2)$

$O = (O_1, O_2)$

$M \cap (N \cup O) = (M \cap N) \cup (M \cap O)$and

$M \cup (N \cap O) = (M \cup N) \cap (M \cup O)$

Proof:

For interior points p_1 and q_1,

Let,$p_1 \in M_1 \cap (N_1 \cup O_1)$

$\Rightarrow p_1 \in M_1$ and $p_1 \in (N_1 \cup O_1)$

$\Rightarrow p_1 \in M_1$ and $(p_1 \in N_1 \; or \; p_1 \in O_1)$

$\Rightarrow (p_1 \in M_1$ and $p_1 \in N_1)$ or $(p_1 \in M_1$ and $p_1 \in O_1)$

$\Rightarrow p_1 \in (M_1 \cap N_1) \cup (M_1 \cap O_1)$

$\therefore M_1 \cap (N_1 \cup O_1) \subseteq (M_1 \cap N_1) \cup (M_1 \cap O_1)$

Again let,$q_1 \in (M_1 \cap N_1) \cup (M_1 \cap O_1)$

$\Rightarrow (q_1 \in M_1$ and $q_1 \in N_1)$ or $(q_1 \in M_1$ and $q_1 \in O_1)$

$\Rightarrow q_1 \in M_1$ and $(q_1 \in N_1$ or $q_1 \in O_1)$

$\Rightarrow q_1 \in M_1$ and $q_1 \in (N_1 \cup O_1)$

$\Rightarrow q_1 \in M_1 \cap (N_1 \cup O_1)$

$\therefore (M_1 \cap N_1) \cup (M_1 \cap O_1) \subseteq M_1 \cap (N_1 \cup O_1)$

$M_1 \cap (N_1 \cup O_1) = (M_1 \cap N_1) \cup (M_1 \cap O_1)$

For exterior points p_2 and q_2 ,

Let,$p_2 \in M_2 \cup (N_2 \cap O_2)$

$\Rightarrow p_2 \in M_2$ or $p_2 \in (N_2 \cap O_2)$

$\Rightarrow p_2 \in M_2$ or $(p_2 \in N_2 \; and \; p_2 \in O_2)$

$\Rightarrow (p_2 \in M_2$ or $p_2 \in N_2)$ and $(p_2 \in M_2$ or $p_2 \in O_2)$

$\Rightarrow p_2 \in (M_2 \cup N_2) \cap (M_2 \cup O_2)$

$\therefore M_2 \cup (N_2 \cap O_2) \subseteq (M_2 \cup N_2) \cap (M_2 \cup O_2)$

Let,$q_2 \in (M_2 \cup N_2) \cap (M_2 \cup O_2)$

$\Rightarrow (q_2 \in M_2 \; or \; q_2 \in N_2) \; and \; (q_2 \in M_2 \; or \; q_2 \in O_2)$

$\Rightarrow q_2 \in M_2$ or $(q_2 \in N_2 \; and \; q_2 \in O_2)$

$\Rightarrow q_2 \in M_2$ or $q_2 \in (N_2 \cap O_2)$

$\Rightarrow q_2 \in M_2 \cup (N_2 \cap O_2)$

$\therefore (M_2 \cup N_2) \cap (M_2 \cup O_2) \subseteq M_2 \cup (N_2 \cap O_2)$

$M_2 \cup (N_2 \cap O_2) = (M_2 \cup N_2) \cap (M_2 \cup O_2)$

Hence,

$M \cap (N \cup O) = (M \cap N) \cup (M \cap O)$ (proved)

Similarly,$M \cup (N \cap O) = (M \cup N) \cap (M \cup O)$

Concept of domain and range of intuitionistic function:

Let, $M = (M_1, M_2)$, $N = (N_1, N_2)$ be two intuitionistic sets where, M_1 and N_1 are interior point and M_2, N_2 are exterior point of the set M and N respectively.

Let, $R = (R_1, R_2)$ be a relation from intuitionistic set M to N is defined as a subset of $M \times N = (M_1 \times N_1, M_2 \times N_2)$.

Example 3:

Let, M and N be two intuitionistic sets where $M = (\{1,2,3\}, \{5,6\})$ and $N = (\{p, q, r\}, \{a, b, c\})$. Here, $M_1 = \{1,2,3\}$, $M_2 = \{5,6\}$, $N_1 = \{p, q, r\}$, $N_2 = \{a, b, c\}$.

Let,$R = (R_1, R_2) \subseteq (M \times N) = (M_1 \times N_1, M_2 \times N_2)$ where, $R = (\{(1, p), (3, p), (3, r)\}, \{(5, b), (5, c)\})$.

When the exterior point is fixed:

Domain of $R = (I_1, M_2) \subseteq (M_1, M_2)$

where, $I_1 = \{i_1 \in M_1 : \exists\, j_1 \in N_1 \; and \; (i_1, j_1) \in R_1\}$
Range of $R = (J_1, N_2) \subseteq (N_1, N_2)$

where, $J_1 = \{j_1 \in N_1 : \exists\, i_1 \in M_1 \; and \; (i_1, j_1) \in R_1\}$

$$I_1 = \{1,3\}$$

$$D_R = (\{1,3\}, \{5,6\})$$

$$J_1 = \{p, r\}$$

$$R_R = (\{p, r\}, \{a, b, c\})$$

When interior point is fixed:

Domain of $R = (M_1, I_2) \subseteq (M_1, M_2)$

where, $I_2 = \{i_2 \in M_2 : \exists\, j_2 \in N_2 \; and \; (i_2, j_2) \in R_2\}$

Range of $R = (N_1, J_2) \subseteq (N_1, N_2)$

$J_2 = \{j_2 \in N_2 : \exists\, i_2 \in M_2 \; and \; (i_2, j_2) \in R_2\}$

$$I_2 = \{5\}$$

$$D_R = (\{1,2,3\}, \{5\})$$

$$J_2 = \{b, c\}$$

$$R_R = (\{p, q, r\}, \{b, c\})$$

When interior and exterior both are changeable:

Domain of $R = (I_1, I_2) \subseteq (M_1, M_2)$

where, $I_1 = \{i_1 \in M_1 : \exists\, j_1 \in N_1 \; and \; (i_1, j_1) \in R_1\}$

$I_2 = \{i_2 \in M_2 : \exists\, j_2 \in N_2 \; and \; (i_2, j_2) \in R_2\}$

Range of $R = (J_1, J_2) \subseteq (N_1, N_2)$

where, $J_1 = \{j_1 \in N_1 : \exists\, i_1 \in M_1 \; and \; (i_1, j_1) \in R_1\}$

$J_2 = \{j_2 \in N_2 : \exists\, i_2 \in M_2 \; and \; (i_2, j_2) \in R_2\}$

$$I_1 = \{1,3\}$$

$$I_2 = \{5\}$$

$$D_R = (\{1,3\}, \{5\})$$

$$J_1 = \{p, r\}$$

$$J_2 = \{b, c\}$$

$$R_R = (\{p, r\}, \{b, c\})$$

Intuitionistic relations

Definition: Here we define four types of special intuitionistic relations. Such as Reflexive, Symmetric, Anti-symmetric, Transitive

Interior Point (M_1): This is a point within the set M where you can find a neighbourhood around M_1 that is completely contained in M.

Exterior Point (M_2): This is a point outside the set M such that you can find a neighbourhood around M_2 that does not intersect .

$$R \subseteq M \times M$$

$$R = (R_1, R_2) \subseteq (M_1 \times M_1 ,\; M_2 \times M_2)$$

Reflexive Relation

Let $R = (R_1, R_2)$ be a binary relation on a set $M = (M_1, M_2)$ which can be define as $(R_1, R_2) \subseteq (M_1 \times M_1, M_2 \times M_2)$.Then the relation$(R_1, R_2)$ is called strongly reflexive if $(a, a) \in R_1, \forall\, a \in M_1$ and $(\mathrm{b}, \mathrm{b}) \notin R_2, \forall\, b \in M_2$

And for weakly reflexive $(a, a) \in R_1, \forall\, a \in M_1$

Example 4: Let, $M = (M_1, M_2) = (\{a, b\}, \{x, y, z\})$ be an intuitionistic set and $R = (R_1, R_2) = (\{(a, a), (a, b), (b, b)\}, \{(x, x), (x, y), (x, z)\})$ be a relation on the set M.

The relation is weakly reflexive since $(a, a), (b, b) \in R_1, \forall\, a, b \in M_1$.But it is not strongly reflexive since $(x, x) \in R_1, \forall\, x \in M_2$

Another relation $R = (\{(a, a), (a, b), (b, b)\}, \{(x, y), (x, z)\})$ is defined on the set M.The relation is strongly and weakly reflexive relation since $(a, a), (b, b) \in R_1, \forall\, a, b \in M_1$ and $(x, x), (y, y), (z, z) \notin R_2, \forall\, x, y, z \in M_2$

Symmetric relation

Let, (R_1, R_2) be a subset of $(M_1 \times M_1, M_2 \times M_2)$ i.e. $R = (R_1, R_2)$ be a relation on an intuitionistic set $M = (M_1, M_2)$.Then the relation R is called a strongly symmetric relation if

$$(a_1, b_1) \in R_1 \Rightarrow (b_1, a_1) \in R_1$$

$$(a_2, b_2) \in R_2 \Rightarrow (b_2, a_2) \notin R_2$$

On the contrary, the symmetric relation is called weakly symmetric relation if it satisfies the condition,

$$(a_1, b_1) \in R_1 \Rightarrow (b_1, a_1) \in R_1$$

Example 5: Let, $M = (M_1, M_2) = (\{a, b\}, \{x, y\})$ be an intuitionistic set and $R = (R_1, R_2) = (\{(a, a), (a, b), (b, a)\}, \{(x, y)\})$ be a relation on the set M.The relation is strongly and weakly symmetric relation since $(a, a), (a, b), (b, a) \in R_1$ and $(x, y) \in R_2$ but $(y, x) \notin R_2$.

Another relation $R = (R_1, R_2) = (\{(a, a), (a, b), (b, a)\}, \{(x, y), (y, x)\})$ is defined on the set M.It is weakly symmetric relation since $(a, a), (a, b), (b, a) \in R_1$.But it is not strongly symmetric relation since $(y, x) \in R_2$.

Anti-symmetric relation

Let, (R_1, R_2) be a relation on an intuitionistic set (M_1, M_2) i.e. (R_1, R_2) be a subset of $(M_1 \times M_1, M_2 \times M_2)$.Then the relation$(R_1, R_2)$ is called a strongly anti-symmetric relation if

$(a_1, b_1) \in R_1$ and $(b_1, a_1) \in R_1 \Rightarrow a_1 = b_1$

$(a_2, b_2) \in R_2 \; and \; (b_2, a_2) \in R_2 \; \Rightarrow \; a_2 \neq b_2$

On the contrary, it is called a weakly anti-symmetric relation if it satisfies the condition,

$(a_1, b_1) \in R_1$ and $(b_1, a_1) \in R_1 \Rightarrow \; a_1 = b_1$

Example 6: Let, $M = (M_1, M_2) = (\{1,2\}, \{b, c\})$ be an intuitionistic set and $R = (R_1, R_2) = (\{(1,1), (1,2), (2,2)\}, \{(b, c), (c, b)\})$ be a relation on the set M.The relation is strongly and weakly anti-symmetric relation since $(1,1), (2,2) \in R_1 \; and \; 1 = 1 \; and \; 2 = 2$ and $(b, c), (c, b) \in R_2$ but $b \neq c$.

Another relation $R = (R_1, R_2) = (\{(1,1), (1,2), (2,2)\}, \{(b, b), (c, c)\})$ is defined on the set M.It is weakly anti-symmetric relation since $(1,1), (2,2) \in R_1 \; and \; 1 = 1 \; and \; 2 = 2$. But it is not strongly anti-symmetric relation since $(b, b), (c, c) \in R_1 \; because \; b = b \; and \; c = c$.

Transitive relation

Let, $R = (R_1, R_2)$ be a relation on an intuitionistic set $M = (M_1, M_2)$ i.e. $R = (R_1, R_2)$ be a subset of $(M_1 \times M_1, M_2 \times M_2)$.The relation (R_1, R_2) is called a strongly transitive relation if

$(a_1, b_1) \in R_1$ and $(b_1, c_1) \in R_1 \Rightarrow (a_1, c_1) \in R_1$

$(a_2, b_2) \in R_2 \; and \; (b_2, c_2) \in R_2 \; \Rightarrow (a_2, c_2) \notin R_2$

On the other hand, it is called a weakly transitive relation if it satisfies the condition,

$(a_1, b_1) \in R_1$ and $(b_1, c_1) \in R_1 \Rightarrow (a_1, c_1) \in R_1$

Example 7: Let, $M = (M_1, M_2) = (\{5,6,7\}, \{p, q, r\})$ be an intuitionistic set and $R = (R_1, R_2) = (\{(5,6), (5,7), (6,7)\}, \{(p, q), (q, r)\})$ be a relation on the set M.The relation is strongly and weakly transitive relation since $(5,6), (6,7) \in R_1 \; and \; (5,7) \in R_1$ and $(p, q), (q, r) \in R_2$ but $(p, r) \notin R_2$.

Another relation $R = (R_1, R_2) = (\{(5,6), (5,7), (6,7)\}, \{(p, q), (p, r), (q, r)\})$ is defined on the set M.It is weakly transitive relation since $(5,6), (5,7), (6,7) \in R_1$.But it is not strongly transitive relation since $(p, q), (p, r), (q, r) \in R_2$.

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Authors contribution

All authors contributed equally to the collection of relevant information, interpretation of the data and preparation of the manuscript.

Conflict of interest

The authors declare that there is no financial or personal relationships that could have influenced the work reported in this paper.

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