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OBLIQUELY PROPAGATING SELF-GRAVITATIONAL SHOCK WAVES IN NON-RELATIVISTIC DEGENERATE QUANTUM PLASMAS

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ABSTRACT

A rigorous theoretical investigation has been carried out on the propagation of non-linear self-gravitational shock waves (SGSHWs) in a magnetized super dense degenerate quantum plasma system (DQPS) composed of inertia-less non-relativistic degenerate electrons and inertial non-degenerate extremely heavy nuclei/element. The nonlinear propagation of these SGSHWs in the plasma system under consideration is studied by the standard reductive perturbation technique, which is valid for a small finite amplitude limit. The nonlinear dynamics of the SGSHWs are found to be governed by the Burgers equation. The Burgers equation is derived analytically and solved numerically. It has been found that the considered plasma model supports positive potential shock waves only. The fundamental properties (amplitude, steepness, etc.) of these SGSHWs are significantly modified by the variation of kinematic viscosity, obliqueness and number density of the plasma species. The results of our present investigation can be applied to astrophysical compact objects like neutron stars.

Keywords: *Degenerate quantum plasma, Shock waves, Nonlinearity, Relativity, Self-gravitational perturbation, Compact objects.*

1. INTRODUCTION

White dwarfs and neutron stars are super-dense astrophysical compact objects, where the number densities are extra-ordinarily very high (Chandrasekhar, 1931; Chandrasekhar, 1931a; Chandrasekhar, 1935; Chandrasekhar, 1939; Chandrasekhar, 1964; Chandrasekhar & Tooper, 1964a; Shapiro and Teukolsky,1983; Koester & Chanmugam, 1990; Garcia-Berro et al., 2010). The particle number density in white dwarf is of the order of 1030cm^{-3} and in neutron star is of the order of 1038cm^{-3} or even more (Shapiro and Teukolsky, 1983; Koester and Chanmugam, 1990). At this high particle number density, classical plasma enters into the regime of quantum plasma (when the quantum nature of its constituent particles starts to affect its macroscopic properties and dynamics) and the state of matter becomes degenerate in the case of astrophysical compact objects. A plasma system which obeys the laws of quantum mechanics and in which the average inter-particle distance becomes comparable to the average de Broglie wavelength of the lightest plasma (viz. electron) particles and the effect of quantum degeneracy of electrons becomes significant (Tyshetskiy et al., 2013) due to Pauli's exclusion principle is known as DQPS. According to Heisenberg's uncertainty principle, as the particle number density increases, the particles are confined in a small space that means the position uncertainty becomes very small and the momentum becomes very large. This very large momentum is responsible for the generation of outward degenerate pressure which prevents dense stars from further gravitational shrinking or collapse. The shock waves are formed due to the balance between the nonlinearity and dissipation. Here, the source of dissipation is the viscous force which is acting on inertial heavy nuclei species. The self-gravitational effect may become important when the heavy nuclei mass is much heavier than the mass of other plasma particles. We should mention here that one may neglect the electrostatic force when one is interested in examining the self-gravitational perturbation potential only, and the quasi-neutral condition is, in general, applicable. Since we have considered here the self-gravitational potential, we have designated the name of the shock wave as self-gravitational shock waves (SGSHWs).

The equation of degeneracy pressure, which is given by Chandrasekhar (Chandrasekhar, 1931; Chandrasekhar, 1931a; Chandrasekhar, 1935) for two limits, namely, the non-relativistic and the ultra-relativistic limits, can be expressed as $P_i = K_i N_i^{\gamma}$, where $\gamma = 5/3$ and $K_i = 3\pi\hbar^2/5m_i$ for non-relativistic limit; $\gamma = 4/3$ and $K_i = 3\hbar c/4$ for ultra-relativistic limit; P_i is the degenerate plasma particle pressure for *i*-species; N_i is the degenerate plasma particle number density for *i*-species; K_i is the constant of proportionality.

A significant number of authors (El-Taibany & Mamun, 2012; Roy et al., 2012; Ema et al., 2015; Hosen et al., 2016, etc.) have examined the propagation of electrostatic and self-gravitational excitations in DQPS by

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considering the Chandrasekhar's degeneracy pressure equation in the case of white dwarfs and neutron stars. Hossen and Mamun, (2014) studied the nonlinear propagation of cylindrical and spherical modified ion-acoustic waves in a degenerate multispecies plasma and applied their results in astrophysical compact objects. Ema et al., (2015) examined the nonlinear propagation of modified electron-acoustic shock waves in an unmagnetized, relativistically degenerate quantum plasma. Hossen and Mamun, (2015) studied the ion-acoustic solitary waves in a degenerate dense plasma with stationary heavy nuclei. Recently, Mamun, (2017) examined the self-gravito-acoustic waves in a three-component degenerate quantum plasma system.

The presence of strong magnetic field (i.e., about 1 Mega Gauss) in white dwarfs has been predicted by Blackett (Blackett, 1947) and also has been observed by Zeeman spectroscopy (Liebert et al., 1977; Euchner et al., 2002). El-Taibany and Mamun, (2012) examined solitary waves in a magnetized degenerate electron-positron plasma and found that the wave amplitude varies with oblique angle (the angle between external magnetic field and the direction of propagation). Shaukat, (2017) studied ion-acoustic solitary waves in an external magnetic field and also observed the change in amplitude with the change in oblique angle. Hosen et al., (2016) studied the nonlinear properties of the ion acoustic waves in a magnetized degenerate quantum plasma. Abdelwahed et al., (2016) studied IASHWs in a pair ion plasma. Haider (Haider, 2016) examined the shock profiles in the presence of degenerate inertial ions and inertia-less electrons and positrons. To the best of the author's knowledge, no investigation has been made of by considering a magnetized super dense DQPS having inertia-less non-relativistic degenerate electrons and inertial non-degenerate extremely heavy nuclei/element. In order to study the fundamental characteristics of the SGSHWs in the plasma system under consideration we have derived here the Burgers equation and have also obtained the associated shock wave solutions.

The manuscript is organized in the following manner: The governing equations of the considered plasma model are stated in Sec. 2. The Burgers equation and the associated shock solutions are derived in Secs. 3. The numerical observations and results are presented in Sec. 4. Finally, a brief conclusion is provided in Sec. 5.

2. GOVERNING EQUATIONS

We have considered a self-gravitating DQPS consisting of extremely high dense degenerate electron (Chandrasekhar, 1931a; Fowler, 1994) species and low dense extremely heavy nuclei/element [viz. 5626Fe, 8537Rd or 9642Mo (Witze, 2014; Vanderburg et al., 2015)] in the presence of an external uniform magnetic field. The magnetic field B exists along the direction of z-axis (B $= B_0\hat{z}$ and $\hat{z}$ is the unit vector along the z direction). At equilibrium, we have $N_{e0} = Z_h N_{h0}$, where Ne_0 (Nh_0) is the electron (nucleus) number density at equilibrium.

The propagation of self-gravitational shock waves (SGSHWs) in the DQPS under consideration is governed by the following equations:

$$\nabla\Psi = -\frac{5}{2}\frac{K_e}{M_e}\nabla N_e^{\frac{2}{3}}, \tag{1}$$

$$\frac{\partial N_h}{\partial T} + \nabla(N_h \mathrm{U_h}) = 0, \tag{2}$$

$$\frac{\partial \mathbf{U_h}}{\partial T} + (\mathrm{U_h}\cdot\nabla)\mathrm{U_h} = -\nabla\Psi + \eta\nabla^2\mathrm{U_h} + \frac{B_0}{c}(\mathrm{U_h}\times\hat{z}), \tag{3}$$

$$\nabla^2\Psi = \omega_{Jh}^2[\left(\frac{N_h}{N_{h0}} - 1\right) + \alpha\left(\frac{N_e}{N_{e0}} - 1\right), \tag{4}$$

where N_h (N_e) is the heavy nuclei (electron) number density; $\mathrm{U_h}$ is the nucleus fluid velocity; M_h (M_e) is the rest mass of a heavy nucleus (an electron); Ψ is the self-gravitational potential; T is the time variable; η is the kinematic viscosity; $K_e = 3\pi\hbar^2/5M_e$; $\alpha = M_e Z_h/M_h$; $\omega_{Jh} = 4\pi G M_h N_{h0}$.

In our considered plasma model, the heavy nucleus provides the inertia and the electron provides the restoring force. Since N_e can be directly found from Eq. (1) it is unnecessary to write the continuity and momentum balance equation for the electron.

The explanation for the validity of Eqs. (1) - (4) describing the dynamics of the SGSHWs in the magnetized DQPS is given below:

(i) Eq. (1) is obtained from the pressure balance equation $\nabla P_e = -M_e N_e \nabla\Psi$ [gravitational shrinking (inward pull due to self-gravitational attraction) counterbalances the outward degenerate electron pressure ($P_e = K_e N_e^{\gamma}$)].

(ii) Eq. (2) represents the continuity equation for non-degenerate heavy nuclei/element.

(iii) The momentum balance equation for non-degenerate extremely heavy nuclei (where the effects of self-gravitational potential, viscous force and static external magnetic field are included) is presented in Eq. (3).

(iv) The Poisson's equation for the self-gravitational potential is presented in Eq. (4), which is obtained from the equation $\nabla^2\Psi = 4\pi G(M_h N_h^{/} + M_e N_e^{/})$, where $N_h^{/}$ and $N_e^{/}$ are the perturbed number densities of the heavy nuclei and degenerate electrons, respectively. $N_h^{/}$ and $N_e^{/}$ can be written in terms of unperturbed number densities N_h and N_e as $N_h^{/} = N_h - N_{h0}$ and $N_e^{/} = N_e - N_{e0}$.

3. BURGERS EQUATION

In order to study the dynamics of SGSHWs, we first assumed the stretched coordinates (Washimi, 1966; Shukla, 1978)

$$\left.\begin{aligned} \xi &= \epsilon\left(l_x x + l_y y + l_z z - V_p T\right), \\ \tau &= \epsilon^2 T, \end{aligned}\right\} \tag{5}$$

where ϵ measures the weakness of the amplitude or dissipation ($0 < \epsilon < 1$); V_p is the phase speed of the wave; l_x, l_y, and $l_z$are the directional cosines of the wave propagation constant *k* along the *x*, *y*, and *z* axes, respectively (where ${l_x}^2 + {l_y}^2 + {l_z}^2 = 1$). We can express the dependent variables in power series of ϵ as (Washimi, 1966; Shukla, 1978;)

$$\left.\begin{aligned} N_h &= N_{h0} + \epsilon {N_h}^{(1)} + \epsilon^2 {N_h}^{(2)} + \cdots\cdots, \\ N_e &= N_{e0} + \epsilon {N_e}^{(1)} + \epsilon^2 {N_e}^{(2)} + \cdots\cdots, \\ U_{hx,y} &= 0 + \epsilon^2\, {U_{hx,y}}^{(1)} + \epsilon^3\, {U_{hx,y}}^{(2)} + \cdots, \\ U_{hz} &= 0 + \epsilon\, {U_{hz}}^{(1)} + \epsilon^2\, {U_{hz}}^{(2)} + \cdots\cdots, \\ \psi &= 0 + \epsilon \Psi^{(1)} + \epsilon^2 \Psi^{(2)} + \cdots\cdots. \end{aligned}\right\} \tag{6}$$

Substituting Eqs. (5) and (6) into Eqs. (1) - (4) and equating the coefficients of ϵ for the lowest order, we obtain

$${U_{hz}}^{(1)} = \frac{l_z}{V_p}\Psi^{(1)}, \tag{7}$$

$${N_h}^{(1)} = \frac{{l_z}^2 N_{h0}}{{V_p}^2}\Psi^{(1)}, \tag{8}$$

$${N_e}^{(1)} = -\beta\, \Psi^{(1)}, \tag{9}$$

$$V_p = l_z\sqrt{\frac{N_{e0}}{\alpha\beta}}, \tag{10}$$

where $\beta = \frac{3}{5}\frac{M_e}{K_e}{N_{e0}}^{1/3}$. Eqs. (7) – (10) represent the z-component of the momentum equation, first order continuity equation, first order electron number density, and the phase speed, respectively. The *x*- and *y*-components of the first order momentum equation can be written as

$${U_{hy}}^{(1)} = \frac{l_x c}{B_0}\frac{\partial\Psi^{(1)}}{\partial\xi}, \tag{11}$$

$${U_{hx}}^{(1)} = -\frac{l_y c}{B_0}\frac{\partial\Psi^{(1)}}{\partial\xi}. \tag{12}$$

If we consider the next higher order for ϵ, we can get the next higher order continuity equation, *z*-component of the momentum equation, and Poisson's equation as

$$\frac{\partial N_h^{(1)}}{\partial\tau} - V_p\frac{\partial N_h^{(2)}}{\partial\xi} + l_x N_{h0}\frac{\partial U_{hx}^{(1)}}{\partial\xi} + l_y N_{h0}\frac{\partial U_{hy}^{(1)}}{\partial\xi} + l_z\frac{\partial}{\partial\xi}\left[N_{h0}{U_{hz}}^{(2)} + N_h^{(1)}{U_{hz}}^{(1)}\right] = 0 \tag{13}$$

$$\frac{\partial {U_{hz}}^{(1)}}{\partial\tau} - V_p\frac{\partial {U_{hz}}^{(2)}}{\partial\xi} + l_z{U_{hz}}^{(1)}\frac{\partial {U_{hz}}^{(1)}}{\partial\xi} + l_z\frac{\partial\Psi^{(2)}}{\partial\xi} - \eta\frac{\partial^2 {U_{hz}}^{(1)}}{\partial\xi^2} = 0, \tag{14}$$

$$\frac{1}{N_{h0}}\left[N_h^{(2)} + \frac{M_e}{M_h}N_e^{(2)}\right] = 0. \tag{15}$$

Now, combining Eqs. (7) – (15) and performing a little mathematics, we obtain the Burgers equation as

$$\frac{\partial\Psi^{(1)}}{\partial\tau}+A\Psi^{(1)}\frac{\partial\Psi^{(1)}}{\partial\xi}=C\frac{\partial^2\Psi^{(1)}}{\partial\xi^2}, \quad (16)$$

where the nonlinear coefficient A and the dissipation coefficient C are given by

$$A=\left[\frac{3l_z^2}{2V_p}+\frac{3}{50}\frac{M_e^3V_p^3}{M_hK_e^2l_z^2N_{h0}N_{e0}{}^{1/3}}\right], \quad (17)$$

$$C=\frac{\eta}{2}, \quad (18)$$

where the direction cosine of the wave vector along the z-axis is considered as $l_z=cos\theta$ with θ being the angle between the directions of the wave propagation vector **k** and the external magnetic field B_0. Considering $T=C\tau$ and R= A/C, Eq. (16) can be written as

$$\frac{\partial\Psi^{(1)}}{\partial T}+R\Psi^{(1)}\frac{\partial\Psi^{(1)}}{\partial\xi}=\frac{\partial^2\Psi^{(1)}}{\partial\xi^2}. \quad (19)$$

4. NUMERICAL OBSERVATIONS AND RESULTS

Now, we look for the stationary shock wave solution of Eq. (16) for analyzing the SGSHWs numerically. By transforming the independent variables ξ and τ to $\zeta=\xi-U_0\tau$ and $\tau=\tau$ (where U_0 is the constant speed of the nucleus fluid) and applying the boundary conditions, viz., $\Psi^{(1)}\to 0$, $\frac{d\Psi^{(1)}}{d\zeta}\to 0$, at $\zeta\to\infty$, we obtain the steady state solution of Eq. (16) with $\Psi^{(1)}=\Psi$ as

$$\Psi=\Psi_m\left[1-Tanh\left(\frac{\zeta}{\Delta}\right)\right], \quad (20)$$

where the amplitude Ψ_m and the width Δ are given by

$$\Psi_m=U_0/A \text{ and } \Delta=2C/U_0. \quad (21)$$

Similarly, we can get the steady state solution of Eq. (19) with $\Psi^{(1)}=\Psi$ as

$$\Psi=\Psi_0\left[1-Tanh\left(\frac{\zeta}{\Delta_1}\right)\right], \quad (22)$$

where the amplitude Ψ_0 and the width Δ_1 are given by

$$\Psi_0=U_0/R \text{ and } \Delta_1=2/U_0. \quad (23)$$

Clearly from Eqs. (20) and (21), the SGSHWs, which are formed due to the balance between the nonlinearity and dissipation, exist because $C>0$. Since $U_0>0$, the SGSHWs with $\Psi>0$ ($\Psi<0$) exist if $A>0$ ($A<0$). It is also clear from Eq. (21) that the amplitude Ψ_m is directly proportional to the shock speed U_0 and inversely proportional to the nonlinear coefficient A whereas the width of the SGSHWs is inversely (directly) proportional to shock speed U_0 (dissipation coefficient C). The nonlinearity of the plasma medium provides the potential of the plasma system. The self-gravitational potential becomes infinite when the non-linear coefficient A is equal to zero and in this case the reductive perturbation technique becomes invalid.

It should be mentioned that our current investigation deals with the basic features of SGSHWs in a magnetized super dense DQPS composed of non-relativistic degenerate electrons and non-degenerate extremely heavy nuclei/element. For numerical analysis, we have used the range of plasma parameters which are comparable to neutron star plasma as well as laboratory plasma.

It should be added here that we have examined the salient features of the self-gravitational shock structures (waves) associated with the self-gravitational perturbation mode of extremely long scale length and slow time scale in astrophysical compact objects like neutron stars, which are at the endpoint stages of stellar evolution (relics of stars), and do not sustain any thermonuclear burning, and therefore can no longer generate thermal pressure to support the gravitational load of their own mass (Mamun, 2017; Asaduzzaman et al., 2020). Since the effect of electrostatic potential is important for the study of short wavelength and comparatively high frequency waves (Mamun, 2017; Asaduzzaman et al., 2020) we have not considered the effect of electrostatic potential for our present investigation because we have studied the shock structures of extremely long wavelength and very low frequency.

In our Introduction section, we have mainly discussed about the electrostatic wave (viz., ion acoustic solitary wave or ion acoustic shock wave) and self-gravitational wave (or self-gravitational acoustic wave or self-gravitational excitations). To form any wave, restoring force and inertia are important parameters. In electrostatic waves, inertia comes from ion and restoring force comes from thermal pressure of electron. But in self-gravitational waves, inertia comes from heavy nuclei and restoring force comes from degenerate pressure of light plasma species (like electrons). Self-gravitational wave is also known as nucleus acoustic wave (the name nucleus acoustic wave is given by the authors A. A. Mamun and M. Asaduzzaman) since heavy nucleus gives the inertia. We have worked with the self-gravitational potential (self-gravitational perturbation) for the first time and our first research article have been published in the American journal "Physics of Plasmas" (Asaduzzaman et al., 2017). The results of our present work are different from other works because our considered particle number density is very high. The degenerate pressure, which is counterbalances by self-gravitational attraction, becomes important at this very high particle number density.

To study the fundamental characteristics of SGSHWs, we have analyzed the solution of the Burgers equation numerically. Figs. 1 – 5 shows the results obtained from the numerical observation.

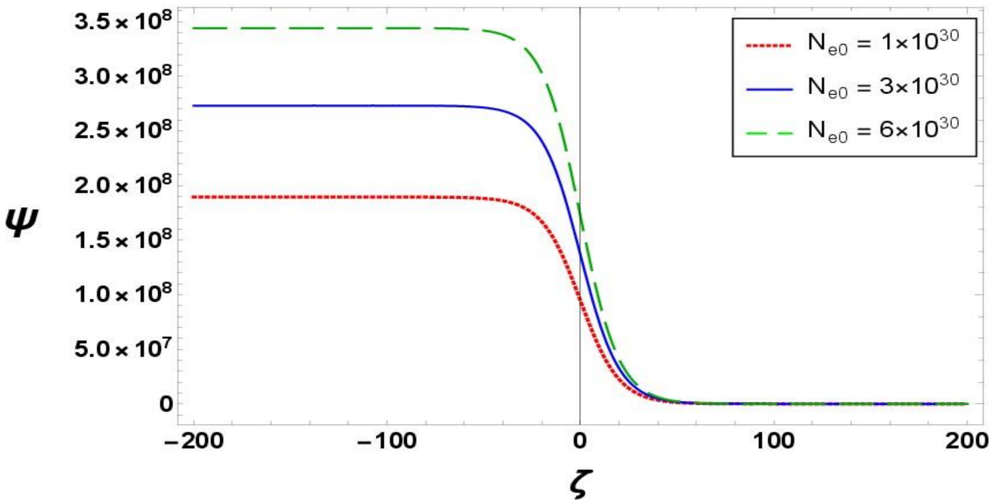

Fig. 1: Varia1tion of shock profile for different values of N_{e0} with $\eta = 1$. The other parameters are U_0 = 0.05 cm/sec, Z_h =37, l_z = 1, and M_h = 85m_p (m_p represents the mass of proton).

The graphical representation of the shock profile for different values of the electron number density is shown in Fig. 1. It is obvious from Fig. 1 that the strength (amplitude) of the SGSHWs decreases as the electron number density increases. It can be observed from Fig. 2 that the strength of the shock profile ($\Psi > 0$) associated with $A > 0$ is independent to the variation of the kinematic viscosity but the steepness of the self-gravitational shock profile ($\Psi > 0$) is largely dependent on the variation of the kinematic viscosity. The steepness of the shock profile decreases with η and this result is similar to the result of Hafez *et al.*, (2017) and Abdelwahed *et al*., (2016).

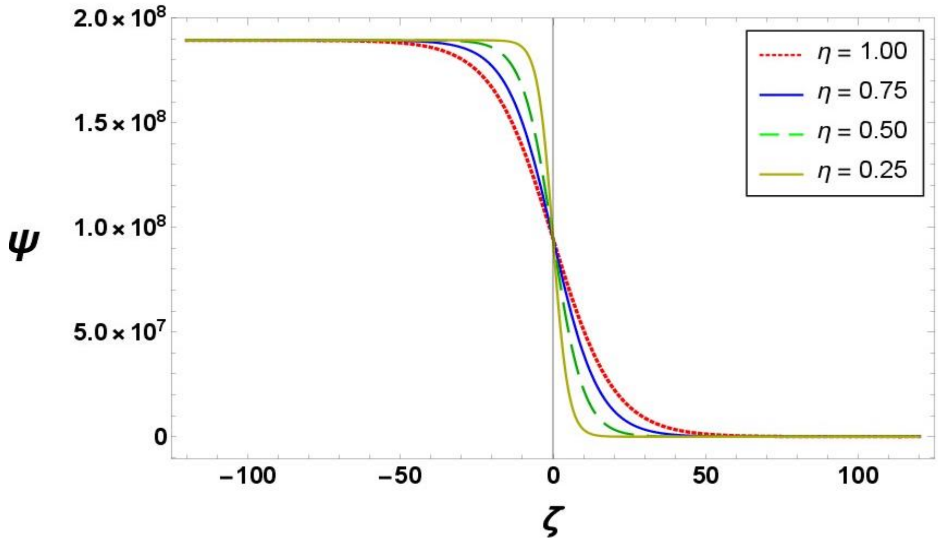

Fig. 2: Variation of the shock profile for different values of η with N_{e0}= 10^{30} cm^{-3}. The other parameters are fixed at $U_0 = 0.05$ cm/sec, Z_h =37, l_z = 1, and M_h = 85m_p.

The variation of nonlinear coefficient with electron number density and the variation of width of the SGSHWs with shock speed are displayed in Fig. 3. It is obvious from Fig. 3 that the nonlinear coefficient A increases as the electron number density decreases (left panel) and the width Δ decreases as U_0 increases (right panel). The variation of the self-gravitational shock potential structure with obliqueness l_z is represented in Fig. 4. It is clear from Fig. 4 that the shock potential increases with the decrease of obliqueness l_z (or with the increase of oblique angle θ). Physically, the shock potential associated with $A > 0$ strongly interact with the external magnetic field when the oblique angle θ increases. Figure 5 shows the variation of the shock profile obtained from Eq. 22 for $R > 1$ ($A > C$), $R = 1$ ($A = C$) and $R < 1$ ($A < C$). Fig. 5 clearly indicates that the amplitude of the SGSHWs increases as R decreases.

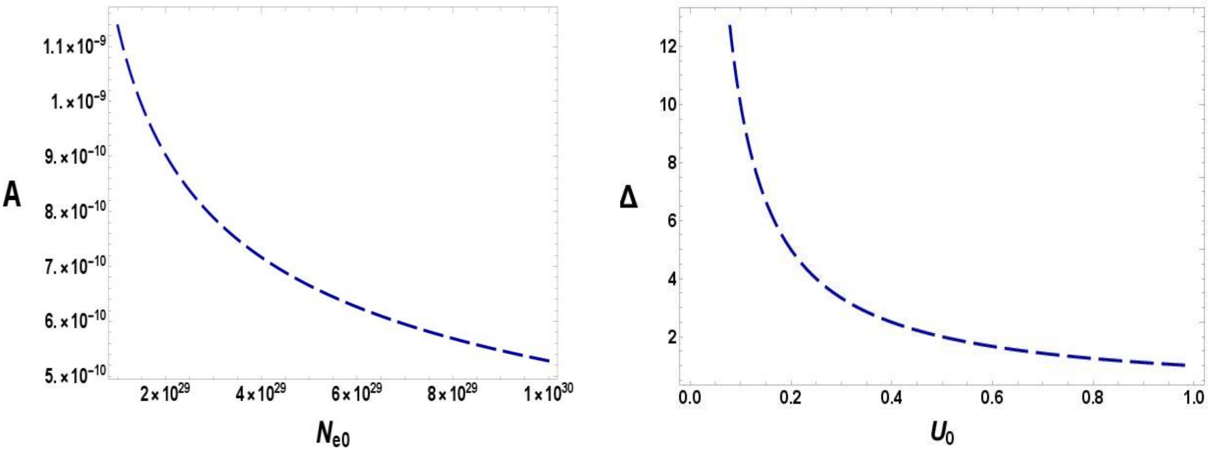

Fig. 3: Plot of A versus N_{e0} for Z_h =37, l_z = 1, $\eta = 1$ and M_h = 85m_p(left panel) and Δ versus U_0 for $\eta = 1$ (right panel).

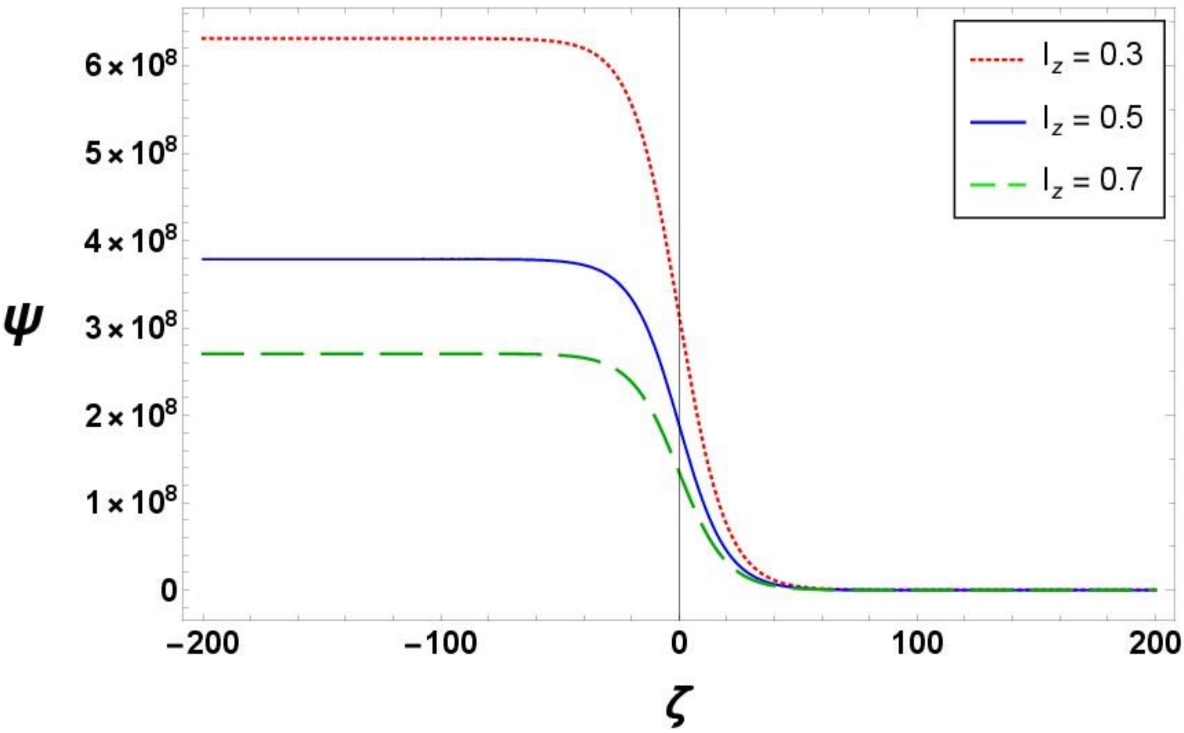

Fig. 4: Showing the variation of the shock profile for different values of l_z. The other parameters are fixed at $U_0 =$ 0.05 cm/sec, Z_h =37, N_{e0}= 10^{30} cm^{-3}, $\eta = 1$ and M_h = 85m_p.

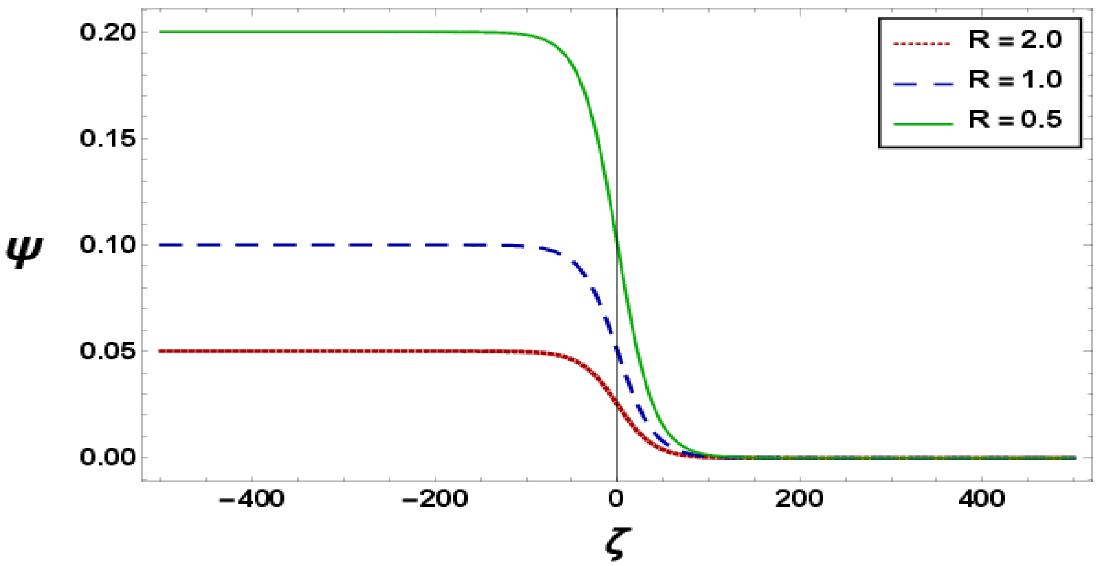

Fig. 5: Variation of the shock profile for different values of R with $U_0 = 0.05$ cm/sec.

5. CONCLUSION

We have studied the salient features of SGSHWs in an extremely high dense DQPS in the presence of a uniform external magnetic field. To obtain the shock structure solution and to perform a numerical analysis, we have derived the Burgers equation by utilizing the reductive perturbation method. Some novel features of the SGSHWs are identified here. The novelty of the SGSHWs is that they are associated with a special type of acoustic waves in which the inertia comes from the heavy nuclear mass density and the restoring force comes from the degenerate pressure of non-relativistic electrons. The basic difference between the results obtained from our model and the results obtained by other authors is that our model gives special type of SGSHWs which has very long wavelength and very low frequency (which has already been shown in Asaduzzaman et al., (2017). The self-gravitational shock structures are formed due to the presence of the kinematic viscosity which is seen to act as the source of dissipation for our considered plasma system. The findings of our investigation can be summarized as follows:

- Our considered plasma system supports only positive potential shock structures (i.e.,$\Psi > 0$) associated with $A > 0$ under consideration of non-relativistic electrons.
- The amplitude of the SGSHWs increases as the electron number density decreases.
- The strength of the shock profile (i.e.,$\Psi > 0$) associated with $A > 0$ is independent on η.
- The steepness of the positive potential shock wave decreases with η.
- The shock potential increases with the increase of oblique angle θ.
- The nonlinear coefficient changes linearly with the electron number density.
- The width of the shock profile decreases with the increase of the shock speed.

We finally stress that the results obtained from our present investigation concerning the SGSHWs and its basic features (amplitude, width, etc.) presented here are correct both numerically and analytically and will be useful in understanding the nonlinear features of the localized self-gravitational disturbances in dissipative space (like white dwarf, neutron star) as well as in laboratory (viz., solid density plasmas [Drake R. P., 2009; Drake R. P., 2010]) degenerate quantum plasmas.

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