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INVESTIGATING THE PITFALLS OF THE LEAST COST AND VOGEL'S APPROXIMATE METHODS: UNDERSTANDING THE IMPACT OF COST MATRIX PATTERNS

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ABSTRACT

To obtain an optimal solution for any Transportation Problem (TP), the first step is to find an Initial Basic Feasible Solution (IBFS). A better IBFS requires fewer iterations to reach an optimal solution. The Least Cost Method (LCM) and Vogel's Approximate Method (VAM) are commonly used approaches to finding IBFS due to their ease of implementation. Researchers frequently propose various methods to discover IBFS, but most of them are modifications of LCM or VAM. While VAM generally performs better, there are instances where it produces worse results compared to LCM and other approaches. Additionally, although researchers have developed new approaches, mainly modified versions of VAM, and demonstrated improved solutions with a few numerical instances, they have not yet identified the causes behind these results. The reasons for LCM and VAM's inability to obtain better IBFS have not been fully determined. This article aims to uncover the causes of pitfalls and the mechanisms of the allocation flow in both LCM and VAM through hypothetical and experimental domains. Several typical numerical instances have been conducted to demonstrate the causes of pitfalls and the allocation flow mechanisms of these methods.

Keywords: *Capacity, Cost Matrix, Least Cost method, Node, Transportation Problem, Vogel's Approximate Method*

1. INTRODUCTION

The Transportation Problem (TP) constitutes an important part of logistics management in the field of Industrial Production Management (IPM) system and it is also the emerging part of Linear Programing Problem (LPP) in the field of Operations Research (OR). TPs have been widely studied not only in Applied Mathematics, but also in Computer Science, Industrial Operations Managements and so on. It is one of the fundamental problems of network flow problem which is usually used to minimize the transportation cost for industries with number of sources and number of destinations while satisfying the supply limits and demand requirements. TPs are frequently encountered in business arena too.

The first and important step of TP is to find Initial Basic Feasible Solution (IBFS). It is well known that Simplex method (developed by G. B. Dantzig) is frequently used to solve LPPs including TPs (Hamdy, 2003). The first input of a Simplex method is an IBFS of TP. The procedure of finding IBFS that G. B. Dantzig used termed as North-West Corner (NWC) Rule by Charnes and Cooper (1954 to 1955) (see also Charnes and Cooper,1962). Dantzig's procedure (NWC) for finding IBFS ignored transportation costs for the flow of allocations. But Dantzig developed an iterative procedure named Simplex method for obtained the optimal solution still to date very important tool to solve TPs as well as LPPs. It is worthwhile to mention here that the better IBFS used in Simplex method produces optimal solution rapidly. Nowadays, several methods are available in the literature for finding IBFS of TP; but among them Least Cost Method (LCM) is relatively very simple to implement. Reinfeld and Vogel (1958) first proposed an algorithm named Vogel's Approximation Method (VAM) on the basis of LCM in which the flow of allocations is determined by Distribution Indicator (DI) which is formed by the manipulation of transportation cost entries. It is known that VAM provides comparatively better IBFS.

After VAM several approaches are developed by manipulating cost entries which are actually somehow the variants of VAM method. Few of them are mentioned here as a state of arts of this research field. Goyal (1984) proposed a modified VAM for run balanced transportation problems in which he modified the procedure to form Distribution Indicator (DI). Kirca and Satir (1990) developed a heuristic, called Total Opportunity-cost Matrix (TOM), to obtain an IBFS for the TP. Mathirajan and Meenakshi (2004) considered several variants of VAM approach for TPs. Korukoglu & Balli (2011) introduced the Total Opportunity-cost Matrix (TOM) rather than DI to control the flow of allocations. Das *et al.* (2014) proposed Logical Development of Vogel's Approximation Method (LD-VAM) to find IBFS for TP. Azad *et al.* (2017) and Amaliah *et al*. (2019) first developed Total Opportunity Cost (TOC) matrix and then they formed DI tableau for allocations by considering TOC. Besides,

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Sharma & Bhadane (2016) proposed a modified NWC approach by using Statistical tool called Coefficient of Range (CoR). Very recently, some other modified approaches based on VAM approach are found in the publications of (Sumathi & Bama, 2018; Karagul & Sahin, 2020; Pratihar *et al.,* 2020; Lekan *et al.,* 2021). On the other hand, Jamali *et al.* (2017) and Jamali & Akhtar (2018) proposed a new technique for controlling the flow of allocation named Weighted Opportunity Cost (WOC) matrix. The weighted opportunity cost matrix is formed by demand and/or supply as a weight factor corresponding to each transportation cost. They also considered some numerical instance to test the efficiency of the proposed algorithms. For finding IBFS of TPs, a good survey is observed in (Mathirajan *et al.,* 2021).

It is observed in the literature that many approaches are available in TPs to find IBFS. Researchers are also continuously working on to develop more efficient approaches to find out better IBFS of TPs. But as far as we know, none of the approaches is the best for finding IBFS to solve all types of TPs. It is also observed in the literature that researchers have just proposed the approaches and shown improvement of the solutions by taking few selected instances. None of them find out the causes of the flow of allocation procedures for superiority of the proposed approach. It is also noticed that all the existing approaches are developed only based on numerical instances.

In this study, two frequently used approaches, namely LCM and VAM, will be considered to analyze the causes of pitfalls and the mechanisms of the flow of allocation procedures. To find out the drawbacks and mechanisms of the flow of allocations in these approaches, we will perform numerical experiments. We will consider some typical numerical instances and investigate their flow of allocation procedures to discover the cases of superiority and/or falling into pitfalls

2. MATHEMATICAL MODEL OF TRANSPORTATION PROBLEM

The mathematical model of TP expressed in LP model as follows:
Minimize

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{j=1}^{m} x_{ij} \le a_i; \quad i = 1, 2, \ldots, m \text{ (supply constraints)} \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} \le b_j; \quad j = 1, 2, \ldots, n \text{ (demand constraints)} \tag{3}$$

$$x_{ij} \ge 0; \quad a_i \ge 0; \quad b_j \ge 0; \quad \forall i = 1, 2, \ldots, m; \quad j = 1, 2, \cdots, n \tag{4}$$

where, Z: Total transportation cost to be minimized, which is the objective function,
c_{ij} : Unit transportation cost of the commodity from each source i to destination j,
x_{ij} : Number of units of commodity sent from source i to destination j,
a_i : Number of commodities to be supplied from source i,
b_j : Number of commodities required to destination j.

In brief, constraints (2) and (3) are called *Capacity constraints* whereas constraints (4) are called *non-negative Restrictions conditions*.

As TP is a special type of LP problems in which commodities are transported from a set of sources to a set of destinations such that the total cost of transportation is minimized. By taking the special characteristics of TP, it can be designed as an especial tableau called Transportation Tableau (TT). The typical view of TT is shown in the Table 1. In the TT, O_i indicates ith source with amount of availability is a_i which is shown in the far-right column. On the other hand, D_j denotes jth destination with demand b_j, which is shown in the bottom row of TT. In this table, there is an $m \times n$ matrix containing cost entries. The cell in ith row and jth column is called (i,j) th cell and is denoted as c_{ij}, which represents the unit shipping cost from ith source to jth destination. So, a TT be can be viewed as a $(m+1) \times (n+1)$ matrix.

Table 1: A TT of a TP with *m* Origins and *n* Sinks.

		Sinks/Destinations					
Origins/Sources		D_1	D_2	⋯	D_{n-1}	D_n	Supply
	O_1	c_{11}	c_{12}	⋯	c_{1n-1}	c_{1n}	a_1
	O_2	c_{21}	c_{22}	⋯	c_{2n-1}	c_{2n}	a_2
	O_3	⋮	⋮	⋯	⋮	⋮	a_3
	⋮	⋮	⋮	⋯	⋮	⋮	⋮
	O_m	c_{1m}	c_{2m}	⋯	c_{mn-1}	c_{mn}	a_m
	Demand	b_1	b_2	⋯	b_{n-1}	b_n	

3. ANALYSIS OF LCM AND VAM

3.1 Algorithm LCM and VAM

Before analyze the algorithms, it is worthwhile to present the algorithms of the LCM and VAM.

Algorithm of Least Cost Method (LCM)

Step 1: Find the smallest transportation cost available in TT and select the corresponding route. (Note: In case, if there are more than one smallest cost, select the cells where maximum allocation can be made)
Step 2: Supply commodity as much as possible to the cell (route) corresponding to that minimum cost.
Step 3: Cross out that row/column (the routes) of the TT which has exhausted. It is noted that cross out row/column (the routes) has no chance to carry goods further as all possible amount of goods has been shipped/ received.
Step 4: Test whether all commodities are shipped or not. If not go to step 1 otherwise
Step 5: Stop.
Step 6: Find IBFS.

Algorithm of Vogel's Approximate Method (VAM)

*Step 1**:** For each row (column), determine Distribution Indicator (DI)/penalty measure by subtracting the smallest unit cost element in the row (column) from the next smallest unit cost element in the same row (column).*
*Step 2**:** Identify the row or column with the largest penalty, breaking ties arbitrarily.*
*Step 3**:** Allocate as much as possible to the variable with the least unit cost in the selected row or column.*
*Step 4**:** Adjust the supply and demand, and cross out the satisfied row or column. If a row and column are satisfied simultaneously, only one of the two is crossed out, and the remaining row (column) is assigned zero supply (demand).*
Step 5:(a) If exactly one row or column with zero supply or demand remains uncrossed out, stop.
(b) If one row (column) with positive supply (demand) remains uncrossed out, determine the basic variables in the row (column) by the least cost method. Stop.
(c) If all the uncrossed-out rows and columns have (remaining) zero supply and demand, determine the zero basic variables by the least cost method. Stop.
(d) Otherwise, go to step-1.
*Step 6**:** Find IBFS.*

3.2 HYPOTHETICAL ANALYSIS OF THE ALGORITHMS OF LCM AND VAM

It is observed that the LCM algorithm is straightforward–what will be the amount of supply/ demand the algorithm does not care about it at all. The algorithm only observes the amount of transportation cost at each cell (route) of the cost matrix. It also does not care about the differences among cost entries in the cost matrix. LCM considers only the current cost of each cell of the reduced cost matrix (if any allocation is done).

So, LCM always seeks the route in which Transportation Cost (TC) is minimum at present, what will be the next scenario it does not care at all. It also does not care what amount of available commodity – small or large – is in the nodes. So, LCM is straightforward. It also needs less computation effort than VAM and other approaches modified from VAM and LCM. But it is observed that VAM gets better IBFSs than LCM in most instances. But why?

There may arise two cases in which LCM give worse IBFS, one is the pattern of cost matrix of transportation another one is the pattern of capacity distribution among the nodes. Here we have discussed about the first one.

(a) Sometimes, LCM is bound to close some better routes, and as a result, it is forced to choose worse routes in later allocation procedures.

When LCM allocates from the source to the sink along the route having the minimum TC, unfortunately, it closes some better routes from that source to other sinks since the source is exhausted and cannot allocate any more to any other sink. Similarly, it closes some better routes to that sink from other sources as the sink is exhausted and cannot receive any more from any other source. That is why the algorithm is bound to choose a route from another source with a large amount of TC compared to the routes closed for previous allocations. The algorithm is also bound to choose a route to another sink that has a large amount of TC compared to routes that have been closed in previous allocations. This situation may occur due to the following circumstances.

Let the minimum value of TC, in the cost matrix (reduced cost matrix), is along the route from source O_i to sink S_j. All the TCs along the routes from O_i to any other sinks are not relatively large compared to that minimum value (i.e., all TCs from O_i are near to minimum value), and all TCs along the routes for any source to sink S_j is not also large. In addition, there are some routes from other sources to some sinks have TCs but vary highly. In this situation, LCM frequently gets pitfalls and produces worse IBFS. Similarly in the reverse case (interchange of sink and source) same situation will be occurred. This phenomenon will be explored numerically in section 4.1.

Now, VAM is an improved version of the LCM that generally, but not always, produces better IBFS. The algorithm of VAM also does not account whatever the demand/supply of each node during the allocation procedures. But VAM does not allocate immediately to the route which has minimum TC like LCM. During allocation procedures it does not observe only TC of each cell but also carefully counts the differences among TCs especially difference between minimum and nearest minimum TCs for each node (i.e., each row and each column) in TT (reduced TT) before each step of allocations. So, in step 1, VAM algorithm first finds out the difference of TCs between minimum and nearest minimum TCs of routes for each node (sink or source) which are denoted as Distribution Indicator (DI).

Then, in step 2, the algorithm finds out the largest DI value among all DI values. It then identifies the route with the minimum TC among the routes of the nodes that have the largest DI value. After identifying the route, the VAM algorithm allocates commodities as much as possible, similar to LCM, along that route. It is noted that this procedure is continued for each step in the reduced TT. That is, the procedure is repeated for each reduced TT until all commodities are allocated. With this modification, VAM can overcome the pitfalls that LCM frequently faces - being bound to close better routes with small TC.

If the route having the lowest TC corresponding to this node is crossed out before allocation through the node that has the largest DI value route, then the algorithm is bound to allocate through the route with a very large TC corresponding to that node, which is frequently faced by the LCM approach. Moreover, if the DI value corresponding to the node having the smallest TC (among all routes of TP) is low and the routes of this node are crossed out, then VAM may be able to allocate the routes corresponding to this node with a smaller TC as its DI values are low. This phenomenon will be explored numerically in section 4.

Though VAM is frequently able to find out better solutions compared to LCM, in some instances, it fails to obtain better solutions compared to LCM and other approaches. This occurs due to the pattern of the cost matrix and the distribution of capacity. Here, we will discuss the pattern of the cost matrix, which may lead to VAM producing worse IBFS compared to LCM.

Let the minimum value of TC, in the cost matrix (reduced cost matrix), is along the route from source O_i to a sink. Also let the largest DI value corresponds to a sink node D_j in which the smallest TC is relatively large enough compared to the overall minimum TC value and which is corresponding to the source node O_i. So, VAM bounds to allocate to the sink node D_j from the source node O_i. In this circumstances source node O_i is never able to allocate along the route in which TC is minimum. Similarly in the reverse case (interchange of sink and source) same situation will be occurred. This phenomenon will be explored numerically in section 4.2.

4. NUMERICAL EXPERIMENTS

4.1 Illustrating How Least Cost Method (LCM) Produces Inferior Initial Basic Feasible Solutions (IBFS) Compared to Vogel's Approximation Method (VAM)

At first, we will numerically demonstrate how LCM get pitfalls for the specific pattern of cost distribution matrix in TT. For this purpose, we have considered a typical numerical instance – Example 1 given in Table 2.

Example 1:

Table 2: Transportation Tableau of a transportation problem, Example 1.

	D_1	D_1	D_1	Supply
O_1	3	10	15	20
O_2	1	4	2	20
O_3	5	6	8	20
Demand	20	20	20	

Before starting the allocation procedure through LCM or VAM approaches, it is worthwhile to analyze the scenario of the typical Example 1 shown in Table 2. The problem has three sources and three sinks. Each node has equal capacity i.e., 20. We choose such capacity to confirm the equal tendency of flow regarding node capacity. That is, each node has no influence regarding the amount of availability/necessity of commodities. As the amount of commodity is equal for each node so differences in total transportation cost for each approach are only due to Transportation Cost (TC).

It is observed in the Table 2 that TC from source O_2 are small and difference for any two sinks are also not so large. It is also noticed that TCs from source O_3 are a bit large but differences are relatively small. On the other hand, the TC along the route from O_1 to D_1 is small but along the routes O_1 to other sinks D_2 and D_3 are very large. Also, from the source O_1, the difference of transportation cost along the routes D_1 and (nearest smaller TC) D_2 is very large.

Now, first, we will apply the LCM approach to find the Improved Basic Feasible Solution (IBFS) and the corresponding total cost. For this purpose, we consider Example 1. The allocation procedures are given step by step for each allocation. It is worthwhile to mention here that, although LCM does not require DI, we will still show DI in each step for the sake of explanations. It is observed that LCM finds that the route from O_2 to D_1 has minimum TC compared to all routes in this TT. So, LCM immediately allocates as much as possible to D_1 from O_2. After first allocation, the scenario is shown in Table 3.

Table 3: LCM approach, after Step 1.

	D_1	D_2	D_3	SUPPLY	DI
O_1	3 ×	10	15	20	7
O_2	1 **20**	4 ×	2 ×	20	1
O_3	5 ×	6	8	20	1
DEMAND	~~20~~	20	20		
DI	2	2	6		

Table 4: LCM approach, after Step 2.

D_1	D_2	D_3	S	DI
3 ×	10 ×	15	20	~~7~~, 5
1 **20**	4 ×	2 ×	20	1
	6 **20**	8 ×	~~20~~	~~1~~, 2
~~20~~	~~20~~	20		
~~2~~	~~2,~~ 4	~~6~~, 7		

Table 5: LCM approach, after Step 3.

	D_1	D_2	D_3	S	DI
O_1	3 ×	10 ×	15 **20**	~~20~~	~~7~~, ~~5~~
O_2	1 **20**	4 ×	2 ×	20	1
O_3	5 ×	6 **20**	8 5	~~5~~	~~1~~, ~~2~~
D	~~20~~	~~20~~	20		
DI	~~2~~	~~2,~~ 4	~~6~~, ~~7~~		

After first allocation, it is observed that all routes from source O_2 are exhausted as its all commodities are allocated to the sink D_1 which is also exhausted. As all the routes from O_2 are exhausted, so sinks D_2 and D_3 need to supply from other sources namely O_1 and O_3. Therefore, the sinks D_2 and D_3 bound to choose worse routes which have relatively larger TC values whereas routes O_2 to D_2 and O_2 to D_3 had smaller TC cost but now exhausted. On the other hand, as sink D_1 is exhausted so the sources O_1 and O_3 have no any possibility to supply to the sink D_1 anymore. Therefore, the sources O_1 and O_3 bound to choose worse routes which have relatively larger TC values whereas routes O_1 to D_1 and O_3 to D_1 had smaller TC cost relative to the present available routes but now exhausted. So, for second allocation LCM choose the route O_3 to D_2 as it corresponds to minimum TC i.e. 6 in reduced TT given it Table 4. So, after second allocation the scenario is shown in Table 4. After second allocation, it is observed that the sources O_2 and O_3 are exhausted. Also, the routes to sinks D_1 and D_2 are exhausted. So, the sink D_3 bounds to choose worst route i.e., O_1 to D_3 with largest TC i.e. 15. As a result, the total transportation costs become very large. After third allocation the reduced TT is shown in the Table 5.

Table 6: LCM approach, Step 3.

	D_1	D_2	D_3	SUPPLY
O_1	3 / 0	10 / 0	15 / **20**	~~20~~
O_2	**1** / **20**	4 / 0	2 / 0	20
O_3	5 / 0	6 / **20**	8 / 5	~~5~~
DEMAND	~~20~~	~~20~~	20	

As all commodities are allocated so LCM algorithm is terminated. Table 6 gives the IBFS of the problem for LCM approach. Therefore, the total transportation cost for this IBFS is:

Total Cost= 1×20+6×20+15×20 =440.

Now we have found IBFS of this problem by VAM. The first Step of VAM algorithm is to find out DI for each row source node and each column sink node. It is noted that DI is the difference of TC between minimum and nearest minimum to each row/column. The DI value corresponding to each node is show in DI row/column of the Table 6.

It is observed that the DI value (difference of transportation cost between two routes) corresponding to the Source O_1 is 7 which is the differences of TC between the two routes (i) O_1 to D_1 and (ii) O_1 to D_2. This is the largest DI value among all DI values either from sources or sinks. It is noticed that among all the routes from all sources to all the sinks, the minimum TC is 1 which along the route from O_2 to D_1. Moreover, the DI value for source O_2 is 1 and for sink D_1 is 2 which small compare to DI value 7. It is also observed that the second smaller transportation cost from source O_2 is 2 which are very near to value 1 and which is from source O_2 to sink D_3. As the DI value for the routes from O_1 (i.e., row 1) is highest, so VAM searches minimum TC among the routes from O_1 (i.e., in row 1). Now, it is observed in the TT that the transportation cost from the single source O_1 to the sinks D_1, D_2 and D_3 are 3, 10 and 15 respectively. So, minimum TC from source O_1 is 3 and its route is from O_1 to D_1. That is why VAM algorithm chooses the route O_1 to D_1 rather than the route O_2 to D_1.

Table 7: VAM approach, finding DI.

	D_1	D_2	D_3	S	DI
O_1	3	10	15	20	7
O_2	1	4	2	20	1
O_3	5	6	8	20	1
D	20	20	20		
DI	2	2	6		

Table 8: VAM approach, after Step 1.

	D_1	D_2	D_3	S	DI
O_1	**3** / **20**	10 / ×	15 / ×	~~20~~	7
O_2	1 / ×	4	2	20	1
O_3	5 / ×	6	8	20	1
D	~~20~~	20	20		
DI	2	2	6		

So, in first step, VAM algorithm allocates commodities as much as possible along the route O_1 to D_1 which is obviously 20 units and whose TC per unit is 3 rather than the route O_2 to D_1 who's TC per unit is 1. So, after first allocation, the reduced TT is shown in Table 8. It is observed in the table 8 that after first allocation, the routes form source O_1 are exhausted and also the sink D_1 does not need any more allocation. It is also observed that the route containing minimum TC i.e., the route from the source O_2 to destination D_1 is now exhausted in which TC is 1. But it is observed that there exist some more routes for the source O_2 with small TC that is near to 1, since DI value for the source O_2 is 1.

Now for preparing second allocation, algorithms develop new DI vectors from reduced TT given in Table 9. It is observed the DI value corresponding to the sink D_3 is highest which is 6. It is also observed that the transportation cost to sink D_3 is lowest along the route O_2 to D_3. So, VAM choose the route O_2 to D_3 and allocates as much as possible along this route which is off course 20 units. After second allocation the reduced TT is shown in Table 10. It is observed in Table 10 that only routes from O_3 to the sink D_3 is unallocated. So, the rest all 20 units are allocated from the source O_3 to the sink D_3. So, the final scenario is shown in Table 11. Therefore, IBFS is shown in Table 12 and corresponding total transportation cost of the IBFS is followed by the table 12.

Table 9: VAM approach, finding DI, after Step 3.

	D_1	D_2	D_3	S	DI
O_1	3 20	10 ×	15 ×	~~20~~	~~7~~
O_2	1 ×	4 ×	2 20	~~20~~	~~1~~, 2
O_3	5 ×	6	8 ×	20	~~1~~, 2
D	~~20~~	20	~~20~~		
DI	~~2~~	~~2~~, 2	~~6~~, 6		

Table 10: VAM approach, after Step 2.

	D_1	D_2	D_3	S	DI
O_1	3 20	10 ×	15 ×	~~20~~	~~7~~
O_2	1 ×	4 ×	2 20	~~20~~	~~1, 2~~
O_3	5 ×	6	8 ×	20	~~1, 2~~
D	~~20~~	20	~~20~~		
DI	~~2~~	~~2, 2~~	~~6, 6~~		

Table 11: VAM approach, after Step 6.

	D_1	D_2	D_3	S	DI
O_1	3 20	10 ×	15 ×	~~20~~	~~7~~
O_2	1 ×	4 ×	2 20	~~20~~	~~1, 2~~
O_3	5 ×	6 20	8 ×	~~20~~	~~1, 2~~
D	~~20~~	~~20~~	~~20~~		
DI	~~2~~	~~2, 2~~	~~6, 6~~		

Table 12: VAM approach, IBFS.

	D_1	D_2	D_3	SUPPLY
O_1	3 20	10 0	15 0	~~20~~
O_2	1 0	4 0	2 20	~~20~~
O_3	5 0	6 20	8 0	~~20~~
DEMAND	~~20~~	~~20~~	~~20~~	

Total Cost = 3x20+2x20+6x20 = 220

Table 13: Comparison between LCM and VAM approaches for finding IBFS.

Allocation Step	1			2			3			Total Cost	Opt. cost	Iteration
Approach	IBFS	DI	cost	IBFS	DI	cost	IBFS	DI	cost			
LCM	x_{21}	1	1	x_{32}	2	6	x_{13}	--	15	**440**	220	3
VAM	x_{11}	7	3	x_{23}	6	2	x_{32}	--	6	220	220	1

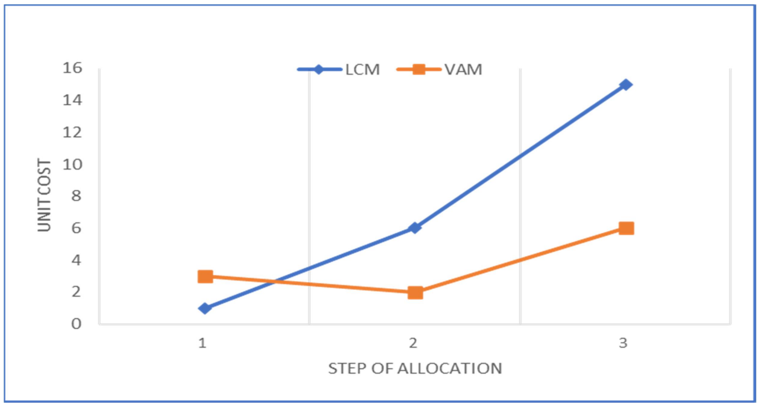

Figure 1: Comparison between LCM and VAM approaches for finding IBFS regarding step of allocations vs. unit cost for Example 1.

The intensive comparison of the allocation procedures of the two approaches is concisely shown in Table 13. It is noticed in Table 12 that the total cost in IBFS of the VAM approach is much cheaper compared to that of the LCM approach. It is observed in the table that there is only one common basic solution for both approaches, which is x32. It is remarked that the DI values of VAM are always higher than those of LCM. Additionally, it is noted that the IBFS obtained by VAM is also the optimal solution, whereas LCM needs three additional iterations to obtain the optimal solution. The comparison of the flow of allocation between LCM and VAM regarding the step of allocation versus unit transportation cost is shown in Figure 1. It is observed in Figure 1 that the unit cost is gradually increasing concerning the number of steps of allocations in the VAM approach, whereas the unit cost is rapidly increasing concerning the number of steps of allocations in the LCM approach. From this numerical analysis it may conclude that when any node contains small TC and also DI value corresponding this node is also small then LCM falls in a pitfall.

4.2 Illustrating How VAM Produces Inferior Initial Basic Feasible Solutions (IBFS) Compared to LCM

Now, Example 2 (see Table 14) will be considered to analyze the performance of the above approaches, as shown in Table 14.

Example 2:

Table 14: Transportation Tableau of another transportation problem, Example 2

	D_1	D_2	D_3	SUPPLY
O_1	2	5	8	20
O_2	6	4	14	20
O_3	15	12	13	20
DEMAND	20	20	20	

Before analyzing the performance of the approaches, we would like to mention the peculiarities of this Example 2. The problem has three sources and three sinks, and each node has an equal capacity of 20. We have deliberately chosen this capacity to ensure that each node has no influence regarding the amount of availability/necessity of commodities. As the amount of commodity is equal for each node, the differences in the total transportation cost for each approach are solely due to the Transportation Cost (TC).

It is observed in the Table 14 that TCs from source O_1 are small and difference of TCs for any two sinks are also not so large. It is also noticed that TCs from source O_2 and O_3 are a bit large but differences of TCs among the routes are not also so large. On the other hand, the TCs along the route to the sink D_3 is relatively large. It is also noticed that the DI corresponding to the sink D_3 is largest and minimum TC along the route to sink D_3 is also relatively larger compared to minimum TC among all the routes in the whole problem.

Now we will find IBFS step by step by using LCM approach. Though LCM does not care about DI values but for the sake of explanations we will show DI in each step. IT is observed in the Table 14 that the minimum TC is 2 along the route O_1 to D_1, so in first LCM allocates as much as possible along the route O_1 to D_1. After 1st allocation the reduced TT is shown in the Table 15.

Table 15: LCM approach, after Step 1.

	D_1	D_2	D_3	S	DI
O_1	**2** **20**	5 ×	8 ×	~~20~~	3
O_2	6	**4**	14	20	2
O_3	15 ×	12 ×	13	20	1
D	~~20~~	20	20		
DI	4	1,8	5,1		

Table 16: LCM approach, after Step 2.

	D_1	D_2	D_3	S	DI
O_1	**2** **20**	5 ×	8 ×	~~20~~	3
O_2	6 ×	**4** **20**	14 ×	~~20~~	2,10
O_3	15 ×	12 ×	**13**	~~20~~	1,1
D	~~20~~	~~20~~	20		
DI	4	1,8	5,1		

It is observed in reduced TT, shown in Table 15, that the minimum TC is 4 along the route O_2 to D_2. So, in second step of allocation LCM allocates as much as possible along the route O_2 to D_2. After second allocation the reduced TT is shown in the Table 16. It is observed that after second allocation, only route from O_3 to D_3 is unallocated so in third step of allocation, LCM allocates along that route. After third step of allocation the, LCM algorithm is terminated. After third allocation the scenario of TT is shown in the Table 17. Therefore, the IBFS of the problem is shown in the Table 18 and corresponding total transportation cost is given below.

Total Cost = 2x20 + 4x20 +13x20 = 380.

Table 17: LCM approach, after Step 3.

	D_1	D_2	D_3	S	DI
O_1	**2** **20**	5 ×	8 ×	~~20~~	3
O_2	6 ×	**4** **20**	14 ×	~~20~~	2,10
O_3	15 ×	12 ×	**13** **20**	~~20~~	1,1
D	~~20~~	~~20~~	~~20~~		
DI	4	1,8	5,1		

Table 18: LCM approach, IBFS.

	D_1	D_2	D_3	S	DI
O_1	**2** **20**	5 0	8 0	~~20~~	3
O_2	6 0	**4** **20**	14 0	~~20~~	2,10
O_3	15 0	12 0	**13** **20**	~~20~~	1,1
D	~~20~~	~~20~~	~~20~~		
DI	4	1,8	5,1		

It is worthwhile to mention here that LCM has allocated commodities to sink D_3 from source O_3 with larger TC, 13 compare to TC 8 but sinks D_1 and D_2 have gotten commodities with very smaller TCs namely 2 and 4 compared other TCs in the TT. Eventually LCM approach is able to find out IBFS with lowest total transportation cost and this is also optimal solution.

Now we will apply VAM approach to find out the IBFS of the problem. The first Step of VAM algorithm is to find out DI for each row (source node) and each column sink node). The DI value corresponding to each node is show in DI row/column of the Table 16. It is observed that the DI value (difference of transportation cost between two routes) corresponding to the sink D_3 is 5 which differences of TC between the two routes (i) O_1 to D_3 and (ii) O_2 to D_3. This the largest DI value among all DI values either from sources or sinks. It is noticed that among all the routes from all sources to all the sinks, the minimum TC is 2 which along the route from O_1 to D_1. Moreover, the DI value for source O_1 is 3 and for the sink D_1 is 4 which small compare to DI value 5. It is also observed that there are many smaller TCs are available {2, 5, 4, 6} in the TT compared to the minimum TC to the route D_3 whose DI value is largest. As the DI value for the sink D_3 (i.e. column 3) is highest, so VAM algorithm searches minimum TC among the routes to the sink D_3 (i.e. in column 3). Now, it is observed in the TT that the transportation cost to the single sink D_3 to the sources O_1, O_2 and O_3 are 8, 13 and 14 respectively. So, minimum TC to the sink D_3 is 8 and its route is from O_1 to D_3. Eventually, VAM algorithm chooses the route O_1 to D_3 whose TC is 8 rather than the route O_1 to D_1 whose TC is only 2.

Table 19: VAM approach, finding DI for each node.

	D_1	D_2	D_3	SUPPLY	DI
O_1	2	5	8	20	3
O_2	6	4	14	20	2
O_3	15	12	13	20	1
DEMAND	20	20	20		
DI	4	1	5		

Table 20: VAM approach, after first allocation -Step 1.

	D_1	D_2	D_3	S	DI
O_1	2 ×	5 ×	**8** **20**	~~20~~	3
O_2	6	4	14 ×	20	2
O_3	15	12	13 ×	20	1
D	20	20	~~20~~		
DI	4	1	5		

Table 21: VAM approach, finding DI in reduced TT.

	D_1	D_2	D_3	S	DI
O_1	2 ×	5 ×	**8** **20**	~~20~~	3
O_2	6	4	14 ×	20	~~2~~, 2
O_3	15	12	13 ×	20	~~1~~, 3
D	20	20	~~20~~		
DI	4, 9	~~1~~, 8	5		

So, in first step, VAM algorithm allocates commodities as much as possible along the route O_1 to D_3 which is obviously 20 units and whose TC per unit is 8. So, after first allocation, the reduced TT is shown in Table 20. It is observed in the Table 20 that after 1st allocation the routes form source O_1 are exhausted and also the sink D_3 does not need any more allocation. It is also observed that the route containing minimum TC i.e., the route from the source O_1 to destination D_1 is now closed for further allocation. Also, the route containing smaller TC i.e., the route from the source O_1 to destination D_2 is also closed for further allocation. It is notice that TC from source O_1 to destination D_1 and D_2 are less than the TC from source O_1 to destination D_3. Now, it is observed that there exist some more routes for the source O_2 to destination D_1 and D_2 with smaller TC namely 6 and 4 but DI value for the source O_2 is only 2. Before second allocation, VAM again find out new DI value from reduced TT which is shown in the Table 21. It is observed that sink D_1 has largest DI value which is 9. So, VAM chooses destination D_1 for next allocation. It is also observed that among all the routes from any source to sink D_1, the route from source O_2 to sink D_1 has a minimum TC, and its amount is 6. So, in the second allocation, VAM allocates as much as possible to sink D_1 from source O_2. After the second allocation, the reduced TT is shown in Table 22.

Table 22: VAM approach, after 2nd allocation – Step 2.

	D_1	D_2	D_3	S	DI
O_1	2 ×	5 ×	**8** **20**	~~20~~	3
O_2	**6** **20**	4 ×	14 ×	~~20~~	~~2~~, 2
O_3	15 ×	12	13 ×	20	~~1~~, 3
D	~~20~~	20	~~20~~		
DI	4, 9	~~1~~, 8	5		

Table 23: VAM approach, after 3rd allocation - Step 3.

	D_1	D_2	D_3	S	DI
O_1	2 ×	5 ×	**8** **20**	~~20~~	3
O_2	**6** **20**	4 ×	14 ×	~~20~~	~~2~~, 2
O_3	15 ×	**12** **20**	13 ×	~~20~~	~~1~~, 3
D	~~20~~	~~20~~	~~20~~		
DI	4, 9	~~1~~, 8	5		

Table 24: VAM approach, IBFS.

	D_1	D_2	D_3	SUPPLY	DI
O_1	2 0	5 0	**8** **20**	~~20~~	3
O_2	**6** **20**	4 0	14 0	~~20~~	~~2~~, 2
O_3	15 0	**12** **20**	13 0	~~20~~	~~1~~, 3
DEMAND	~~20~~	~~20~~	~~20~~		
DI	4, 9	~~1~~, 8	5		

It is observed in the Table 21 that, though the minimum TC is 4 along the route O_2 to D_2 but VAM chooses the route O_2 to D_1 for second allocation whose TC is larger. After second allocation only the route O_3 to D_2 is open. So, in third allocation, VAM allocates all the remain commodities along this route. After third allocation the algorithm is terminated. After third allocation the scenario of TT is shown in the Table 20. Therefore, the IBFS of the problem is shown in the Table 21 and corresponding total transportation cost is given below.

Total Cost = 8x20 +6x20 +12x20 = 520

Table 22: Comparison between LCM and VAM approaches for finding IBFS.

Allocation Step	1			2			3			Total Cost	Opt. cost	Iteration
Approach	IBFS	DI	cost	IBFS	DI	cost	IBFS	DI	cost			
LCM	x_{11}	4	2	x_{22}	10	4	x_{33}	--	13	**380**	**380**	1
VAM	x_{13}	5	8	x_{21}	9	6	x_{32}	--	12	**520**	**380**	3

The intensive comparison of the allocation procedures of the two approaches is concisely shown in Table 22. It is noticed in Table 22 that the total cost in the IBFS of the LCM approach is much cheaper compared to that of the VAM approach. It is also observed in the table that there is no common basic solution for both approaches. Additionally, it is noticed that the DI values of VAM are not always higher than those of LCM. It is remarked that the IBFS obtained by LCM is also the optimal solution, whereas VAM needs 3 more iterations to obtain the optimal solution.

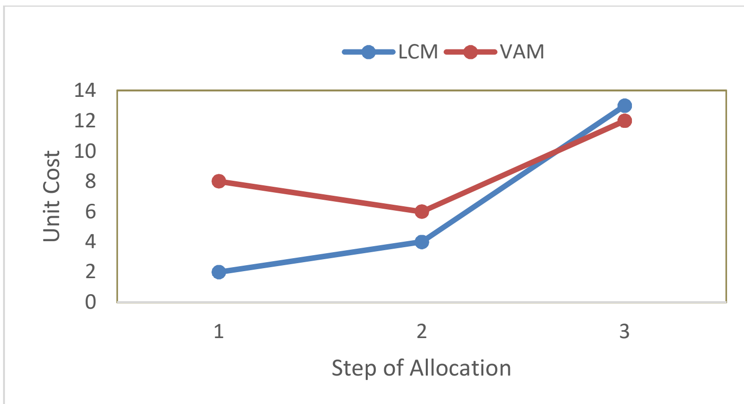

Figure 2: Comparison between LCM and VAM approaches for finding IBFS regarding step of allocations vs. unit cost for Example 1.

The comparison of the flow of allocation between LCM and VAM regarding the step of allocation versus unit transportation cost is also shown in Figure 2. It is observed in Figure 2 that the unit transportation cost is gradually increasing with respect to the number of steps of allocations in LCM. Additionally, in the initial two steps among the three steps, the TC of the LCM approach is much smaller than those of VAM. It is also noticed that in both approaches, the unit cost rapidly increases. However, in the last step, the unit cost of LCM is larger than that of VAM, but both TCs are very close.

From this numerical analysis, it may be concluded that when any node contains a larger TC, and the corresponding DI value corresponding to this node is also the largest, then VAM may fall into a pitfall. Additionally, from this numerical analysis, it may be concluded that when the differences among the TCs are similar, then LCM is better as it needs less computational cost compared to VAM and/or its variants.

5. CONCLUSION

LCM and VAM are the most frequently used methods to find out Improved Basic Feasible Solutions (IBFS). LCM is very easy to implement and computationally much cheaper. On the other hand, though in general, VAM performs better, sometimes it produces worse results too. In this article, we have discussed the inside views of the flow of allocations of both LCM and VAM algorithms roughly. We have also pointed out why and when LCM and VAM fall into pitfalls and produce worse IBFS. Numerically, we have also demonstrated why and when LCM and VAM encounter pitfalls to find IBFS. From these hypothetical as well as numerical analyses, it may be concluded that LCM may fall into a pitfall when:

(i) There exist some routes from (or to) any node containing the smallest TCs, and the DI value corresponding to this node is also small.
(ii) There exist some other routes from (or to) any (or more) node(s) that have large DI values with smaller TC but not the smallest.

Again, VAM may fall into a pitfall when:
The minimum value of TC (in the cost matrix/reduced cost matrix), is along the route from source O_i to a sink other than sink D_j and the largest DI value corresponds to a sink node D_j in which the smallest TC is relatively large enough compared to the overall minimum TC value and this smallest TC is along the route from O_i to D_j. Then VAM bounds to allocate to the sink node D_j from the source node O_i. In these circumstances, source node O_i is never able to allocate along the route in which TC is minimum. Similarly, in the reverse case (interchange of sink and source), the same situation will occur.

It is noted that when all DI values of the TT are similar, both LCM and VAM almost always produce similar IBFS. Since LCM is very easy to implement and computationally much cheaper than any other existing approaches (except the NWC approach, which produces a worse solution), researchers may exploit these phenomena to develop much better modified LCM algorithms in the future. On the other hand, VAM, in general, obtains improved IBFS, so by exploiting these phenomena, researchers may also develop much better modified VAM approaches in the future.

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