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SELF-GRAVITATIONAL SHOCK POTENTIAL IN DEGENERATE QUANTUM PLASMAS

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ABSTRACT

A rigorous theoretical investigation has been made on the propagation of nonlinear self-gravito-acoustic shock structures (SGASSs) in a quantum plasma system consisting of non-inertial degenerate non-relativistic electron species and inertial non-degenerate heavy nucleus species. The nonlinear behaviors for this self-gravitational perturbation (SGP) mode in planar geometry has been examined. The Burgers equation has been derived by employing the standard reductive perturbation technique. To analyze the Burgers equation numerically, the solution of the Burgers equation has been obtained for stationary localized condition. The dissipative force, which is effective on heavy elements, plays vital role for the formation of SGASSs in the plasma system under consideration. The non-relativistic limit has great effect on the basic properties (amplitude, width, etc.) of the SGASSs. The obtained results are applicable in white dwarfs and neutron stars which are the most common examples of astrophysical compact objects.

Keywords: *Shock structures, Nonlinearity, Self-gravitational potential, Dissipative force.*

1. INTRODUCTION

Classical plasmas are those in which the macroscopic dynamics of the plasma is not affected by the quantum characteristics of the plasma particles. More precisely, the plasmas which shows macroscopic behaviour and also abide by the classical mechanical laws are known as classical plasmas. Classical plasmas become quantum plasma when the particle number density increases or the temperature of the plasma decreases and in this case the quantum behaviour of the plasma particles plays important role for the macroscopic properties and dynamics. Degenerate matter, where quantum mechanical effects is the dominating factor, exist in the cores of dead stars. Degenerate matter is strictly followed by the Pauli's exclusion Principle, which gives the rule that two fermionic particles can not stay at the same energy (quantum) state. Degeneracy pressure arises during the contraction of a giant star because during reduction of volume, particles with same energy forces into the upper state to follow the Pauli's exclusion principle. Such plasmas (which contains degenerate matter) are known as degenerate quantum plasmas.

In astrophysical compact objects, matter exists under extreme conditions (Chandrasekhar, 1931; Chandrasekhar, 1931a; Chandrasekhar, 1935; Chandrasekhar, 1939; Chandrasekhar, 1964; Chandrasekhar and Tooper, 1964a). In such plasma systems, particle (e.g., electron) number density is very high. The dense stars like neutron star prevents gravitational shrinking due to the presence of degenerate pressure. The expression for degenerate pressure (Chandrasekhar, 1931; Chandrasekhar, 1931a; Chandrasekhar, 1935) for non-relativistic limit is given by $P_s = K_s n_s^{\gamma}$, where $\gamma = 5/3$ and $K_s = 3\pi\hbar^2/5m_s$; P_s represents degenerate pressure; n_s represents particle number density; K_s represents proportionality constant; $s = e$ for electron species.

Many authors (Shukla and Eliasson, 2006; Marklund and Brodin, 2007; Mahmood *et al.*, 2003; Haas, 2007; Michael *et al.,* 2007; Misra and Samanta, 2008; Misra *et al.*, 2010; Hossen and Mamun, 2015; Hossen and Mamun, 2014; Roy *et al.*, 2012; Mamun, 2017; Ema *et al.*, 2015; El-Taibany and Mamun, 2012; Hosen *et al.*, 2016) have studied the propagation of nonlinear waves in quantum plasma system. Marklund and Brodin (Marklund and Brodin, 2007) generates the expressions for spin- 1/2 electron plasmas. Mahmood *et al.* (Mahmood *et al.*, 2003) investigated the nonlinear propagation of ion acoustic wave in a homogeneous magnetized plasma. Michael *et al.* (Michael *et al.,* 2007) studied ion-acoustic waves in a five component plasma. Misra and Samanta (Misra and Samanta, 2008) observed the existence of small but finite amplitude quantum electron-acoustic double layers in a magnetized plasma system having two distinct groups of cold and hot electrons by using a quantum magnetohydrodynamic model. Hossen and Mamun (Hossen and Mamun, 2015) examined nonlinear structures in a dense degenerate plasma. Roy *et al.* (Roy *et al.*, 2012) have investigated the nonlinear propagation of electrostatic waves in a plasma system having cold ion fluid and ultra-relativistic degenerate electrons. Mamun (Mamun, 2017) studied nonlinear waves in a three component plasma.

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So far we know, a plasma system (which describes gravito-nucleus acoustic waves) having inertia-less non-relativistic degenerate electrons and inertial heavy element has not been considered by any authors. Actually, we have first disclosed the concept of gravito-nucleus acoustic wave (self-gravitational wave/perturbation), where we have introduced the self-gravitational potential in place of electrostatic potential for the first time (Asaduzzaman *et al.,* 2017). In our work, self-gravitational potential is very important because we have studied the self-gravitational perturbation mode of very large scale length and slow time scale in white dwarfs and neutron stars, which are relics of stars and have no thermonuclear burning and therefore thermal pressure is not present here (Chandrasekhar, 1931; Chandrasekhar, 1931a; Shukla and Eliasson, 2011) to support the gravitational load of their own mass. Since our work based on very long wavelength and low-frequency perturbation mode, all the interactions of the particles excluding interaction with the self-gravitational field of such compact stars have been neglected. We have done a series of work based on this model and the present work is also important because no authors have investigated the basic characteristic of shock potential by considering only the heavy element as the source of dissipation and the non-relativistic electron as the source of restoring force with the consideration of self-gravitational potential only. Therefore, in our present work, we have investigated the SGASSs by considering a plasma system consisting of heavy element and inertia-less non-relativistic degenerate electrons.

We have organized the paper as follows: Section 2 contains the governing equations. Section 3 represents the methodology. Section 4 includes the parametric investigation and results and finally section 5 represents the conclusion.

2. GOVERNING EQUATIONS

We consider the nonlinear propagation of SGASSs in a degenerate plasma containing non-relativistic electrons and inertial heavy element/nuclei. At equilibrium, $n_{e0} = Z_h n_{h0}$ (where n_{e0} is the electron number density and n_{h0} is the heavy nuclei number density).

The nonlinear propagation of SGASSs in the plasma system under consideration can be expressed mathematically by Equations (1)-(4) as:

$$\frac{\partial \psi}{\partial x} = -K\,\frac{\partial n_e^{\gamma-1}}{\partial x}, \tag{1}$$

$$\frac{\partial n_h}{\partial t} + \frac{\partial}{\partial x}(n_h u_h) = 0, \tag{2}$$

$$\frac{\partial u_h}{\partial t} + u_h\frac{\partial u_h}{\partial x} = -\frac{\partial \psi}{\partial x} + \eta\,\frac{\partial^2 u_h}{\partial x^2}, \tag{3}$$

$$\frac{\partial^2 \psi}{\partial x^2} = 4\pi G[m_h n_h^{/} + m_e n_e^{/}]. \tag{4}$$

In the above equations, the wave potential (self-gravitational potential) is denoted by ψ; the nucleus fluid speed is expressed by u_h; n_e represents electron number density; heavy nucleus number density is represented by n_h; the rest mass of electron and heavy nucleus are represented by m_e and m_h respectively; x represents space variable; t represents time variable; η represents kinematic viscosity; $K = \frac{K_e}{m_e}\left(\frac{\gamma}{\gamma-1}\right)$; $\gamma = \frac{5}{3}$; $K_e = 3\pi\hbar^2/5m_e$; G represents gravitational constant; $n_h^{/} = n_h - n_{h0}$; $n_e^{/} = n_e - n_{e0}$; $n_h^{/}$ and $n_e^{/}$ represents perturbed particles; n_h and n_e represents unperturbed particles.

In the plasma model under consideration, heavy element is the source of inertia and electron (degenerate) is the source of restoring force. We have not written the equation of continuity and equation of momentum for electron because we get the value of n_e directly from Eq. (1).

3. METHODOLOGY

To study the fundamental characteristics of shock potential, we need to analyse the solution of the Burgers equation which can be derived as follows:

We consider the stretched coordinates (Shukla and Yu, 1978; Washimi and Taniuti, 1966)

$$\left.\begin{aligned} \xi &= \epsilon(x - V_p t)\,, \\ \tau &= \epsilon^2 t\,, \end{aligned}\right\} \tag{5}$$

where ϵ measures the weakness of amplitude or dissipation ($0 < \epsilon < 1$); V_p represents propagation speed (phase velocity) of the SGASSs. We can expand n_h, u_h, and ψ in terms of ϵ as

$$\left.\begin{aligned} n_h &= n_{h0}+\epsilon n_h^{(1)}+\epsilon^2 n_h^{(2)}+\cdots\cdots, \\ u_h &= 0+\epsilon\, u_h^{(1)}+\epsilon^2\, u_h^{(2)}+\cdots\cdots, \\ \psi &= 0+\epsilon\psi^{(1)}+\epsilon^2\psi^{(2)}+\cdots\cdots. \end{aligned}\right\} \tag{6}$$

We can get various sets of equation by using Eqs. (5) and (6) into Eqs. (1) - (4). If we take the lowest order coefficient of ϵ, we obtain

$$u_h^{(1)} = \frac{\psi^{(1)}}{V_p}, \tag{7}$$

$$n_h^{(1)} = \frac{n_{h0}}{V_p^{\,2}}\,\psi^{(1)}, \tag{8}$$

$$V_p = \sqrt{\frac{\gamma-1}{\beta F}}, \tag{9}$$

where $F = \frac{1}{K n_{e0}^{\gamma-1}}$ and $\beta = \frac{m_e Z_h}{m_h}$.

Here, equation (7) is the momentum equation, Eq. (8) is the first order continuity equation, and Eq. (9) is the expression for phase speed.

For the next higher order coefficient of ϵ, we obtain a set of equations as

$$\frac{\partial n_h^{(1)}}{\partial\tau} - V_p\frac{\partial n_h^{(2)}}{\partial\xi} + \frac{\partial}{\partial\xi}\left[n_{h0}u_h^{(2)} + n_h^{(1)}u_h^{(1)}\right] = 0 \tag{10}$$

$$\frac{\partial u_h^{(1)}}{\partial\tau} - V_p\frac{\partial u_h^{(2)}}{\partial\xi} + u_h^{(1)}\frac{\partial u_h^{(1)}}{\partial\xi} + \frac{\partial\psi^{(2)}}{\partial\xi} - \eta\frac{\partial^2 u_h^{(1)}}{\partial\xi^2} = 0, \tag{11}$$

$$n_h^{(2)} - \beta n_{h0}\frac{F\psi^{(2)}}{(\gamma-1)} + \beta n_{h0}\frac{(2-\gamma)F^2}{2(\gamma-1)^2}[\psi^{(1)}]^2 = 0\,. \tag{12}$$

We deduce the Burgers equation by using Eqs. (7) – (12) and by performing some mathematical calculation, which is expressed as

$$\frac{\partial\psi^{(1)}}{\partial\tau} + A\psi^{(1)}\frac{\partial\psi^{(1)}}{\partial\xi} = C\frac{\partial^2\psi^{(1)}}{\partial\xi^2}, \tag{13}$$

where the nonlinear coefficient A and the dissipation coefficient C are

$$A = \left[\frac{3}{2V_p} + \frac{V_p(2-\gamma)F}{2(\gamma-1)}\right], \tag{14}$$

$$C = \frac{\eta}{2}. \tag{15}$$

Assuming $T = C\tau$ and $R = A/C$ we can write Eq. (13) as

$$\frac{\partial\psi^{(1)}}{\partial T} + R\psi^{(1)}\frac{\partial\psi^{(1)}}{\partial\xi} = \frac{\partial^2\psi^{(1)}}{\partial\xi^2}. \tag{15a}$$

4. PARAMETRIC INVESTIGATION AND RESULTS

We have considered a dissipative plasma system having inertia-less electrons (non-relativistically degenerate) and nondegenerate heavy nuclei/element and have also studied the nonlinear propagation of SGASSs in the considered plasma system.

In order to get the shock structure solution of Eq. (13), which is needed for the numerical analysis of the SGASSs, we transform ξ and τ to $\zeta = \xi - u_0\tau$ (u_0 is the constant velocity) and $\tau = \tau$. After using the conditions, viz., $\psi^{(1)} \to 0$, $\frac{d\psi^{(1)}}{d\zeta} \to 0$, at $\zeta \to \infty$, the shock structure solution of Eq. (13) with $\psi^{(1)} = \psi$ can be written as

$$\psi = \psi_0\left[1 - Tanh\left(\frac{\zeta}{\Delta}\right)\right], \tag{16}$$

where $\psi_0 = u_0/A$ and $\Delta = 2C/u_0$. (17)

Similarly, Eq. (15a) has the solution

$$\psi = \psi_m \left[1 - Tanh\left(\frac{\zeta}{\Delta_1}\right)\right], \quad (18)$$

with $\psi_m = u_0/R$ and $\Delta_1 = 2/u_0$. (19)

Equation (17) reveals that the amplitude ψ_0 of the SGASSs has linear relation with shock speed (u_0) and has inverse relation with nonlinear coefficient (A). The width Δ has inverse (linear) relation with u_0 (C). We have analyzed the solution of the Burgers equation for investigating the salient features of the SGASSs. Figs. 1-6 represents the results which we have found from the numerical calculations.

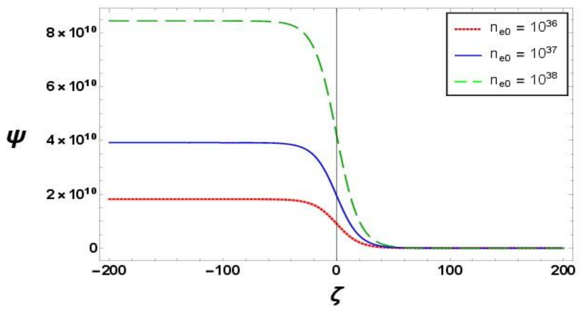

Figure 1: Showing the formation of self-gravito-acoustic shock structures for various n_{e0} with $\eta = 1$, where u_0 = 0.05 cm/sec, Z_h =79, and $m_h = 197m_p$ (m_p is the mass of proton).

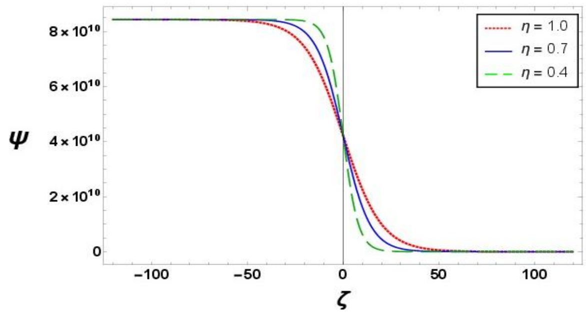

Figure 2: Showing the change of nonlinear profile with η for n_{e0}= 10^{38} cm^{-3}, u_0 = 0.05 cm/sec, Z_h =79, and $m_h = 197m_p$.

Equation (16) and (17) clearly discloses that the SGASSs exists when $C > 0$. Since U_0 has positive values, the SGASSs with positive potential exist for positive values of A and the SGASSs with negative potential exist for negative values of A. The nonlinear coefficient A has the domonant role for the potential of our plasma model. It is obvious from Fig. 1 that the SGASSs are formed due to the presence of nonlinearity and dissipation and the strength (amplitude) of the SGASSs increases as the particle (electron) number density increases, which is similar to the results of Asaduzzaman and Mamun (Asaduzzaman and Mamun, 2020), Islam *et al.* (Islam *et al.*, 2021) and Ema *et al.* (Ema *et al.*, 2015). It is also observed from Fig. 1 that the amplitude of the SGASSs increases largely for very large particle number density. .Fig. 2 clearly indicates that for positive values of self-gravitational potential (ψ), the shock strength associated with $A > 0$ has no variation with the kinematic viscosity but the steepness of the SGASSs changes largely with the kinematic viscosity. The steepness of the SGASSs increases when the kinematic viscosity decreases, that means the shock potential sharply falls to minimum value for lower kinematic viscosity. The result obtained from Fig. 2 has the similarity with the result of Abdelwahed *et al.* [Abdelwahed *et al.*, 2016], Hafez *et al.* [Hafez *et al.*, 2017], Asaduzzaman and Mamun [Asaduzzaman and Mamun, 2020] and Islam *et al.* [Islam *et al.*, 2021].

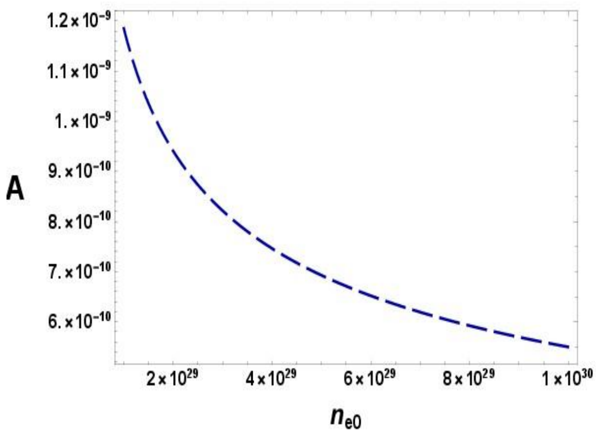

Figure 3: A versus n_{e0} curve for Z_h =79, $\eta = 1$ and $m_h = 197m_p$.

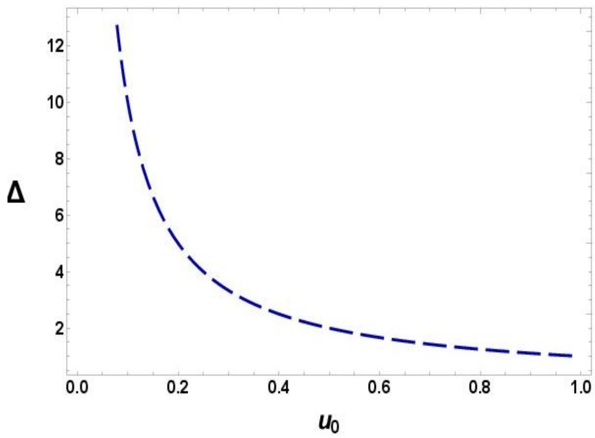

Figure 4: Width **Δ** versus $\boldsymbol{u_0}$ plot when $\boldsymbol{\eta = 1}$.

Fig. 3 shows the changing of nonlinear coefficient A with the number density of degenerate electron. It is clear from Fig. 3 that the nonlinear coefficient A decreases as the electron number density increases. The change of width Δ of the SGASSs with the speed of propagation (U_0) is shown in Fig. 4 and it is also obvious from Fig. 4 that U_0 has inverse effect on the width of the SGASSs.

Fig. 5 displays the variation of the shock potential found from Eq. (18) for $\mathbf{R > 1}$ **(A > C),** $\mathbf{R = 1}$ **(A = C)** and $\mathbf{R < 1}$ **(A < C).** Fig. 5 reveals that the strength of the shock potential decreases as R increases. We also observed from Fig. 5 that the shock potential increases when the dissipation coefficient C becomes larger than the nonlinear coefficient A, that means Fig. 5 clearly indicates that the dissipation coefficient C is responsible for the formation of the self-gravito-acoustic shock structures. The variation of phase velocity with the electron number density is displayed in Fig. 6, which discloses that phase velocity increases exponentially with the electron number density. Fig. 6 also indicates that the phase velocity becomes very large for very large particle number density and for a certain very large particle number density the phase velocity becomes maximum. After a certain very large particle number density the phase velocity becomes larger than the speed of light and which is not acceptable.

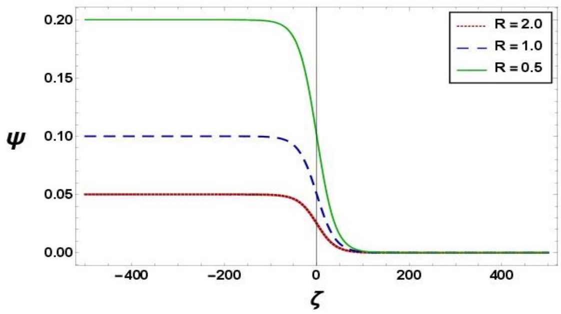

Figure 5: Formation of SGASSs for various R with $u_0 = 0.05$ cm/sec and $\eta = 1$.

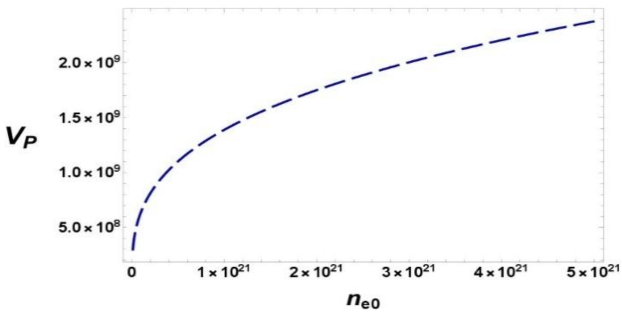

Figure 6: Phase velocity V_P versus electron number density (n_{e0}) curve with Z_h =79 and $m_h = 197m_p$.

5. CONCLUSIONS

We have studied the propagation of SGASSs associated with the self-gravitational potential in a degenerate quantum plasma system consisting of non-relativistically degenerate electrons and inertial heavy nuclei/element. We have obtained the solution of the Burgers equation and have also numerically analyzed to found the basic properties of SGASSs. The findings of our research work can be pinpointed as follows:

- the SGASSs have positive potential when the nonlinear coefficient has positive value (i.e., when $A > 0$).
- the strength of SGASSs varies linearly with the electron number density that means the amplitude (strength) decreases as the number of particles/cm^3 decreases.
- the amplitude of the SGASSs (i.e.,$\psi > 0$) remains unchanged with kinematic viscosity (η).
- The SGASSs become more abrupt when η decreases.
- The number density of electron has inverse relation with nonlinear coefficient.
- the width of the SGASSs and the shock speed are inversely related.
- when the ratio of nonlinear coefficient and dissipation coefficient increases (i.e., when R increases) then the amplitude of the SGASSs decreases.
- the phase velocity changes exponentially with the electron number density.

We finally stress that the results obtained from our current study should be helpful for understanding the fundamental characteristics of the self-gravitational potential structures in astrophysical compact objects like white dwarfs and neutron stars where matters exist under extreme conditions.

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