COMPUTATION OF FLOWS AROUND THE TRANSOM STERN HULL BY THE MODIFIED RANKINE SOURCE PANEL METHOD

Goutam Kumar Saha and **Md. Shahjada Tarafder***
Department of Naval Architecture and Marine Engineering
Bangladesh University of Engineering and Technology, Dhaka-1000
*Corresponding e-mail: mshahjadatarafder@name.buet.ac.bd

Abstract: *The paper presents a numerical method for calculating a potential flow around a ship of transom stern with respect to the double-body potential. The method of solution is based on the distribution of Rankine sources on the hull as well as its image and on the free surface. An iterative algorithm is used for determining the free surface and wave resistance using Dawson's upstream finite difference operator. A verification of numerical modeling is made using NPL- 4A model and the validity of the computer program is examined by identifying the transverse and diverging wave patterns of AMECRC model moving in infinite depth of water.*

***Keywords**: Wave resistance, wave pattern, Rankine source panel method, transom hull*

INTRODUCTION

A large number of vessels currently being used in Bangladesh in passenger service share the characteristics of possessing a cut-off or transom stern. This feature of the vessel defies simple hydrodynamic analysis because the extent and detailed shape of the flow behind the transom is unknown and must be a part of the mathematical solution to the problem. In principle, the pressure acting on the free surface behind the transom must be atmospheric and the flow must separate from the transom tangentially.

High speed displacement vessels are frequently used as patrol boats because of their speed capability combined with good sea-keeping characteristics. These vessels have a high length-beam ratio and a transom stern. A practical computational method for such transom stern vessels is of special interest to ship hydrodynamicists due to the peculiar property of the flow pattern. If the ship speed is high enough ($F_n \geq 0.4$), the transom clears the surrounding water and the entire transom area is exposed to the air. The transom flow detaches smoothly from the undesired of the transom and a depression is created on the free surfaced behind the transom.

Tulin and Hsu (1986) developed a theory for high-speed displacement ships with transom sterns. The flow was assumed to be smooth at the aft waterline and to have a trailing wake. The trailing wake resulted in substantial residuary resistance at high speeds for normal waterline ships. Cheng (1989) presented a practical computational method for 3-dimensional transom stern flows. The boundary condition for dry transom stern was derived within the framework of a free surface potential flow. The transom was treated as an inflow boundary and the transom boundary condition was used to specify the starting values of linearized free surface conditions.

Telste and Reed (1994) presented a method of calculating the flow near a transom stern ship moving forwarded at a moderate to high steady speed into otherwise undisturbed water. Modified free-stream linearization was used to obtain a Neumann-Kelvin boundary value problem in which the usual linearization about the main free surface level was replaced in the area behind the transom stern by linearization about a surface originating at the hull-transom intersection. A Rankine singularity integral equation was presented for obtaining the solution of the resulting mathematical boundary value problem.

Wang et al. (1995) calculated the wave resistance of a fast displacement hull with a transom stern using several different theoretical methods. The results of the calculations were compared with experimental data for a hull from the National Physical Laboratory (NPL) round-bilge hull series.

Sireli and Insel(1999) investigated the effects of transom stern on the wave resistance of high speed marine craft by using a series of five monohull forms by both experimental and numerical methods. The wave resistance of the hull was calculated by potential flow theory based on Dawson's algorithm and the viscous resistance by boundary layer calculations.

Doctors and Day (2000) used the linearized near field solution for the flow past a vessel with a transom stern. The hollow in the water behind the stern was represented by an extension to the usual centre-plane source distribution employed to model the hull itself. The resistance, sinkage and trim were computed by means of an integration of the resulting pressure distribution over the wetted surface of the vessel.

Millward et al. (2003) developed a numerical method to calculate the flow past light displacement ships hulls with a transom stern for Froude numbers ranging from 0.4 to approximately 1.0. The hull was allowed to trim and heave in calculations and the results of the calculations were compared with experimental data available for the Model 100A of NPL round bilge hull series.

The objective of the present research is to develop a numerical model for calculating the flow around the transom stern of a high speed vessel by the modified Rankine source panel method. The special boundary condition is applied to the transom and to the portion of the free surface downstream of the stern. The physical constraints imposed by this transom boundary condition require that the static pressure be atmospheric and that the flow leave tangentially at the transom. The computational method has been applied to NPL- 4A hull and AMECRC model.

MATHEMATICAL FORMULATION

Let us consider a ship moving in an infinite depth of water with a constant speed U in the direction of the negative x-axis as shown in Figure 1. The z-axis is vertically upwards and the y-axis extends to starboard. The origin of the co-ordinate system is located in an undisturbed free surface at amidships.

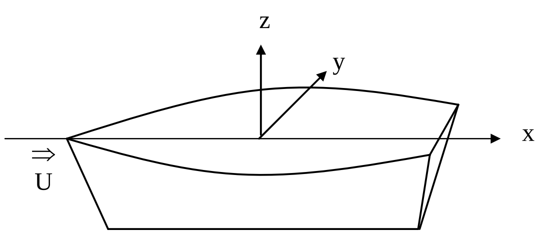


Figure 1. Definition sketch of co-ordinate system of ship hull with transom

The total velocity potential ϕ is the sum of the double body velocity potential Φ and the perturbed velocity potential φ representing the effect of free surface wave.

$$\phi = \Phi + \varphi \tag{2.1}$$

Now the problem for a ship can be constructed by specifying the Laplace equation

$$\nabla^2(\Phi + \varphi) = 0 \tag{2.2}$$

with the following boundary conditions:

(a) Hull boundary conditions: The normal velocity component on the hull surface must be zero.

$$\nabla(\Phi + \varphi)\cdot n = 0 \tag{2.3}$$

Where $n = n_x i + n_y j + n_z k$ represents a normal to the hull surface in the outward direction.

(b) Free surface condition: The velocity potential needs to satisfy the dynamic and the kinematic conditions on the free surface

$$g\zeta + \frac{1}{2}\nabla\phi\cdot\nabla\phi = \frac{1}{2}U^2 \text{ on } z = \zeta(x, y) \tag{2.4}$$

$$\varphi_x\zeta_x + \varphi_y\zeta_y - \varphi_z = 0 \quad on\ z = \zeta(x, y) \tag{2.5}$$

Eliminating ζ from equations (2.4) and (2.5)

$$\left.\begin{aligned} &\frac{1}{2}\varphi_x\left(\nabla\varphi\cdot\nabla\varphi\right)_x \\ &+\frac{1}{2}\varphi_y\left(\nabla\varphi\cdot\nabla\varphi\right)_y \\ &+g\varphi_z = 0 \end{aligned}\right\} on\ z = \zeta(x, y) \tag{2.6}$$

The free surface condition equation (2.6) is nonlinear in $\nabla\phi$ and should be satisfied on the free surface at z = ζ(x, y), which is unknown and can be linearized about the double body solution Φ by neglecting the non-linear terms of φ. After linearization the free surface boundary condition (2.6) can finally be expressed as

$$\left.\begin{aligned} &\Phi_l{}^2\phi_{ll} + 2\Phi_l\Phi_{ll}\phi_l + \\ &g\phi_z = -\Phi_l{}^2\Phi_{ll} \end{aligned}\right\} on\ z = 0 \tag{2.7}$$

where the subscript l denotes the differentiation along a streamline of double body potential Φ on the symmetry panel *z = 0*. The free surface boundary condition given by Eq. (2.7) involves the gradient of the velocity potential along a stream-wise direction designated by l and differentiation is carried out along the corresponding double body streamlines. Finally it is necessary to impose a radiation condition to ensure that the free surface waves vanish upstream of the disturbance.

The solution of the Laplace equation in connection with the boundary conditions Eqs. (2.3), Eqs. (2.7) and the radiation condition for the flow around the cruiser stern hull is given by Tarafder and Suzuki (2008).

NUMERICAL SOLUTION OF FREE SURFACE FLOW AROUND TRANSOM HULL BY RANKINE SOURCE PANEL METHOD

The transom stern solution begins with the cruiser stern solution. The free surface of the transom stern is considered to be consisted of two parts. The main section of the free surface is handled in the same way as for cruiser sterns. The special section of the free surface behind the transom is treated as an inflow boundary and the starting values of a free surface calculation are specified at the transom.

Implementation of boundary condition at the transom

Besides the boundary conditions given by Eqs. 2.3 to 2.6, the boundary condition which is to be satisfied along the intersection curve between the transom and the free surface is that the static pressure is atmospheric p_∞.

$$\left.\begin{aligned} &p_T = p_\infty \quad \text{at} \quad x = x_T \\ &\text{and} \\ &z = z_T \text{ for a given } y \end{aligned}\right\} \tag{3.1}$$

where, the subscript T denotes the transom and x_T and z_T the longitudinal and vertical co-ordinates of the transom. The effect of an air wake behind the transom is neglected and the atmospheric pressure is considered a global constant. A steady-state balance of kinetic and potential energy is obtained from Bernoulli's equation as:

$$\frac{1}{2}\left[\phi_x^2+\phi_y^2+\phi_z^2\right]+gz_T=\frac{1}{2}U_\infty^2 \tag{3.2}$$

The above equation can be rearranged as follows:

$$\frac{\phi_x^2+\phi_y^2+\phi_z^2}{U_\infty^2}=1-\frac{2g}{U_\infty^2}z_T \tag{3.3}$$

Thus the kinetic energy for the water at the transom which lies below the mean water level and takes on a negative value can be determined by the right hand side of Eq. (3.3). This equation is the first constraint for transom stern flows. Another constraint to be satisfied at the transom is the exit flow that must be tangential to the hull surface. The flow direction is a part of the free surface solution and is determined by the transom geometry which can specified by a local tangential unit vector:

$$\left.\begin{aligned}\tau_x&=\frac{\varphi_x}{\sqrt{\varphi_x^2+\varphi_y^2+\varphi_z^2}}\\ \tau_y&=\frac{\varphi_y}{\sqrt{\varphi_x^2+\varphi_y^2+\varphi_z^2}}\\ \tau_z&=\frac{\varphi_z}{\sqrt{\varphi_x^2+\varphi_y^2+\varphi_z^2}}\end{aligned}\right\} \tag{3.4}$$

A first approximation of the tangential unit vector is obtained by replacing the potential in Eq. (3.4) by the double model potential to give the tangential unit in the direction of the double model flow:

$$\left.\begin{aligned}\tau_x&=\frac{\Phi_x}{\sqrt{\Phi_x^2+\Phi_y^2+\Phi_z^2}}\\ \tau_y&=\frac{\Phi_y}{\sqrt{\Phi_x^2+\Phi_y^2+\Phi_z^2}}\\ \tau_z&=\frac{\Phi_z}{\sqrt{\Phi_x^2+\Phi_y^2+\Phi_z^2}}\end{aligned}\right\} \tag{3.5}$$

Where, the vector Φ_x, Φ_y and Φ_z represents the velocity for a hull panel whose centroid lies just forward of the sharp corner at the transom. Now the three velocity components at the transom in the x, y and z directions are approximated by:

$$\left.\begin{aligned}u_T&=U_\infty\sqrt{1-\frac{2g}{U_\infty^2}z_T}\quad\tau_x\\ v_T&=U_\infty\sqrt{1-\frac{2g}{U_\infty^2}z_T}\quad\tau_y\\ w_T&=U_\infty\sqrt{1-\frac{2g}{U_\infty^2}z_T}\quad\tau_z\end{aligned}\right\} \tag{3.6}$$

The quantity $\frac{U_\infty}{gz_T}$ represents the square of the Froude number based on the transom depth. Eq. (3.6) is applied as the transom boundary condition for the free surface calculations in case of a dry transom. In so doing, the static pressure is forced to be atmospheric and the free surface flow is forced to leave the transom tangentially in a direction specified by the double model flow.

Implementation of Free Surface Boundary Condition behind the Transom

For the case of a transom stern, the main section of the free surface is handled in the same way as for cruiser sterns. The free surface boundary condition in the special section of the free surface behind the transom uses the conventional linearization about a uniform flow:

$$\varphi_{xx}+\frac{g}{U^2}\varphi_z=0$$

$$u_x+\frac{g}{U^2}w=0 \quad \text{at} \qquad z=0 \tag{3.7}$$

Computations are performed at the centroid of the panel. To eliminate upstream propagating waves, a two-point upstream finite difference operator is used to start the computation for the transom stern flows:

$$u_{x,i}=CA_i\,u_i+CB_i\,u_t \tag{3.8}$$

where

$$CB_i=\frac{1}{x_t-x_i}=-CA_i$$

The index i denotes the foremost point behind the transom and the index t the upstream point at the transom. The quantity u_i is an unknown to be determined and the specified velocity at upstream point u_t is to be taken from the transom boundary condition Eq. (3.6). However, the total velocity u_T is the sum of the uniform velocity and the perturbation velocity u_t. Thus

$$u_t=U-u_T \tag{3.9}$$

Combining Eqs. (3.8), (3.9) and (3.7) we obtain

$$CA_i\,u_i+CB_i\,(U-u_T)+\frac{g}{U^2}w_i=0$$

$$CA_i\,u_i+\frac{g}{U^2}w_i=-CB_i\,(U-u_T) \tag{3.10}$$

For the downstream value at the same y value with index i+1, the above equation can be written as

$$CA_{i+1}\,u_{i+1}+CB_{i+1}u_i+\frac{g}{U^2}w_i=0 \tag{3.11}$$

The resulting system of equations is solved using the modified Rankine source panel method as given by

Tarafder and Suzuki (2008).

WAVE-MAKING RESISTANCE

The pressure on the hull surface can be calculated from the perturbation velocity potential by using a linearized version of the Bernoulli equation which is consistent with the linearized Dawson free-surface boundary condition:

$$\mathrm{p} + \rho \mathrm{g}\mathrm{z} + \frac{1}{2}\rho\nabla\varphi\cdot\nabla\varphi = \mathrm{p}_\infty + \frac{1}{2}\rho \mathrm{U}^2$$

$$p - p_\infty = \begin{bmatrix} \frac{1}{2}\rho U^2 - \rho g z - \\ \frac{1}{2}\rho\nabla(\Phi+\phi)\cdot\nabla(\Phi+\phi) \end{bmatrix}$$

$$p - p_\infty = \frac{1}{2}\rho\begin{bmatrix} U^2 - 2gz - \Phi_x^2 - \Phi_y^2 - \Phi_z^2 \\ -2\Phi_x\phi_x - 2\Phi_y\phi_y - 2\Phi_z\phi_z \end{bmatrix}$$

Now the pressure co-efficient

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2} = \frac{1}{U^2}\begin{bmatrix} U^2 - 2gz - \Phi_x^2 \\ -\Phi_y^2 - \Phi_z^2 - 2\Phi_x\phi_x \\ -2\Phi_y\phi_y - 2\Phi_z\phi_z \end{bmatrix}$$

Assuming that the pressure is constant within a hull surface panel, the wave resistance can be determined by

$$C_w = \frac{R_w}{1/2U^2L^2} = \frac{1}{L^2}\sum_{i=1}^{N_B/2} C_p(i)\, n_{xi}\, \Delta S_i \qquad (4.1)$$

where ΔS_i is the area of the hull surface panel and n_{xi} is the x-component of the unit normal on a surface panel. The wave profile can be obtained from Eq. (2.4) as

$$\zeta(x,y) = \frac{1}{2g}\begin{bmatrix} U^2 - \Phi_x^2 - \Phi_y^2 \\ -2\Phi_x\phi_x - 2\Phi_y\phi_y \end{bmatrix} \qquad (4.2)$$

RESULTS AND DISCUSSIONS

To investigate the effect of the transom stern on the wave resistance and wave pattern, the method has been applied first for NPL- 4A hull. Since the body is symmetric one-half of the computational domain is used for numerical treatment. The extent of free surface domain is 1.5 ship-lengths upstream to 6.5 ship-lengths downstream. The transverse extension of the free surface domain is about 2.0 ship-lengths. The NPL- 4A hull is discretized by 2×30×7 and is shown in Figure 2. The one-half of the free surface is discretized by 1515 panels (main domain: 70×18, domain behind stern: 51×5) and is given in Figure 3.

Figure 4 gives a side-view of the computed wave profiles behind the transom for NPL- 4A hulls at various Froude numbers. The vertical and horizontal axes are normalized by L and are plotted to the same scale. From the Figure 4 it can be observed that as the Froude number increases the bow wave grows and the first trough becomes deeper with the phase shifting slightly towards the bow. The transom clears the surrounding water and the assumption of a dry transom stern is valid.

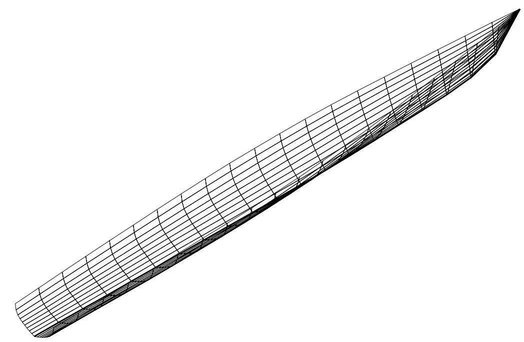


Figure 2. Discretization of NPL- 4A hull by 2×30×7 panels

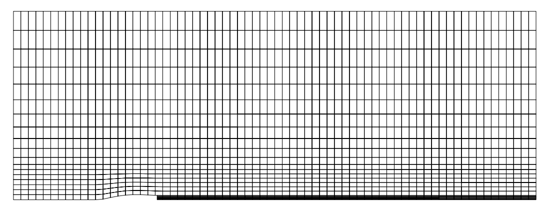


Figure 3. Discretization of one-half of free surface for NPL- 4A by 1515panels (main domain: 70×18, domain behind stern: 51×5)

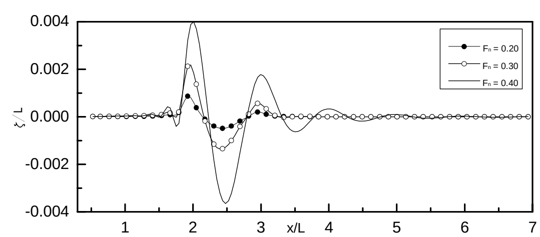


Figure 4. Wave profile at the central line behind the transom stern of NPL- 4A hull at various Froude numbers

The comparison of computed wave resistance is drawn in Figure 5 by taking 1515, 1695 and 1875 panelson one-half of the free surface respectively. The computations were performed for Froude numbers ranging from 0.2 to 1.0 at increment of 0.05. The hull form was held in fixed at the even keel position for this computation. The calculated wave resistance shows a general agreement in respect of hump and hollow but the significant difference is found especially in the high speed range for 1515 panels.

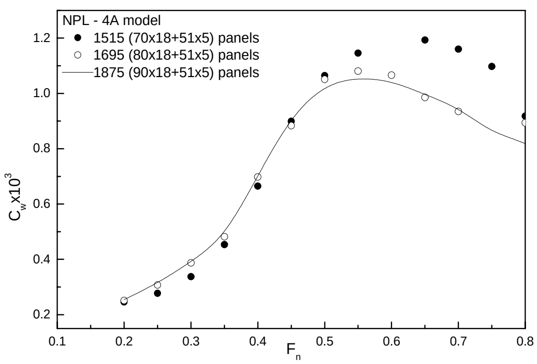


Figure 5. Comparison of calculated wave resistance for various numbers of panels on the free surface of NPL-4A hull

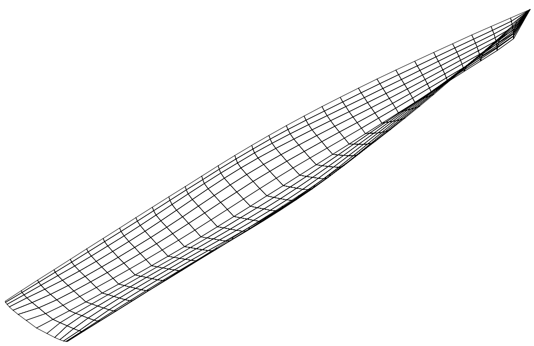


Figure 6. Discretization of AMECRC model by 2×30×7 panels

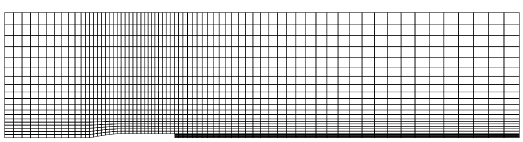


Figure 7. Discretization of one-half of free surface for AMECRC model (main domain: 70×19, domain behind stern: 51×5)

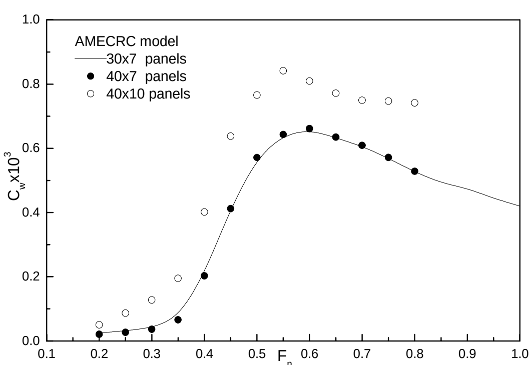


Figure 8. Wave-making resistance of AMECREC model for various numbers of panels on body surface

The discretization of AMECRC model and its corresponding free surface are given in Figure 6 and 7 respectively. The calculated wave-making resistance of is compared in Figure 8 by taking 30×7, 40×7 and 40×10 panels on one-half of AMECREC model and significant difference is found for 400 panels.

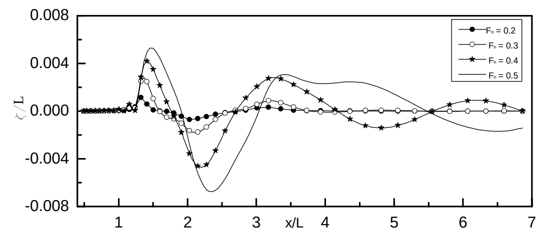
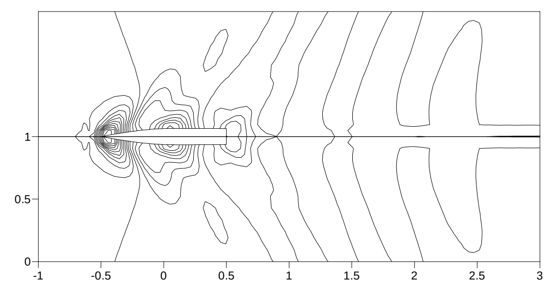
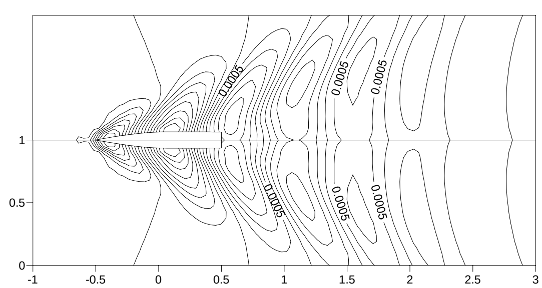


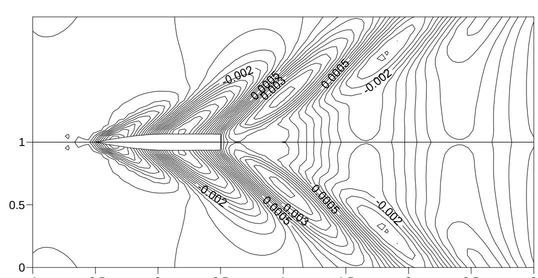
Figure 9. Wave profile at the central line behind the transom stern of AMECRC model at various Froude numbers.



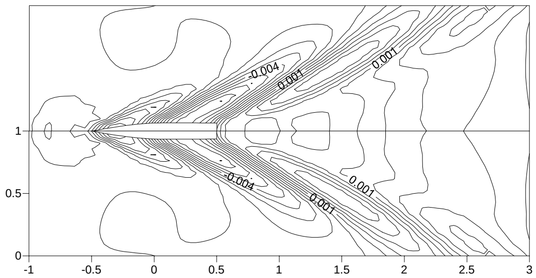
(a) Froude No. F_n = 0.2



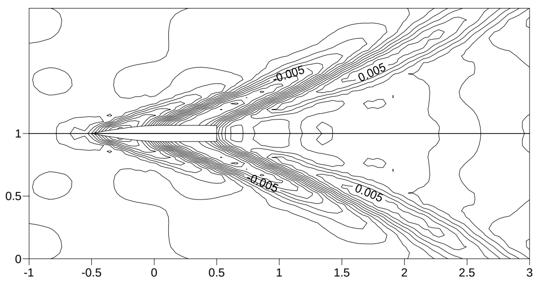
(b) Froude No. Fn = 0.4



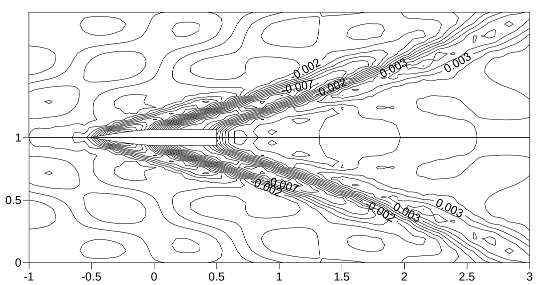
(c) Froude No. F_n = 0.5



(d) Froude No. $F_n = 0.6$



(e) Froude No. $F_n = 0.7$



(f) Froude No. $F_n = 0.8$

Figure 10. Wave pattern around the AMECRC model (a – f)

Figure 9 shows a side-view of the computed wave profiles behind the transom of AMECRC model at various Froude numbers. Figure 10 show the wave pattern around AMECRC hull at various speeds. As can be seen in these figures the wave pattern consists of transverse and diverging waves radiating from the bow and the distance between the successive transverse waves depends on the speed of the ship.

CONCLUSIONS

The paper presents the modified Rankine source panel method for calculating the flow of transom stern ship using double body linearization of the free surface boundary condition. The following conclusions can be drawn from the present numerical analysis:

- The present method could be an efficient tool for evaluating the flow field, wave pattern and wave resistance for practical ship forms.
- The trend of the calculated wave resistance curve is quite satisfactory. The wave pattern consists of transverse and diverging waves radiating from the bow and the distance between the successive transverse waves depends on the speed of the ship.
- The calculated results depend to a certain extent on the discretization of the hull and the free surface. Similar panel arrangement should therefore be used if relative merits of different competing ship designs are to be judged.

REFERENCES

[1] Tulin, M. P. and Hsu, C. C., 1986, “Theory of high-speed displacement ships with transom sterns”, The Journal of Ship Research, Vol. 30, No. 3, pp. 186-193.

[2] Cheng, B. H., 1989, “Computation of 3D transom stern flows”, 5th International Conference on Numerical Ship Hydrodynamics, pp. 581-591.

[3] Telste, J. G. and Reed, A. M., 1994, “Calculation of transom stern flows”, 6th International Conference on Numerical Ship Hydrodynamics, pp. 79-92.

[4] Wang, Y., Sproston, J. L. and Millward, A., 1995, “A theoretical determination of the wave resistance of a fast displacement hull with a transom stern, The Journal of Ship Research, Vol. 39, No. 2, pp.103-107.

[5] Sireli, E. M. and Insel, G. M., 1999, “The effects of transom stern on the resistance of high speed craft”, Naval Technical Congress, IMAM, Naples.

[6] Doctors, L. J. and Day, A. H., 2000, “The squat of a vessel with a transom stern”, 15th International Workshop on Water Waves and Floating Bodies (15 IWWWFB), Caesarea, Israel, pp. 40-43.

[7] Millward, A., Nicolaou, D. and Rigby S. G., 2003, “Numerical modeling of the water flow around a fast ship with a transom stern”, International Journal of Maritime Engineering, The Royal Institute of Naval Architects, pp. 21-34.

[8] Tarafder, M. S. & Suzuki, K., 2008, “Numerical Calculation of Free Surface Potential Flow around a Ship Using the Modified Rankine Source Panel Method”, Ocean Engineering, Elsevier Science, UK, Vol. 35, No. 5-6, pp. 536-544.

[9] Dawson, C. W., 1977, “A practical computer method for solving ship-wave problems”, Proceedings of Second International Conference on Numerical Ship Hydrodynamics, pp.30-38.