SORET EFFECTS DUE TO NATURAL CONVECTION BETWEEN HEATED INCLINED PLATES WITH MAGNETIC FIELD

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Abstract: *The effect of small uniform magnetic field on separation of a binary mixture for the case of a fully developed natural convection between two heated inclined plates has been investigated in this paper. Neglecting the induced electric field the equations governing the motion, temperature and concentration are solved by simple perturbation technique, in terms of dimensionless parameter measuring buoyancy force. The expressions for velocity, temperature and concentration are obtained. The effects of Hartmann number (M), thermal diffusion number (t_d), the buoyancy force parameter (N) and the inclination angle (ψ) of the plates with the horizontal are studied on the flow and heat transfer quantities.*

Keywords: *Natural convection, thermal diffusion, magnetic field, incompressible fluid, binary mixture.*

INTRODUCTION

Separation process of components of a fluid mixture, wherein one of the components is present in extremely small proportion, is of much interest due to their various applications in science and technology. Besides, environmental engineering applications involving heat and mass transfer constitute the backbone of many operations in chemical industry. Many authors cited a wide variety of these applications involving convective transport phenomena. Several others investigated the intricate nature of solution structure from a fundamental point of view in idealizing settings. Nield and Bejan[1] and Bejan and Kraus[2] performed detailed reviews of the subject including exhaustive lists of references.

Few studies are found when the porous medium is thermally stratified i.e. the ambient temperature is not uniform and it varies linearly in the stream-wise direction. This phenomenon has its applications in hot dike complexes, in volcanic region for heating ground water, development of advanced technologies for nuclear waste management, separation process in chemical engineering etc. Rees and Lage[3], Takhar and Pop[4] and Tewari and Singh[5] analytically analyzed free convection from a vertical plate immersed in a thermally stratified porous medium under boundary layer assumptions. On the other hand, Angirasa and Peterson[6] and Kumar and Singh[7] numerically investigated the natural convection process in a thermally stratified porous medium.

It is customary to consider one of the components of the binary mixture as solvent and the other component as solute. Groot and Mazur[8] showed that if separation due to thermal diffusion occurred, it might even render an unstable system to a stable one. This effect is also quite small, but devices can be arranged to produce very steep temperature gradients so that separations of mixtures are affected. Perhaps, Sharma and Anzew[9] studied the problem of baro-diffusion in a binary mixture of compressible viscous fluids set in motion due to an infinite disk rotation.

Nomenclature

A	Aspect ratio
$\overline{B}$	Magnetic flux
B_o	Applied magnetic field
c	Concentration function
c_o	Saturation concentration
c_1	Concentration of the rarer component
c_2	Concentration of the abundant component
C	Non dimensional concentration function
C_o	Non dimensional saturation concentration
C_1	Non dimensional rarer concentration
C_p	Specific heat
d	Half of the distance between the plates
D	Diffusion coefficient
g	Acceleration due to gravity
$\overline{J}$	Current density vector
k	Thermal conductivity of the fluid
M	Hartmann number
N	Constant which measures the buoyancy force
p	pressure
$\overline{q}$	Velocity vector
S_T	Soret coefficient
T	Fluid temperature
T_o	Ambient temperature
T_1	Uniform constant temperature at the wall
t_d	Thermal diffusion number
u	velocity
U_o	Perturbed velocity
U, V	Dimensionless x and y velocity components
V_1	Velocity of the rarer component
V_2	Velocity of the abundant component
x, y, z	Cartesian co-ordinates
X, Y	Dimensionless Cartesian co-ordinates

Greek Symbols

θ	Dimensionless temperature
θ_o	Perturbed temperature
ρ	Density of the binary fluid mixture
ρ_o	Reference density
ψ	Inclination angle of the plates with the horizonta
υ	Kinematic viscosity
σ	Electrical conductivity
ε	Perturbed velocity
μ	Dynamic viscosity
φ	Heat due to viscous dissipation
β	Thermal expansion coefficient
∇	Vector differential operator

They obtained results on separation action in this configuration for small baro-diffusion number while taking the Schmidt number to be the order unity and included the effect of separation at the disk. Hurle and Jakeman[10] discussed the effect of a temperature gradient on diffusion of a binary mixture. Other investigators[11-14] analyzed the effects of baro-diffusion and thermal diffusion on separation of a binary mixture in different geometry. In all these investigations, it has been found that an increase in the pressure gradient or the temperature gradient or both could enhance the separation process.

In many practical cases, the fluid mixture is found to be electrically conducting. Maleque and Alam[13] numerically studied free convection and mass transfer characteristics for a unsteady magneto hydrodynamic flow of an electrically conducting viscous incompressible fluid past an infinite vertical porous plate with Dufour and Soret effects. Alam *et al.*[14] numerically studied Dufour and Soret effects on combined free-forced convective and mass transfer flow past a semi infinite vertical flat plate under the influence of transversely applied magnetic field.In general, the thermal diffusion (Soret) effects are of a smaller order of magnitude than the effects described by Fourier's or Fick's laws and are often neglected in heat and mass transfer processes. However, the Soret effect, for instance, has been utilized for isotope separation and in mixtures between gases with very light molecular weight (H_2, He). Therefore, the main objective of this paper is to study the Soret effects due to natural convection between heated inclined parallel plates with magnetic field. In this work, a binary mixture of incompressible viscous thermally and electrically conducting fluids sheared between two inclined parallel plates in presence of a constant uniform transverse magnetic field has been considered. Using the expressions for velocity and temperature distribution as derived by Osterle and Young[12], the effect of magnetic field on the concentration distribution of the rarer component of a binary fluid mixture has been investigated.

GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

Let us consider the case when one of the components of the binary mixture of incompressible thermally and electrically conducting viscous fluids is present in small quantity between the inclined parallel plates as shown in Figure 1. Hence, the density and viscosity of the mixture is independent of the distribution of the components. The mass concentration is given by $c_2 = 1- c_1$. The flow problem of the binary mixture is identical to that of a single fluid but the velocity is to be understood as the mass average velocity $V = (\rho_1 V_1 + \rho_2 V_2) / \rho$ and the density $\rho = \rho_1 + \rho_2$, where the subscripts 1 and 2 denote the rarer and the more abundant components respectively. Under the above assumptions, the basic equations relevant to the problem are:

Continuity equation

$$\nabla \cdot \overline{q} = 0 \tag{1}$$

Momentum equation

$$\rho(\overline{q} \cdot \nabla)\overline{q} = -\nabla p + \mu \nabla^2 \overline{q} \sin\psi + \overline{J} \times \overline{B} \tag{2}$$

Energy equation

$$\rho C_p (\overline{q} \cdot \nabla) T = k \nabla^2 T + \mu\phi + \frac{\overline{J}^2}{\sigma} a \tag{3}$$

Concentration equation

$$(\overline{q} \cdot \nabla) C_1 = D\left[\nabla^2 C_1 + A\nabla \cdot (C_1 \nabla p) + S_T \nabla \cdot (C_1 \nabla T)\right] \tag{4}$$

Now, since the velocity distribution is symmetrical about x-axis, the corresponding boundary conditions are:

$$\overline{q} = 0 \text{ and } \frac{d\overline{q}}{dy} = \frac{dT}{dy} = 0 \text{ at } y = 0 \tag{5}$$

FORMULATION OF THE MATHEMATICAL MODEL

The binary fluid mixture of thermally and electrically conducting viscous incompressible fluids is sheared between two infinitely wide inclined plates at y = -d and y = d separated by distance 2d. The plates are maintained at

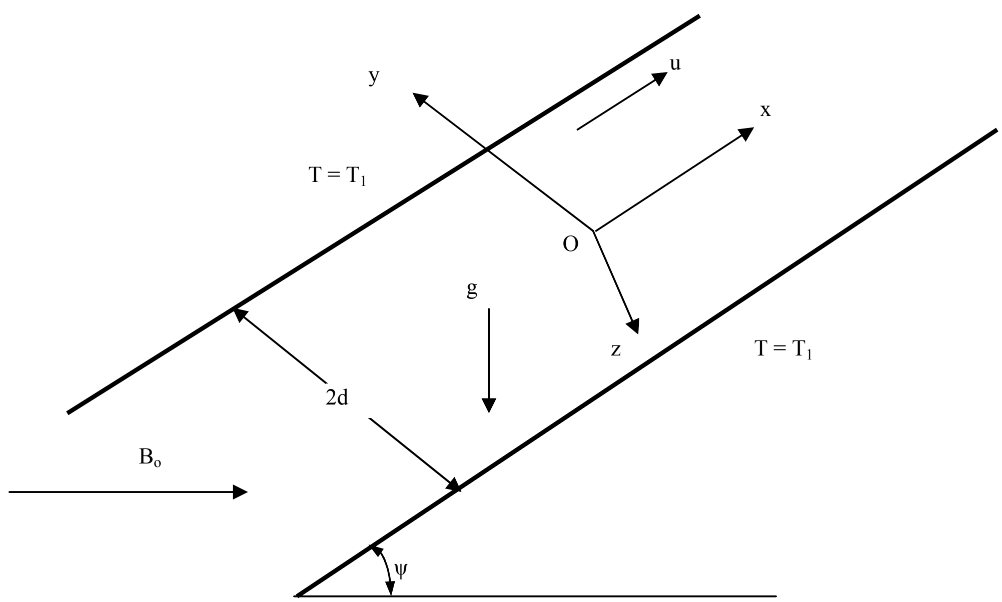

Figure 1: Physical Configuration and Coordinate Systems.

uniform temperature T_1 which exceeds the ambient temperature T_o ($T_o < T_1$). A transverse magnetic field of uniform strength is applied perpendicular to the plates. The flow of the fluid due to buoyancy force is in the direction parallel to the plates and is of magnitude 'u', hence, it is considered to be symmetric about the origin. The induced magnetic field is of the order of the product of magnetic Reynolds number and imposed magnetic field. As the fully developed natural convection flow of a fluid with very small electrical conductivity is considered here, it is the case of low magnetic Reynolds number and hence the induced magnetic field due to the weak applied magnetic field may be neglected. In fully developed flow, the pressure distribution must be hydrostatic, hence

$$\frac{\partial p}{\partial x} = -\rho_o g \tag{6}$$

In this case, the density (ρ) varies slightly from point to point because of the variation in temperature T and can be expressed as

$$\rho = \rho_o\left[1-\beta(T-T_o)\right] \tag{7}$$

which is well known Boussinesq approximation, where

$$\beta = -\frac{1}{\rho}\left(\frac{\partial p}{\partial T}\right)_\rho \tag{8}$$

For the above assumptions, the governing equations (2), (3) and (4) for the steady flow of a binary mixture of incompressible thermally and electrically conducting viscous fluids sheared between two electrically non conducting inclined parallel flat plates in the presence of uniform magnetic field become

$$\upsilon\frac{\partial^2 u}{\partial y^2} + \beta g(T-T_o)\sin\psi - \left(\frac{\sigma B_o^2}{\rho}\right)u = 0 \tag{9}$$

$$k\frac{\partial^2 T}{\partial y^2} + \rho\upsilon\left(\frac{\partial u}{\partial y}\right)^2 + \sigma B_o^2 u^2 = 0 \tag{10}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial C_1}{\partial y} + S_T C_1\frac{\partial T}{\partial y}\right) = 0 \tag{11}$$

The boundary conditions on velocity, temperature and concentration in terms of dimensionless quantities are

$U = 0$, $\theta = 1$ and $C = 1$ at $Y = 1$ and

$$\frac{dU}{dY} = \frac{d\theta}{dY} = \frac{dC}{dY} = 0 \text{ at } Y = 0 \tag{17}$$

SOLUTION OF THE PROBLEM

The solution of equations (14) and (15) under the boundary conditions (17) have been developed by Osterle and Young[12], by perturbing the velocity and temperature as

$$U = U_O + \phi N \text{ and } \theta = \theta_O + \varepsilon N \tag{18}$$

i. Zeroth order equations:

$$\frac{d^2U_o}{dY^2} + \theta_o\sin\psi - M^2U_o = 0 \tag{19}$$

$$\frac{d^2\phi}{dY^2} + \varepsilon\sin\psi - M^2\phi = 0 \tag{20}$$

ii. First order equations:

$$\frac{d^2\theta_o}{dY^2} = 0 \tag{21}$$

$$\frac{d^2\varepsilon}{dY^2} + M^2U_o^2 + \left(\frac{dU_o}{dY}\right)^2 = 0 \tag{22}$$

Corresponding boundary conditions are

$$U_o = 0,\ \phi = 0,\ \theta_o = 1,\ \varepsilon = 0 \text{ at } Y = 1 \tag{23}$$

and

$$\frac{dU_o}{dY} = \frac{d\phi}{dY} = \frac{d\theta_o}{dY} = \frac{d\varepsilon}{dY} = 0 \text{ at } Y = 0 \tag{24}$$

Solving Eqs. (19) to (22) under the boundary conditions (23) and (24), it is obtained that

$$U_o = \frac{\sin\psi}{M^2}(1-\mathrm{sech}M\times\cosh MY) \tag{25}$$

$$\theta_o = 1 \tag{26}$$

$$\varepsilon = \frac{\sin^2\psi}{M^2}\left[\frac{1}{2} - \frac{2}{M^2} - \frac{Y^2}{2} + \frac{\cosh(2M)}{4M^2\cosh^2 M} - \frac{\cosh(2MY)}{4M^2\cosh^2 M} + \frac{2\cosh MY}{M^2\cosh M}\right] \tag{27}$$

$$\phi = \frac{\sin^3\psi}{M^4}\left\{\left[\frac{3}{M^2\cosh M} + \frac{\sec hM\tanh M}{M} - \frac{\cosh 2M}{3M^2\cosh^3 M}\right]\cosh MY + \left(\frac{1}{2} - \frac{3}{M^2} - \frac{Y^2}{2} + \frac{\cosh 2M}{4M^2\cosh^2 M} - \frac{Y\sinh MY}{M\cosh M} + \frac{\cosh 2MY}{12M^2\cosh^2 M}\right)\right\} \tag{28}$$

The equation of continuity (1) suggests that

$$u = u(y) \tag{12}$$

Let is introduce the following non dimensional parameters

$$U = \frac{\upsilon u}{g\beta d^2(T_1-T_o)}, \theta = \frac{T-T_o}{T_1-T_o}, C_1 = C_o\times C, Y = \frac{y}{d} \text{ and },$$

$$M = B_o d\sqrt{\frac{\sigma}{\rho\upsilon}}, t_d = S_T(T_1-T_o), N = \frac{\rho g^2\beta^2 d^4(T_1-T_o)}{\upsilon k} \tag{13}$$

The non-dimensional form of equations (9) to (11) are

$$\frac{d^2U}{dY^2} + \theta\sin\psi - M^2U = 0 \tag{14}$$

$$\frac{d^2\theta}{dY^2} + M^2NU^2 + N\left(\frac{dU}{dY}\right)^2 = 0 \tag{15}$$

$$\frac{d}{dY}\left(\frac{dC}{dY} + t_d C\frac{d\theta}{dY}\right) = 0 \tag{16}$$

Using the above expressions for Eq. (16) under the boundary conditions (17) the concentration function

$$C = e^{t_d\ (1-\theta)} \tag{29}$$

is obtained. For non-magnetic case, the concentration for the rarer component of the binary mixture cannot be obtained directly from Eq. (29) because M is present as a factor in the denominator for the expression of θ. So by putting M = 0 in Eqs (14) to (16) and solving under the boundary conditions (17), the concentration function is obtained as follows:

$$C = e^{t_d N(1-Y^4)/12} \tag{30}$$

RESULTS AND DISCUSSION

Numerical computations have been carried out for different values of Hartmann number (M), thermal diffusion number (t_d), the buoyancy force parameter (N) and the inclination angle (ψ). With the above mentioned flow parameters, the results are displayed in Figures 2-10 in terms of the concentration, temperature and velocity profiles. As the problem is carried out for the case of higher viscous force in compare to inertia force, the values of M are taken very small which results low intensity of magnetic field in the fluid flow as well as heat and mass transfer characteristics. Besides the values of t_d is chosen arbitrarily.

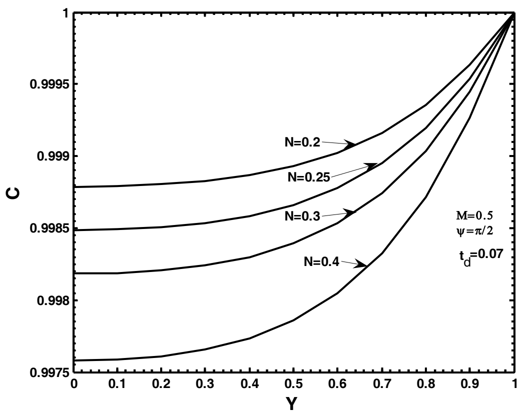

Figure 2: Concentration profiles with the variation of N

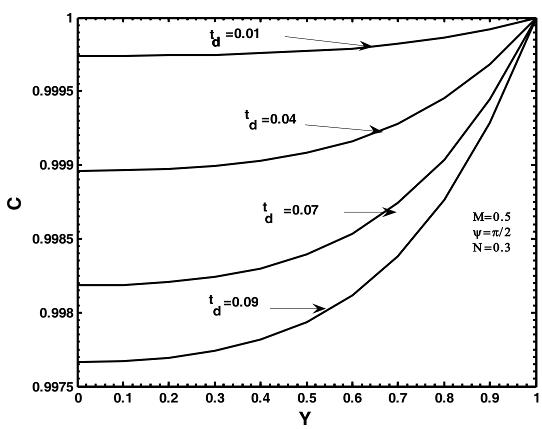

Figure 3: Concentration profiles with the variation of t_d

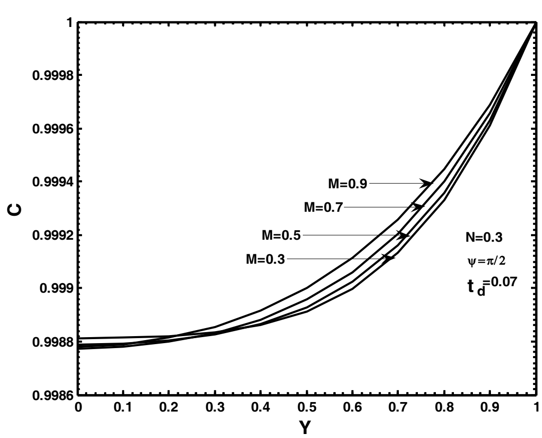

Figure 4: Concentration profiles with the variation of M

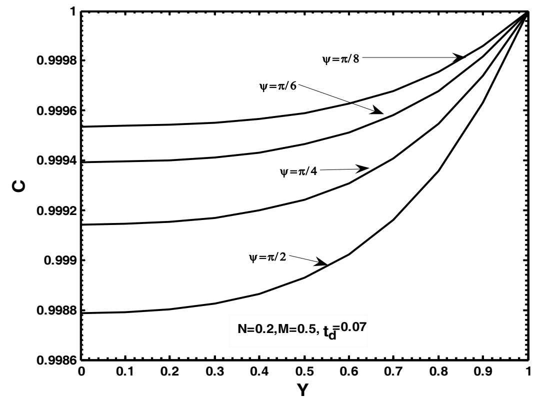

Figure 5: Concentration profiles with the variation of ψ

Figure 2 displays the concentration profiles for different values of N. From this figure, it is seen that rate of species separation decrease with the increase of N for fixed values of M = 0.5, t_d = 0.07 and $\psi = \pi/2$. Similar effect has been observed in Figure 3 for different values of t_d and fixed values of M = 0.5, N = 0.3, and $\psi = \pi/2$. This suggests that the rate of separation of species can be enhanced by decreasing the temperature difference between the plates. In Figure 4, concentration profiles are shown for various values of M with fixed values of N = 0.3, $\psi = \pi/2$ and t_d = 0.07. Interestingly, it is found in this figure that the rate of species separation enhances by the increase of magnetic field near the plates but reverse action takes place in the central part of the plates. As the magnetic field retards the fluid flow, the rarer and lighter components of the binary mixture increase in the central part of the plates with the decreasing strength of the magnetic field. Moreover, variations in concentration profiles with ψ are shown in Figure 5. From this figure, it is noticed that the concentration decreases with the increase of ψ.

In Figure 6, temperature profiles are displayed for different values of N with fixed values of M = 0.5, $\psi = \pi/2$ and t_d = 0.07. From this figure, it is observed that the increase of N leads to the increase of temperature indicating that the rate of separation of species decreases with the increase of N. In Figure 7, the effects of magnetic

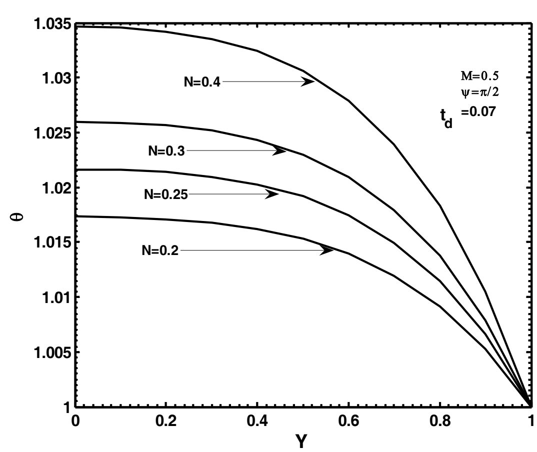

Figure 6: Temperature profiles with the variation of N

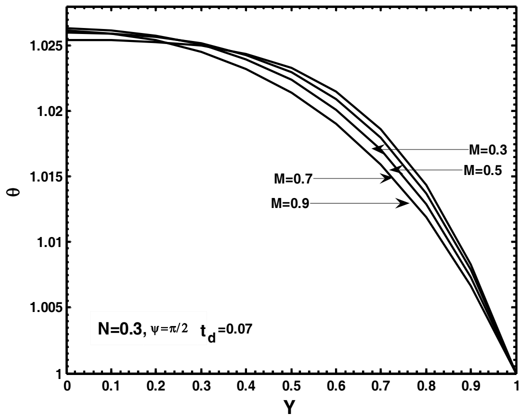

Figure 7: Temperature profiles with the variation of M

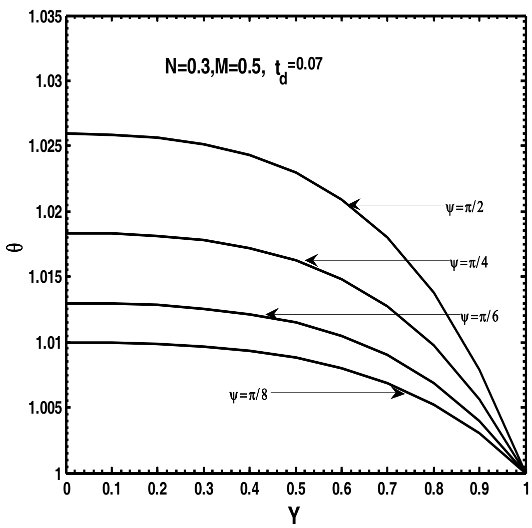

Figure 8: Temperature profiles with the variation of ψ

field parameter on temperature are shown. From this figure, it is found that temperature decreases with the increase of M near the plates, but reverse action takes place at the centre. For $\psi = \pi/2$, the channel becomes vertical and the walls are maintained at constant temperature, hence maximum temperature is found at the center for higher M. In Figure 8, variations of temperature are shown with the variation of ψ. From this figure, it is achieved that temperature increases with the increase of ψ.

Finally, the influences of M and ψ on velocity are shown in Figures 9 and 10 respectively. It is seen in Figure 9 that velocity decreases as M increases whereas velocity increases with the increase of ψ for fixed values of N = 0.3, M = 0.5 and td = 0.07 as shown in Figure 10.

CONCLUSION

In this paper, Soret effects due to natural convection between heated inclined plates under the influence of transverse magnetic field has been studied numerically on separation of a binary mixture. Neglecting the induced magnetic field, the equations governing the motion, temperature and concentration are solved by simple perturbation technique, in terms of dimensionless parameter measuring the buoyancy force. From the present study, the following conclusions are summarized:

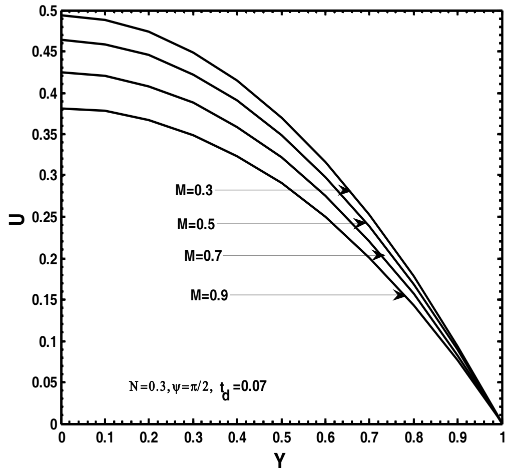

Figure 9: Velocity profiles with the variation of M

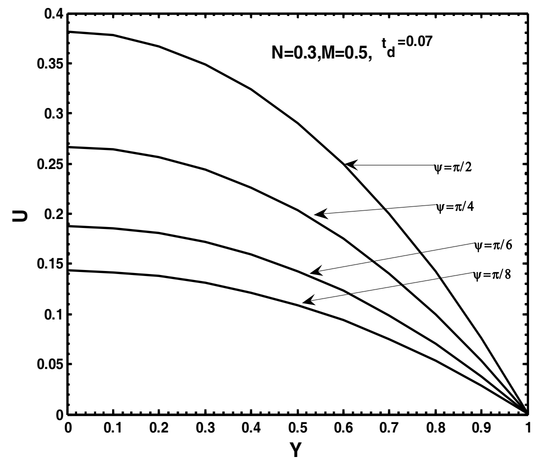

Figure 10: Velocity profiles with the variation of ψ

i) The temperature gradient and the buoyancy force have the reverse effect on the separation of the binary fluid mixture.
ii) The rarer and lighter component of the binary mixture increases in the central part of the plates with the decrease in the strength of the magnetic field, but a reverse action takes place near the plates.
iii) Separation of the binary fluid mixture increases with the increase of the angle ψ that the plates make with the horizontal.

In an engineering application, harmful gases can be separated through gas separation process. Hence, this work may help on separating two gases, especially at huge plants in Oil and Natural Gas Corporation of India (ONGC) etc.

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