EFFECTS OF THERMAL CONDUCTIVITY OF FLUID ON FREE CONVECTION FLOW ALONG A VERTICAL FLAT PLATE WITH TRANSVERSE CONDUCTION

A K M Safiqul Islam[1*], **M A Alim**[2], **Md Rezaul Karim**[3] and **ATM M Rahman**[4]
[1]Department of Mathematics, Gowripur Govt. College, Mymensingh, Bangladesh.
[2]Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh.
[3]Department of Mathematics, Jagannath University, Dhaka-1100, Bangladesh
[4]Department of Computer Science and Engineering, Dhaka International University, Banani, Dhaka-1213, Bangladesh.
[*]Corresponding e-mail: asafiqis@gmail.com, safiqis@yahoo.com

Abstract: *This paper reports the effects of transverse conduction variation with thermal conductivity on free convection flow along a vertical flat plate. The governing equations with associated boundary conditions reduce to local non-similarity boundary layer equations for this phenomenon are converted to dimensionless forms using a suitable transformation. The transformed non-linear equations are then solved using the implicit finite difference method together with Keller-box technique. Numerical results of the velocity and temperature profiles, skin friction and surface temperature profiles for different values of the thermal conductivity parameter, the Prandtl number and the transverse conduction variation parameters are presented graphically. Detailed discussion is given for the effect of the aforementioned parameters. Mentionable effect is found in skin friction and surface temperature for the transverse conduction variation parameter.*

Keywords: *thermal conductivity, transverse conduction variation, Grashof number, Prandtl number.*

NOMENCLATURE

C_{fx}	Local skin friction coefficient
C_p	Specific heat at constant pressure
f	Dimensionless stream function
g	Acceleration due to gravity
h	Dimensionless temperature
T	Temperature of the interface
T_b	Temperature at outside surface of the plate
T_f	Temperature of the fluid
T_∞	Temperature of the ambient fluid
$\bar{u}$	Velocity component in x- direction
$\bar{v}$	Velocity component in y- direction
u	Dimensionless velocity component in x- direction
v	Dimensionless velocity component in y- direction
$\bar{x}$	Cartesian co-ordinates
$\bar{y}$	Cartesian co-ordinates
x	Dimensionless Cartesian co-ordinates
y	Dimensionless Cartesian co-ordinates

GREEK SYMBOLS

γ	Thermal conductivity of the fluid
λ	Transverse conduction variation of plate
∇	Vector differential operator
η	Similarity variable
κ_∞	Thermal conductivity of the ambient fluid
κ_s	Thermal conductivity of the solid
κ_f	Thermal conductivity of the fluid
	Viscosity of the fluid
μ	Kinematic viscosity
ν	Density of the fluid inside the boundary layer
ρ	Shearing stress
τ_w	
Ψ	Stream function

INTRODUCTION

Effects of thermal conductivity of fluid and transverse conduction variation problems on free convection flow are important from the technical point of view and such types of problems have received much attention by many researchers in electronics and in physics broadly.

Miyamoto et al.[1] studied the effect of axial heat conduction in a vertical flat plate on free convection heat transfer. Pozzi and Lupo[2] investigated the coupling of conduction with laminar convection along a flat plate. Khan[3] analyzed the conjugate effect of conduction and convection with natural convection flow from a vertical flat plate and in an inclined square cavity. Mamun[4] studied the effects of conduction and convection on magnetohydrodynamic flow with and without viscous dissipation from a vertical flat plate. Rahman et al.[5] investigated the effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat conduction. Nasrin and Alim[6] studied the combined effects of viscous dissipation and temperature dependent thermal conductivity on MHD free convection flow with conduction and joule heating along a vertical flat plate. Alim et al.[7] analyzed the combined effect of viscous dissipation & joule heating on the coupling of conduction & free convection along a vertical flat plate. Alim et al.[8] investigated Joule heating effect on the coupling of conduction with MHD free convection flow from a vertical flat plate. Chowdhury and Islam[13] analyzed MHD Free Convection Flow of Visco-elastic Fluid past an Infinite Porous Plate. Alam et al.[14] studied viscous dissipation effects on MHD natural convection flow over a sphere in the presence of heat generation.

The present study is to incorporate the idea of the effects of thermal conductivity of fluid on natural convective boundary layer flow along a vertical flat plate with transverse conduction variation.

MATHEMATICAL FORMULATION

At first we consider a steady two-dimensional laminar natural convection flow of a viscous and incompressible fluid along a vertical insulated top and bottom tip flat plate of length l and thickness b (Figure-1). It is assumed that the temperature at the outer surface of the plate is maintained at a constant temperature T_b, where $T_b > T_\infty$, the ambient temperature of the fluid. In this work $\bar{y}$ -axis i.e. normal direction to the surface and $\bar{x}$ -axis is taken along the flat plate. The coordinate system and the configuration are shown in Fig. 1.

The governing equations of such laminar flow with thermal conductivity and also transverse conduction variation along a vertical flat plate under the Boussinesq approximations $\rho = \rho_\infty[1-\beta(T_b - T_\infty)]$, where ρ_∞ and T_∞ are the density and temperature respectively outside the boundary layer. For the present problem for continuity, momentum and energy equations take the following forms

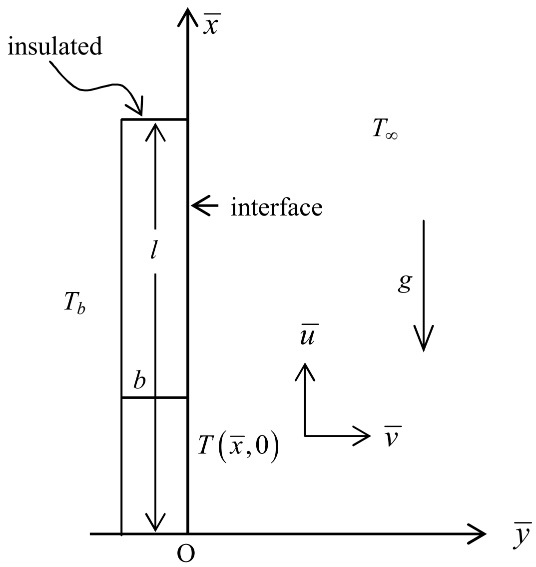

Figure 1. Physical model and coordinate system.

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \qquad (1)$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T_f - T_\infty) \qquad (2)$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}} + \bar{v}\frac{\partial T_f}{\partial \bar{y}} = \frac{1}{\rho\ C_p}\frac{\partial}{\partial \bar{y}}(\kappa_f \frac{\partial T_f}{\partial \bar{y}}) \qquad (3)$$

Here β is coefficient of volume expansion. The temperature dependent thermal conductivity, which is used by Rahman[5] as follows

$$\kappa_f = \kappa_\infty[1 + \delta(T_f - T_\infty)] \qquad (4)$$

Where κ_∞ is the thermal conductivity of the ambient fluid and δ is a constant, defined as

$$\delta = \frac{1}{\kappa_f}\left(\frac{\partial \kappa}{\partial T}\right)_f .$$

The appropriate boundary condition to be satisfied by the above equations are

$$\left.\begin{aligned} &\bar{u}=0,\ \bar{v}=0 \\ &T_f = T(\bar{x},0),\ \frac{\partial T_f}{\partial \bar{y}} = \frac{\kappa_s}{b\kappa_f}(T_f - T_b) \end{aligned}\right\} on\ \bar{y}=0,\ \bar{x}>0 \qquad (5)$$
$$\bar{u}\to 0,\ T_f \to T_\infty\ \ as\ \bar{y}\to\infty,\ \bar{x}>0$$

The non-dimensional governing equations and boundary conditions can be obtained from equations (1) - (3) using the following dimensionless quantities

$$x = \frac{\bar{x}}{l},\ y = \frac{\bar{y}}{l}Gr^{\frac{1}{4}},\ u = \frac{\bar{u}l}{\upsilon}Gr^{-\frac{1}{2}},\ v = \frac{\bar{v}l}{\upsilon}Gr^{-\frac{1}{4}}, \qquad (6)$$
$$\theta = \frac{T_f - T_\infty}{T_b - T_\infty},\ Gr = \frac{g\beta l^3(T_b - T_\infty)}{\upsilon^2}$$

where l is the length of the plate, Gr is the Grashof number, θ is the dimensionless temperature.

Now from equations (1)-(3), we get using the following dimensionless equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (7)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta \qquad (8)$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}(1+\gamma\theta)\frac{\partial^2 \theta}{\partial y^2} + \frac{\gamma}{\Pr}\left(\frac{\partial \theta}{\partial y}\right)^2 \qquad (9)$$

where $\Pr = \frac{\mu\ C_p}{\kappa_\infty}$ is the Prandtl number, $\gamma = \delta(T_b - T_\infty)$ is the non-dimensional thermal conductivity variation parameter for fluid. The corresponding boundary conditions (5) then take the following form

$$u=0, v=0, \theta-1=(1+\gamma\theta)\lambda\frac{\partial\theta}{\partial y} \quad on \quad y=0,\ x>0 \tag{10}$$
$$u\to 0,\ \theta\to 0 \quad as \quad y\to\infty,\ x>0$$

here $\lambda=\left(\frac{\kappa_\infty b}{\kappa_s l}\right)Gr^{\frac{1}{4}}$ is the transverse conduction variation parameter. The described problem is governed by this parameter λ.

To solve the equations (8) and (9) subject to the boundary conditions (10) the following transformations are proposed by Merkin & Pop[9]

$$\psi = x^{\frac{4}{5}}(1+x)^{-\frac{1}{20}}f(x,\eta)$$
$$\eta = y\ x^{-\frac{1}{5}}(1+x)^{-\frac{1}{20}} \tag{11}$$
$$\theta = x^{\frac{1}{5}}(1+x)^{-\frac{1}{5}}h\ \ (x,\eta)$$

here η is the similarity variable and ψ is the non-dimensional stream function which satisfies the continuity equation and is related to the velocity components in the usual way as

$u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$.

Moreover, $h\ (x,\eta)$ represents the non-dimensional temperature. The momentum and energy equations are transformed for the new co-ordinate system. At first, the velocity components are expressed in terms of the new variables for this transformation. Thus the following equations

$$f''' + \frac{16+15x}{20(1+x)}ff'' - \frac{6+5x}{10(1+x)}f'^2 + h = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{12}$$

$$\frac{1}{\Pr}h'' + \frac{\gamma}{\Pr}\left(\frac{x}{1+x}\right)^{\frac{1}{5}}hh'' + \frac{\gamma}{\Pr}\left(\frac{x}{1+x}\right)^{\frac{1}{5}}h'^2 + \frac{16+15x}{20(1+x)}fh' - \frac{1}{5(1+x)}f'h = x\left(f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}\right) \tag{13}$$

where prime denotes partial differentiation with respect to η. The boundary conditions as mentioned in equation (10) then take the following form

$$f(x,0) = f'(x,0) = 0$$
$$h'(x,0) = \frac{x^{\frac{1}{5}}(1+x)^{-\frac{1}{5}}h(x,0)-1}{\lambda(1+x)^{-\frac{1}{4}} + \gamma x^{\frac{1}{5}}(1+x)^{-\frac{9}{20}}h(x,0)} \tag{14}$$
$$f'(x,\infty)\to 0,\quad h(x,\infty)\to 0$$

From the process of numerical computation, in practical point of view, it is important to calculate the values of the surface shear stress in terms of the skin friction coefficient. This can be written in the non-dimensional form as Molla et al.[10]

$$C_f = \frac{Gr^{-\frac{3}{4}}l^2}{\mu\upsilon}\tau_w \tag{15}$$

where $\tau_w[=\mu(\partial\ \bar{u}/\partial\ \bar{y})_{\bar{y}=0}]$ is the shearing stress. Using the new variables described in (6), the local skin friction can be written as

$$C_{fx} = x^{\frac{2}{5}}(1+x)^{-\frac{3}{20}}f''(x,0) \tag{16}$$

In practical point of view, it is important to calculate the values of the surface temperature. The numerical values of the surface temperature are obtained from the relation. This can written in the non-dimensional form as

$$\theta(x,0) = x^{\frac{1}{5}}(1+x)^{-\frac{1}{5}}h(x,0) \tag{17}$$

ACCURACY TEST OF THE RESULT

Table 1 and Table 2 depict the comparisons of the present numerical results of the skin friction C_{fx} and the surface temperature $\theta(x,0)$ with those obtained by Pozzi & Lopo[2] and Merkin & Pop[9] respectively. Here, the thermal conductivity and transverse conduction variation parameters are ignored (i.e. γ = 0 & λ = 0) and the Prandtl number Pr = 0.733 with $x^{\frac{1}{5}}=\xi$ is chosen. It is clearly seen that there is an excellent agreement among the present results with the solutions Pozzi & Lopo[2] and Merkin & Pop[9].

Table 1. Comparison of the skin friction C_{fx} with Prandtl number Pr = 0.733, γ = 0 and λ = 0.

C_{fx}			
$x^{\frac{1}{5}}=\xi$	Pozzi & Lupo[2]	Merkin & Pop[9]	Present work
0.1	0.014	0.014	0.015
0.4	0.172	0.172	0.170
0.5	0.250	0.250	0.272
0.6	0.337	0.337	0.340
0.7	0.430	0.430	0.423
0.8	0.530	0.530	0.528
0.9	0.635	0.635	0.633
1.0	0.741	0.745	0.748
1.2	0.817	0.972	0.972

NUMERICAL METHOD OF SOLUTION

This paper investigates the effects of thermal conductivity of fluid on natural convective boundary layer flow along a vertical flat plate with transverse conduction variation. The set of equations (12) and (13) together with the boundary conditions (14) are solved by applying implicit finite difference method with Keller-box elimination technique[11], which is well documented by Cebeci and Bradshaw [12].

Table 2. Comparison of the surface temperature $\theta(x,0)$ with Prandtl number $Pr = 0.733$, $\gamma = 0$and $\lambda = 0$.

$\theta(x,0)$			
$x^{\frac{1}{5}} = \xi$	Pozzi & Lupo[2]	Merkin & Pop[9]	Present work
0.1	0.177	0.177	0.204
0.4	0.493	0.493	0.481
0.5	0.557	0.557	0.580
0.6	0.608	0.608	0.615
0.7	0.651	0.651	0.651
0.8	0.684	0.686	0.687
0.9	0.708	0.715	0.716
1.0	0.717	0.741	0.742
1.2	0.640	0.781	0.781

RESULTS AND DISCUSSION

The main objective of the present study is to analyze the effect of transverse conduction variation due to temperature on free convective flow along a vertical flat plate with thermal conductivity. In this simulation the values of the Prandtl number *Pr* are considered to be 0.733, 1.00, 1.50, 2.00 and 2.50 that corresponds to hydrogen, steam, water, methyl chloride and sulfur dioxide respectively.

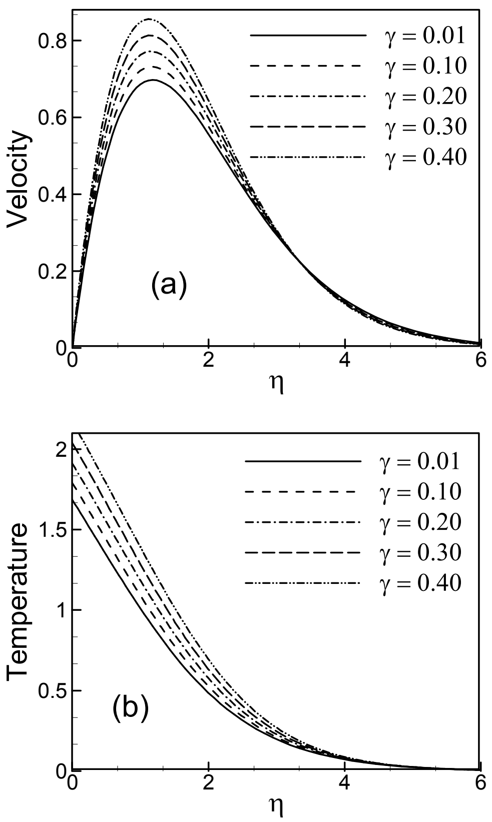

Figure 2. (a) Velocity and (b) Temperature profiles for different values of γ with $Pr = 0.733$ and $\lambda = 1.0$.

The velocity and the temperature profiles obtained from the solutions of equations (12) and (13) are depicted in Figures 2 to 4. Also the local skin friction and the surface temperature obtained from the solutions of equations (16) and (17) are depicted in Figs 5 to 7. Numerical computation are carried out for a range of thermal conductivity variation parameter γ = 0.01, 0.10, 0.20, 0.30, 0.40 and transverse conduction variation parameter λ = 0.30, 0.50, 0.70, 1.00, 1.30.

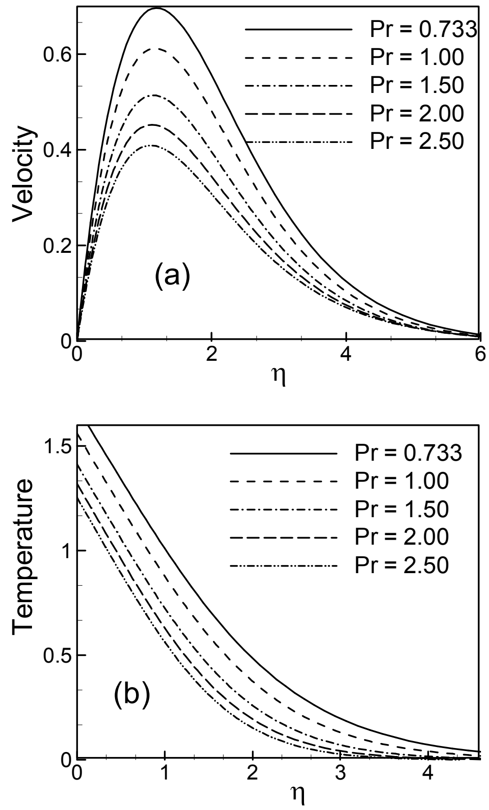

Figure 3. (a) Velocity and (b) Temperature profiles for different values of *Pr* with $\gamma = 0.01$ and $\lambda = 0.3$.

The effect of thermal conductivity variation parameter γ for fluid on the velocity and temperature profiles against η within the boundary layer with Pr = 0.733 and λ = 1.00 are shown in Fig. 2(a) and 2(b), respectively. It is seen from Fig. 2(a) and 2(b) that the velocity and temperature increase within the boundary layer with the increasing values ofγ. It means that the velocity boundary layer and the thermal boundary layer thickness expand for large values ofγ.

Fig. 3(a) and 3(b) illustrate the velocity and temperature profiles against η for different values of Prandtl number *Pr* with γ= 0.01 and λ = 0.30. From Fig. 3(a), it can be observed that the velocity decreases as well as its position moves toward the interface with the increasing *Pr*. From Fig. 3(b), it is seen that the temperature profiles shift downward with the increasing values of *Pr*.

In Figs 4(a) and 4(b) describe the velocity and temperature profiles against η for different values of transverse conduction variation parameter λ with γ = 0.01 and Pr = 0.733. From Fig. 4(a), it can be observed that the velocity increases as well as its position moves outward the interface with the increasing values ofλ. From Fig. 4(b), it is seen that the temperature profiles also the same as increasing within the boundary layer. It means that the velocity boundary layer and the thermal boundary layer thickness increase for large values of λ from 0.30 to 1.30. There is an inverse trend visible in velocity profiles.

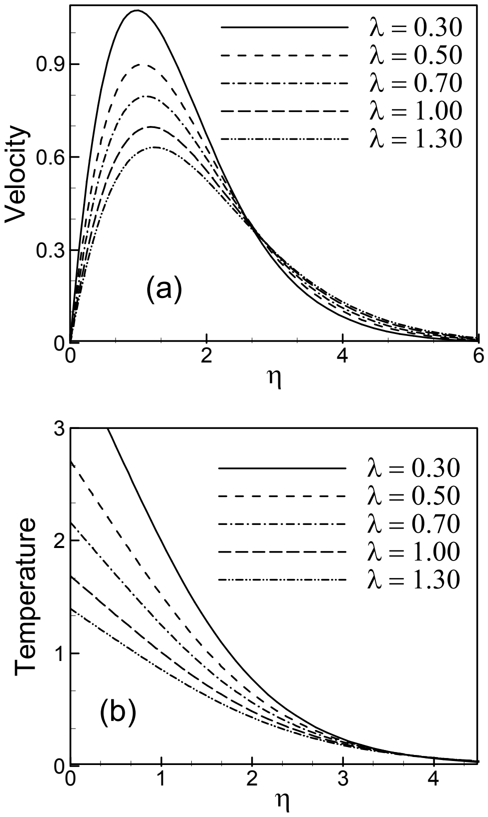

Figure 4. (a) Velocity and (b) Temperature profiles for different values of λ with γ = 0.01 and Pr = 0.733.

If we compare the velocity and temperature for the three parameters we find the velocity boundary layer and the thermal boundary layer thickness are much more clear and large for Pr and λ but thin forγ.

Figure 5(a) and Figure 5(b) illustrate the effect of the thermal conductivity variation parameter on the skin friction coefficient C_{fx} and surface temperature $\theta(x,0)$ against x with Pr = 0.733 and λ = 1.00. It is seen from Figure 5(a) that the skin friction increases monotonically along the upward direction of the plate for a particular value ofγ. It is also seen that the local skin friction coefficient increases for the increasing values ofγ. From Figure 5(b), it can be seen that the surface temperature increases due to the increasing value along the positive x direction for a particularγ.

Figure 6(a) and Figure 6(b) deal with the effect of Prandtl number Pr on the local skin friction and surface temperature against x with γ= 0.01 and λ = 0.30. It can be observed from Figure 6(a) that the skin friction decreases monotonically for a particular value of Pr. It can also be noted that the skin friction decreases for the increasing values of Pr. From Figure 6(b), it can be seen that the surface temperature distribution decreases due to the increases along the positive x direction for a particular value of Pr.

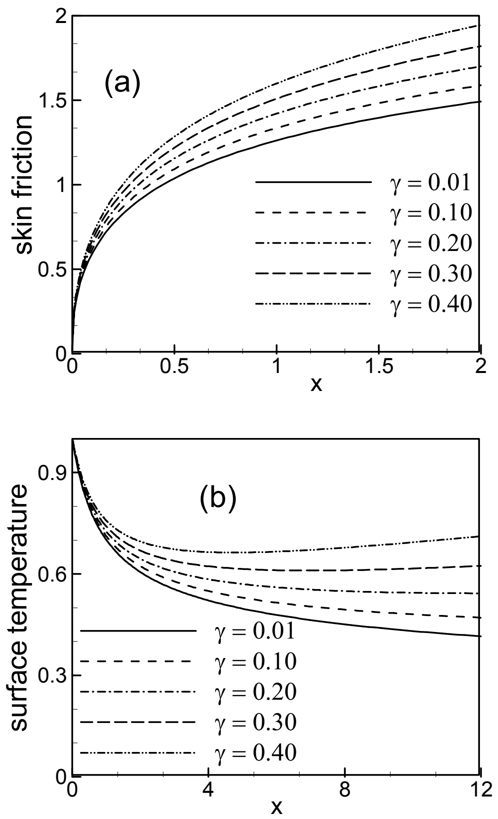

Figure 5. (a) Local skin friction coefficient and (b) Surface temperature distribution for different values of γ with Pr = 0.733 and λ = 1.0.

The variation of the local skin friction coefficient C_{fx} and surface temperature distribution $\theta(x,0)$ for different values of $\lambda$with γ = 0.01 and Pr = 0.733 at different positions are illustrated in Figures 7(a) and 7(b), respectively. It can also be noted from Figure 7(a) that the skin friction increases monotonically for a particular value ofλ. Again Figure 7(b) shows that the surface temperature $\theta(x,0)$ increases for increasing values of λ.

In compare with skin friction for the thermal conductivity variation, Prandtl number and transverse conduction variation parameters we find this is very much clear and larger forλ. The skin friction for γ is clearly opposite trend with Pr andλ. But in surface temperature we seeγ and Pr acts inversely withλ. The effect is very clear and larger forλ.

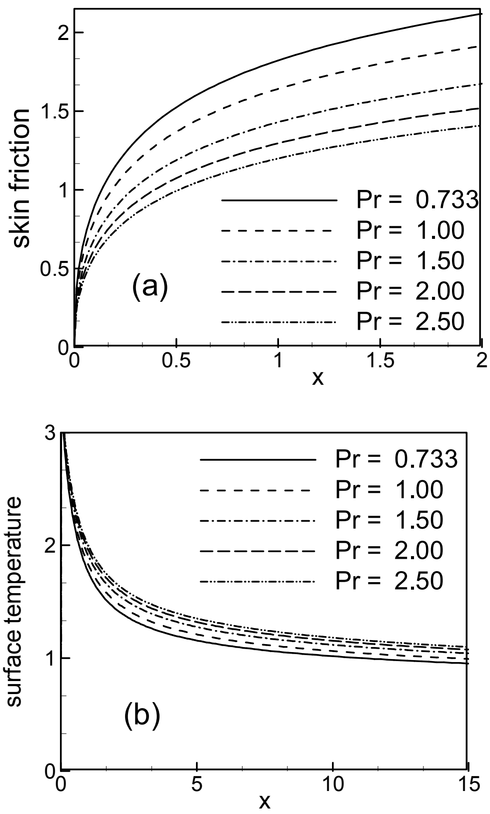

Figure 6. (a) Skin friction and (b) Surface temperature for different values of *Pr* with $\gamma = 0.01$ and $\lambda = 0.3$.

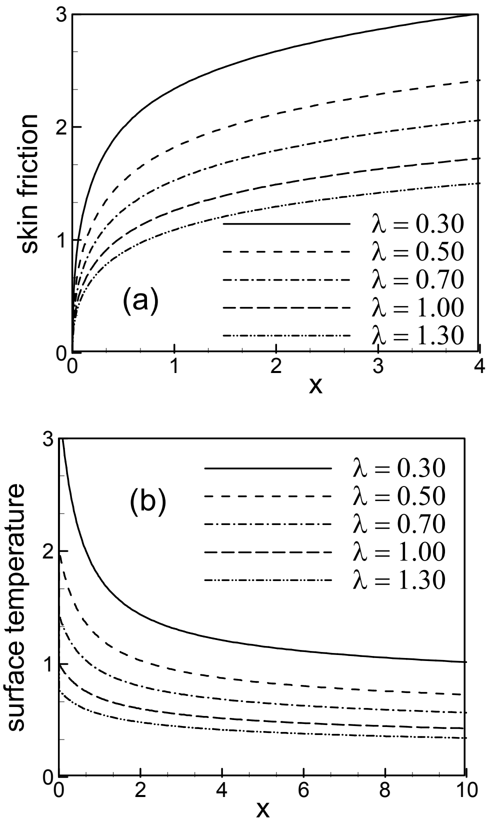

Figure 7. (a) Skin friction and (b) Surface temperature for different values of λ with $\gamma = 0.01$ and $Pr = 0.733$.

CONCLUSIONS

The effects of thermal conductivity of fluid on free convective flow along a vertical flat plate with transverse conduction variation have been studied in this paper. From the present investigation the following conclusions may be drawn

- The velocity within the boundary layer increases for increasing values of γ and decreases for increasing values of *Pr* and λ.
- Significant effect is found in velocity for transverse conduction variation parameter. An inverse trend is noted in velocity profile.
- The temperature within the boundary layer increases for increasing values of γ and decreases for increasing values of *Pr* and λ.
- The local skin friction decreases for the increasing values of *Pr* andλ. On the other hand, this increases for increasing values ofγ . Remarkable effect is found in skin friction forλ.
- An increase in the values of γ and *Pr* leads to an increase in surface temperature. On the other hand, this decrease for increasing values ofλ. Important effect is found in surface temperature forλ. The surface temperature for λ is opposite trend withγ and *Pr,* which is mentionable.

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