HEAT TRANSFER IN A POROUS MEDIUM OVER A STRETCHING SURFACE WITH INTERNAL HEAT GENERATION AND SUCTION OR INJECTION IN THE PRESENCE OF RADIATION

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Abstract: *Heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection has been analyzed numerically in the presence of radiation. In this analysis, the governing equations are transformed into a system of ordinary differential equations and solved them numerically using Nachtsheim-Swigert shooting iteration technique. The local similarity solutions for the flow and the heat transfer characteristics are presented graphically for various material parameters entering into the problem. The effects of the pertinent parameters on the local skin friction coefficient (viscous drag) and the Nusselt number (rate of heat transfer) are also displayed graphically.*

Keywords: *Internal heat generation, suction, injection, radiation, Nusselt number.*

INTRODUCTION

Boundary layer flow on continuous moving surface is a significant type of flow occurring in several engineering applications. Aerodynamic extrusion of plastic sheets, cooling of an infinite metallic plate in a cooling bath, the boundary layer along a liquid film in condensation processes and a polymer sheet or filament extruded continuously from a dye, or a long thread traveling between a feed roll and a wind-up roll, are examples for practical applications of continuous flat surface. As examples on stretched sheets, many metallurgical processes involve the cooling of continuous strips or filament by drawing them through a quiescent fluid and that in the process of drawing, when these strips are stretched.

The flow field of a stretching wall with a power-law velocity variation was discussed by Banks[1]. Ali[2] and Elbashbeshy[3] extended the work of Banks[1] for a porous stretched surface with different values of the injection parameter. Gupta and Gupta[4] analyzed the stretching problem with constant surface temperature. Sriramula *et al.*[5] studied steady flow and heat transfer of a viscous incompressible fluid flow through porous medium over a stretching sheet. Pop and Na[6] studied free convection heat transfer of non-Newtonian fluids along a vertical wavy surface in a porous medium. Ali *et al.*[7] studied radiation effect on natural convection flow over a vertical surface in a gray gas. Following Mansour[8] studied the interaction of mixed convection with thermal radiation in laminar boundary layer flow over a horizontal, continuous moving sheet with constant suction/injection. Bakier and Gorla[9] considered the effect of thermal radiation on the mixed convection from horizontal surfaces in saturated porous media.

Elbashbeshy and Bazid[10-14] re-analyzed the stretching problem discussed earlier by Elbashbeshy[3] including variable viscosity, internal heat generation, suction/injection and porous medium. Subhas and Veena[15] analyzed visco-elastic fluid and heat transfer in a porous medium over a stretching surface. In the present study, the thermal radiation interaction of the boundary layer flow over a stretching surface embedded in a porous medium with internal heat generation and suction or injection has been investigated. The similarity solutions have been

Nomenclature

C_f	Skin friction coefficient
c_p	Specific heat at constant pressure
$h(x)$	Local heat transfer coefficient
K	Permeability parameter
K'	Permeability of the porous medium
N	Radiation parameter
Nu_x	Nusselt number
Pn	Radiative Prandtl number
Pr	Prandtl number
Q	Volumetric rate of heat generation
q_r	Rediative heat flux
q_w	Heat flux
Re_x	Local Reynolds number
T	Temperature within the boundary layer
T_w	Temperature at the plate
T_∞	Temperature outside the boundary layer
u_o	Velocity of the plate
v_w	Suction/Injection parameter
u, v	x and y velocity components
x, y	Cartesian co-ordinates
Greek Symbols	
θ	Dimensionless temperature
κ	Thermal conductivity
ρ	Density of the fluid
κ_1	Mean absorption coefficient
Ψ	Stream function
μ	Coefficient of the dynamic viscosity
σ_1	Stefan-Boltzmann constant
η	Similarity parameter
λ	Heat generation or absorption parameter
μ_e	Effective viscosity
α_c	Thermal diffusivity of the porous medium
τ_w	Shear stress
υ_a	Apparent kinematic viscosity

numerically for various parameters entering into the problem using shooting method and interpreted the obtained data from the physical point of view.

MATHEMATICAL FORMULATION

Let us consider a steady, two-dimensional laminar flow of a viscous, incompressible fluid through a porous medium of permeability K' over a stretching surface with a uniform temperature T_w and velocity $u_o x$, moving axially through a stationary fluid. Two equal and opposite forces are introduced along x- axis so that the wall is stretched keeping the origin fixed and y- axis is perpendicular to it. The fluid is considered to be gray; absorbing-emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x-direction is considered negligible in comparison to the y-direction. The conservation equations of the laminar boundary layer are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu\frac{\partial^2 u}{\partial y^2}-\frac{\mu_e}{K'}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_c\frac{\partial^2 T}{\partial y^2}+Q\left(T-T_\infty\right)-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

where, u, v are the velocity components along x, y co-ordinates respectively, ρ is the density of the fluid, K' is the permeability of the porous medium, μ is the coefficient of the dynamic viscosity, μ_e is the effective viscosity, T is the temperature of the fluid in the boundary layer, T_∞ is the temperature of the fluid outside the boundary layer, α_c is the effective thermal diffusivity of the saturated porous medium and Q is the volumetric rate of heat generation.

By using Rosseland approximation, q_r takes the form

$$q_r=-\frac{4\sigma_1}{3\kappa_1}\frac{\partial T^4}{\partial y} \tag{4}$$

where, σ_1 is the Stefan-Boltzmann constant and κ_1 is the mean absorption coefficient.

We assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of temperature. This is accomplished by expanding T^4 in a Taylor's series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong T_\infty^4+(T-T_\infty)\times 4T_\infty^3=4T_\infty^3 T-3T_\infty^4 \tag{5}$$

By using Eqs. (4) and (5), Eq. (3) gives

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+Q\left(T-T_\infty\right)+\frac{16\sigma_1 T_\infty^3}{3\rho c_p\kappa_1}\frac{\partial^2 T}{\partial y^2} \tag{6}$$

The corresponding boundary conditions for the above problem are given by

$$\left.\begin{aligned}&u=u_o x,\ v=\pm v_w,\ T=T_w \text{ at } y=0\\&u=0,\ T=T_\infty \text{ as } y\to\infty.\end{aligned}\right\} \tag{7}$$

Positive and negative values for v_w indicate blowing and suction respectively, while $v_w = 0$ corresponds to an impermeable plate.

The following non-dimensional variables are introduced in order to obtain the non-dimensional governing equations:

$$\eta=y\sqrt{\frac{u_o\rho}{\mu}},\ \psi(x,y)=\sqrt{\frac{\mu u_o}{\rho}}xf(\eta),\ \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{8}$$

where, ψ is the stream function.

Since $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, we have from Eq. (8) that

$$u=u_o x f'(\eta)\ and\ v=-\sqrt{\frac{\mu u_o}{\rho}}f(\eta) \tag{9}$$

Here f is non-dimensional stream function and prime denotes differentiation with respect to η. Now substituting Eqs. (8)-(9) into Eqs. (1)-(2) and (6) we obtain,

$$f'''+ff''-f'^2-Kf'=0 \tag{10}$$

$$\theta''+Pn\left(\lambda\theta+f\theta'\right)=0 \tag{11}$$

where,

$K=\frac{\mu_e}{\rho K' u_o}$ is the permeability parameter

$\lambda=\frac{Q}{u_o}$ is the heat generation or absorption parameter

$Pn=\frac{3N\,\mathrm{Pr}}{3N+4}$ is the radiative Prandtl number

$N=\frac{\kappa\kappa_1}{4\sigma_1 T_\infty^3}$ is the radiation parameter

$\mathrm{Pr}=\frac{\rho\upsilon_a c_p}{\kappa}$ is the Prandtl number

The corresponding boundary conditions (7) becomes,

$$\left.\begin{aligned}&f=f_w,\ f'=1,\ \theta=1 \text{ at } \eta=0\\&f'=0,\ \theta=0 \text{ as } \eta\to\infty\end{aligned}\right\} \tag{12}$$

where, $f_w=\mp v_w\sqrt{\frac{\rho}{\mu u_o}}$ is the suction and injection velocity at the plate for $f_w > 0$ and $f_w < 0$ respectively.

SKIN FRICTION COEFFICIENT

The quantities of main interest in such problem are the skin friction coefficient and the Nusselt number (rate of heat transfer). The shearing stress on the surface is defined by

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{13}$$

The local skin friction coefficient is defined as

$$C_f=\frac{\tau_w}{\frac{1}{2}\rho u_o^2}=2x^{3/2}\,\mathrm{Re}_x^{-1/2}f''(0) \tag{14}$$

where $\mathrm{Re}_x=\frac{u_0 x}{\upsilon_a}$ is the local Reynolds number.

Thus from Eq. (14) we see that the local values of the skin friction coefficient C_f is proportional to $f''(0)$.

NUSSELT NUMBER

The local heat flux may be written by Fourier's law as

$$q_w(x)=\kappa\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{15}$$

The local heat transfer coefficient is given by

$$h(x)=\frac{q_w(x)}{T_w-T_\infty} \tag{16}$$

The local Nusselt number may be written as

$$Nu_x=\frac{h(x)}{\kappa}=-(\mathrm{Re}_x)^{1/2}\theta'(0) \tag{17}$$

Thus from Eq. (17) we see that the local Nusselt number Nu_x is proportional to $-\theta'(0)$.

NUMERICAL SOLUTION

The set of nonlinear ordinary differential equations (10)-(11) with boundary conditions (12) have been solved by using sixth order Runge-Kutta method along with Nachtsheim-Swigert[16] shooting iteration technique with f_w, K, λ, Pr, N as prescribed parameters. A step size of $\Delta\eta$ = 0.001 is selected to be satisfactory for a convergence criterion of 10^{-6} in all cases. The value of η_∞ is found for each iteration loop by the statement $\eta_\infty = \eta_\infty + \Delta\eta$. The maximum value of η_∞ to each group of parameters f_w, K, λ, Pr, N is determined when the value of the unknown boundary conditions at $\eta_\infty = 0$ does not change to successful loop with error less than 10^{-6}.

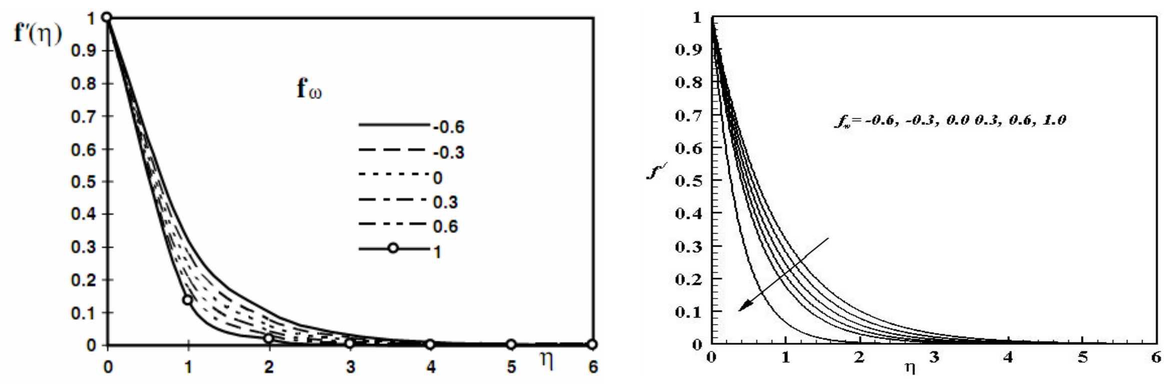
CODE VERIFICATION

To assess the accuracy of our code, we calculated the velocity for the case of injection and suction. Setting $N = 0$, we have compared these with that of Elbashbeshy and Bazid[14]. Figure 1 shows the comparison of the data produced by the present code and those of Elbashbeshy and Bazid[14]. In fact, the results show a close agreement, hence an encouragement for the use of the present code.

RESULTS AND DISCUSSION

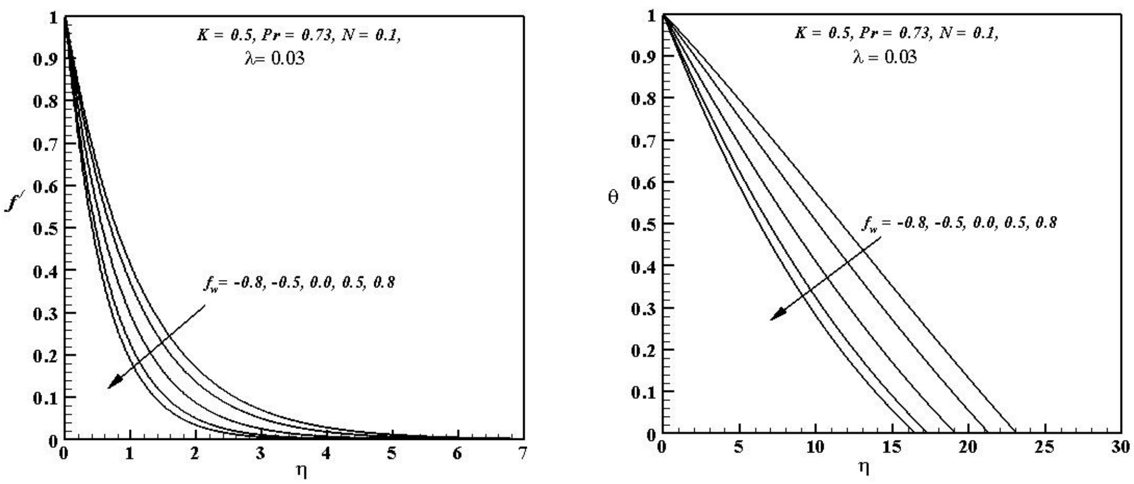
For discussing the results, the numerical calculations are presented in the form of non-dimensional velocity and temperature profiles. In the calculations, the values of permeability parameter K, suction parameter f_w, Prandtl number Pr, radiation parameter N, Heat generation or absorption parameter λ are chosen arbitrarily.

Figure 2(a) shows the dimensionless velocity profiles for different values of injection and suction parameter f_w. It can be seen that the velocity profiles decrease monotonically with the increase of injection parameter as well as suction parameter. Figure 2(b) also shows the similar behavior (like as the velocity profiles) of the effect of suction and injection parameter on the temperature profiles. Figures 3(a)-(b) show the velocity profiles for different values of permeability parameter K in case of suction and injection. From these figures, we see that the velocity decreases with the increase of the permeability parameter K in both case of suction and injection.



Elbashbeshy and Bazid[14] Present

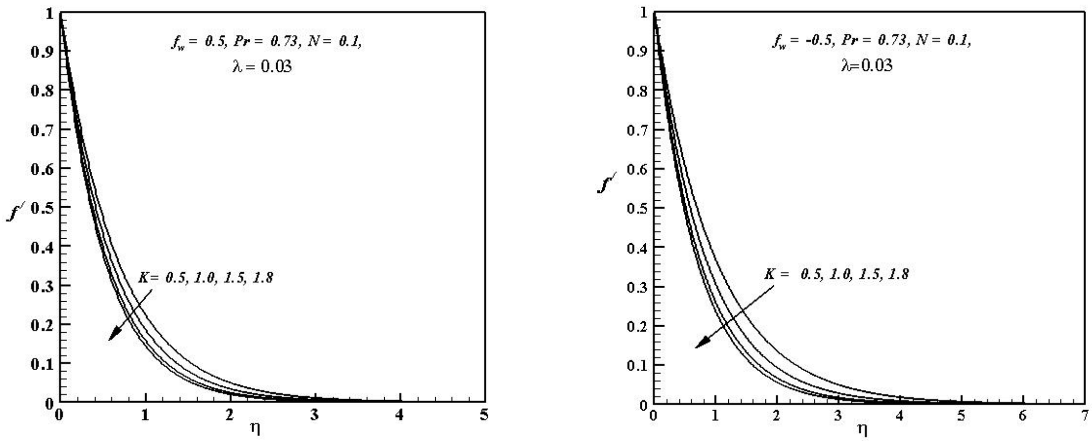
Figure 1. Comparison of Velocity and Temperature Profiles for Code Validation



(a) (b)

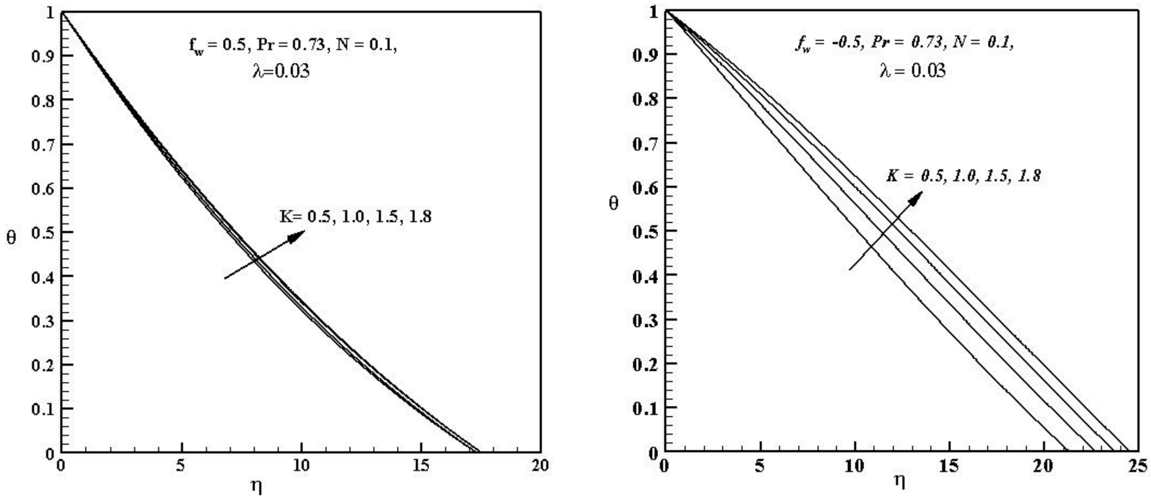
Figure 2. Velocity and Temperature Profiles for Different Values of Injection and Suction

Figures 4(a)-(b) show that dimensionless temperature profiles increase with increasing K in case of both suction and injection. From these figures, we see that the temperature profile increases very slowly for the case of suction; whereas for the case of injection its increasing behavior is significant. The effect of the heat generation and absorption parameter λ on the temperature field is shown in the Figures 5(a)-(b). From these figures, we observe that temperature profiles increase with the increasing λ in case of both suction and injection. We also observe that after a certain distance from the plate the profiles overlap and decrease monotonically for both the cases of fluid suction and injection.



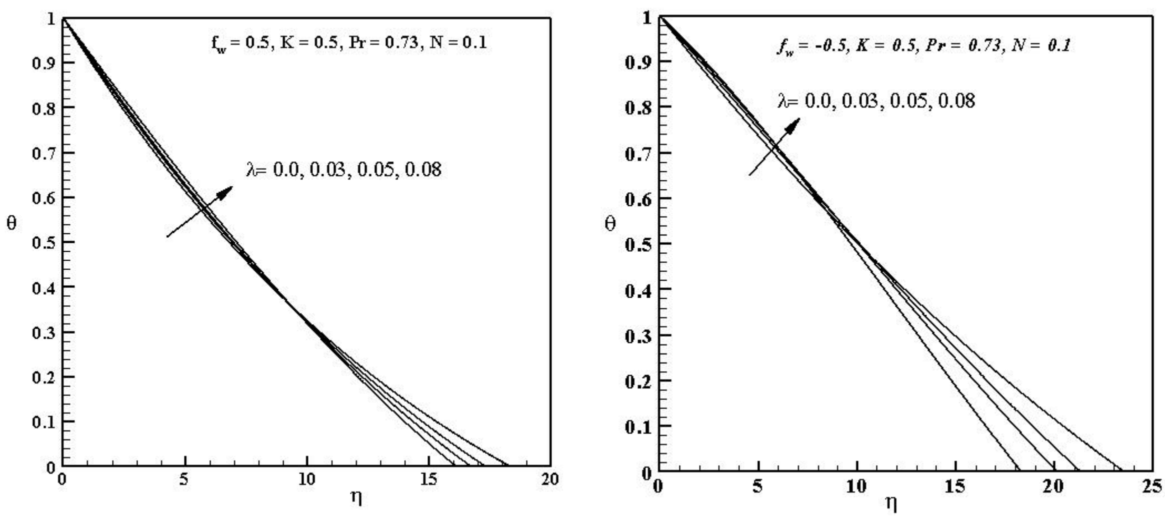
(a) (b)

Figure 3. Velocity Profiles for Various Values of K at: (a) f_w= 0.5 and (b) f_w= -0.5



(a) (b)

Figure 4. Temperature Profiles for Various Values of K at: (a) f_w= 0.5 and (b) f_w= -0.5

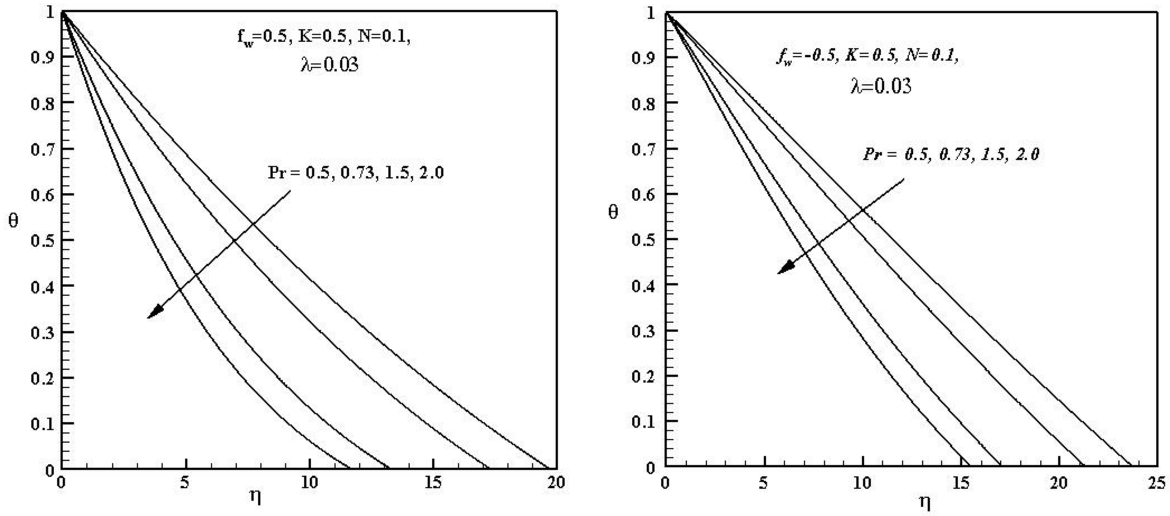


(a) (b)

Figure 5. Temperature Profiles for Various Values of λ at: (a) f_w= 0.5 and (b) f_w= -0.5

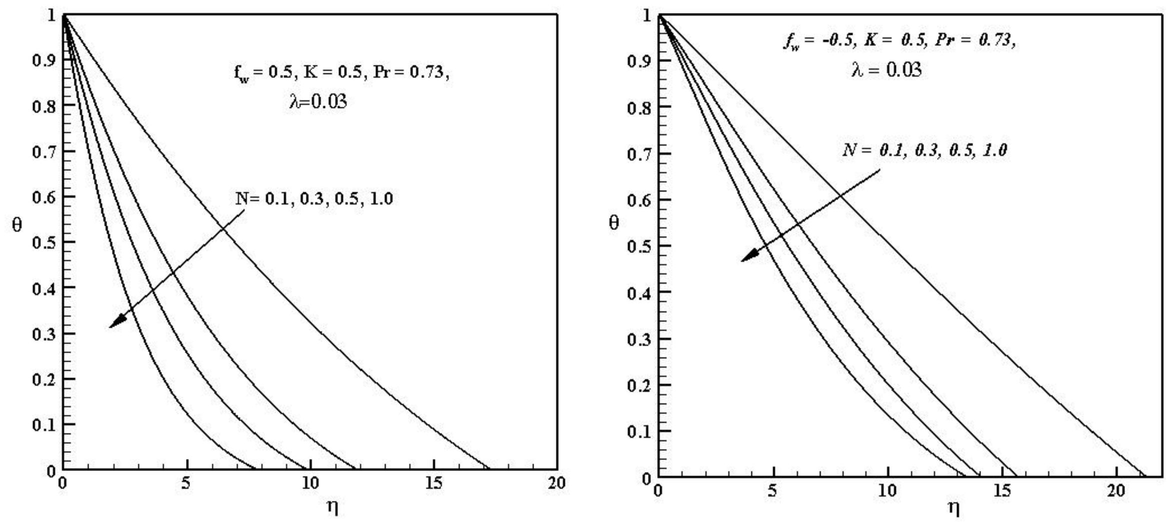
Figures 6(a)-(b) show the temperature profiles for different values of Prandtl number *Pr*. From these figures we see that the dimensionless temperature profile decreases with increasing *Pr* in case of suction and injection. All the above calculations have been carried out for a fixed radiation parameter *N*. Therefore, the effects of radiation parameter *N* on velocity and temperature profiles are not clear from the earlier discussions. Figures 7(a)-(b) show the effect of radiation parameter *N* on the temperature profiles in the case of suction and injection. We observe from these figures that temperature profile decreases with the increase of radiation parameter for both cases. For large *N*, it is clear that temperature decreases more rapidly with the increase of *N*. Therefore using radiation, we can control the temperature distributions.

Figures 8(a)-(b), respectively, show that skin friction coefficient for different values of permeability parameter *K* and the heat generation and absorption parameter *λ*. Here we found skin friction coefficient decreases with increasing permeability in the case of suction and injection and increases with increasing heat source parameter in the case of suction and injection. Figures 9(a)-(b) show that dimensionless rate of heat transfer coefficient decreases with increasing heat source parameter and permeability parameter in the case of suction and injection respectively. Figures 10(a)-(b), respectively, show the effects of Prandtl number *Pr* and radiation parameter *N* on the Nusselt number. The Nusselt number Nu_x increases with the increase of *Pr* as well as with *N* in case of suction and injection.



(a) (b)

Figure 6. Temperature Profiles for Various Values of *Pr* at: (a) f_w= 0.5 and (b) f_w= -0.5



(a) (b)

Figure 7. Temperature Profiles for Various Values of *N* at: (a) f_w= 0.5 and (b) f_w= -0.5

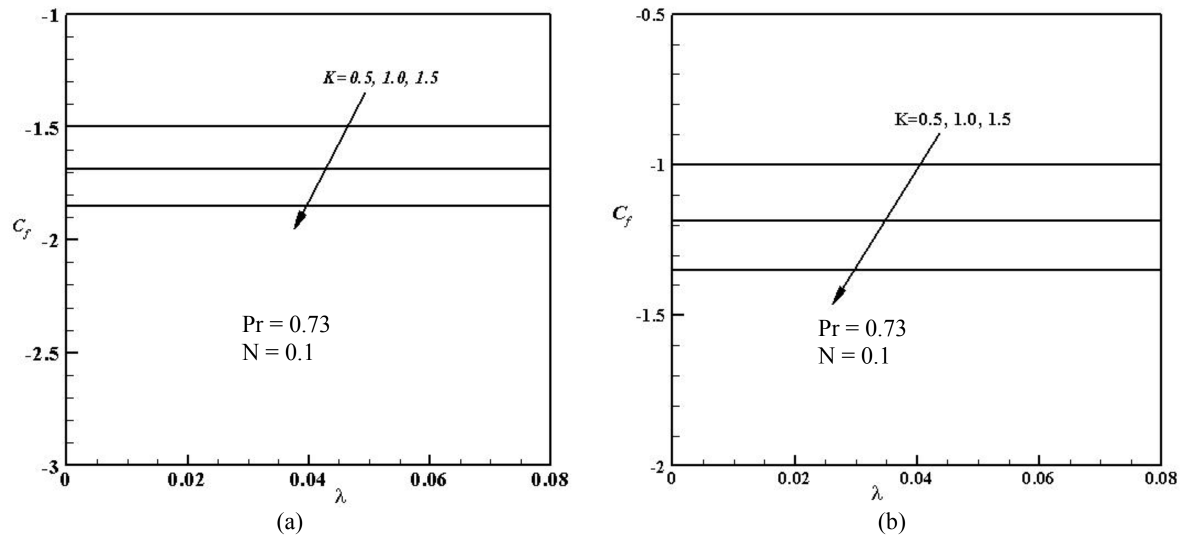


Figure 8. Variation of Skin Friction Coefficient as a Function of λ for Various Values of K at: (a) f_w= 0.5 and (b) f_w= -0.5

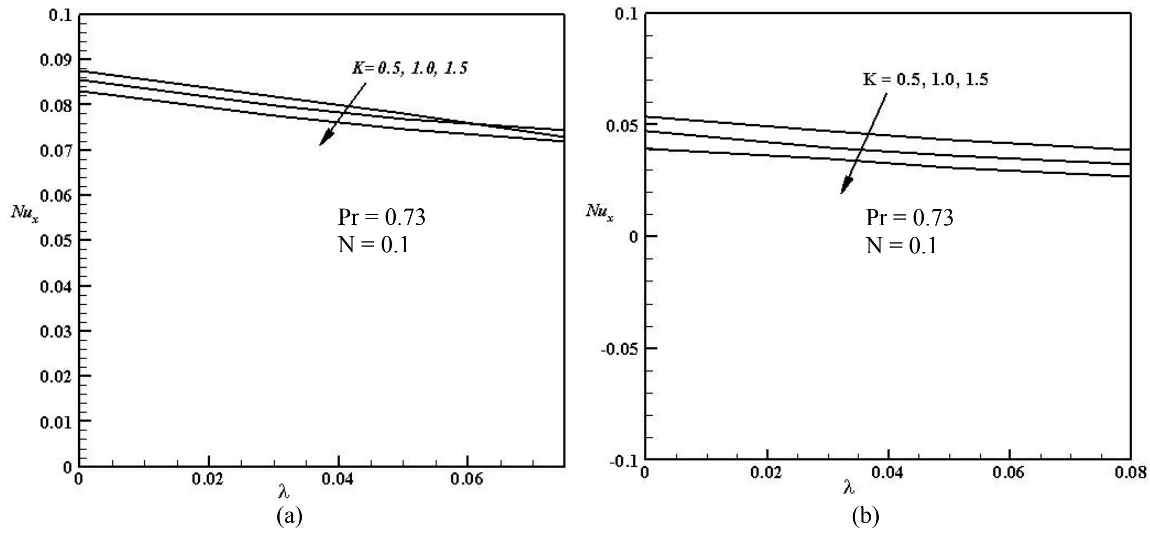


Figure 9. Variation of Nusselt Number as a Function of λ for Various Values of *K* at: (a) f_w= 0.5 and (b) f_w= -0.5

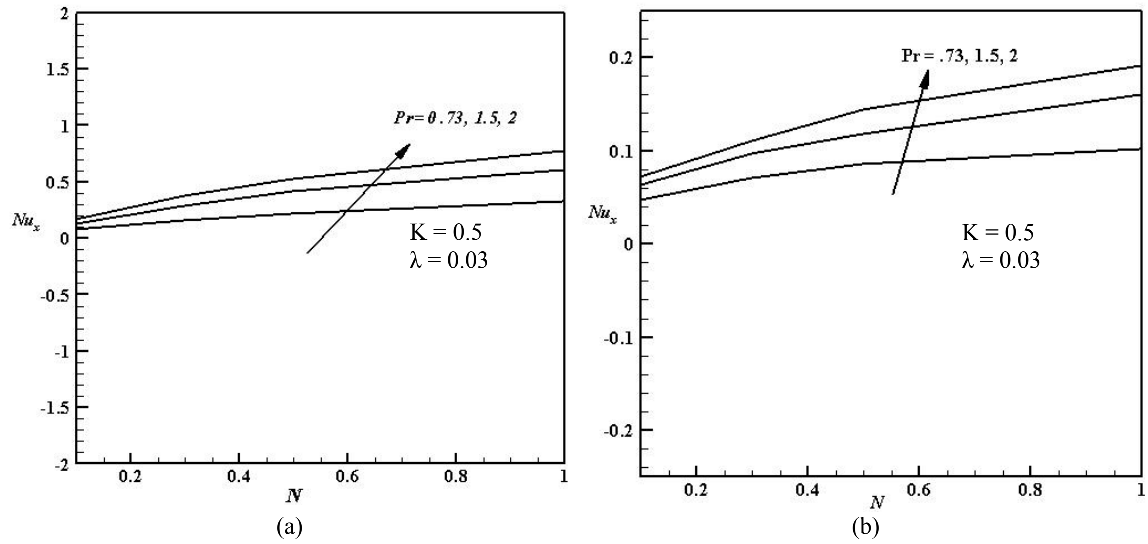


Figure 10. Variation of Nusselt Number as a Function of *N* for Various Values of *Pr* at: (a) f_w= 0.5 and (b) f_w= -0.5

CONCLUSIONS

In the present work, we have analyzed heat transfer in a porous medium over a stretching surface with internal heat generation and suction or injection. Using usual similarity transformations, the governing equations have been transformed into a system of non-linear ordinary differential equations and are solved for similar solutions by using Nachtsheim-Swigert shooting iteration technique. A numerical study has been performed to investigate the influence of permeability parameter, heat source (sink) parameter, Prandtl number and radiation parameter on the skin friction coefficient, the rate of heat transfer coefficient, dimensionless velocity and temperature profiles. From the above investigation, we may draw the following conclusions:

(i) Radiation has significant effect on temperature profiles.
(ii) Velocity profiles decrease with increasing injection as well as suction.
(iii) The temperature profiles increase with increasing permeability and heat source parameter and decreases with increasing Prandtl number and radiation parameter.
(iv) The skin friction coefficient increases with heat source and decreases with permeability.
(v) The rate of heat transfer coefficient decreases with heat source parameter as well as permeability and increases with Prandtl number as well as radiation parameter.

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