ON A STEPPING METHOD FOR THE NON STEADY ROLLING CONTACT RESOLUTION

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Abstract: *The importance of contact and surface problems in industrial machining requires specific studies by tribological researchers to help engineering developments. During cyclic rolling, mechanical components may fail from wear fatigue and it is necessary to develop numerical tools based on simplified approaches to quantify their life time. Numerous wear equations reported in literature have shown that the wear rate is in most cases linked to the traction and the velocities which occur in the contact area. The knowledge of these parameters at every time enables us to follow the wear evolution in the softer material. In this paper, we suggest a stepping method to solve the non steady rolling contact problems. This method is based on the well known approach Fastsim of Kalker and integrates a numerical finite difference scheme to describe the evolution of transient phenomena occurring during non steady rolling contact.*

Key Words: *Rolling Contact, Non Steady, Stepping Approach, Fastsim.*

INTRODUCTION

The progressive degradation of the bodies rolling on each other under normal loads generates wear. Considering industrial applications such as cam and roller for example, it is obvious that the surface degradation affects the mechanical efficiency of the mechanisms and may lead to deterioration of the machinery. In order to quantify the loss of material, many investigations were carried out to determine the wear types and their mechanisms.

Nomenclature

C	The CFL number
E	Young's modulus (Pa)
F	Normal contact load (N)
L	Equivalent flexibility (Pa^{-1})
L_1	Flexibility associated to ν_x (Pa^{-1})
L_2	Flexibility associated to ν_y (Pa^{-1})
L_3	Flexibility associated to ϕ (Pa^{-1})
M_x	Spatial discretization along x
M_y	Spatial discretization along y
N	Discretization number
P	Contact pressure ((Pa)
P_l	Dissipated power per unit length (W/m)
$R_{1x,y}$	Roller radius curvature respectively in the planes *(xoz)* and *(yoz)* (m)
$R_{2x,y}$	Path radius curvature respectively in the planes *(xoz)* and *(yoz)* (m)
V	Rolling velocity (m/s)
a	Semi axis of the contact ellipse along x (m)
a_i	Local geometry of the elliptic contact area along x (m)
b	Semi axis of the contact ellipse along y (m)
t	Time (s)
w_x	Sliding velocity along x (m/s)
w_y	Sliding velocity along y (m/s)
x, y, z	Spatial coordinates
y_i	Local geometry of the elliptic contact area along y (m)
ϕ	Spin (m^{-1})
μ	Friction ratio
ν_x	Longitudinal creepage
ν_y	Lateral creepage
τ	Shear forces (Pa)
υ	Poisson's ratio
ω_1	Roller rolling rotation (Rad/s)
ω_2	Path rolling rotation (Rad/s)
Δx	Spatial step along x (m)
Δy	Spatial step along y (m)
Δt	Temporal step (s)
$\underline{c}$	Creepage vector (m/s)
$\underline{s}$	Sliding velocities vector (m/s)

Bi-dimensional and tri-dimensional models were suggested to compute the wear rate, that is, the worn volume per unit area of surface per unit time[1]. In this context, we can mention the works of Ding et al.[2] in which a numerical approach to simulate fretting wear is developed, the studies of Yang[3] which aim to predict a standard steady state wear coefficient, the analysis of Enblom et al.[4] to simulate the railway wheel profile due to wear in a steady state case, etc.

It has been shown in Chevalier et al.[5], for instance, that the wear rate can be linked to the dissipated power per unit length in the contact area formed by the contacting bodies. This parameter is computed by summing the product of shear forces by the sliding velocities in a strip of the contact area along the rolling direction. The determination of tangential forces and the slip requires the resolution of the complete rolling contact problem. In the particular case of quasi identity, the resolution of normal and tangential problems of contact can be done separately[6]. First, we determine the contact area and the pressure distribution and then we can solve the tangential problem using the algorithm "Fastsim" of Kalker to compute tangential forces and the slip occurring during rolling.

The theory of "Fastsim" is essentially based on kinematic equations of the contact problem and it was shown its accuracy by comparison with the exact theory of contact[7]. Most researchers treat the problem assuming steady state conditions, omitting the transient terms in the kinematical equations. In industrial problems, transient phenomena are common and should be considered for a realistic estimation of wear evolution in machinery. This will allow industrialists to predict the life time of their systems and seek for solutions to make use of them as much as possible.

In this paper, we present a stepping approach to solve a transient rolling contact problem. Based on the finite differences method and the algorithm "Fastsim", this approach is efficient to describe the evolution of normal pressure, tractions and slip velocities which occur in the contact path versus time. Tested on a severe contact case, the transient simulation gives good results by comparison with Fastsim.

STATEMENT OF THE TRANSIENT ROLLING CONTACT PROBLEM

We consider two elastic bodies in contact under a normal load F. The lower solid of a radius R_{x2} is in rotation around the axis y, and the upper one is carried to rotation by adherence. R_x and R_y are respectively the curvatures radii in the planes (xoz) and (yoz). The contact renewal velocity of the path V in the x-direction is equal to $R_{x2}\omega_2$ where ω_2 is the angular velocity.

It was shown by kalker[7] that in a general way, the sliding velocity components w_x and w_y are given by:

$$\begin{cases} w_x = V(\nu_x - \phi y - \dfrac{\partial u}{\partial x}) + \dfrac{\partial u}{\partial t} \\ w_y = V(\nu_y + \phi x - \dfrac{\partial v}{\partial x}) + \dfrac{\partial v}{\partial t} \end{cases} \tag{1}$$

Where ν_x, ν_y and ϕ are respectively the longitudinal creepage, transversal creepage and the spin. u and v are the elastic displacement respectively along the directions x and y.

When the normal load is applied, the contacting bodies are deformed and a contact path is generated in the tangential plane (xoy). If the contacting bodies' curvatures remain constant in vicinity of contact, the contact area and pressure distribution can be easily determined by the Hertz theory[8]. Otherwise, the semi hertzian approach with diffusion (SHAD for short) developed in a previous work[9-11] can be used. This simplified method aims to the resolution of the stationary contact problem between two solids of unspecified geometry.

Kalker[6] supposes that in the contact area relative displacements are proportional to shear forces by a flexibility L depending on the rigidity module of solids and the coefficients C_{ij} of the linear theory of Kalker[6]. This is called the "bed springs" hypothesis. The comparison of the total tangential forces given by the linear theory and the simplified approach Fastsim[11,12] showed that we can distinguish three flexibilities L_1, L_2 and L_3 associated respectively to the creepages ν_x, ν_y and ϕ. Thus, Eq. (1) becomes:

$$\begin{cases} w_x = VL\left(\dfrac{\nu_x}{L_1} - \dfrac{\phi y}{L_3} - \dfrac{\partial \tau_{xz}}{\partial x}\right) \\ w_y = VL\left(\dfrac{\nu_y}{L_2} + \dfrac{\phi x}{L_3} - \dfrac{\partial \tau_{yz}}{\partial x}\right) + L\dfrac{\partial \tau_{yz}}{\partial t} \end{cases} \tag{2}$$

τ_{xz} and τ_{yz} are the tractions in the contact area and L the mean flexibility of the contact given by:

$$L = \frac{L_1|\nu_x| + L_2|\nu_y| + L_3|\phi ab|}{\sqrt{\nu_x^{\,2} + \nu_y^{\,2} + \phi^2 ab}} \tag{3}$$

System (2) can be written in a compacted way:

$$\frac{\underline{s} - \underline{c}}{L} = \frac{\partial \underline{\tau}}{\partial t} + (-V)\frac{\partial \underline{\tau}}{\partial x} \tag{4}$$

Where $\underline{s}$, $\underline{c}$ and $\underline{\tau}$ are vectors defined as follow:

$$\underline{s} = \left(w_x, w_y\right)^T \tag{5}$$

$$\underline{c} = VL\left(\frac{\nu_x}{L_1} - \frac{\phi y}{L_3}; \frac{\nu_y}{L_2} - \frac{\phi x}{L_3}\right)^T \tag{6}$$

$$\underline{\tau} = \left(\tau_{xz}, \tau_{yz}\right)^T \tag{7}$$

The unknown parameters of Eq. (4) are $\underline{s}$ and $\underline{\tau}$. These quantities should be determined at each time step and for each position $\underline{x}$ in order to estimate the dissipated power at the interface and to simulate wear.

Next, we will use the finite difference scheme to solve Eq. (4) of partial derivatives.

Numerical resolution

In the following, we consider that x is the spatial coordinate of the problem since we put $v_y = \phi = 0$. The finite difference method is used to approximate the sliding velocities and tractions at each point of coordinates ($\underline{x}$, t) belonging to a finite discretised space (Fig. 1).

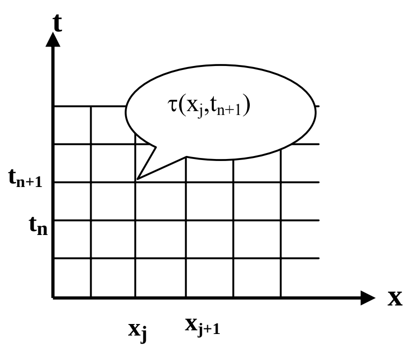

Figure 1. Grid space / time

Validation

Description of the rolling case

To illustrate this scheme, we consider two revolution bodies (Roller/Path) in contact under a normal load F equal to 1500 N. We impose the rotation velocity of the roller starting from zero and increasing with an important slope until the velocity reaches a constant value, that is the stationary rolling final state (Fig. 2).

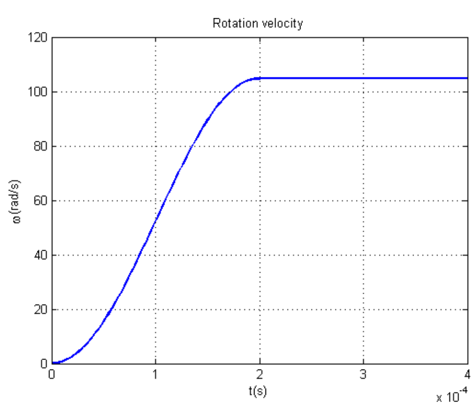

Figure 2. Transient rolling velocity

Geometric and elastic characteristics of both contacting solids are given in table 1.

Table 1. Geometric and elastic characteristics of contacting bodies

	Roller	*Path*
Geometry	R_{x1} = 20 mm R_{y1} = 500 mm	R_{x2} = 25 mm $R_{y2} \rightarrow \infty$
Elastic characteristics	E = 210 000 MPa υ = 0.28	

Results and discussion

The contact area is elliptic of semi axis lengths a and b equal respectively to 0.229 mm and 2.627 mm. The pressure distribution is ellipsoidal with a maximum at the centre of the contact area equal to 1190 MPa (Fig. 3).

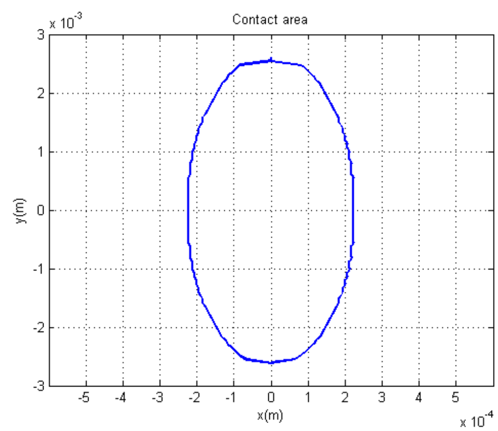

Figure 3(a). Normal problem results: contact area

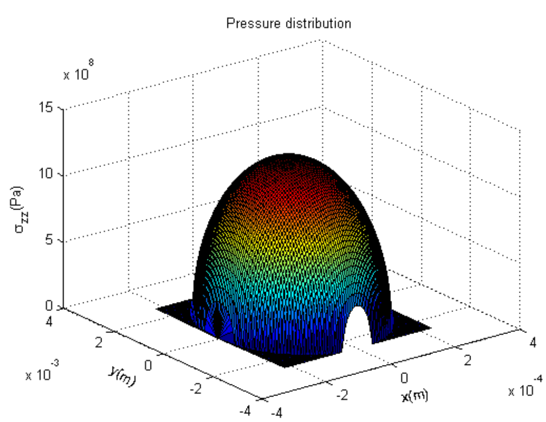

Figure 3(b). Normal problem results: pressure distribution

The local geometry of the contact area and the pressure distribution expressions are given by:

$$a_i = a\sqrt{\left(1-\left(\frac{y_i}{b}\right)^2\right)}\ ,\qquad P(x,y)=\frac{3F}{2\pi ab}\sqrt{1-(\frac{x}{a})^2-(\frac{y}{b})^2} \tag{10}$$

Now it is required to solve the tangential contact problem. So, let's consider a constant longitudinal creepage ν_x equal to 0.001 so that slip occurs only in the x direction. To solve Eq. (4), we make use of the algorithm Fastsim to determine tangential forces and sliding velocities at a given moment. The resolution procedure can be summarised as follow:

- First, we put at $t = t_1$ all the tractions and the slip to zero.
- Second, we cut the contact area into stripes along x and y and we assume the adherence at the leading edge (Fig. 4). Consequently, the slip is zero and tractions at $t = t_2$ can be found using the approximation: $\tau(x,t_2)\approx\tau_j^2$.
- The Coulomb law is then used to check the tractions saturation:

$$\begin{cases} \vec{w}_x=\vec{0}\Rightarrow|\vec{\tau}|\le\mu P \\ \vec{w}_x\ne\vec{0}\Rightarrow|\vec{\tau}|=\mu P \quad\text{and}\quad \vec{\tau}=-\dfrac{\mu P}{|\vec{w}_x|}\vec{w}_x \end{cases} \tag{11}$$

- Once tractions are determined in the contact area at the time t_2, Eq. (4) allows us to calculate the sliding velocities $s(x,t_2)$ and we can go to the next time t_3.

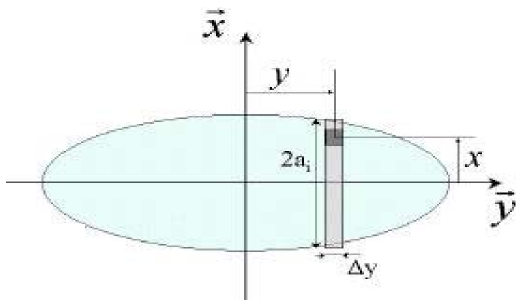

Figure 4. Discretization of the contact area into strips

These steps remain unchanged for the determination of tractions and slip at each time step. We develop by this way a Stepping Approach for transient rolling contact problems (Satran for short) in the following. We present below some results of this method in the case of the present transient contact example.
Tractions and dissipated power evolutions are depicted in Figs. 5 and 6 at different time steps. We can notice the increase of tangential forces along time starting from nearly zero to reach an invariant shape when the rolling velocity becomes steady.

It's also the case for the distribution of the dissipated power per unit length which is very small at the beginning and increasing progressively. This quantity is calculated by integrating the product of tangential forces and sliding velocities along a strip.

$$P_l=\int_{-a_i}^{a_i}(\tau_{xz}w_x+\tau_{yz}w_y)dx \tag{12}$$

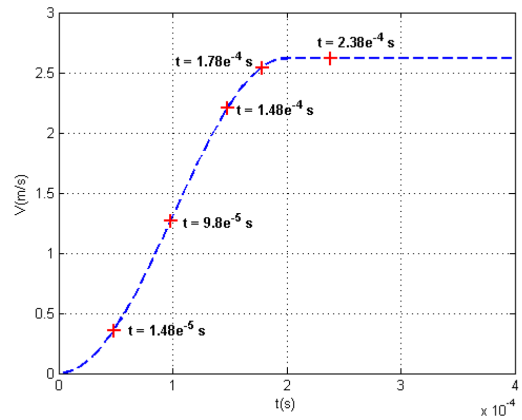

Figure 5(a). Different time steps

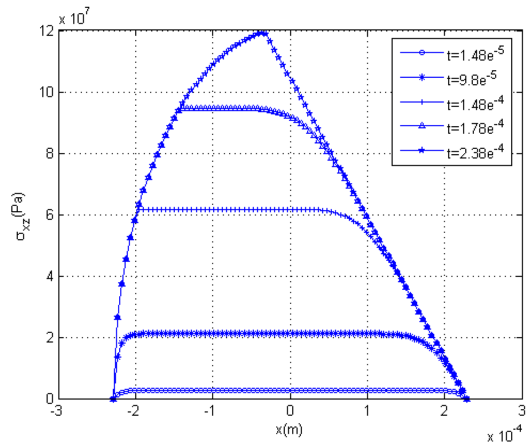

Figure 5(b). Evolution of tractions versus time steps

The evolution of tangential forces allows us to distinguish two phases. In one phase, tractions are linear; this corresponds to the adhesion state with no slip. The second phase is characterised by an elliptic distribution similar to the pressure one, slip is then occurring. In fact, when tractions increase linearly, the Coulomb's bound could be exceeded and we have to set the great tractions to μp (μ is the friction coefficient).

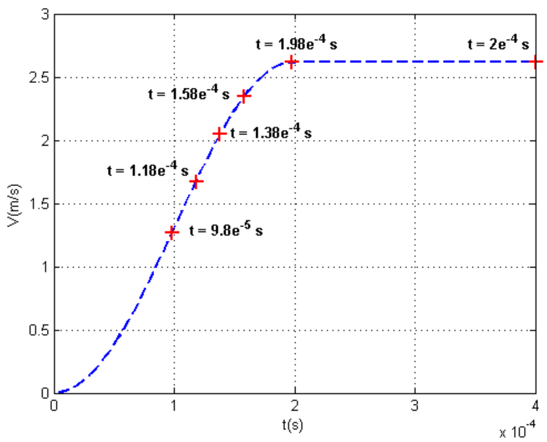

Figure 6(a). Different time steps

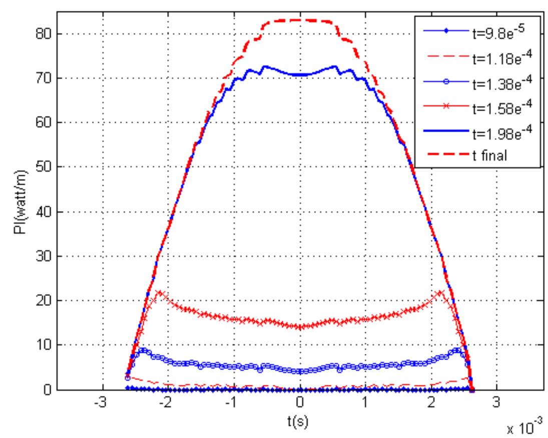

Figure 6(a). Evolution of dissipated power per unit length

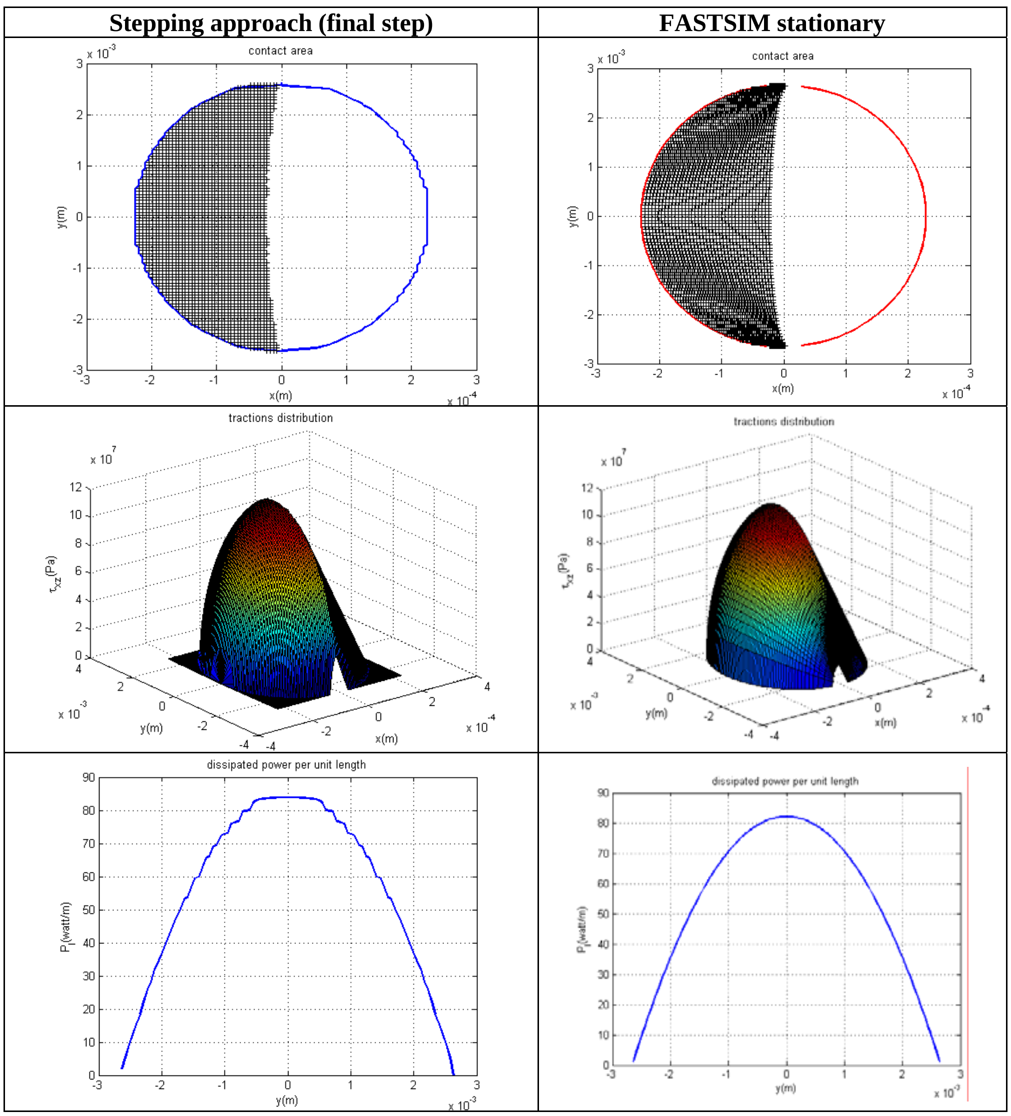

Figure 7. Comparison Satran at final step/Fastsim
(1st row: contact areas, 2nd row: Tractions, 3rd row: Dissipated power)

The time step Δt used in the calculation is equal to 2.10^{-6} s so that the CFL condition is satisfied. The numerical cost of time is of a few seconds. For instance, in this calculation we consider a spatial discretisation M_x x M_y = 80 x 80 respectively along x and y and N = 200 for temporal discretisation that is 128.10^4 calculation steps and the CPU time is equal to 42 s. This observation is one of the advantages of this method mainly when it's required to carry out many calculations taking into account the dispersion of problem parameters.

When rolling velocity is no longer transient and steady state rolling is reached, we expect to have the same results by both Satran and Fastsim. In Fig. 7, we present the results of Satran at the last time step in comparison with those of Fastsim in case of stationary rolling contact.

Results of the stepping approach Satran show a favourable agreement with the steady solution of Fastsim. The contact areas are divided similarly into a stick region on the right and a slip region in black). In the stick region, tractions present a linear aspect as it was mentioned above. We can notice that tractions distributions are the same by both approaches, theirs magnitudes are also identical.

The sliding velocity given by Fastsim is zero in the stick area but is singular in the region of the trailing edge (it increases asymptotically to infinity). This is due to the ellipsoidal distribution pressure chosen in the modeling and discussed in a previous work Eddhahak[9]. The total dissipated power found by both approaches is of the same magnitude and equal to 0.29 watt.

In this example, inertia of the roller has not been taken into account. As the roller is initially in rest and suddenly compelled to follow the path motion a great rotating acceleration is transmitted to the roller via the tangential component of the contact load. Therefore, rolling appears progressively after a first step of sliding. In this step, the tangential forces acting in the contact area are saturated and equal to Coulomb's traction bound μp.

CONCLUSIONS

Friedrisch numerical scheme to solve the non steady rolling contact problem in a short CPU time has been developed. Validation of this method was proved in the particular case of a steady state rolling by comparison with Fastsim. The presented method is efficient to describe the evolution versus time of the sliding velocities and the tractions distributions generated by revolution bodies in contact under load. This finding enables us to estimate the dissipated power during rolling. The latter parameter is often met in the literature dealing with the wear models to estimate the variation of the wear rate under different wear mechanisms. This approach gives access to follow the evolution of the worn profile of the softer material with time and therefore to predict the life time of the technological component.

REFERENCES

[1] Goryacheva, I.G., 1998, "Contact mechanics in tribology", Springer – Verlag.

[2] Ding, J., Leen, S.B. and MCColl, I.R., 2004, "The effect of slip regime on fretting wear-induced stress evolution", Int. Journal of Fatigue, Vol.26, pp. 521-531.

[3] Yang, L.J., 2005, "A methodology for the prediction of standard steady state wear coefficient in an aluminium-based matrix composite reinforced with alumina particles," Journal of Materials Processing Technology, 162-163, pp. 139-148.

[4] Enblom, R. and Berg, M., 2005, "Simulation of railway wheel profile development due to wear influence of disc braking and contact environment", Wear, 258, pp. 1055-1063.

[5] Chevalier, L., Cloupet, S., and Soize, C., 2005, "Probabilistic approach for wear modelling in steady state rolling contact", Wear, 258, pp. 1543-1554.

[6] Kalker, J.J., 1991, "Three dimensional elastic bodies in rolling contact", Kluwer academic publishers, Boston, London.

[7] Kalker, J.J., 1982, "A Fast Algorithm for the Simplified Theory of Rolling Contact", Vehicle System Dynamics Vol. 11, pp.1-13.

[8] Johnson, K.L., 1985, "Contact mechanics", Cambridge University Press, Cambridge, UK.

[9] Chavalier, L. Eddhahak-Ouni, A. and Cloupet, S., 2008, "On a Simplified Model for Numerical Simulation of Wear During Dry Rolling Contacts", Journal of Tribology, Vol.131.

[10] Eddhahak-Ouni, 2006, "Simulation de l'usure superficielle dans les contacts roulants. Mise en place des méthodes adaptées au cas non stationnaire des contacts came/galet", Thesis, Université de Marne La Vallée.

[11] Eddhahak, A., Chevalier, L., and Cloupet. S., 2006, "On a simplified method for wear simulation in rolling contact problems", *Proc. III European Conference on Computational Mechanics*.

[12] Chevalier, L., Cloupet, S., and Eddhahak-Ouni, A., 2006, "Contributions à la modélisation simplifiée de la mécanique des contacts roulants ", Mécaniques & Industries, Vol. 7, pp.155-168.