SINGULARITY ANALYSIS OF A 3-PRRR KINEMATICALLY REDUNDANT PLANAR PARALLEL MANIPULATOR

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Abstract: *Finding Singular configurations (singularities) is one of the mandatory steps during the design and control of mechanisms. Because, in these configurations, the instantaneous kinematics is locally undetermined that causes serious problems both to static behavior and to motion control of the mechanism. This paper addresses the problem of determining singularities of a 3-PRRR kinematically redundant planar parallel manipulator by use of an analytic technique. The technique leads to an input –output relationship that can be used to find all types of singularities occurring in this type of manipulators.*

Key Words: *Planar parallel manipulators, Redundant manipulators, Singularity analysis, Jacobian matrices.*

INTRODUCTION

A parallel manipulator can be defined as a closed-loop mechanism composed of an end-effector and a fixed base, linked together by at least two independent kinematic chains. This type of manipulators has been an area of interest for researchers in the past decade and has found various industrial applications in the recent years. These manipulators can offer higher stiffness, higher accuracy and higher payload-to-weight ratio with regard to serial ones[1].

However, like every other type of manipulators, these manipulators have some drawbacks. Smaller workspace, reduced dexterity and more complex kinematic and dynamic models are problems that parallel manipulators have as compared to their serial counterparts.

Most of studies for parallel manipulators, regarding kinematics, dynamics and design, have been applied to non-redundant parallel manipulators [e.g. 2]. Redundant parallel manipulators have been introduced to alleviate some of the shortcomings of parallel manipulators that were mentioned before.

Redundancy in parallel manipulators was first introduced by Lee et al [3] and Merlet [4]. Merlet [4] divided the redundancy into three types:

1. Redundancy caused by putting additional active joints in branches (limbs) of the existing system, which is the same as redundancy in serial manipulators, Fig. 1a.

2. Redundancy caused by replacing passive joints of the branches of the existing system with active ones which represents redundancy in joint actuation, Fig. 1b.

3. Redundancy caused by putting additional branches, each having instantaneous motion space (IMS) greater than or equal to the current IMS, which represents redundancy in parallelism, Fig. 1c.

In some other studies [5, 6], redundancy is divided into two main types: Actuation Redundancy and Kinematic Redundancy in which all types of redundancies stated by Merlet [4] can be obtained.

Actuation Redundancy is defined as replacing existing passive joints of a manipulator by active ones. Actuation redundancy does not change mobility or reachable workspace of a manipulator but entails the manipulator having more actuators than are needed for a given task and may be used to reduce singularities within the manipulator's workspace[7-9]. Redundancy in parallel manipulators has been investigated by some researchers in the last decade, but most of these studies have focused on actuation redundancy [10, 11].

Kinematic redundancy increases mobility and actuated-joint degrees of freedom (ADOFs) of parallel manipulators. Kinematic redundancy is obtained when extra active joints and links (if needed) are added to manipulators. For instance, by adding one extra active prismatic joint to one limb of a 3-RRR planar parallel manipulator, it is converted into a kinematically redundant parallel manipulator, see Fig. 2. In this example, the resulting redundant parallel manipulator has 4- ADOFs, one more than the planar task space. Shaded area in Fig. 2 is the area that point q_1 can cover when the prismatic actuator slides within a certain range and linkage p_1q_1 rotates around the point p_1. All possible locations of q_1 as a part of linkage p_1r_1 are located on a full circle centered at r_1. As a result, an infinite number of solutions for inverse displacement problem are on the intersection of the two aforementioned regions, i.e., these are located on the arc mq_1n. Note that the inverse displacement problem for each limb of the original non-redundant 3-RRR has at most two solutions.

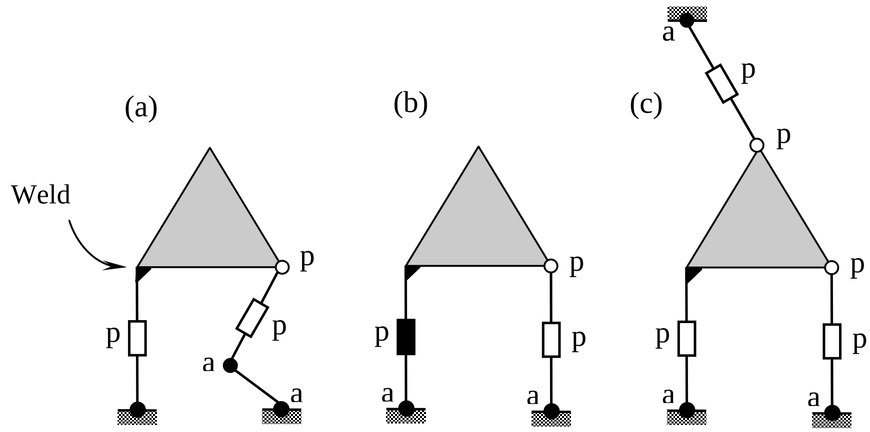

Figure 1. Three redundant planar parallel manipulators in which “a” and “p” denote active and passive joints respectively.

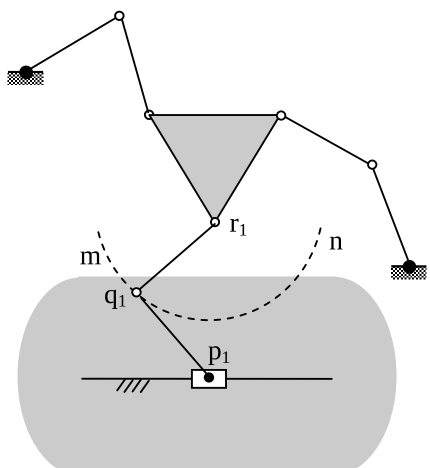

Figure 2. The 4-ADOFs kinematically redundant planar parallel manipulator (2-RRR + 1-PRRR).

One of the most important problems in dealing with parallel manipulators is the presence of singular configurations in their workspace. At these points, manipulator looses or gains one or more degrees of freedom and will become scarcely controllable, so these configurations must be found and avoided during the design step, trajectory planning and control stages of manipulator.

Having an infinite number of solutions for the inverse displacement problem is an important advantage of kinematically redundant manipulators that can be used to avoid singular points.

In addition, larger workspace and improved dexterity are other advantages of kinematic redundancy. On the other hand, kinematic redundancy results in more controlling parameters than required for a set of given tasks [12].

Singularity analysis of non redundant planar parallel manipulators has been investigated by many researchers [e.g. 13, 14]; but in the case of redundant ones, this analysis is few [e.g. 15, 16].

This paper focuses on determination of singular configurations of a 3-PRRR kinematically redundant planar parallel manipulator, shown in Fig. 3, and finding all types of singularities occurring in it. However, the offered method can be used in singularity analysis of all types of relevant kinematically redundant planar parallel manipulators.

JACOBIAN MATRICES

First, Jacobian matrices of the 3-PRRR kinematically redundant planar parallel manipulator are derived. A 3-PRRR is originally a 3-RRR that has three redundant actuated prismatic joints resulting in a 6-ADOFs planar parallel manipulator. The manipulator, in general form, is depicted in Fig. 3.

With reference to Fig. 3, velocity of an arbitrary point of the end-effector or its extension, C, can be written for the ith leg as

$$\dot{\mathbf{C}} = \mathbf{v}_{pi} + (\mathbf{v}_{qi} - \mathbf{v}_{pi}) + (\mathbf{v}_{ri} - \mathbf{v}_{qi}) + (\dot{\mathbf{C}} - \mathbf{v}_{ri}) \quad (1)$$

In which i=1, 2, 3; moreover we have

$$\mathbf{v}_{pi} = \dot{\rho}_i \mathbf{e}_i$$

$$(\mathbf{v}_{qi} - \mathbf{v}_{pi}) = \dot{\theta}_i \mathbf{E}\mathbf{b}_i$$

$$(\mathbf{v}_{ri} - \mathbf{v}_{qi}) = \dot{\gamma}_i \mathbf{E}\mathbf{d}_i$$

$$(\dot{\mathbf{C}} - \mathbf{v}_{ri}) = \omega \mathbf{E}\mathbf{s}_i \quad , i=1, 2, 3$$

where $\dot{\mathbf{C}}$ is the velocity of point C. $\dot{\rho}_i$ and $\dot{\theta}_i$ are rates of ith prismatic and revolute actuated joints respectively; while $\dot{\gamma}_i$ is the rate of ith unactuated joint with respect to the base. $\mathbf{e}_i$ is a unit vector along the direction of ith prismatic joint; ω is angular velocity of the end-effector. $\mathbf{b}_i$ and $\mathbf{d}_i$ are vectors directed from p_i to q_i and from q_i to r_i respectively and finally **E** is a 2×2 orthogonal matrix rotating vectors in a plane through an angle of 90 counterclockwise, i.e.

$$\mathbf{E} \equiv \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Substitution of the above relations in Eq. (1) and simplification of the obtained expression results in

$$\dot{\rho}_i \mathbf{e}_i + \dot{\theta}_i \mathbf{E}\mathbf{b}_i + \dot{\gamma}_i \mathbf{E}\mathbf{d}_i + \omega \mathbf{E}\mathbf{s}_i = \dot{\mathbf{C}}\text{ , } i = 1,2,3 \qquad (2)$$

Since $\dot{\gamma}_i$ is an unactuated joint, it should be eliminated. To this end, we multiply both side of Eq. (2) by $\mathbf{d}_i^T$, then we have

$$\dot{\rho}_i \mathbf{d}_i^T \mathbf{e}_i + \dot{\theta}_i \mathbf{d}_i^T \mathbf{E}\mathbf{b}_i + \omega \mathbf{d}_i^T \mathbf{E}\mathbf{s}_i - \mathbf{d}_i^T \dot{\mathbf{C}} = 0\text{ ,}$$
$$i = 1,2,3 \qquad (3)$$

Moreover, writing Eq. (3) for i=1,2,3 produces

$$\mathbf{J}\dot{\boldsymbol{\theta}} + \mathbf{K}\mathbf{t} = \mathbf{0} \qquad (4)$$

Where

$$\dot{\boldsymbol{\theta}} = [\dot{\rho}_1, \dot{\theta}_1, \dot{\rho}_2, \dot{\theta}_2, \dot{\rho}_3, \dot{\theta}_3]$$

$$\mathbf{t} = [\omega, \dot{C}_x, \dot{C}_y]$$

t and $\dot{\boldsymbol{\theta}}$ are twist and input variable vectors respectively. Matrices **J** and **K** are

$$\mathbf{J} = \begin{bmatrix} u_1 & v_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & u_2 & v_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & u_3 & v_3 \end{bmatrix} \qquad (5)$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{d}_1^T \mathbf{E}\mathbf{s}_1 & -\mathbf{d}_1^T \\ \mathbf{d}_2^T \mathbf{E}\mathbf{s}_2 & -\mathbf{d}_2^T \\ \mathbf{d}_3^T \mathbf{E}\mathbf{s}_3 & -\mathbf{d}_3^T \end{bmatrix} \qquad (6)$$

in which

$$u_i = \mathbf{d}_i^T \mathbf{e}_i$$

$$v_i = \mathbf{d}_i^T \mathbf{E}\mathbf{b}_i$$

So we can consider the manipulator as an input–output device

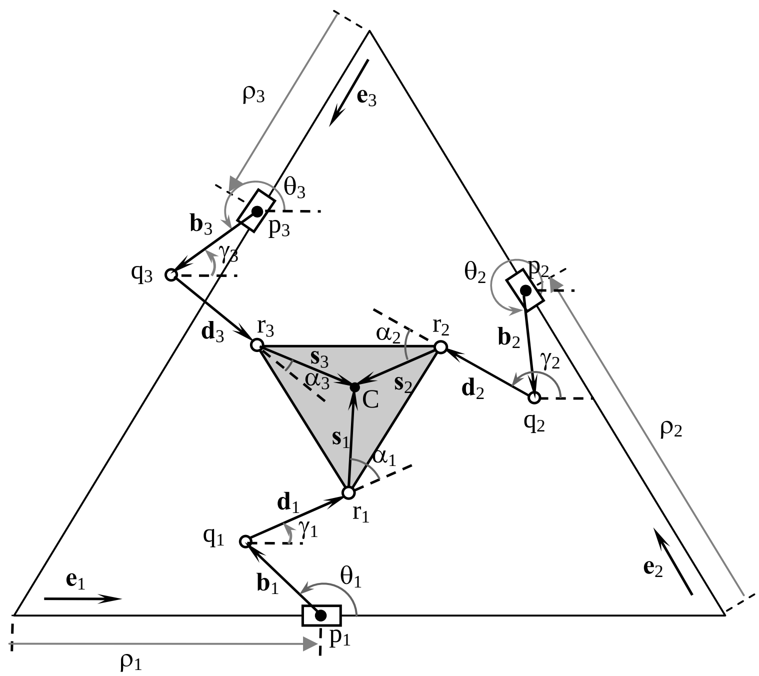

Figure 3. A 6-ADOFs 3-PRRR kinematically redundant planar parallel manipulator.

SINGULARITY ANALYSIS

Gosselin and Angeles[17] identified three different types of singularities for closed kinematic chains, based on the associated direct and inverse Jacobian matrices (**J** and **K** respectively). In this section, the three types of singularities discussed in [17] are investigated for the case of manipulator under study.

FIRST TYPE OF SINGULARITIES

First type of singularities (Inverse kinematic singularities) consists of a point or a set of points where different branches of the inverse kinematic problem meet. In redundant parallel manipulators, matrix **J** is not square, therefore the inverse kinematic singularities can be said to occur when rank of **J** is lower than degrees of freedom of the end-effector that is number of rows of **J**. Therefore a kinematically redundant parallel manipulator is in an inverse kinematic singularity when any minor square matrix extracted from **J** is singular. This degeneracy can also be identified as a condition that sets determinant of $\mathbf{JJ}^T$ to zero[4].

$$\left|\mathbf{JJ}^T\right| = \begin{vmatrix} u_1^2+v_1^2 & 0 & 0 \\ 0 & u_2^2+v_2^2 & 0 \\ 0 & 0 & u_3^2+v_3^2 \end{vmatrix} = 0 \tag{7}$$

Eq. (7) is equivalent to

$$\prod_{m=1,3}(u_m^2+v_m^2)=0 \tag{8}$$

Condition (8) is correct when

$$\mathbf{e}_i \perp \mathbf{b}_i \text{ and } \mathbf{b}_i \parallel \mathbf{d}_i\ ,\ i=1 \text{ or 2 or 3} \tag{9}$$

In other words, first type of singularities occurs when one or some of the legs are fully extended or folded and direction of correspondent prismatic joint(s) is perpendicular to the direction of extended or folded leg(s), Fig. 4. Then, motion of actuators of the leg(s) does not produce any motion of end-effector.

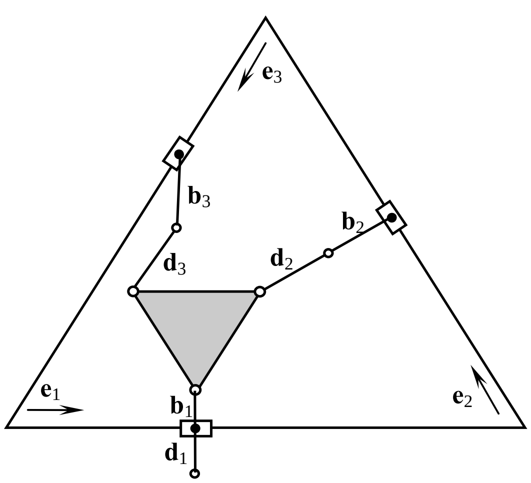

Figure 4. An example of first type of singularities in which legs one and two are in singular poses.

Note that as long as manipulator is not at the workspace boundary, the inverse kinematic singularities of the manipulator are avoidable as it is possible to choose at least a set of solutions which is free of inverse singularities.

SECOND TYPE OF SINGULARITIES

Second type of singularity, occurring only in parallel manipulators, consists of a point or a set of points where different branches of direct kinematic problem meet. This type of singularities occurs when determinant of **K** vanishes. The corresponding configuration can be inferred by imposing linear dependence of columns or rows of **K**.
With reference to Fig. 3, one can write

$$\begin{aligned} \mathbf{d}_1^T\mathbf{Es}_1 &= -d_1 s_1 \sin\alpha_1 \\ \mathbf{d}_2^T\mathbf{Es}_2 &= -d_2 s_2 \sin\alpha_2 \\ \mathbf{d}_3^T\mathbf{Es}_3 &= -d_3 s_3 \sin\alpha_3 \end{aligned} \tag{10}$$

Where d_i and s_i are the magnitude of vectors $\mathbf{d}_i$ and $\mathbf{s}_i$, respectively, and α_i is the angle between vectors $\mathbf{d}_i$ and $\mathbf{s}_i$. In addition, vector $\mathbf{d}_i$ can be written as

$$\mathbf{d}_i = d_i(\cos\gamma_i, \sin\gamma_i)^T \tag{11}$$

Introducing Eqs. (10) and (11) into Eq. (6) leads to

$$\mathbf{K} = \begin{bmatrix} d_1 s_1 \sin\alpha_1 & d_1\cos\gamma_1 & d_1\sin\gamma_1 \\ d_2 s_2 \sin\alpha_2 & d_2\cos\gamma_2 & d_2\sin\gamma_2 \\ d_3 s_3 \sin\alpha_3 & d_3\cos\gamma_3 & d_3\sin\gamma_3 \end{bmatrix} \tag{12}$$

Inspection of Eq. (12) reveals three instances of direct kinematics singularities. The first case occurs when elements of the second and third columns are linearly dependent, i.e. when

$$\frac{\sin\gamma_1}{\cos\gamma_1} = \frac{\sin\gamma_2}{\cos\gamma_2} = \frac{\sin\gamma_3}{\cos\gamma_3} = A \tag{13a}$$

or

$$\tan\gamma_1 = \tan\gamma_2 = \tan\gamma_3 = A \tag{13b}$$

where A is a constant. Eq. (13) shows that the first case of direct kinematics singularities occurs when distal links are parallel.

Then nullspace of **K** represents a set of pure translations of end-effector along a direction normal to $\mathbf{d}_i$, indicated by vector **n** in Fig. 5i. End-effector can move in that direction even if actuators are locked; likewise, a force applied to end-effector in that direction cannot be balanced by the actuators.

The second case in which **K** is singular occurs when elements of the first column are zero, i.e. when

$$\sin\alpha_i = 0\ ,\ i=1, 2, 3 \tag{14}$$

Considering Fig. 3, in this case, extensions of the three vectors $\mathbf{d}_i$, i=1, 2, 3, intersect at the common point *C*. Since point C is an arbitrary point of the end-effector or its extension, these direct singularities take place when all three distal links meet at a common point regardless of where it is located (Fig. 5ii).

Then, nullspace of **K** represents a set of pure rotations of end-effector about the common intersection point. The end-effector can rotate about that point even if all actuators are locked; likewise, a moment applied to end-effector cannot be balanced by the actuators, so manipulator gains some additional uncontrollable degrees of freedom.

The third case occurs when rows of matrix **K** are linearly dependent. Linear dependency between the rows has the same meaning as between the columns. Linear dependency between any two rows happens when two distal links are aligned with the side of the end-effector that is between them. Fig. 6 illustrates such a configuration for the first and second legs in which point C is located on the line passing through the vectors $\mathbf{d}_1$ and $\mathbf{d}_2$. For this configuration we have

$$\begin{aligned} \sin\gamma_1 &= -\sin\gamma_2 \\ \cos\gamma_1 &= -\cos\gamma_2 \\ \sin\alpha_1 &= \sin\alpha_2 = 0 \end{aligned} \tag{15}$$

Therefore, in this case, first and second rows of **K** are linearly dependent.

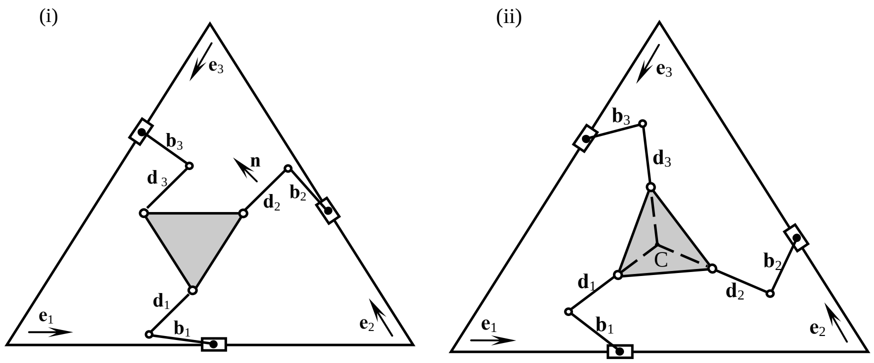

Figure 5. Examples of second type of singularities in which (i) distal links are parallel (ii) extension of distal links intersect at a common point.

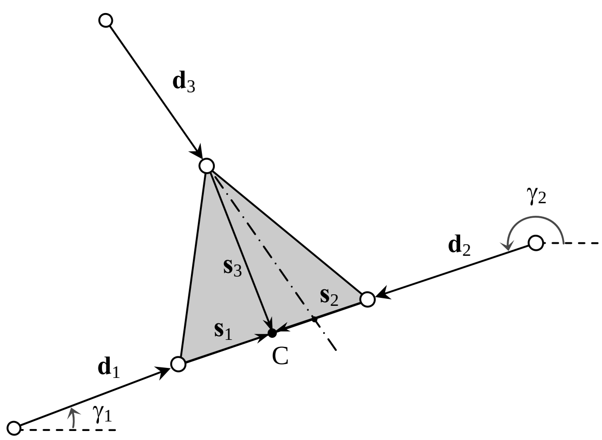

Figure 6. A direct kinematics singularity in which first and second legs are aligned with a side of the end-effector.

THIRD TYPE OF SINGULARITIES

Third type of singularities (combined singularities) occurs when determinants of $\mathbf{JJ}^T$ and **K** both vanish, such that none of the rows of **K** vanishes. In this type of singularities, the two previous types of singularities occur simultaneously. We have these singularities whenever extensions of three vectors $\mathbf{d}_i$, i=1, 2, 3, are either parallel or concurrent at a common point and at least one leg is fully extended or fully folded and the correspondent prismatic joint is perpendicular to it. At these configurations, motion of actuators of at least one leg does not produce any cartesian velocity along the corresponding leg axis. As well, end-effector can move freely in one or more directions even if all actuators are locked and some forces or torque applied to end-effector cannot be balanced by the actuators. Figure 7 shows the manipulator in such a configuration in which all the three legs are in the pose of inverse kinematic singularity and extension of distal links intersect at a common point.

CONCLUSIONS

A summary of redundancy and its advantages was presented. It was shown that most of singularities are avoidable by use of kinematic redundancy, i.e. by considering poses that are singular-free. That is, of course, as long as the pose is not on the outer boundary of workspace, which is caused by physical restriction of the manipulator and it is inevitable; a 3-PRRR kinematically redundant planar parallel manipulator was regarded and its Jacobin matrices were obtained. Using these matrices, all singular configurations are found. The proposed method which is quick, simple and systematic allows a user to find loci of singular configurations of any relevant manipulator, and thus the user will be able to decide whether singularities are acceptable or not.

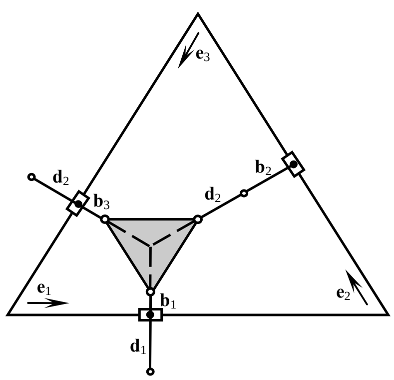

Figure 7. An example of third type of singularities in which two first singularities occur simultaneously.

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