MODIFICATION OF SURFACE MESH FOR THE GENERATION OF KNIFE ELEMENT FREE HEXAHEDRAL MESH

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Abstract: *Hexahedral elements provide greater accuracy and efficiency over tetrahedral elements for finite element analysis of solids and for this reason the all-hexahedral element auto meshing has a growing demand. The whisker-weaving based plastering algorithm developed by the authors can generate hexahedral mesh (HM) automatically. In this method the prerequisite for generating HM is quadrilateral surface mesh (SM). From the given SM, combinatorial dual cycles or whisker sheet loops for whisker weaving algorithm are generated to produce HM. Generation of good quality HM does not depend only on the quality of quadrilaterals of the SM but also on the quality of the dual cycles generated from it. If the dual cycles have self-intersection, it could cause the formation of degenerated hexahedron called knife element, which is not usable in finite element analysis. In this paper a detailed method is proposed to modify the SM to remove self-intersections from its dual loops. The SM modification procedure of this proposed method has three basic steps. These steps are (a) face collapsing, (b) new face generation and (c) template application. A fully automatic computer program is developed on the basis of this proposed method and a number of models are analyzed to show the effectiveness of the proposal.*

Key Words: *Surface mesh, hexahedral mesh, self-intersection, knife element, whisker- weaving*

INTRODUCTION

The finite element method is at present the most important tool for industrial engineering (shipbuilding, automobile, aerospace etc.) design and analysis. At the beginning stage of finite element method, most users were satisfied to simulate vastly simplified forms of their final design utilizing only tens or hundreds of elements. Painstaking preprocessing was required to subdivide domains into usable elements. Market forces have now pushed meshing technology to a point where users now expect to mesh complex domains with thousands or millions of elements with no more interactions than the push of a button. Increasingly larger and more complex designs are being simulated using the finite element method. With its increasing popularity comes the incentive to improve automatic meshing algorithms.

For three-dimensional model, hexahedral meshing is preferred. The advantages of hexahedral mesh are: 1) it needs fewer hexahedrons to fill the domain, so needs less analysis time, 2) hexahedrons fit man-made object better and 3) better numerical behavior of these elements in some problems e.g. stress analysis.

Review of previous work of hexahedral meshing

Generating surface mesh (SM) first and then constructing hexahedral mesh (HM) inward from the SM has several benefits. In many finite element analysis, high quality mesh is needed near the boundary of a solid than deep inside the volume. So if a SM can be used, it is possible to generate good quality hexahedrons near the boundary.

For a large and complicated solid, it may sometimes be needed to decompose the whole region into sub domains to make the meshing procedure easier and these sub domains must have compatible or same mesh in the common boundary of the adjacent regions. This also necessitates having SM to generate HM. The following two methods generate HM from SM.

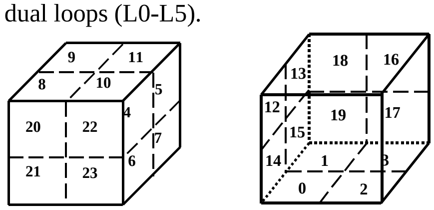
With plastering[1] method, elements are first placed starting with the boundaries and then advances towards the center of the volume. As the algorithm advances, complex interior voids may result, which in some cases are impossible to fill with hexahedrons. The remaining unplastered regions are then filled with tetrahedrons.

Whisker weaving algorithm[2] also starts generating HM from a quadrilateral SM inward. To generate HM, this algorithm produces a set of loops from the SM. These loops (dual of SM) represent the outer boundary of a set of two-dimensional surfaces called whisker sheets. Then the algorithm seeks to complete the sheet diagrams by a set of rules[2]. Each complete whisker sheet represents a layer of hexahedron. Although the knife element generation problem exists in the whisker-weaving algorithm, it can be considered that whisker weaving based plastering algorithm[3] associated with post processing works is able to reliably generate all hexahedral mesh for large and complex geometries.

The above discussion justifies the selection of whisker weaving algorithms[2, 3] for generating HM for the study. The present paper is intended to fix the knife element generation problem associated with these algorithms.

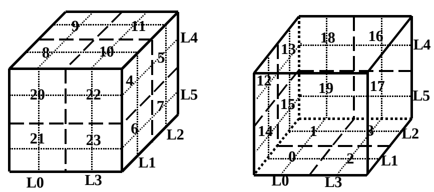
Procedure of constructing combinatorial duals (loops of whisker sheets) of the given quadrilateral surface mesh

The procedure is explained with a very simple example. Fig. 1 shows a cube with quadrilateral surface mesh. The combinatorial dual loops of surface mesh start from an edge of a surface quadrilateral (face) and then continue moving to the opposite edge until it returns to the starting edge. In Fig. 2 the dotted lines on the surface quads represent the dual loops (L0-L5).



(a) Top, front and right sides (b) left, back and bottom sides

Figure 1. A simple block structure with surface mesh



(a) Top, front and right sides (b) left, back and bottom sides

Figure 2. A simple block with surface mesh and dual loops.

Self-intersection of dual loops and knife element

Fig. 3 shows a simple block with SM. In this Fig. the solid line other than the mesh represents a dual loop. The loop starts from the edge marked by star and it crosses itself on its way. This crossing is called self-intersection (SI).

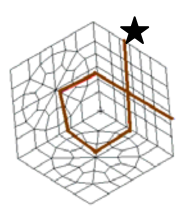
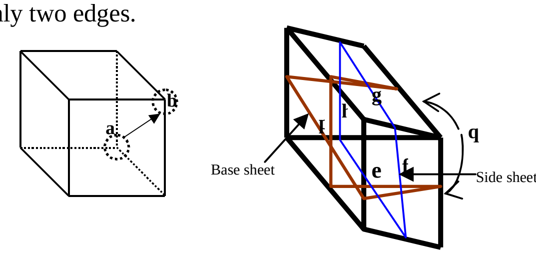
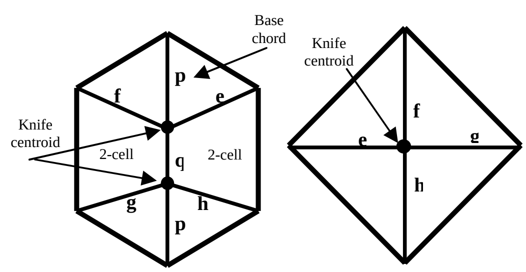


Figure 3. Self-intersection of the dual loop

Presence of self-intersections (SI) on these loops may cause the formation of degenerated hexahedron called 'knife element' while whisker weaving[2,3]. When same face (having SI) is connected twice to make a hexahedron, a face (not of surface mesh) of that hexahedron is collapsed and knife element is formed. In Fig. 4(a), if node *a* of the hexahedron is merged to the node *b* of the same (to collapse the face containing both node *a* and *b*), a knife element is formed. Fig. 4(b) shows a knife element in three dimensions and the corresponding two whisker sheets where it appears. In that figure, the four side faces of the knife element from bottom in anticlockwise directions are *e*, *f*, *g* and *h*. The outer face (on surface mesh) on which a sheet (base sheet) passes twice is marked by *p* (base face). The degenerated face is *q*, which contains only two edges. The whisker sheets can describe the formation of this degeneracy. In whisker sheets, faces *e*, *f*, *g*, *h*, *p* and *q* will represent corresponding STC edges (as they are dual to faces). The knife element appears (its centroid) twice in the base sheet in Figure 4(c) as it passes twice through face *p*. The other sheet is called side sheet in Figure 4(d). In the base sheet, the right hand side STC 2-cell contains STC edges *e*, *q* and *h* which means the faces represented by them will share a common hexahedral edge (since a 2-cell represents a mesh edge). Similarly due to the left hand side STC 2-cell, faces *f*, *q* and *g* will share a common hexahedral edge. The result of this situation is one degenerated face (represented by *q*) having only two edges.



(a) Hexahedron (b) Knife element



(c) Base sheet (d) Side sheet

Figure 4. Definition of knife element

It is important to mention here that knife elements can be removed by post processing operations called collapsing or driving[3]. Driving a knife element destroys it and creates a new one next to it. This operation is continued until a geometric boundary is reached and a new face on the surface mesh is created. However, driving is not a general solution. It may work only in a fairly regular mesh whereas in irregular mesh the condition to apply this technique may not reach. Collapsing moves knife element in the backward direction by collapsing the base face until the surface mesh is reached. Although collapsing always removes the knife, another type of degeneracy called a doublet can result. As these complicated methods are not much effective and also make changes in surface mesh, surface mesh modification approach to avoid knife element in the first place is chosen.

Fig. 5(a) shows a model, which has two self-intersecting loops. The self-intersecting faces are shown as shaded. If a column of hexahedrons can be arranged between these faces, the formation of knife element can be avoided[4]. Such a column has to be arranged manually, which will decompose the domain. Each decomposed domain will then have to be meshed separately. If the situation like Fig. 5 (b) and (c) appears, the column of hexahedrons will cause a hole in the domain, which is not possible to mesh by whisker weaving. This will again necessitate decomposing the domain. As whisker weaving is intended to mesh large structures without decomposition, it is intended in this study to remove all self-intersections of the surface mesh.

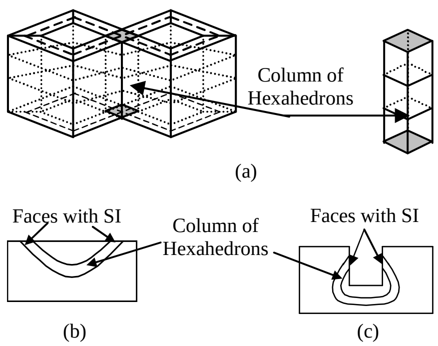


Figure 5. Column of hexahedrons to avoid knife elements

Proposal of the whole procedure of HM generation

The present study proposes a method to develop hexahedral mesh (HM) from two-dimensional surface mesh (SM) such that no knife element forms inside the domain. The surface mesh is generated using automatic quadrilateral surface mesh generator such as paving method[5]. The generated surface mesh should be of high quality and contain even number of quadrilaterals. From this SM, combinatorial duals are formed. These duals are then checked for the presence of any self-intersection. If self-intersection is present then SM is modified to remove it. In the following sections the surface mesh modification procedure will be discussed in details. After the modification of SM, the dual loops are then created again which have no SI. From these loops, using whisker-weaving based plastering technique[3], the HM is generated which has no knife element.

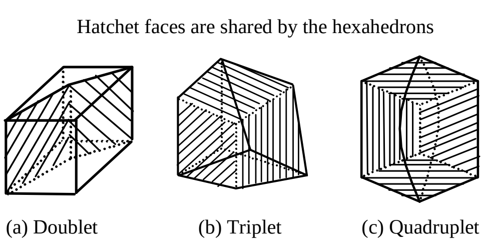


Figure 6. Invalid elements

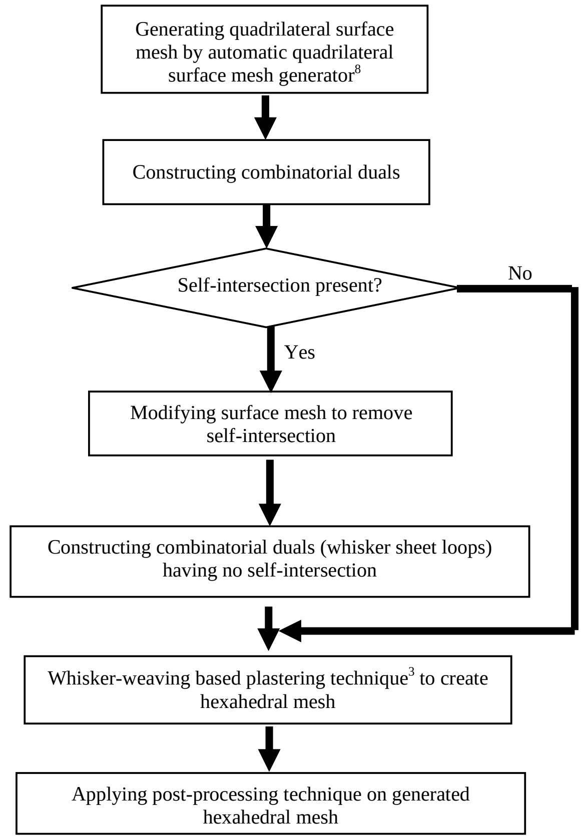


Figure 7. Flow chart for hexahedral mesh generation

As the whisker weaving based plastering technique[3] is fully automatic, sometimes invalid elements (Fig. 6) like doublets (two hexahedrons sharing two faces between them), triplets (two hexahedrons sharing three faces between them) and quadruplets (two hexahedrons sharing four faces between them) generate and also some distorted elements may be produced. For this reason a post-processing program is developed which includes 1) removing doublets, triplets and quadruplets, 2) handling node/edge problems for inverted elements 3) applying three-dimensional Laplacian smoothing[6] and 4) applying three-dimensional optimization based smoothing[7]. Fig. 7 shows the proposed strategy of the whole hexahedral mesh generation procedure in a flow chart.

SOME INTRODUCTORY CONCEPTS

In this section two important topics are discussed in brief.

Checking the quality of quadrilateral surface mesh using internal angles of the quadrilaterals

The quality of the surface mesh should be good to generate good quality HM. To judge if a mesh is of sufficient good quality, it is needed to define a standard.

Zhu et al.[8] deemed a quadrilateral element satisfactory if all its internal angles θ fall within 90°±45° and was considered as unsatisfactory if θ exceeds the limit 90°±60°. Lo and Lee[9] found that the first condition appeared to be too strict, so a more flexible range of 90°±52.5° was used for quadrilateral interior angles. In the present study Lo and Lee's range is chosen for acceptable element. Any element exceeding this range is considered unacceptable. The optimum shape for quadrilateral is a square with interior angles 90°. The following equations were used to measure the distortion factor of quadrilaterals.

The deviation of each interior angle of a quadrilateral $\delta\theta_i$ is defined as

$$\delta\theta_i = \left|\frac{\pi}{2} - \theta_i\right| \quad i = 1, 2, 3, 4. \tag{1}$$

The distortion factor for quadrilateral F_q is defined as

$$F_q = \sqrt{\sum_{i=1}^{4} (\delta\theta_i)^2} \tag{2}$$

It can be seen that F_q would attain a minimum value of zero for a perfect square and the acceptable range of 90°±52.5° defined by Lo and Lee[9] would correspond to $F_q \leq 105°$.

Edge valence of a node

Edge valence of a node is defined as the number of nodes or edges connected to that particular node. In Fig. 8, edge valence of node *i* is 5. The concept of edge valence is used in face collapsing operation.

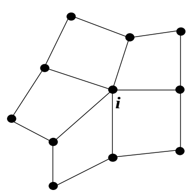


Figure 8. Edge valence of node *i*

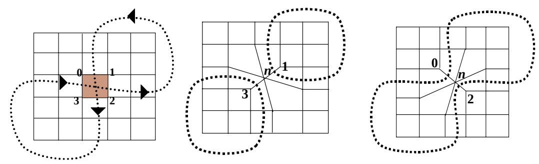
SURFACE MESH MODIFICATION TECHNIQUES

The goal of the present study is to modify the surface mesh in such a way that not only SI is removed but the final surface mesh has requirement quality as well. To overcome the difficulties associated with other researchers method[4, 10-12], three steps are proposed to remove the self-intersection of the dual loops. These steps are (1) face collapsing, (2) new face generation and (3) template application. The first and last steps make changes on the face containing SI, whereas step 2 makes change in surface mesh to remove self-intersection from any particular face.

Face collapsing

The most desirable technique to eliminate self-intersection is face-collapsing operation proposed by Folwell and Mitchell[10]. Here face collapsing technique is introduced which performs positive collapsing before negative collapsing and is guided by the proposed technique of quality checking. By collapsing a face (quadrilateral of a surface mesh) the redirection of the loop occurs which ultimately removes self-intersection. The detailed description of the process is given below.

Face collapsing is done by merging a pair of nodes of any face to a new node *n*. If a case like Fig. 9 (a) appears, the shaded face (containing SI) can be collapsed either by merging nodes 0 and 2 (Fig. 9 (b)), or by merging nodes 1 and 3 (Fig. 9 (c)). When nodes 0 and 2 of that face are merged, it is seen that two loops are formed from the original one. Such type of collapsing which breaks the original loop into two is called positive collapsing. If nodes 1 and 3 of the face are merged instead, the original loop remains intact. This type of collapsing is called negative collapsing. The neighboring quadrilaterals of the collapsed face, which shared the merging nodes, will then be provided with the new node *n* (which has the average of the two merging nodes coordinates) in exchange of the merging nodes.



(a) SI (b) Positive collapsing (c) Negative collapsing

Figure 9. Face collapsing

A negative face collapsing reduces the number of self-intersections by only one, whereas a positive face collapsing leaves two closed lines and in this way some of the previous self-intersections may automatically remove. This is shown in Fig. 10. After removal of one SI (the circled one) positively, two separate loops are created and in this way two additional self-intersections are removed as these are no longer SI. Therefore all positive face collapsing are performed before negative collapsing to get the advantage of removing SI with minimum number of collapsing.

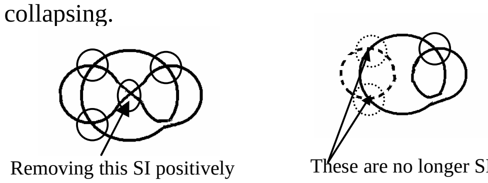


Figure 10. Effect of positive collapsing

Face collapsing certainly eliminates self-intersection but in some case it may cause the formation of unacceptable elements (Fig. 11). For this reason two reliable checking procedures are developed. If these checking procedures detect the formation of bad quality quadrilaterals then collapsing of that particular quadrilateral must be postponed. The checking procedures developed in this study are quadrilateral quality checking and edge valence checking. Both of the procedures are discussed next.

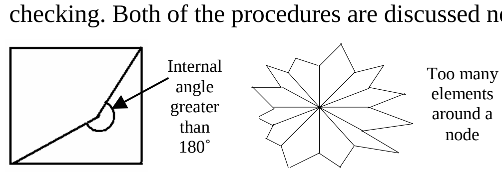


Figure 11. Unacceptable elements

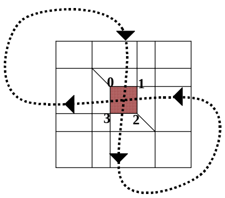
Face collapsing with quad quality checking

This developed and presented procedure, which is very simple to understand and implement is also very effective. After a face is collapsed, all the corresponding quadrilaterals (which had the merging nodes) are checked to find if any one has quality less than desirable. If face of such quality is found then that particular collapsing is postponed and the next quadrilateral having self-intersection is tested. After each collapsing, Laplacian smoothing[6] on the local surface mesh of the collapse is performed to keep the mesh as smooth as possible to perform the next collapsing.

Face collapsing with edge valence checking

This is another way of evaluation if a particular face collapsing would be allowed or not. In Fig. 12(a), a situation is depicted where the shaded quadrilateral has self-intersection. For positive collapsing, node 1 and 3 should be merged to a new node. If the positive collapsing is performed then the surface mesh will be like Fig. 12(b).

Edge valence of a new node, formed by merging two nodes, is the sum of the merging nodes edge valence minus 2. Edge valence of the other nodes of the collapsed face will be reduced by 1.



(a) SI in shaded face (b) Mesh after positive collapsing

Figure 12. Edge valence checking

After node 1 and 3 are merged, the edge valence of nodes *n*, 0 and 2 thus becomes 6, 2 and 2 respectively. Edge valence of a node equal to 2 means the angle (360°) around that node will be shared by only two quadrilaterals (Fig. 11). It causes formation of unacceptable elements. In Fig. 12(b) it can be seen that the quadrilaterals having nodes 0 or 2 are unacceptable as internal angles are 180°. So, after a face collapsing, if edge valence of any node becomes 2, then that collapsing must be postponed. There is no problem in merging nodes 0 and 2 to collapse the same face of the Fig. 12(a).

If new node *n* gets edge valence greater than 11, there will again generate bad shaped elements (average sharing angle becomes very small).

So by face collapsing, the nodes should have edge valence more than 2 but less than 12, so that the collapsing does not result formation of distorted quadrilateral elements.

New face generation

The users could regard some faces in stress concentration areas (shaded faces of Fig. 13) as important because regular hexahedral mesh is needed. When template is applied on a face, the surface mesh becomes unfavorable for generating good quality hexahedron. Collapsing can also change the arrangement of nodes. For this reason, some faces may not be permitted by the user to be collapsed and templated. So this study proposes the idea of constrained faces. In this study, a way of avoiding SI from any such constrained faces (constrained for both templating and collapsing) is presented.

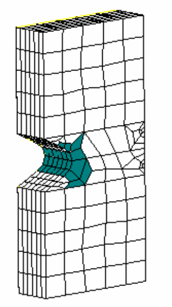
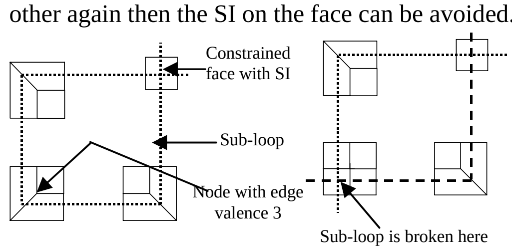


Figure 13. Constrained faces (shaded ones)

For any particular SI, the dual loop can be considered as a combination of two sub-loops.

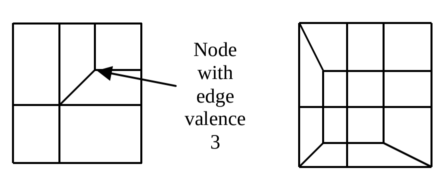
If the mesh can be modified in such a way that both of these sub-loops are broken, then the SI on the original face might be avoided. In Fig. 14(a), one sub-loop and the face with SI is shown. Fig. 14(b) shows the modification of SM, which breaks the loop. The modification is made in the region of the face (on the loop), which has node with edge valence 3. Such modification on the other sub-loop has also to be done. If the broken links do not connect each other again then the SI on the face can be avoided.



(a) A sub-loop with SI (b) Modified SM

Figure 14. Modified SM to avoid SI in a particular face

Fig. 15 (a) shows an example of the original mesh around the node with edge valence equal to 3, selected to modify the mesh to break the sub-loop. The total change of mesh due to new face insertion and also to resolve connectivity problem is shown in Fig. 15 (b). If appropriate region is found for applying this operation, then the self-intersection can be successfully avoided.



(a)Original mesh around node with edge valence 3 (b) Modified mesh

Figure 15. Meshing to resolve connectivity

The surface mesh quadrilaterals having self-intersection and placed on geometric edges and corners (Fig. 13) are not allowed to collapse as these collapsing cause distortions of the geometry. If such faces are near stress concentration zone, then can be considered as constrained.

Application of template

The dotted lines in Fig. 16 shows the shape of the template, which is applied on a face to remove the self-intersection it has. The original face has the shape represented by the solid lines. The dotted lines show that the original face is divided into 12 new faces. Hannemann[11] first proposed this particular type of template to apply on all the faces having self-intersection.

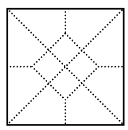
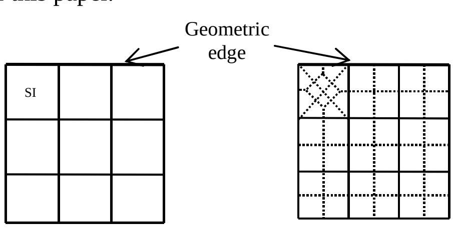


Figure 16. Template

When face collapsing is not possible, only then the proposed method applies template on all the faces having self-intersection and other faces (without SI) of the mesh are divided into four. These two operations perform four important tasks. These are a) removal of all the remaining self-intersections, b) resolving connectivity problems due to template application, c) keeping the sizes of the faces as even as possible and d) also keeping the final number of quadrilaterals in the mesh even. In Figure 17(a), the face labeled as SI has self-intersection but cannot be collapsed as it is on the geometric edge. The dotted lines in Fig. 17(b) show the changes to be done to the faces of the mesh. One face is templated and the others are divided into four for the reasons discussed above. The advantages of this proposed technique over Hannemann's method are discussed in later part of this paper.



(a) Face collapsing not possible (b) Effect of templating

Figure 17. Example of template application

PROCEDURE OF APPLICATION OF SM MODIFICATION STEPS

The method of application of the surface mesh modification steps is proposed here. Fig. 18 shows the flow chart of the surface mesh modification procedure.

The faces of SM on geometric edges are never allowed to collapse. Depending on the geometry of the model and the region of interest for finite element analysis, some faces can be constrained by the user for both face collapsing and templating. In this paper the constrained faces means faces constrained for both collapsing and template application.

Face collapsing technique is applied on the unconstrained faces. The positive collapsing is applied first. In this way, with a few number of collapsing, lots of SI (including SI on constrained faces) is possible to remove.

If still self-intersection is present on constrained faces, new face generation step is applied. Face collapsing is applied next without collapsing the newly generated faces (in step 2) as well as the constrained faces. This loop of new face generation and face collapsing is continued until all the self-intersections in the constrained faces are possible to remove.

After removing SI by using face collapsing and new face generation step, if self-intersection still exists then template is applied on all faces having self-intersection, and all the remaining faces (not having self-intersections) are divided into four. If there is no self-intersection then only subdivision of faces is performed. If template application is not needed and if the mesh has even number of faces then subdivision of faces could be avoided if needed.

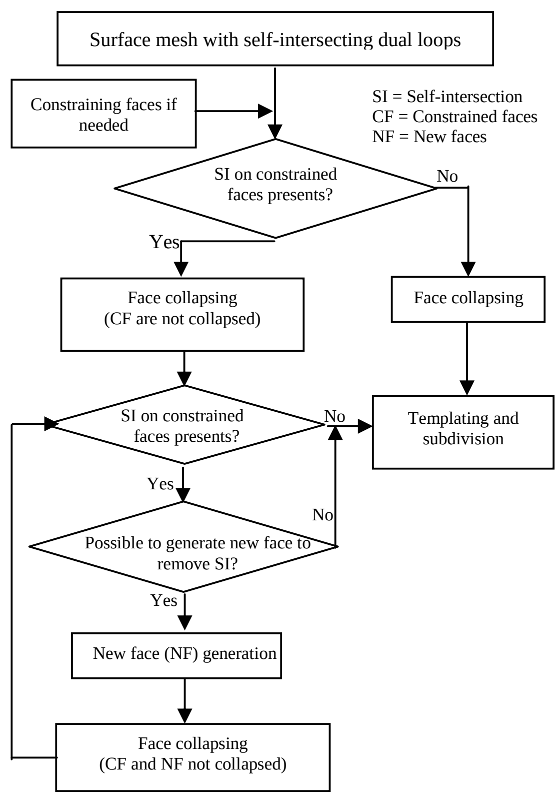


Figure 18. Flow-chart of surface mesh modification

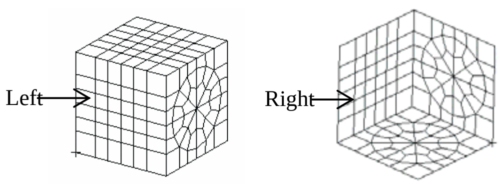
The face collapsing step can use either face quality checking or edge valence checking method. The final output found is even numbered uniform surface mesh, which has dual loops without any self-intersections.

The output (modified surface mesh) will have approximately four times the original number of face elements. So the SM with faces four times (lengthwise two times) larger than the final mesh size should be provided as input.

RESULTS AND DISCUSSION

A number of models of different shape and surface mesh are tested with the proposed method of SM modification.

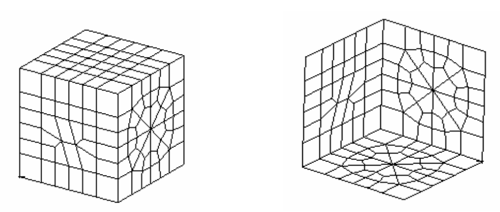
Model 1: At the center of this model's three surfaces, circular mesh is made. For this asymmetrical surface mesh, 4 self-intersections occur. Fig. 19 (a) shows meshes on top, front and left surfaces of the model and Fig. 19 (b) shows the meshes on the other three surfaces. The original mesh has 240 faces.



(a) Top, front and left (b) Right, back and bottom

Figure 19. Mesh on model-1

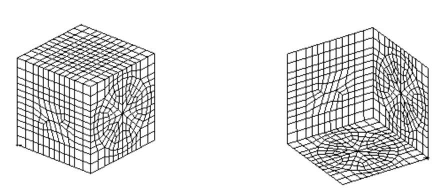
Face collapsing with face quality checking procedure collapse 2 faces each by positive and negative collapsing. At the end of this step the total number of faces left is 236. Face collapsing with edge valence checking produces the same result. As no self-intersection is left to remove with templating and the number of faces is also even, subdivision of mesh is optional. Fig. 20 shows the model with modified surface mesh when subdivision is not applied.



(a) Top, front and left (b) Right, back and bottom

Figure 20. Modified mesh on model-1(no subdivision)

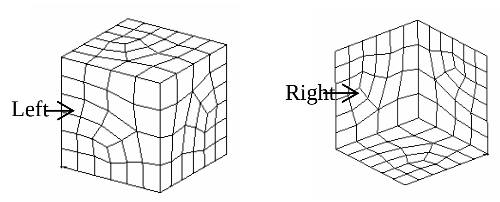
Fig. 21 shows the model when subdivision is applied. After subdivision the number of faces in SM is 944.



(a) Top, front and left (b) Right, back and bottom

Figure 21. Modified mesh on model-1 (after subdivision)

Model 2: The second model is a block having surface mesh consisting of 156 quadrilateral elements. This mesh produces 30 self-intersections in its dual cycles. Fig. 22 shows the original SM.



(a) Top, front and left (b) Right, back and bottom

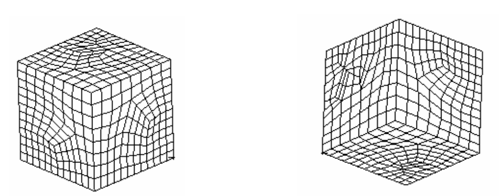
Figure 22. Mesh on model-2

Using face collapsing with face quality checking method, positive collapsing collapses 6 faces, which removes all 30 self-intersections. Fig. 23 shows the table where the effect of each face collapsing on the number of total SI of the SM is presented. The data shows each positive collapsing causing the removal of a number of other SI automatically.

Type of collapsing	Number of faces in SM	Number of SI
	156 (Original)	30
Positive	155	23
Positive	154	14
Positive	153	11
Positive	152	6
Positive	151	3
Positive	150	0

Figure 23. Effect of positive face collapsing on total SI

At the end of the process, 150 faces remain. The modified mesh is shown in Fig. 24 where subdivision is applied. This mesh has a total of 600 faces. Face collapsing with edge valence checking procedure produces the same result.



(a) Top, front and left (b) Right, back and bottom

Figure 24. Modified mesh on model-2

Model 3: This model has 574 faces and 4 self-intersections. Fig. 25 shows the model with original SM.

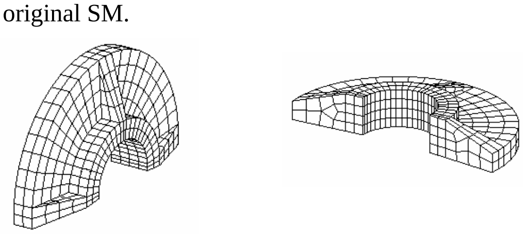


Figure 25. Original mesh on model-3

Face collapsing with face quality checking, causes one positive collapsing, which removes three SI. The remaining SI is removed by applying template. The final mesh is shown in Fig. 26 and it has 2300 faces. Face collapsing with edge valence checking produces same result.

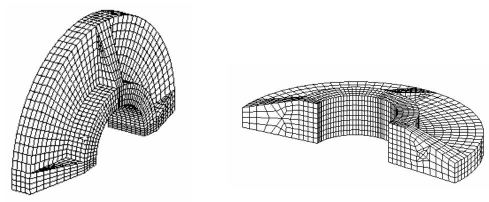


Figure 26. Modified mesh on model-3

Model 4: This model is an example of having extrusion. Fig. 27 shows the top and bottom view of the surface mesh of the model. The surface mesh contains 184 quadrilaterals. 4 of these have self-intersections. Template is applied on only one quadrilateral.

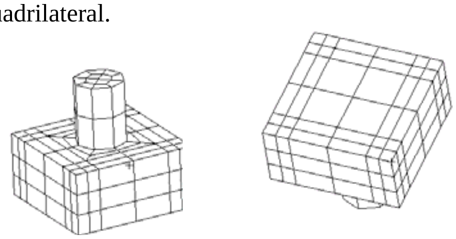


Figure 27. Original mesh on model-4

The modified surface mesh is shown in Fig. 28. This mesh contains 732 quadrilaterals.

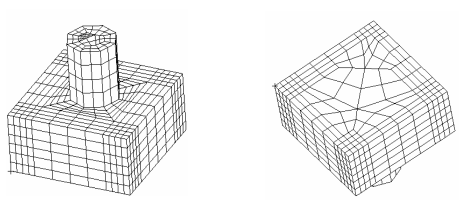
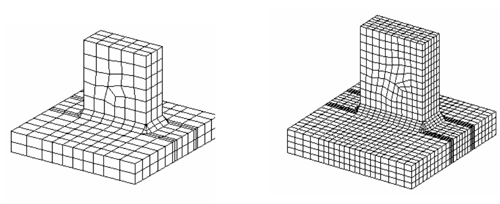


Figure 28. Modified mesh on model-4 (edge valence checking)

Model 5: All the sides of the vertical plate of the next model are connected to the base. Fig. 29(a) shows the model. It contains 516 nodes and 514 surface quadrilaterals.

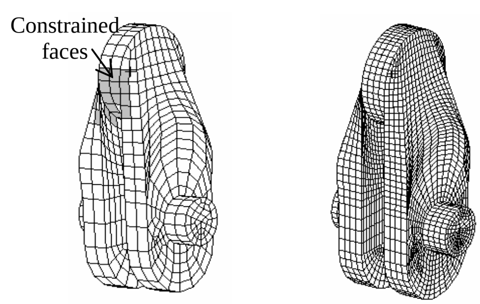


(a) Original mesh (b) Modified mesh

Figure 29. Mesh on model-5

Of these, 4 quadrilaterals have self-intersection. After removing the self-intersections and applying subdivisions, the modified surface mesh shown in Fig 29(b) contains 2050 nodes and 2048 surface quadrilaterals.

Model 6: This model is a part of a crankshaft. The original surface mesh has 1008 quadrilateral elements and 84 of them have self-intersections. The reasons of forming SI are the presence of cylindrical extrusions in the lower parts of the model where SM is made asymmetric. Constraints are applied on the faces near the neck of the model to avoid face collapsing and templating there. The original mesh is shown in Fig. 30(a). By collapsing (face quality checking) 13 unconstrained faces it is possible to remove 71 SI. The remaining 13 SI are removed by templating. It is noted that none of the constrained faces are templated by the proposed method. The modified mesh has 4084 faces (Fig. 30(b)). Face collapsing with edge valence checking produces similar result.



(a) Original mesh (b) Modified mesh

Figure 30. Mesh on model-6

Model 7: The next model has 2584 quadrilateral elements and 88 of these have self-intersection. Some of these self-intersections take place near the inner edge of the model. Fig. 31 (a) shows the model with original SM. Some faces in the region shown in Fig. 31 (b) are constrained (shaded faces) to avoid distortion of the final SM.

It is possible to remove 87 SI by collapsing only 18 unconstrained faces in the first step. The remaining SI is in a constrained face. If step 2 (new face generation) is not applied, the mesh pattern in the region of interest is shown in Fig. 32(a).

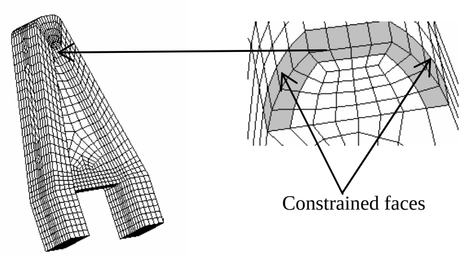
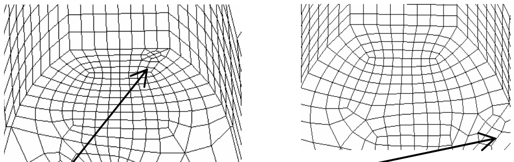


Figure 31. Original mesh on model-7

If step 2 is applied after the first step, then it is possible to avoid the SI in the constrained face and for that, templating is needed on 6 unconstrained faces. Fig. 32(b) shows the region of interest with the modified SM when step 2 is applied.



(a) Step 2 not applied (b) Step 2 applied

Figure 32. Modified mesh on model-7

This example shows that the proposed new face generation method can effectively remove self-intersection from a face. Face collapsing with face quality checking produces similar result.

DISCUSSION ON RESULTS

Face collapsing operation can use edge valence checking or face quality checking. Sometimes these two methods generate different results. A particular face collapsing, which is not allowed by one method but allowed by the other, is the reason for this difference. The advantage of getting two results instead of one (converged) is that the better one can be chosen. By changing value of the acceptable maximum and minimum angles in face quality checking, it is possible to get same results as edge valence checking procedure. So the developed technique of face quality checking is maneuverable and an effective method.

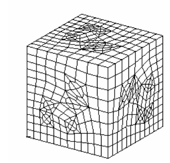
If self-intersection is not possible to remove by face collapsing, application of template is the only available solution. Generating fewer templates is desirable for better quality mesh. The new face generation step cannot always provide solution, as appropriate condition must be reached to apply this step. Constraining a large number of faces compared to the total mesh size should be avoided to get the best result.

COMPARATIVE STUDY

The surface mesh modification technique proposed in this study is effective and reliable. Here the advantage of this method over the conventional methods of surface mesh modification is discussed.

Hannemann[11] proposed a method to remove self-intersection, which is similar to the step three (template application and subdivision) proposed in this paper. It is seen that to remove self-intersection by only that step, template has to be applied on all the faces having self-intersection, which degrades the quality of the surface mesh considerably and constrained condition cannot be applied also. Fig. 33 shows the result found by Hannemann's method of the model 2 (template applied on all 30 self-intersections) of Fig. 22. The proposed method produces far better result (distortion factor 22.1) and is shown in Fig. 24.

Three elements have distortion factor more than 105



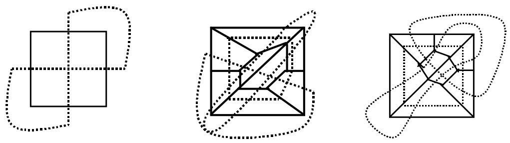
Average distortion factor is 36.9

Figure 33. The result by Hannemann's method

Folwell and Mitchell[10] proposed a method of removing SI by face collapsing but the present method provides a better and more detailed study. The present study proposes the checking procedures (face quality and edge valence) to allow face collapsing only when it results acceptable mesh quality. Moreover, the presented technique proposes a new idea about constraining faces, which avoids collapsing and templating in user-defined areas. Present method provides an output surface mesh, which has even sized elements. This technique can also guarantee even number of quadrilateral elements, which is a basic requirement for hexahedral mesh generation even if the original surface mesh has odd number of surface quadrilaterals.

Egorova[12] et al. proposed a method to produce surface mesh without self-intersection. From given geometry of the model, this method decomposes the surface of the model into a number of triangular or quadrilateral polygons. The number of nodes on each boundary edge is given as input and using this information, the polygons are surface meshed by pre-defined templates of quadrilaterals. Then by a heuristic method the number of nodes on each edge is modified to produce dual cycles without self-intersection. At present, very reliable and good quality commercial surface mesh generators are available and in use for practical/industrial application. The developed mesh modification technique can effectively work with these mesh generators and needs a very little change in the original mesh produced by these tools to remove self-intersection. Thus a good quality surface mesh is found in almost all cases by our proposed technique. This is the advantage of this proposed method over the method of Egorova et al.

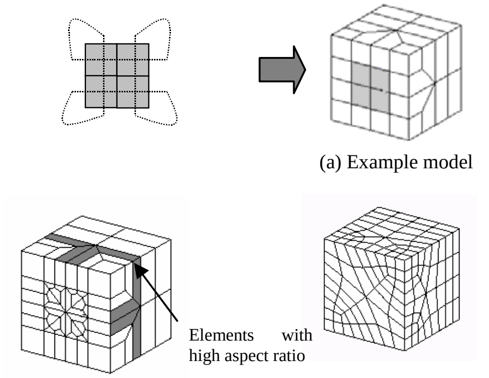
Wenjie[4] et al. presented a study, which states that if any dual sheet contains even number of self-intersections, it is possible to generate hexahedral mesh by making particular connectivity among the self-intersected quadrilaterals. It is discussed previously with Fig. 5 that column of hexahedrons has to be arranged between such connected faces and this will necessitate decomposing the domain. As whisker weaving is intended to mesh large structures without decomposition, it is intended in this study to remove all self-intersections of the surface mesh. Wenjie et al. mentioned that removing self-intersection by face collapsing causes distortion of geometry. In the present study, as face collapsing is totally avoided on the quadrilaterals on geometric edges, it is possible to successfully avoid such kind of distortion. Fig. 34 (a) and (b) show two templates proposed by Wenjie et al.



(a) (b)

Figure 34. Templates of Wenjie et al.

Removing self-intersection only by using template needs all the faces having self-intersection to be templated which degrades the quality of surface mesh. The situation worsens when some of theses faces (with SI) are located nearby. Fig. 35 (a) shows such an example model, which has self-intersection on four neighbor quads (shaded quads of Fig. 35 (a)). After investigation it is found that the first template proposed by Wenjie et al. is practically impossible to apply. If the second template is applied, the mesh becomes like Fig. 35 (b), which has average distortion factor 37.62. It is also seen that elements with high aspect ratio are generated (shaded) as this template causes the other elements (not having SI) on the same dual loop to be divided into two in only one direction. The proposed new method is applied on the same model of Fig. 35(a) and the mesh result is shown in Fig. 35 (c). Average distortion factor found is only 19.1, which is far better from the result found by using Wenjie et al. proposed template.

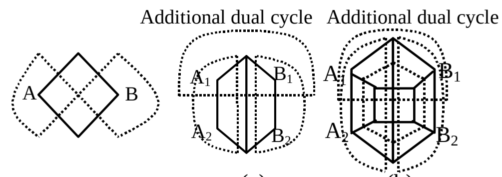


(a) Example model

(b) 2nd template of Wenjie et al. (c) Applying new method

Figure 35. Comparison between new method and the method of Wenjie et al.

Wenjie et al. also proposed structure insertion to remove self-intersection. Fig. 36 (a) shows if a face has self-intersection, and if no edge of the face is on geometric edge, then node A and B can be opened to edges (A to A_1 and A_2 and B to B_1 and B_2). The effect of this change on the neighbor faces is examined and seen that this method also generates elements with high aspect ratio and also a lot of bad quality elements generate when applied to models like Fig. 35 (a) (when some self-intersecting faces are located nearby).



(a) (b)

Figure 36. Structure insertion method proposed by Wenjie et al.

ADVANTAGES OF THE PROPOSED STUDY

1. The proposed method of modifying surface mesh for removing self-intersection is a combined method of face collapsing and template application. As a result it is possible to get advantages of both of these techniques and also to avoid the disadvantages associated with these.
2. This study first proposes the idea of constraining faces to avoid collapsing and templating a particular face. This idea is especially important for stress concentration zone and also for common surface shared by two decomposed domains where same surface mesh is the requirement for merging the domains. Positive collapsing and new face generation technique are proposed in this paper to remove self-intersections from these constrained faces.
3. After removing all self-intersection, this method subdivides all faces of the surface mesh. This makes the quality of final surface mesh even sized (better aspect ratio), which is not possible by using the conventional method[4,11]. This action also reduces the distortion factor.
4. The presented method is applied to a number of complicated and practical problems and the results achieved are quite satisfactory.
5. A fully automatic computer program (object oriented C++) is developed on the basis of this proposed method, which proves the effectiveness of the proposal.
6. The time required for this surface modification technique is negligible.

The whisker weaving based plastering algorithm[3] has several benefits over the conventional whisker weaving technique[2]. These are summarized as follows:

1. As soon as a set of whiskers is weaved a hexahedron is generated by the proposed method. So it is possible to check the quality of it.
2. Even before the generation of a hexahedron, the quality of it can be assumed from the angles between the faces associated with the whiskers. Using this advantage it is possible to skip any particular weaving to avoid forming bad shaped hexahedrons.
3. All the steps in whisker weaving can be followed or checked in real 3-D space, which is very important for improving the rules of whisker weaving.
4. As the database contains actual nodal coordinates, the post processing work like smoothing also becomes easier to apply.
5. This technique has no disadvantages over the

conventional whisker weaving.
6. Primal construction algorithm[2] is not needed.

CONCLUSION

A total HM generation procedure is proposed in this paper. A detailed discussion with figure is presented on how knife elements generate. The reasons are also explained why post processing of knife elements are not adopted. Finally to prevent the formation of knife elements in the HM, a surface mesh modification method is presented. This method has three steps. The first step collapses the surface quadrilaterals to remove self-intersections. This collapsing is carried out such that no unacceptable elements are formed. For this, two types of checking procedure are proposed. The second step breaks of the dual loops to avoid self-intersection by forming new faces. The third and final step does two works. First it applies templates on all the faces having self-intersections (if present), and finally it divides all other faces into four. The third step, in this way, not only removes the self-intersection and resolve connectivity problem but also keeps the number of faces even, which is an essential condition for hexahedral mesh generation. All these steps use smoothing operation to keep the whole process more applicable and to produce a surface mesh with good quality faces. Moreover, a detailed comparative study is made with works of other researchers, which proves that the proposed technique is effective and better. The output surface mesh is ready to be used in HM generation by whisker weaving based plastering algorithm[3] which eliminates the possibility of the generation of knife element. The advantages of whisker weaving based plastering algorithm over conventional whisker weaving algorithm is also provided.

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