VISCOUS DISSIPATION EFFECTS ON MHD NATURAL CONVECTION FLOW ALONG A SPHERE

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Abstract: *In this paper, we describe the viscous dissipation effects in magnetohydrodynamic (MHD) natural convection flow on a sphere. The natural convection laminar flow from a sphere immersed in a viscous incompressible optically thin fluid in the presence of magnetic field has been investigated. The governing boundary layer equations are first transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations are then solved numerically using a very efficient finite-difference method with Keller-box scheme. Here we have focused our attention on the evolution of shear stress in terms of the local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity profiles as well as temperature profiles for some selected parameters consisting of magnetic parameter M, viscous dissipation parameter N and the Prandlt number Pr.*

Keywords: *Viscous dissipation, magnetohydrodynamics, natural convection, Nusselt number.*

INTRODUCTION

A study of the flow of electrically conducting fluid in presence of magnetic field is important from the technical point of view and such types of problems have received much attention by many researches. The specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to the surface. The surface is maintained at a uniform temperature T_w, which may either exceed the ambient temperature T_∞ or may be less than T_∞. With $T_w > T_\infty$, an upward flow is established along the surface due to free convection; while with $T_w < T_\infty$, there is a downward flow. Additionally, a magnetic field of strength β_0 acts normal to the surface. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force, which tends to oppose the fluid motioning edge. The velocity is very small so that the magnetic force, which is proportional to the magnitude of the longitudinal velocity and acts in the opposite direction, is also very small. Consequently, the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from the free stream pressure gradient. Kuiken[1] studied the problem of magnetohydrodynamic free convection in a strong cross field. Also the effect of magnetic field on the free convection heat transfer has been studied by Sparrow and Cess[2]. MHD free convection flows of visco-elastic fluid past an infinite porous plate considered by Chowdhury and Islam[3]. Raptis and Kafousias[4] have investigated the problem of MHD free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Elbashbeshy[5] also discussed the effect of free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field. But Hossain[6] introduced the viscous and Joule heating effects on MHD-free convection flow with variable plate temperature. Moreover, Hossain *et al*.[7,9] discussed both forced and free convection boundary layer flows of an electrically conducting fluid in presence of magnetic field.

Molla *et al.*[10] investigated the MHD natural convection flow on a sphere in presence of heat generation. The problems of free convection boundary layer flow over or on bodies of various shapes discussed by many researchers. Amongst them, Nazar *et al*.[11, 12] considered the free convection boundary layer on an isothermal sphere and on an isothermal horizontal circular cylinder in a micropolar fluid. To our best of knowledge, viscous dissipation effects on MHD free convection flow from an isothermal sphere has not been studied yet.

Nomenclature

C_p	: Specific heat at constant pressure.
C_{fX}	: Local skin friction coefficient.
f	: Dimensionless stream function
g	: Acceleration due to gravity
Gr	: The local Grashof number.
N	: Viscous dissipation parameter
Nu_X	: The local Nusselt number coefficient.
Pr	: Prandtl number.
P	: Fluid pressure.
q_w	: Surface heat flux.
T	: Temperature of the fluid.
T_w	: Temperature at the surface.
T_∞	: Temperature of the ambient fluid.
U	: Velocity component in the X-direction.
V	: Velocity component in the Y-direction.
X	: Measured from the leading edge.
Y	: Distance normal to the surface.
x	: The dimensionless coordinate.
y	: The pseudo-similarity variable.

Greek symbols

β	: Co-efficient of volume expansion
β_0	: Magnetic field strength.
ν	: Kinematic viscosity
μ	: Viscosity of the fluid
θ	: Dimensionless temperature
ρ	: Density of the fluid inside the boundary layer.
ψ	: Stream function
σ_0	: The electrical conduction
κ	: Thermal conductivity of the fluid.

The present work considers the natural convection boundary layer flow on a sphere of an electrically conducting and steady viscous incompressible fluid in presence of strong magnetic field. The governing partial differential equations (PDE) are reduced to locally non-similar PDE by adopting appropriate transformations. The transformed boundary layer equations are solved numerically using implicit finite difference scheme together with the Keller box technique. Here, we focused on the evolution of the surface shear stress in terms of the local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity and temperature distributions for a set of parameters consisting of viscous dissipation parameter *N*, magnetic parameter *M* and Prandtl number, *Pr*.

FORMULATION OF THE PROBLEM

Natural convection boundary layer flow on a sphere of an electrically conducting and steady two-dimensional viscous incompressible fluid in presence of strong magnetic field and heat generation is considered. It is assumed that the surface temperature of the sphere, $T_w > T_\infty$, T_∞ being the ambient temperature of the fluid. Under the usual Boussinesq and boundary layer approximation, the basic equations are

$$\frac{\partial}{\partial X}(rU) + \frac{\partial}{\partial Y}(rV) = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \nu\frac{\partial^2 U}{\partial Y^2} + g\beta(T-T_\infty)\sin\left(\frac{X}{a}\right) - \frac{\sigma_0\beta_0^2}{\rho}U \tag{2}$$

$$U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y} = \frac{\kappa}{\rho C_P}\frac{\partial^2 U}{\partial Y^2} + \frac{\nu}{\rho C_P}\left(\frac{\partial U}{\partial Y}\right)^2 \tag{3}$$

The boundary conditions for the equations (2) to (3) are

$$U = V = 0,\ T = T_w \ \ on\ Y = 0$$

$$U \to 0,\ T \to T_\infty \ \ at\ \ Y \to \infty \tag{4}$$

$$r = a\sin\left(\frac{X}{a}\right) \tag{5}$$

where, $r = r(X)$, r is the radial distance from the symmetrical axis to the surface of the sphere, g is the acceleration due to gravity, β is the coefficient of thermal expansion, ν is the kinematics viscosity, T is the local temperature, C_p is the specific heat at constant pressure, ρ is the density, σ_0 is the electrical conduction and Pr is the Prandtl number. To non-dimensionalise the above equations, the following dimensionless variables are introduced:

$$x = \frac{X}{a},\ \ y = G_r^{1/4}\frac{Y}{a},\ u = \frac{a}{\nu}G_r^{-1/2}U,$$

$$v = \frac{a}{\nu}G_r^{-1/4}V,\ \ \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

Where $Gr = g\beta(T_w - T_\infty)a^3/\nu^2$ is the Grashof number and θ is the non-dimensional temperature. Thus equation(5) becomes, $r = a \sin x$ (7)

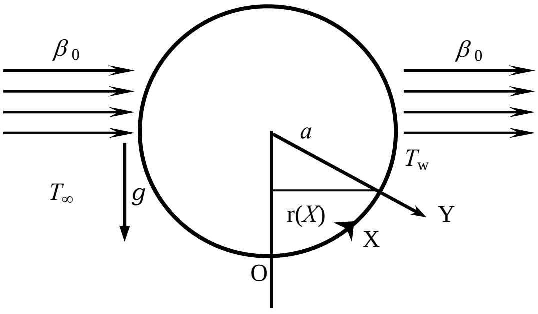

Fig. 1: Physical model and coordinate system

Using the above values, equations (1) to (3) take the following forms:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta\sin x - \frac{\sigma_0\beta^2 a^2}{\rho\nu Gr^{1/2}}u \tag{9}$$

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2\theta}{\partial y^2} + \frac{Gr}{a^2 C_P(T_w - T_\infty)}\left(\frac{\partial u}{\partial y}\right)^2 \tag{10}$$

Where, $M = \left(\sigma_0\beta^2 a^2/\rho\nu Gr^{1/2}\right)$ is the magnetic parameter and $N = Gr/a^2 C_P(T_w - T_\infty)$, is the viscous dissipation parameter. Therefore, momentum and energy equation [eq.(9)-(10)] can be written as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta\sin x - Mu \tag{11}$$

$$u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \frac{1}{p_r}\frac{\partial^2\theta}{\partial y^2} + N\left(\frac{\partial u}{\partial y}\right)^2 \tag{12}$$

The boundary conditions (4) become

$u = v = 0,\ \ \theta = 1$ at $y = 0$

$u \to 0,\ \theta \to 0$ as $y \to \infty$ (13)

To solve equations (11) and (12) subject to the boundary conditions (13), we assume $\psi(x,y) = x\ r(x) f(x,y)$, where $\psi(x,y)$ is a non-dimensional stream function, which is related to the velocity components u and v in the usual way as

$$u = \frac{1}{r}\frac{\partial\psi}{\partial y} \quad \text{and} \quad v = -\frac{1}{r}\frac{\partial\psi}{\partial x} \tag{14}$$

$$u = x(\partial f/\partial y),\ (\partial u/\partial y) = x(\partial^2 f/\partial y^2),$$
$$(\partial^2 u/\partial y^2) = x(\partial^3 f/\partial y^3)$$
$$v = -[(1 + x\cos x/\sin x)f(x,y) + x(\partial f/\partial x)], \tag{15}$$
$$(\partial u/\partial x) = (\partial f/\partial y) + x(\partial^2 f/\partial x\partial y)$$

Using the above transformed values in equations (11) and (12) and simplifying, we have the following:

$$\frac{\partial^3 f}{\partial y^3} + \left(1 + \frac{x}{\sin x}\cos x\right)f\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{\theta}{x}\sin x - M\frac{\partial f}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial y\partial x} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) \tag{16}$$

and

$$\frac{1}{p_r}\frac{\partial^2\theta}{\partial y^2} + \left(1 + \frac{x}{\sin x}\cos x\right)f\frac{\partial\theta}{\partial y} + Nx^2\left(\frac{\partial^2 f}{\partial y^2}\right)^2 = x\left(\frac{\partial f}{\partial y}\frac{\partial\theta}{\partial x} - \frac{\partial f}{\partial x}\frac{\partial\theta}{\partial y}\right) \tag{17}$$

The corresponding boundary conditions are

$f = f'(y) = 0,\ \theta = 1$ at $y = 0$

$f'(y) \to 0,\ \theta \to 0$ as $y \to \infty$ (18)

It has been seen that at the lower stagnation point of the sphere ie $x \approx 0$, using limiting value equations (16) and (17) reduce to the following ordinary differential equations:

$$\frac{d^3 f}{dy^3} + 2f\frac{d^2 f}{dy^2} - \left(\frac{df}{dy}\right)^2 + \theta - M\frac{df}{dy} = 0 \quad (19)$$

and $\frac{1}{p_r}\frac{d^2\theta}{dy^2} + 2f\frac{d\theta}{dy} = 0$ (20)

along with the boundary conditions

$f = f'(y) = 0,\ \theta = 1$ at $y = 0$
$f'(y) \to 0,\ \theta \to 0$ as $y \to \infty$ (21)

In practical application, the physical quantities of principal interest are the heat transfer and the skin- friction coefficient, which can be written in non- dimensional form as

$$Nu_X = \frac{aGr^{-1/4}}{k(T_w - T_\infty)}q_w \quad \text{and} \quad C_{fX} = \frac{Gr^{-3/4}a^2}{\mu\nu}\tau_w \quad (22)$$

Where, $q_w = -k\left(\frac{\partial T}{\partial Y}\right)_{Y=0}$ and $\tau_w = \mu\left(\frac{\partial U}{\partial Y}\right)_{Y=0}$, k being the thermal conductivity of the fluid. Using the new variables (6), we have

$$Nu_X = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} \quad \text{and} \quad C_{fX} = \frac{Gr^{-3/4}a^2}{\mu\nu} \times \frac{\mu\nu}{a^2 Gr^{-3/4}}\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (23)$$

$$\therefore C_{fX} = x\left(\frac{\partial^2 f}{\partial y^2}\right)_{y=0} \quad (24)$$

RESULTS AND DISCUSSION

Here we have investigated the effects of viscous dissipation with magnetohydrodynamic natural convection flow on a sphere. Solutions are obtained for the fluid having Prandtl number Pr = 1.00, 1.74, 2.00, 3.00, viscous dissipation parameter N = 0.10, 0.30, 0.50, 0.70, 1.00 against y at any position of x and for a wide range of values of magnetic parameter M. Also the results for local skin friction coefficient and local rate of heat transfer have been obtained for fluids having Prandtl number Pr = 1.00, 1.74, 2.00, 3.00 at different positions of x for a wide range of values of magnetic parameter M.

Here, it is found that from Fig. 2(a), velocity distribution increases as the values of viscous dissipation parameter N increase in the region $y \in [0, 12]$ but near the surface of the sphere velocity increases significantly and then decreases slowly and finally approaches to zero. The maximum values of the velocity are 0.48450, 0.51282, 0.53527, 0.55384 and 0.56949 for N = 0.10, 0.30, 0.50, 0.70 and 1.00 respectively which occur at y = 1.23788 for the first, second and third maximum values, y = 1.30254 for the fourth and fifth maximum values. Here it is observed that the velocity increases by 17.54179 % as N increases from 0.10 to1.00. From Fig. 2(b), it is seen that when the values of viscous dissipation parameter N increases in the region $0 \le y \le 12$, the temperature distribution also increases. We also observed that the maximum temperature has been found at the surface on the sphere.

The effects of magnetic parameter M for Pr = 0.72 and N = 0.90 on the velocity and temperature profiles are shown in figures 3(a) to 3(b). Figures 3(a) and 3(b) present the effects of magnetic parameter M on the velocity and temperature profiles. From these figures, it is seen that the velocity decreases with the increasing values of magnetic parameter M and the temperature increases with the increasing values of M.

Figs. 4(a) and 4(b) indicate the effects of the Prandtl number Pr with M = 0.50 and N = 0.40 on the velocity and the temperature profiles. From figure 4(a), it is observed that the increasing values of Prandtl number Pr leads to the decrease of the velocity. The maximum values of the velocity are 0.52815, 0.42524, 0.38592 and 0.36155 for Pr = 1.00, 1.74, 2.00 and 3.00, respectively, which occur at y = 1.23788 for the first maximum value and y = 0.99806 for the second, third and fourth maximum values. Here, it is depicted that the velocity decreases by 31.544068 % as Pr increases from 1.00 to 3.00. Again from Fig. 4(b) it is observed that the temperature decreases with the increasing values of Prandtl number, Pr. But near the surface of the sphere temperature is maximum and then decreases away from the surface and finally takes asymptotic value.

The effects of magnetic parameter M for Pr = 0.72 and N = 0.90 on the skin friction coefficient C_{fX} and local heat transfer coefficient Nu_X are shown in figures 5(a) and 5(b). From figures 5(a) and 5(b), it is observed that the increasing values of magnetic parameter M leads to the decrease in the skin friction co-efficient C_{fX} and the local heat transfer co-efficient Nu_X.

Figures 6(a) and 6(b) show that skin friction coefficient C_{fX} and heat transfer coefficient Nu_X decrease for increasing values of Prandtl number Pr while magnetic parameter M = 0.50 and viscous dissipation parameter N = 0.40. The values of skin friction coefficient C_{fX} and Nusselt number Nu_X are recorded to be 0.33987, 0.33486, 0.32018, 0.31108 and 0.27366, 0.25235, 0.15165, 0.02378 for Pr

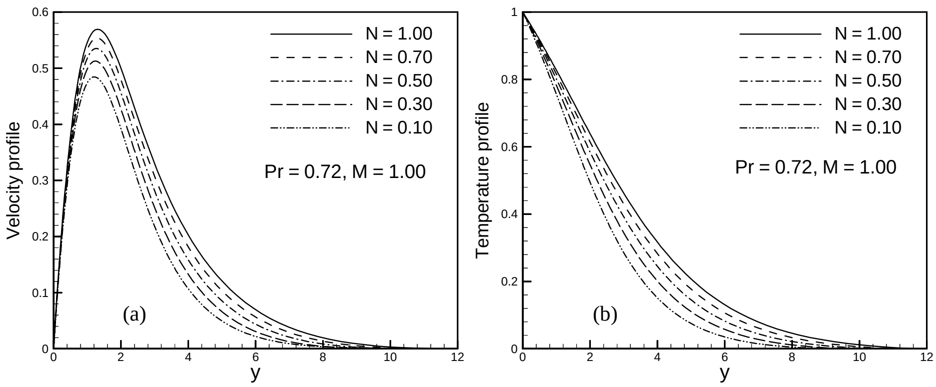

Fig. 2: (a) Velocity and (b) Temperature profiles for different values of viscous dissipation parameter N.

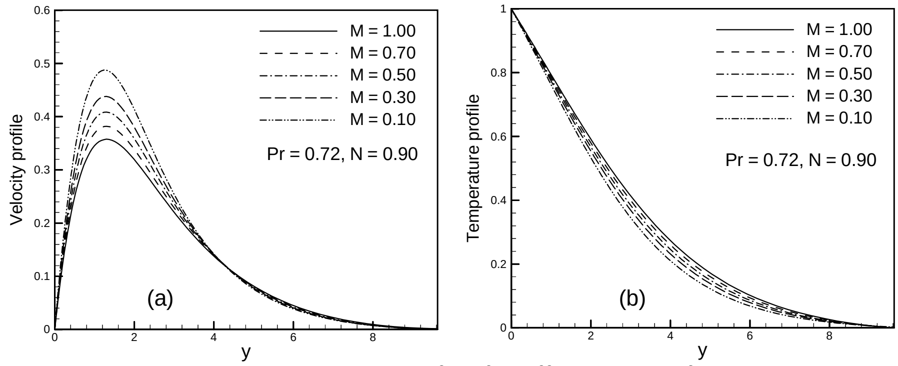

Fig. 3: (a) Velocity and (b) Temperature profiles for different values of magnetic parameter *M*.

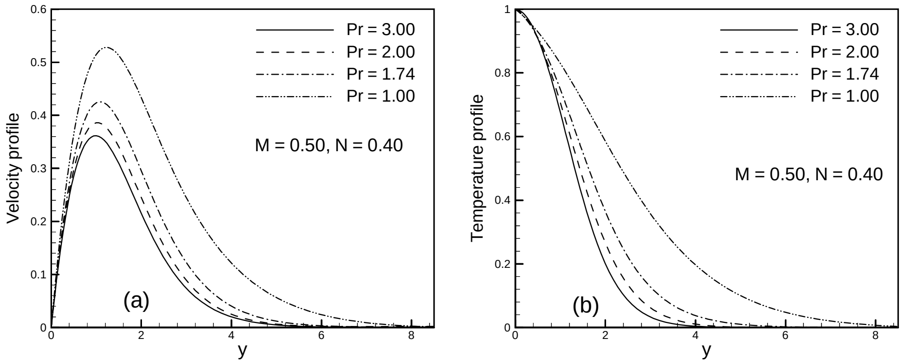

Fig. 4: (a) Velocity and (b) Temperature profiles for different values of Prandtl number *Pr*.

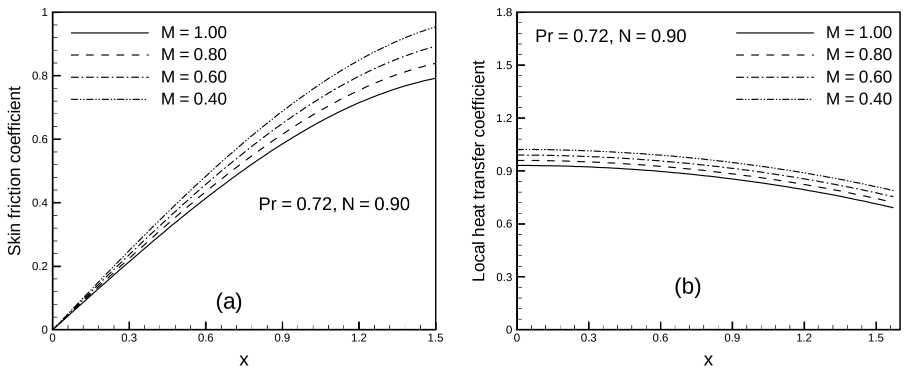

Fig. 5: (a) Skin friction coefficient and (b) Local heat transfer coefficient for different values of magnetic parameter *M*.

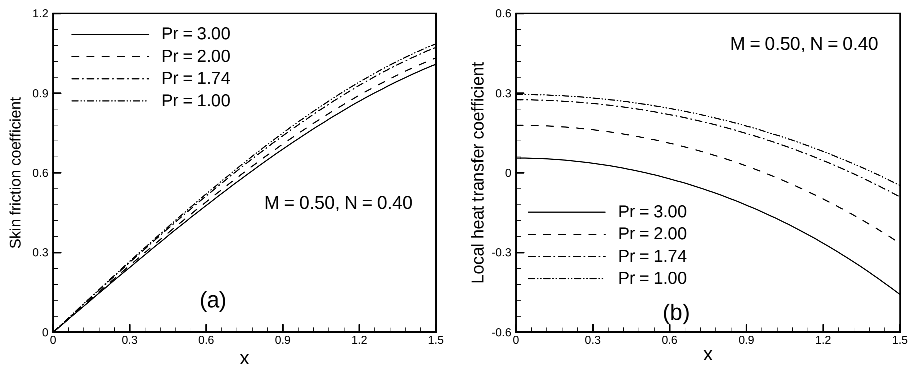

Fig. 6: (a) Skin friction coefficient and (b) Local heat transfer coefficient for different values of Prandtl number *Pr*.

=1.00, 1.74, 2.00, 3.00 respectively which occur at the same point x = 0.38397. Here, it observed that at x = 0.38397, the skin friction coefficient decreases by 8.470885% and Nusselt number Nu_X decreases by 91.310385% as the Prandtl number Pr changes from 1.00 to 3.00.

In Table 1 are given the tabular values of the local skin friction coefficient Cf_x and local Nusselt number Nu_X for different values of magnetic parameter M while Pr = 0.72 and N = 0.90. Here we found that the values of local skin friction coefficient C_{fX} decreases at different position of x for magnetic parameter M = 0.40, 0.60, 0.80, 1.00. The rate of the local skin friction coefficient C_{fX} is decrease by 14.7153% as the magnetic parameter M changes from 0.40 to 1.00 and x = 0.80285. Furthermore, it is seen that the numerical values of the local Nusselt number Nu_X decrease for increasing values of magnetic parameter M. The rate of decrease the local Nusselt number Nu_X is 9.64356% at position x = 0.80285 as the magnetic parameter M changes from 0.40 to 1.00.

CONCLUSIONS

The effect of viscous dissipation in magnetohydrodynamics natural convection flow on a sphere has been investigated for different values of relevant physical parameters. The governing boundary layer equations of motion are transformed into a non-dimensional form and the resulting non-linear systems of partial differential equations are reduced to local non-similarity boundary layer equations, which are solved numerically by using implicit finite difference method together with the Keller-box scheme. From the present investigation the following conclusions may be drawn:

Significant effects of magnetic parameter M on velocity and temperature profiles as well as on skin friction coefficient C_{fX} and the rate of heat transfer Nu_X have been found in this investigation but the effects of magnetic parameter M on the rate of heat transfer is more significant.

An increase in the values of magnetic parameter M causes both the local skin friction coefficient C_{fX} and the local rate of heat transfer Nu_X to decrease, the velocity to decreases but the temperature to increases.

As viscous dissipation parameter N increases, both velocity and temperature increase significantly.

For increasing values of Prandtl number Pr leads to decrease velocity, temperature, local skin friction coefficient C_{fX} and also local rate of heat transfer Nu_X.

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Table 1: Skin friction coefficient and rate of heat transfer against x for different values of magnetic parameter M with Pr = 0.72 and N =0.90.

	M = 0.40		M = 0.60		M = 0.80		M = 1.00	
x	C_{fX}	Nu_X	C_{fX}	Nu_X	C_{fX}	Nu_X	C_{fX}	Nu_X
0.00000	0.00000	1.02248	0.00000	0.99028	0.00000	0.96003	0.00000	0.93160
0.10472	0.08795	1.02141	0.08340	0.98919	0.07933	0.95893	0.07569	0.93045
0.20944	0.17522	1.01838	0.16613	0.98612	0.15800	0.95582	0.15073	0.92731
0.40143	0.33132	1.00768	0.31395	0.97528	0.29842	0.94485	0.28455	0.91625
0.50615	0.41318	0.99902	0.39132	0.96651	0.37181	0.93597	0.35437	0.90730
0.80285	0.62615	0.96344	0.59178	0.93047	0.56121	0.89952	0.53401	0.87053
1.01229	0.75407	0.92818	0.71112	0.89478	0.67310	0.86343	0.63938	0.83415
1.20428	0.85072	0.88812	0.80021	0.85425	0.75569	0.82249	0.71638	0.79291
1.30900	0.89399	0.86301	0.83950	0.82887	0.79161	0.79685	0.74945	0.76711
1.50098	0.95406	0.81068	0.89264	0.77603	0.83900	0.74355	0.79206	0.71350
1.57080	0.96930	0.78954	0.90550	0.75471	0.84993	0.72206	0.80142	0.69192