A HEURISTIC SOLUTION OF MULTI-ITEM SINGLE LEVEL CAPACITATED DYNAMIC LOT-SIZING PROBLEM

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Abstract: *The multi-item single level capacitated dynamic lot-sizing problem consists of scheduling N items over a horizon of T periods. The objective is to minimize the sum of setup and inventory holding costs over the horizon subject to a constraint on total capacity in each period. No backlogging is allowed. Only one machine is available with a fixed capacity in each period. In case of a single item production, an optimal solution algorithm exists. But for multi-item problems, optimal solution algorithms are not available. It has been proved that even the two-item problem with constant capacity is NP (nondeterministic polynomial)-hard. That is, it is in a class of problems that are extremely difficult to solve in a reasonable amount of time. This has called for searching good heuristic solutions. For a multi-item problem, it would be more realistic to consider an upper limit on the lot-size per setup for each item and this could be a very important parameter from practical point of view. The current research work has been directed toward the development of a model for multi-item problem considering this parameter. Based on the model a program has been executed and feasible solutions have been obtained.*

Keywords*: Heuristics, inventory, lot-sizing, multi-item, scheduling.*

INTRODUCTION

The multi-item single level capacitated dynamic lot-sizing problem consists of scheduling N items over a horizon of T periods. Demands are given and should be satisfied without backlogging. The objective is to minimize the sum of setup costs and inventory holding costs over the horizon subject to a constraint on total capacity in each period. Mathematically, the problem can be stated as:

Minimize $$Z(X) = \sum_{i=1}^{N}\sum_{j=1}^{H}(S_i\delta(x_{ij}) + h_i I_{ij})$$

Subject to

$$I_{ij} = I_{i,j-1} + x_{ij} - D_{ij} \qquad i = 1, 2,\dots, N \text{ and } j = 1, 2,\dots, H$$

$$\sum_{i=1}^{N} k_i x_{ij} \le C_j \qquad j = 1, 2, \dots, H$$

$$x_{ij},\ I_{ij} \ge 0 \qquad i = 1, 2, \dots, N \text{ and } j = 1, 2, \dots, H$$

where, N = the number of items, H = the time horizon, D_{ij} = the given demand for item i in period j, I_{ij} = the inventory of item i at the end of period j, x_{ij} = the lot-size of item i in period j, S_i = the setup cost for item i, h_i = the unit holding cost for item i, k_i = the capacity absorption rate for item i, C_j = the capacity in period j and $\delta(x_{ij})$ is a binary setup variable indicating whether a setup cost must be incurred for item i in period j or not.

$$\delta(x_{ij}) = 1 \quad if\ x_{ij} > 0 \quad and \quad 0 \quad if\ x_{ij} = 0$$

A multi-item, multi-echelon inventory problem, with stochastic variables is extremely difficult to solve in a realistic time period, which leads to NP (nondeterministic polynomial) -hardness, quite similar to scheduling problem[1]. Hence, it appears highly unlikely that an efficient optimal algorithm will ever be developed. So the search for a good heuristic method is definitely warranted. As a consequence, many heuristics were developed for this problem. Eisenhut's procedure[2] could be called period-by-period heuristic. His procedure was later extended by many, including Dixon and Silver[3]. Basic assumptions of the Dixon-Silver model are: (i) the requirements for each product are known period by period, (ii) for each product there is a fixed setup cost incurred each time production takes place, (iii) unit production and holding costs are linear, (iv) the time required to set up the machine is negligible, (v) all costs and production rates can vary from product to product but not with respect to time, and (vi) in each period there is a finite amount of machine time available that can vary from period to period. The objective is to determine lot-sizes so that (i) costs are minimized, (ii) no backlogging occurs, and (iii) capacity is not exceeded. It would be more realistic to assume an upper limit, a maximum value of the lot-size from a machine. This restriction may be imposed per setup and this could be a very important parameter from practical point of view for several reasons. Situations like (1) machine's inability to run continuously, and (2) machine may not be available for indefinite period for a particular product, (3) there may be storage limitation for WIP inventory can be considered in this regard. The current research work has thus been directed toward an extension of the Dixon-Silver model considering the above mentioned situation. It is to be noted that Dixon-Silver heuristic allows only one setup for each item in each period. But the limitation on lot-size may need more than one setup in a particular period. So should this limitation be incorporated into Dixon-Silver heuristic, each time an item processed in a new setup is to be considered a new item. This may call for splitting an item into several new items in a particular period. However, the maximum number of the new splitted items will be restricted by the maximum periodical demand of the item. Let the maximum periodic demand and the limited lot-size for the ith item be $d_{\max i}$ and $x_{\max i}$, respectively. Then the number of new items for the ith item will be $n_i = d_{\max i} / x_{\max i} - 1$. Thus the total number of new items will be $\sum_{i=1}^{N} n_i$. So after meeting the lot-size limitation, the total number of items to be considered in the model should be .

$$N' = N + \sum_{i=1}^{N} n_i$$

In view of the above discussions, the model may now be presented as follows.

Mathematical Model:

Minimize $Z(X) = \sum_{i=1}^{N'} \sum_{j=1}^{H} (S_i \delta(x_{ij}) + h_i I_{ij})$

Subject to

$I_{ij} = I_{i,j-1} + x_{ij} - D_{ij}$ $\quad i = 1, 2, \ldots, N'$ and $j = 1, 2, \ldots, H$

$I_{i0} = I_{iH} = 0$ $\quad i = 1, 2, \ldots, N'$

$\sum_{i=1}^{N'} k_i x_{ij} \le C_j$ $\quad j = 1, \ldots, H$

$0 \le x_{ij} \le x_{\max i.}$ $\quad i = 1, 2, \ldots, N'$ and $j = 1, 2, \ldots, H$

$I_{ij} \ge 0$ $\quad i = 1, 2, \ldots, N'$ and $j = 1, 2, \ldots, H.$

The unit production cost is assumed to be constant for each item. Therefore, the total production cost (excluding setup costs) will be a constant and hence is not included in the model. If initial inventory exists, or if positive ending inventory is desired, then the net requirements should be determined. That is, use the initial inventory to satisfy as much demand as possible in the first few periods. The net requirements, will be that demand not satisfied by the initial inventory. Hence, an equivalent problem is created with zero starting inventory. Now increase the demand in the last period, *H*, by the desired ending inventory. Now the equivalent problem satisfies the starting and ending inventory constraints.

THE STRUCTURE OF THE HEURISTIC

For a detailed statement of the algorithm the reader is referred to the original publication by Dixon and Silver[3]. Several other mathematical models have been developed to solve these types of NP-hard inventory problem, those are computationally harder and thus require more time in information processing. Often, they become near NP-hard problem, with global search options[4-6]. This research thus concentrates on basic Dixon-silver heuristic. The purpose of this section is to outline the structure of the heuristic.

The Lot-Sizing Technique

Dixon-Silver heuristic is period-by-period heuristic which is unidirectional in that they proceed by constructing a schedule period by period, starting with period 1. To determine which production lots should be scheduled in each period a priority indices is used. Consider a period *R* in the process: one certainly has to produce max{0, $d_{iR} - I_{i,R-1}$} for all *i* in order to avoid stock outs in the current period. The remaining capacity (if any) can be used to produce future demands for some future setup costs may be saved at the expense of some added inventory holding costs. Consider items which need a setup in the current period (i.e., $d_{iR} > I_{i,R-1}$). Priority indices which indicate the viability of producing future demands for these items in the current period are then computed. A very simple priority index for the next period's demand would be ($S_i - h_i d_{i,R+1}$).

The actual priority indices (U_i) used by the heuristic are more sophisticated in that they try to capture potential savings per time period. In fact they are derived from well-known heuristics for the single level uncapacitated dynamic lot-sizing problem, e.g., the Silver-Meal criterion[7].

In any case, future demands are included into the current production lot based on the priority index in a greedy fashion until either no lots with a positive index remain or until the capacity constraint is hit. The heuristics then proceed to the next period and the process is repeated.

Ensuring Feasibility

If the total capacity demanded exceeds the capacity available in some period, then some or all of the requirements of that period must be satisfied by production in preceding periods and by such pre-production the infeasibility can be removed.

Consider the determination of lot-sizes in period *R*. Let AP_j be the amount of production (in capacity units) in period *R* that will be used in future period *j*. If $I'_{i,j}$ is the inventory at the end of period *j* for item *i* which is resulted from only the currently scheduled production in period *R*, then

$$AP_j = \sum_{i=1}^{N} k_i \left(I'_{i,j-t} - I'_{i,j} \right) \tag{1}$$

Let CR_j be the total demand (in capacity units) in period *j*. Then

$$CR_j = \sum_{i=1}^{N} k_i \, d_{ij} \tag{2}$$

The production plan for period *R* is feasible if and only if the following condition is satisfied for $t = 2, \ldots, H$.

$$\sum_{j=R+1}^{R+t-1} AP_j \ge \sum_{j=R+1}^{R+t-1} (CR_j - C_j) \quad , \tag{3}$$

where C_j is the capacity in period *j*. That is, the production in period *R* for periods *R*+1 to *R*+*t*–1 must exceed the total amount that demand exceeds capacity in those periods, and this must be the case for all *t*. This set of constraints can be used to guide the selection of which time supplies to increase. It is now the case though that a lot-size may be forced to be increased when $U_i < 0$. Furthermore, it may be necessary to schedule lots which do not exactly satisfy an integer number of periods' requirements. A simple approach to rectifying this difficulty is to increase the lot-sizes until the feasibility conditions are satisfied, while minimizing the additional costs incurred.

Implementation of the Heuristic

The original multi-item problem with constant capacity is NP-hard. In the present work a new constraint on upper limit of the lot-size is considered. With this new constraint the problem is also NP-hard. Therefore, a simple heuristic has been developed which guarantees a feasible solution.

Step 1 Create an equivalent demand matrix.

- Convert the initial demand matrix into equivalent demand matrix with the use of initial inventory, ending inventory and safety stock.
- Use the initial inventory to satisfy as much demand as possible in the first few periods. The net requirements will be that demand not satisfied by the initial inventory. During the calculation of the net demands, the amount of the safety stock should be maintained.

Let Iin_i = initial inventory for item *i*,
$Iend_i$ = ending inventory for item *i*,
$Irem_i$ = remaining initial inventory for item *i*,
SS_i = safety stock for item *i*, and
d_{ij} = equivalent demand for product *i* in period *j*.

Initially set $Irem_i = Iin_i - SS_i$ and period $j = 1$.

Then set $d_{ij} = \begin{cases} 0 & \text{if } Irem_i > D_{ij} \\ D_{ij} - Irem_i & \text{if } Irem_i \le D_{ij} \end{cases}.$

Compute $Irem_i = Irem_i - D_{ij}$. Set $j = j + 1$ and recycle till $Irem_i > 0$.

- Since the amount of the safety stock is always maintained, the demand in the last period H would be partially satisfied by the safety stock of the period H-1. If ending inventory is desired, then the requirements in period H should be increased by the desired ending inventory. Then $d_{iH} = D_{iH} + Iend_i - SS_i$.
- Compute the net demands for all $i = 1, 2, \ldots, N$.

Step 2 Check the feasibility of the problem.

Feasibility Condition: . $\sum_{j=1}^{H} CR_j \le \sum_{j=1}^{H} C_j$

If the feasibility condition is not satisfied, the problem is infeasible, i.e., all demands cannot be met with the available capacity.

Step 3 Convert the multi-setup problem into single setup problem.

Step 3.1

- Find the maximum demand $d_{\max i}$ for each item i by $d_{\max i} = \max\{d_{ij} \mid j = 1, 2, \ldots, H\}$.
- Find the number of new items n_i to be considered to satisfy demand $d_{\max i}$ using the formula $n_i = d_{\max i} / x_{\max i} - 1$. Then the number of total items after limiting the lot-size is $N' = N + \sum_{i=1}^{N} n_i$. Item i is splitted into n_i + 1 items. Let the new items be. i_0, i_1 ,.i_{n_i}. Initially set $d_{\text{rem } ij} = d_{ij}$ and $l = 0$.

Then set . $d_{i_l, j} = \begin{cases} d_{rem\ ij} & if\ d_{rem\ ij} \le x_{\max\ i} \\ x_{\max\ i} & if\ d_{rem\ ij} > x_{\max\ i} \end{cases}$

Compute . $d_{rem\ ij} = \begin{cases} 0 & if\ d_{rem\ ij} \le x_{\max\ i} \\ d_{rem\ ij} - x_{\max\ i} & if\ d_{rem\ ij} > x_{\max\ i} \end{cases}$

Set $l = l + 1$ and recycle up to $l = n_i$. Now the equivalent demand matrix $N \times H$ is converted into a new demand matrix $N' \times H$.

Step 3.2

- Initialize the values of setup cost, holding cost and capacity absorption rate for the N' new items from that of the N items by using the formulas $S_{i_0} = S_{i_1} = \ldots = S_{i_{ni}} = S_i$, $h_{i_0} = h_{i_1} = \ldots = h_{i_{ni}} = h_i$ and $k_{i_0} = k_{i_1} = \ldots = k_{i_{ni}} = k_i$.

Step 4 Apply the heuristic with inclusion of the limited lot-size per setup [through Steps 4.1 to 4.12]

Step 4.1

- Start at period 1, i.e., set $R = 1$.

Step 4.2

- Initialize lot-size x_{ij} by equalizing to demand d_{ij} , i.e., set $x_{ij} = d_{ij}$ $i = 1, 2, \ldots, N'$ and $j = 1, 2, \ldots, H$.
- Calculate remaining allowable amount $x_{\text{rem } ij}$ that can be produced if x_{ij} is produced at period j by $x_{\text{rem } ij} = x_{\max i} - x_{ij}$ $i = 1, 2, \ldots, N'$ and $j = 1, 2, \ldots, H$.

Step 4.3

- Initially set the value of time supply to one i.e. $T_i = 1$, where $i = 1, 2, \ldots, N'$. Time supply T_i denote the integer number of periods requirements that this lot will exactly satisfy.

Step 4.4

- For each item i, $i = 1, 2, \ldots, N'$, produce d_{iR} (> 0) in the lot-sizing period R.
- After producing d_{iR} calculate remaining capacity in period R, denoted by, RC_R by

$$RC_R = C_R - \sum_{i=1}^{N'} k_i\, d_{iR}$$

- Initialize I'_{ij} with zero, i.e., set $I'_{ij} = 0$, $i = 1, 2, \ldots, N'$ and $j = 1, 2, \ldots, H$.

Step 4.5

- Calculate AP_j and CR_j by the following formulas

$$AP_j = \sum_{i=1}^{N'} k_i \left(I'_{i,j-1} - I'_{i,j}\right) \quad \text{and} \quad CR_j = \sum_{i=1}^{N'} k_i\, d_{ij} \ .$$

- Determine the earliest period t_c at which the feasibility constraint (3) is not satisfied, i.e., set

$$t_c = \min\left\{ t \mid \sum_{j=R+1}^{R+t-1} AP_j < \sum_{j=R+1}^{R+t-1} (CR_j - C_j) \right\} .$$

If there is no infeasibility, set $t_c = H + 1$.

Step 4.6

- Consider only items i' with (1) $T_{i'} < t_c$, (2) $x_{can} > 0$, where $x_{can} = \min\{d_{i',R+T_{i'}}, x_{rem\ i'R}\}$ and (3) RC_R is sufficient to produce x_{can}. Among these find the item i that has the largest U_i, where

$$U_i = \frac{AC(T_i) - AC(T_i + 1)}{k_i d_{i,T_i+1}}$$

and

$$AC(T_i) = \left\{ S_i + h_i \sum_{j=R}^{R+T_i-1} (j - R) d_{ij} \right\} / T_i \ .$$

Step 4.7

(a) If $U_i > 0$, then it is economic to produce x_{can} in period R. Increase the value of lot-size, x_{iR} inventory I'_{ij} for $j = R+1, \ldots, R+T_i$, and $x_{\text{rem } i,R+Ti}$ by x_{can}. Decrease the value of lot-size $x_{i,R+T_i}$, demand $d_{i,R+T_i}$, remaining capacity RC_R and $x_{\text{rem } iR}$ by x_{can}. Set $T_i = T_i + 1$ and continue from Step 4.5.

(b) If $U_i \le 0$, then it is not economic to increase T_i of any item (total cost increases).

- Check the value of t_c.

(i) If $t_c > H$, then no infeasibilities left and lot-sizing of the current period is complete. Go to Step 4.12.

(ii) If $t_c < H$, there are infeasibilities and production of one or more item is to be increased and it is done through Steps 4.8 to 4.11.

Step 4.8

- Calculate the amount of production Q still needed in the current period to eliminate infeasibilities in the later period by the following formula

$$Q = \max_{R+t_c-1 \le t \le H} \left[\sum_{j=R+1}^{t} \left(CR_j - C_j - AP_j\right) \right] \ .$$

Step 4.9

- Consider only items i' with (1) $T_{i'} < t_c$ (2) $x_{can} > 0$, where $x_{can} = \min\{d_{i',R+T_{i'}}, x_{rem\ i'R}\}$ and (3) RC_R is sufficient to produce x_{can}. To decide the best item (from a cost standpoint) to be produced in period R, calculate the priority index $\Delta_{i'}$ for all of these items, where

$$\Delta_{i'} = \frac{AC(T_{i'} + 1) - AC(T_{i'})}{k_{i'} d_{i',T_{i'}+1}} \ .$$

- Among these find the one, denoted by i, that has the smallest Δ_i .

Steps 4.10

- Let $W = k_i x_{can}$.
- **If** $Q > W$ **then**

Increase the value of lot-size x_{iR} , inventory I'_{ij} for $j = R+1, \ldots, R+T_i$, and $x_{rem\ i',R+T_i}$ by x_{can}. Decrease the value of lot-size $x_{i,R+T_i}$,demand $d_{i,R+T_i}$,remaining capacity RC_R

and $x_{\text{rem } iR}$ by x_{can}. Set $Q = Q - W$ and $T_i = T_i + 1$, and continue from Step 4.9.

else

Set $IQ = [Q/K_i]$. Increase the value of lot-size, x_{iR} inventory I'_{ij} for $j = R+1, \ldots, R+T_i$, and $x_{rem\ i',R+T_i}$ by IQ. Decrease the value of lot-size $x_{i,R+T_i}$, demand $d_{i,R+T_i}$ and $x_{rem\ i'R}$ by IQ.

Step 4.11

- Set $R = R + 1$.

(a) If $R < H$, then continue from Step 4.3.

(b) If $R > H$, lot-sizing is complete up to period H for N' items.

Step 4.12

- Convert the $N' \times H$ lot-sizing matrix into $N \times H$ lot-sizing matrix by applying the formula $x_{i,j} = \sum_{l=0}^{n_i} x_{i_l,j}$.

Step 5

- Calculate the values of
  - i. Forecasted machine time required/period.
  - ii. Total expected setup cost.
  - iii. Total expected inventory holding cost.
  - iv. Total expected safety stock cost.
- Stop.

RESULTS WITH THE LIMITED LOT-SIZE PER SETUP

The algorithm developed to generate feasible solution for multi-item single level capacitated lot-sizing problem with limited lot-size was tested in PC version with of a programming language. A near optimal solution was obtained. This section presents the results obtained from the modified model.

The algorithm has been tested with hypothetical data. It is assumed that entire production to meet demands is done in the plant and no subcontracting is permissible. Moreover, a further assumption is made that plant capacity could not be increased.

Product Data

The relevant product data (e.g., holding cost, setup cost, production rate, safety stock, initial inventory and ending inventory) has been depicted in Table 1. The problem size has been restricted at 12 products and 12 time periods; each time period corresponds to a month.

Product Demand and Plant Capacity

Product demands are quite seasonal and the same seasonal indices are used for all the products. Forecasted demand and the capacity of the machine are shown in Table 2. It has been assumed that the capacity per month is the total number of hours available per month. Two percent of the capacity is reserved as a buffer to guard against uncertainty in the actual production rate. In this hypothetical problem, Period 1 corresponds to the month of June, Period 2 corresponds to the month of July. Thus the machine capacity in Period 1 is 98% of the total hours in June, i.e., 30 × 24 × 0.98 = 706 hours. To be in the safe side, it has been assumed that the number of days in February is 28. Then the machine capacity in Period 9 is 28 × 24 × 0.98 = 660 hours. Similarly the machine capacity for the other periods has been calculated.

Table 1: Relevant product data for the hypothetical machine.

Item No (i)	Holding Cost (h_i)	Setup Cost (S_i)	Maximum Lot-Size ($x_{\max i}$)	Production Rate ($1/k_i$)	Safety Stock (SS_i)	Initial Inventory (Iin_i)	Ending Inventory ($Iend_i$)
01	0.0167	322.0	6000	524	0	19320	18893
02	0.0167	81.0	60000	349	10602	200180	124225
03	0.0167	124.0	68000	245	4577	24460	43294
04	0.0167	124.0	29000	172	1974	23260	21757
05	0.0167	81.0	49000	349	7581	55489	92168
06	0.0167	124.0	68000	245	4861	-2727	44394
07	0.0167	124.0	44000	172	2026	9659	8466
08	0.0167	105.0	41000	847	11117	29705	40273
09	0.0167	105.0	32000	464	9533	11362	84717
10	0.0167	106.0	185000	575	20417	242944	227344
11	0.0167	105.0	150000	1261	16634	324215	271627
12	0.0167	105.0	97000	663	9794	45439	69068

Table 2: Forecasted demand and capacity of the hypothetical machine.

Item	Period											
No	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
01	11456	11456	10501	13365	13365	11456	8592	1909	1909	1909	4773	4773
02	53124	53124	48697	61977	61977	53124	39842	8854	8854	8854	22135	22135
03	18099	18099	16591	21116	21116	18099	13574	3016	3016	3016	7541	7541
04	9250	9250	8480	10792	10792	9250	6938	1542	1542	1542	3854	3854
05	39546	39546	36250	46137	46137	39546	29659	6591	6591	6591	16478	16478
06	18363	18363	16833	21423	21423	18363	13772	3060	3060	3060	7651	7651
07	4976	4976	4562	5806	5806	4976	3732	829	829	829	2074	2074
08	41690	41690	38216	48638	48638	41690	31267	6948	6948	6948	17371	17371
09	32816	32816	30081	38285	38285	32816	24612	5469	5469	5469	13673	13673
10	96745	96745	88683	112868	112868	96745	72559	16124	16124	16124	40310	40310
11	119220	119220	109285	139088	139088	119220	89415	19870	19870	19870	49675	49675
12	27715	27715	25405	32333	32333	27715	20786	4619	4619	4619	11548	11548
	Available Machine Hours											
	706	729	729	706	729	706	729	729	660	729	706	729

Table 3: Equivalent demand with the use of initial inventory, ending inventory and safety stock.

Item No	Period											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
01	0	3592	10501	13365	13365	11456	8592	1909	1909	1909	4773	23666
02	0	0	0	27344	61977	53124	39842	8854	8854	8854	2135	135758
03	0	16315	16591	21116	21116	18099	13574	3016	3016	3016	7541	46258
04	0	0	5694	10792	10792	9250	6938	1542	1542	1542	3854	23637
05	0	31184	36250	46137	46137	39546	29659	6591	6591	6591	16478	101065
06	25951	18363	16833	21423	21423	18363	13772	3060	3060	3060	7651	47184
07	0	2319	4562	5806	5806	4976	3732	829	829	829	2074	8514
08	23102	41690	38216	48638	48638	41690	31267	6948	6948	6948	7371	46527
09	30987	32816	30081	38285	38285	32816	24612	5469	5469	5469	13673	88857
10	0	0	59646	112868	112868	96745	72559	16124	16124	6124	0310	247237
11	0	0	40144	139088	139088	119220	89415	19870	19870	19870	9675	304668
12	0	19785	25405	32333	32333	27715	20786	4619	4619	4619	1548	70822

Table 4: Final lot-sizes and forecasted machine time requirements for the heuristic with the limited lot size per setup.

Item No	Period											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
01	6000	12000	14730	12000	7549	0	8592	6000	0	23666	4500	0
02	0	0	27344	1977	60000	53124	39842	26562	0	75758	22135	60000
03	32906	0	21116	21472	17743	0	13574	16589	0	0	46258	0
04	0	29000	0	0	7528	0	8480	6938	0	0	23637	0
05	31184	36250	49000	43274	0	39546	29659	36251	0	3065	15866	82134
06	44314	38256	0	21423	2041	16322	13772	3060	60955	0	0	0
07	12687	0	0	0	10782	0	4561	0	12246	0	0	0
08	41690	41000	56966	20318	41000	41000	31267	41000	0	43742	0	0
09	32816	32000	45386	32000	32000	29068	24612	32000	0	86937	0	0
10	0	59646	7205	105663	112868	96745	72559	88682	0	48215	14022	185000
11	0	40144	150000	128176	0	119220	89415	109285	0	4668	300000	0
12	45190	0	64666	0	27715	0	25405	20786	0	70822	0	0
	Forecasted Machine Requirements (hours)											
	677.9	704.5	727.1	706.0	729.0	706.0	728.4	701.6	320.0	704.4	706.0	729.0

Table 5: Inventories at the end of each period for all items.

Item No	Period											
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
01	13864	14408	18637	17272	11456	0	0	4091	2182	23939	23666	18893
02	147056	93932	72579	12579	10602	10602	10602	28310	19456	86360	86360	124225
03	39267	21168	25693	26049	22676	4577	4577	18150	15134	12118	50835	43294
04	14010	33760	25280	14488	11224	1974	3516	8912	7370	5828	25611	21757
05	47127	43831	56581	53718	7581	7581	7581	37241	30650	27124	26512	92168
06	23224	43117	26284	26284	6902	4861	4861	4861	62756	59696	52045	44394
07	17370	12394	7832	2026	7002	2026	2855	2026	13443	12614	10540	8466
08	29705	29015	47765	19445	11807	11117	11117	45169	38221	75015	57644	40273
09	11362	10546	25851	19566	13281	9533	9533	36064	30595	112063	98390	84717
10	146199	109100	27622	20417	20417	20417	20417	92975	76851	108942	82654	227344
11	204995	125919	166634	155722	16634	16634	16634	106049	86179	70977	321302	271627
12	62914	35199	74460	42127	37509	9794	14413	30580	25961	92164	80616	69068

Equivalent Demand Schedule

Table 3 depicts the equivalent demand after considering initial inventory, ending inventory and safety stock.

Results of the Heuristic

Table 4 shows the final lot-sizes and forecasted machine hour requirements for each period, and Table 5 shows the inventories at the end of each period for all items. The following results have also been found after applying the heuristic with the limited lot-size per setup.

Total available machine time ($\sum_{t=1}^{H} C_t$) : 8587.0 hour
Total setup time : 0 hour
Total forecasted machine time : 8139.8 hour

Total inventory holding cost, $C_{inv} = \sum_{i=1}^{N}\sum_{t=1}^{H}(I_{it} - SS_i)$: \$ 83162.35

Total expected safety-stock cost, $C_{SS} = \sum_{i=1}^{N} SS_i$: \$ 19862.85

Total expected setup cost, $C_{set} = \sum_{i=1}^{N} n_i S_i$: \$ 15733.00

Total expected cost ($C_{inv} + C_{SS} + C_{set}$) : \$ 118758.20

Effect of the limitation on the lot-size is dependent on the extent of reduction of the lot-size. It is obvious that the smaller the allowable lot-size, the greater will be the number of setup which will eventually lead to more splitted items. Thus when the lot-size was reduced by 90%, the

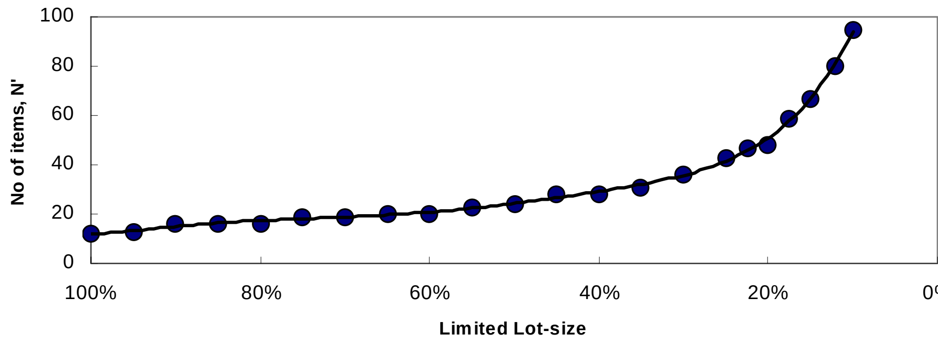

Figure 1: The growth rate of number of items with the limited lot-size.

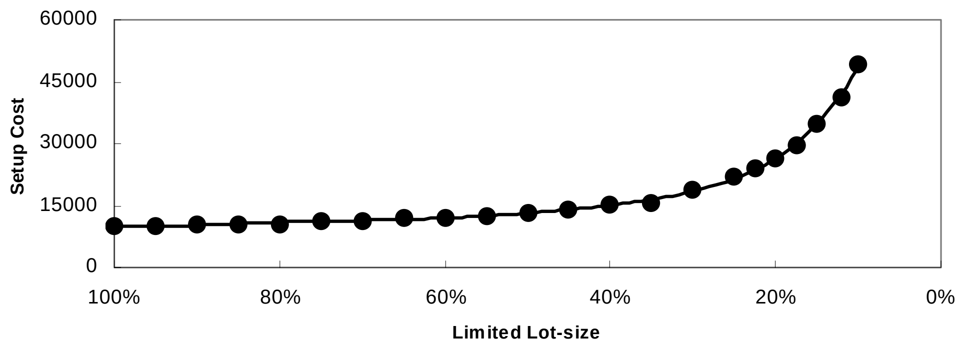

Figure 2: The variation of setup cost with the limited lot-size.

model yielded the total number of splitted items of 95 from the original twelve items. This in turn led to the increase number of required setups.

Costs due to implementation of this restriction on lot-size went up quite significantly- the extent of which was found to be more than 23%. Further decrease in lot-size would obviously result in higher costs. But at the lower range of allowable lot-size, there has been a trend of slight increase in setup costs.

To see the effect of the limited lot-size to different parameters, the first value of the limited lot-size of each item has been chosen as shown below. These values have been chosen so that the number of total items after limiting the lot-size remains unchanged and a little decrease in these values will increase the number of total items.

Item No	Maximum Lot-Size
01	40000
02	150000
03	70000
04	40000
05	130000
06	50000
07	9000
08	90000
09	150000
10	250000
11	400000
12	90000

Next the value of the limited lot-size of each item is reduced step by step. With the variation of the limited lot-size, the change of the values of the number of total items, the machine utilization time, total inventory cost, total setup cost, total safety stock cost and total cost has been shown in the following figures.

Figure 1 shows the growth rate of number of items as a function of the limited lot-size. This growth rate is increasing with the decrease of the limited lot-size. The decrease in the limited lot-size decreases the amount of production quantity per setup of an item. This decrease in production quantity results in an increase in the number of items.

Figure 2 shows the variation of setup cost with the limited lot-size. With the decrease of the limited lot-size, the setup cost increases significantly. If the limited lot-size per setup is decreased, then the number of setup needed is increased accordingly. Therefore the setup cost is also increased.

Figure 3 shows the variation of total inventory holding cost with the limited lot-size. With the decrease of the limited lot-size, the variation of the total inventory holding cost is fluctuating. This nature of the variation needs to be more investigation.

Figure 4 shows the variation of total cost with the limited lot-size. With the decrease of the limited lot-size, total cost increases, since the setup cost increases significantly, the inventory holding cost is fluctuating and safety stock cost remains almost unchanged.

CONCLUSION

Lot-sizing problem has been recognized to be one of the most important functions in industrial units. Thus efforts have been given to develop usable optimizing routines but within limited boundary conditions. Various models have been developed with restricted applications in real-life settings because of their demanding computational enormity. Thus heuristic models have been evolved. These heuristics produce optimal and near optimal

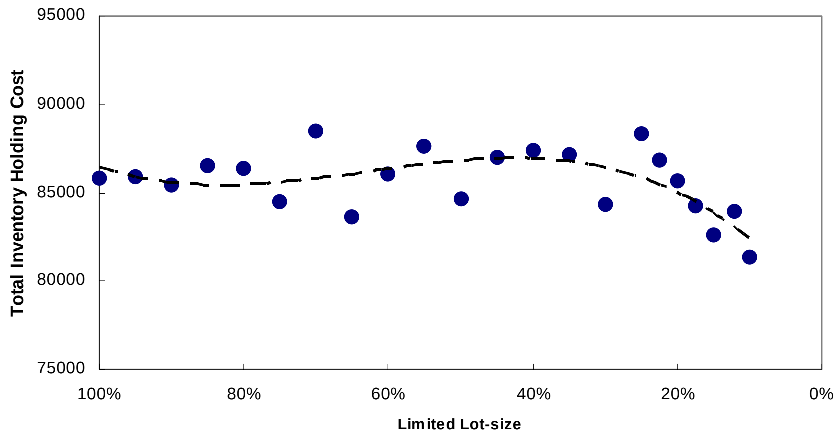

Figure 3: The variation of total inventory holding cost with the limited lot-size.

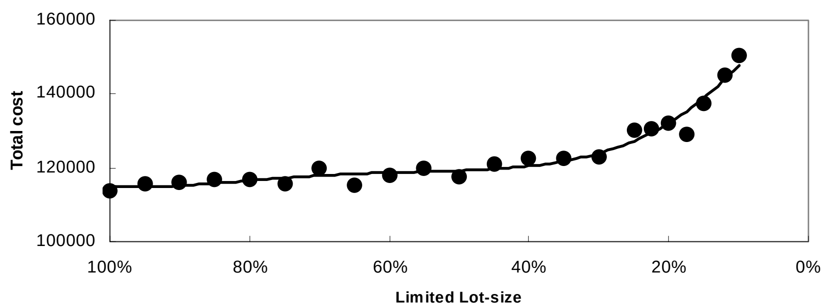

Figure 4: The variation of total cost with the limited lot-size.

solutions. In the present work the Dixon-Silver heuristic was extended to include a very important parameter, maximum limit of production lot-size from a machine. From analysis and results, the present work has demonstrated that feasible solutions could be obtained with competitive computer usage to a realistic lot-sizing problem. The heuristic is based on a lot-sizing technique and a set of feasibility conditions which should be intuitively appealing to managers. This paper has been concerned with a single stage process. Extension of the heuristic for multiple production stages could be a significant contribution.

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