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EFFECT OF INPLANE LOADING ON SOUND RADIATION OF A FLOATING RUNWAY WHEN AN AIRPLANE IS TAKING OFF

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Abstract:

This work is focused on effect of inplane loading on sound generated by an airplane taking off from a floating runway. Assuming runway as a simple, infinitely long beam, modeled as a Timoshenko-Mindlin plate floating on water, an expression for sound radiation incorporating inplane loading is developed, for a wave number ratio of 0.1 to 2.2, to study effect of varying take off speeds and inplane loading. In developing this expression, a Fourier transformation in space in wave number domain is utilized rather than using wave propagation method to reduce analysis to a substructure.

Keywords: Moving load; Timoshenko-Mindlin plate; floating runway; inplane loading.

NOMENCLATURE

ξ	Wave number variable	C_0	Sound speed in the acoustic medium, m / s
γ	Wave number ratio	E	Elastic modulus, N / m^2
ν	Poisson's ratio	G	Complex shear modulus
ρ_v	Mass density of the material, kg / m^3	$H(x)$	Heavy side step function
ρ_0	Mass density of the acoustic medium, kg / m^3	I	The cross sectional moment of inertia per unit width
κ^2	Cross sectional shape factor or the shear correction factor	K_0	Acoustic wave number
ζ	Non dimensional wave number variable	K_B	Free bending wave number
α_0	Fluid loading parameter	M	Mach number, non-dimensional number
$\delta(x-Vt)$	Delta function	P	Sound pressure on the beam surface
f_0	Strength of external force per unit width, Nm / s	Q	Compressive inplane loading
h	Height of the beam, m	V	Subsonic speed of moving force of length 2L, m / s
$p(x, y=0, t)$	Acoustic pressure acting on the beam's surface	Z_a	Acoustic impedance operator
$u(x,t)$	Transverse displacement of the beam, m	Z_m	Beam impedance operator
C_L	Longitudinal wave speed, m / s		

1. Introduction

Because of their relatively simple construction and ease of maintenance, pontoon-type *Very Large Floating Structures* (VLFS) are considered to be one of the most promising designs for a floating airport or runway,

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particularly in sheltered areas. Sound generated by moving loads on such structures is an area of concern as it causes acoustic pollution for marine life and has not been addressed to date. Effect of berthing, plate connections, initial plate deformation, corrosion, hogging, sagging are some forms of inducing additional loads in form of compression / tension to plating of a floating runway and need to be considered for analyzing their effect on sound radiation by taking off of an airplane from such floating runways.

Study of effect of compression goes back to the time of Bryan (1891) who undertook the first theoretical examination of plate under uniform compression. After his work numerous researchers have investigated local instability in plates under a wide variety of loading and boundary conditions using many different methods of analysis. Excellent textbooks by Timoshenko and Gere (1961) and Bulson (1970) describe main results of these investigations.

Study of sound radiation using beams due to moving loads has been investigated by Keltie and Peng (1989). Results show that for beams under light fluid loading, coincidence sound radiation peak for a stationary force is split into two coincidence peaks due to effects of Doppler shift, while for beams under heavy fluid loading there are no pronounced sound radiation peaks. Following study of Keltie *et al.*, (1999) formulated vibration response of periodically simply supported beam on the whole structure in wave number domain through Fourier transform. This problem was an advance on traditional substructure methods. For an air-loaded beam subjected to a stationary line force, they showed that radiated sound power exhibited peaks at certain wave number ratios. Wave number ratios at which radiation peaks occur nearly coincide with lower bounding wave number ratios of odd number of propagation zones. However, Cheng's formulation did not include presence of numerous wave number components induced from elastic supports and is subject to restriction that external force is located on one of the elastic supports. Cheng *et al*. (2000; 2001) introduced a “wave number harmonic series” to discuss vibro-acoustic response of a fluid-loaded beam on periodic elastic supports subjected to a moving load. Results show that response of a beam on an elastic foundation can be approximated using a periodically, elastically supported beam when support spacing is small compared with flexural wavelength. For such beams when force is stationary a single radiation peak occurs which splits into two peaks due to Doppler shift when force becomes traveling.

Aim of this study is to propose a simple methodology for calculating sound radiation from a floating airport subjected to longitudinal bending due to inplane loading (compression or tension) when subjected to moving loads such as airplanes. An expression for sound radiation for a floating platform is developed for a wave number ratio of 0.1 to 2.2. In developing the expression, Fourier transform methodology is utilized as suggested in Keltie and Peng (1989). To simplify a three-dimensional runway with time varying loading caused by airplane take-off, the runway is assumed to behave as a simple, infinitely long beam supported by buoyancy. Hence the model is assumed to be a simple beam, described by a one dimensional Timoshenko-Mindlin beam equation. A compressive inplane load of magnitude Q per unit width or a tensile inplane load of magnitude $-Q$ per unit width is considered to account for additional loads.

2. Mathematical Formulation

Axially loaded beams are often called beam-columns. To eliminate boundary effect of finite length of beam, these beams are assumed to extend to infinity. Hence we assume that the finite length floating runway behaves as a simple, infinitely long beam floating on water. The geometry and material properties are assumed to be linearly elastic. Structural damping is ignored since there is no apparent resonant mechanism in this problem. Water is assumed to be inviscid, and flow resulting from the airplane take-off is irrotational. The x-axis is aligned with the length of runway and y-axis is directed vertically upwards, as seen in Fig. 1. Because the floating runway is very narrow compared to its length, as a simplification, we assume that deformation and loading assumed do not vary across the runway. The structure is assumed to behave like a beam, described by the one dimensional Timoshenko-Midlin plate equation. An excitation force of length 2L moving at a subsonic speed V is assumed to be acting on the runway. The space $y>0$ is filled with an acoustic medium such as water. Other side of the plate is assumed to be vacuum.

A uniform distributed moving line force considered to be acting on the floating runway is, given by

$$f(x,t) = \frac{f_0}{2L}[H(x - Vt + L) - H(x - Vt - L)]e^{j\omega t}$$

where f_0 is strength of external force per unit width, $H(x)$ is Heavy side step function, and $\delta(x-Vt)$ is a Delta function. We consider a distributed load with a constant advance velocity instead of a point load because moving loads in practice have normally a finite area over which they are distributed and a point load represents only an extreme case.

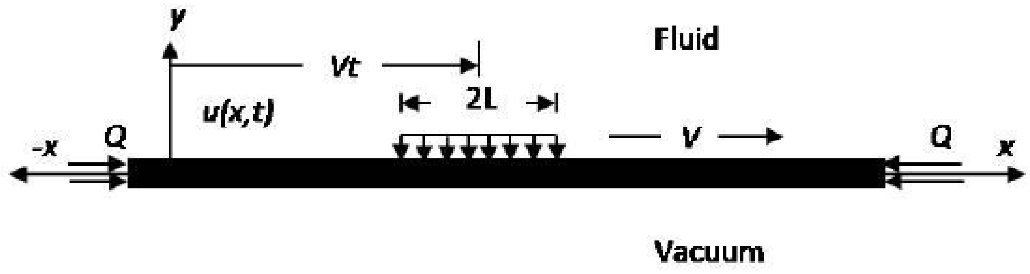


Fig. 1: Schematic representation of problem geometry

A compressive inplane load of magnitude Q per unit width is considered to be present. If the inplane load is tensile then it attains a magnitude -Q. Vibration equation for the one dimensional elastic plate, including rotational inertia, transverse shear effects and inplane loading, is given by the Timoshenko-Mindlin beam equation as

$$D\frac{\partial^4 u(x,t)}{\partial x^4} + Q\frac{\partial^2 u(x,t)}{\partial x^2} + \rho_v h\frac{\partial^2 u(x,t)}{\partial t^2} - \rho_v I\left(1 + \frac{D\rho_v}{\kappa^2 G}\right)\frac{\partial^4 u(x,t)}{\partial x^2 \partial t^2} + \rho_v I\frac{\rho_v}{\kappa^2 G}\frac{\partial^4 u(x,t)}{\partial t^4}$$
$$= \left(1 - \frac{D}{\kappa^2 Gh}\frac{\partial^2}{\partial x^2} + \frac{\rho_v h^2}{12\kappa^2 G}\frac{\partial^2}{\partial t^2}\right)[f(x,t) - p(x,y=0,t)] \quad (1)$$

where $u(x,t)$ is transverse displacement of the plate, $D = Eh^3/12(1-\nu^2)$ is flexural rigidity of the plate, E the elastic modulus, $G = E/2(1+\nu)$ is the shear modulus, $I = h^3/12$ is the cross sectional moment of inertia per unit width, h is height of the plate, ν the Poisson's ratio, ρ_v is mass density of the plate, $\kappa^2 = \pi^2/12$ is cross sectional shape factor or shear correction factor and $p(x, y-0, t)$ is acoustic pressure acting on the plate's surface.

Pressure distribution induced by the vibrating plate in acoustic medium is denoted by $p(x,y,t)$ and satisfies the wave equation in two-dimensional space, given by

$$\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial x^2} - \frac{1}{C_0^2}\frac{\partial^2}{\partial t^2}\right)p(x,y,t) = 0 \quad (2)$$

where C_0 is sound of speed in acoustic medium.

If ρ_0 is mass density of the acoustic medium, the boundary condition at $y = 0$ is given by

$$\rho_0\frac{\partial^2 u}{\partial t^2} = -\frac{\partial p}{\partial y}|_{y=0} \quad (3)$$

By applying spatial Fourier transformation $FT() = \int_{-\infty}^{\infty}()e^{i\xi x}\,dx$, with ξ as the wave number variable, the force function for a harmonic line force in wave number domain may be written as

$$\tilde{F}(\xi,t) = f_0(\frac{\sin\xi L}{\xi L})e^{j(\omega+\xi V)t} \quad (4a)$$

the transformed displacement as

$$\widetilde{U}_s(\xi,t) = U_s(\xi)e^{j(\omega+\xi V)t} \tag{4b}$$

and the transformed pressure as

$$\tilde{P}(\xi,y,t) = P(\xi,y)e^{j(\omega+\xi V)t} \tag{4c}$$

Upon substitution of equation (4a), (4b) and (4c) in the relevant beam equation and the combination of Equations (2) and (3), we get

$$U_S(\xi) = \frac{Z_F F(\xi)}{Z_m + Z_F Z_a} \tag{5}$$

and

$$P(\xi, y=0) = \frac{j\rho_0(\omega+\xi V)^2}{K_y} U_s(\xi) \tag{6}$$

where the acoustic impedance operator (Z_a) is given by

$$Z_a = \frac{j\rho_0(\omega+\xi V)^2}{K_y} \tag{7}$$

the beam impedance operator (Z_m) as

$$Z_m = D\xi^4 - Q\xi^2 - \rho_v h(\omega+\xi V)^2 - \xi^2\left(\rho_v I + \frac{D\rho_v}{\kappa^2 G}\right)(\omega+\xi V)^2 + \rho_v I\frac{\rho_v}{\kappa^2 G}(\omega+\xi V)^4 \tag{8}$$

the Z_F by

$$Z_F = 1 + \frac{D}{\kappa^2 Gh}\xi^2 - \frac{\rho_v h^2}{12\kappa^2 G}(\omega+\xi V)^2 \tag{9}$$

and K_y is given by

$$K_y = \begin{cases} -j\sqrt{\xi^2 - (K_0+M\xi)^2} & for\ \xi^2 > (K_0+M\xi)^2 \\ \sqrt{(K_0+M\xi)^2 - \xi^2} & for\ \xi^2 < (K_0+M\xi)^2 \end{cases} \tag{10}$$

where $M(=V/C_0)$ is the Mach number and $K_0(=\omega/C_0)$ the acoustic wave number.

Since we need to calculate total acoustic power, we first calculate time averaged sound intensity as given by Morse and Ingrad (1986) and then use this sound intensity to calculate the total acoustic power. Hence

$$\bar{I} = \frac{1}{T}\int_0^T \overline{PV}dt \quad or \quad \bar{I} = \frac{1}{2}Re[P\dot{U}_s^*]$$

where $\bar{I}$ is time averaged sound intensity, P is sound pressure on beam surface, $\dot{U}_s^*$ is beam surface velocity of conjugation and $\dot{U}_s = \frac{dU_s(\xi)}{dt} = j(\omega+\xi V)U_s(\xi)$.

To find total acoustic power (Π), surface acoustic intensity distribution is integrated over the infinite length of the beam as

$$\Pi = \int_{-\infty}^{\infty} \frac{1}{2}Re[P(x,y=0,t)\dot{U}_s^*(x,t)]dx$$

Upon substituting sound pressure (6) and surface velocity (5) of the beam, sound power radiated per unit width of the beam can be simplified as

$$\Pi = \frac{\rho_0}{4\pi}Re\left[\int_{-\infty}^{\infty}\frac{(\omega+\xi V)^3}{K_y}|U_s(\xi)|^2\,d\xi\right] \tag{11}$$

Limiting the study to subsonic motion of the moving load, i.e, the taking off airplane, the limits within which K_y is real is given by

$$\xi_1 = \frac{-K_0}{1+M} \le \xi \le \xi_2 = \frac{K_0}{1-M}$$

This allows us to rewrite the expression for sound power as

$$\Pi = \frac{\rho_0}{4\pi} Re\left[\int_{\xi_1}^{\xi_2} \frac{(\omega + \xi V)^3}{K_y} |U_s(\xi)|^2 \, d\xi\right] \tag{12}$$

Equation (12) gives the total acoustic power for a Timoshenko-Mindlin beam subjected to a inplane loading.

In order to present numerical results, concept of non-dimensional parameters as discussed by Keltie and Peng (1989) is used. Hence the following non-dimensional parameters are defined

Wave number variable $\zeta = \frac{Wavenumber \quad variable \quad (\xi)}{Acoustic\ wavenumber \quad (K_0)}$ (13a)

Free bending wave number $[K_B] = \left[\frac{\rho_v h \omega^2}{D}\right]^{\frac{1}{4}}$ (13b)

Wave number ratio $[\gamma] = \frac{K_0}{K_B}$ (13c)

Longitudinal wave speed $[C_L] = \sqrt{\frac{E}{\rho_v}}$ (13d)

Fluid loading parameter $[\alpha_0] = \frac{\rho_0 C_L}{\sqrt{12}\rho_v C_0}$ (13e)

Power per unit width $[W] = \frac{4\pi\omega\,(\rho_v h)^2}{\rho_0 f_0^2}\Pi$ (13f)

Substituting (13) in Equation (12) gives the dimensionless radiated sound power per unit width as

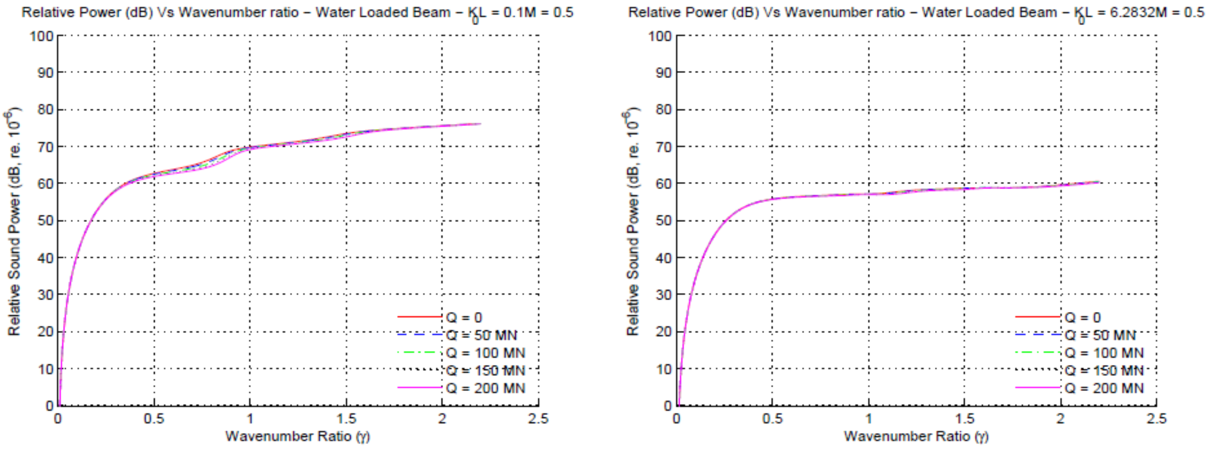
$$W = \int_{\zeta_1}^{\zeta_2} \alpha^3 \beta \left| Z_F \frac{sin(\zeta K_0 L)}{\zeta K_0 L} \right|^2 | D_w |^{-2} \tag{14}$$

where

$$\zeta_1 = \frac{-1}{1+M} \le \zeta \le \zeta_2 = \frac{1}{1-M}$$
$$\alpha = 1 + M\zeta$$
$$\beta = \sqrt{\alpha^2 - \zeta^2}$$
$$D_w = \beta(D_1 - D_2 + D_3 - D_5) + jD_4$$
$$Z_F = 1 + \frac{2(1+\nu)\gamma^4}{\kappa^2}\left(\frac{C_0}{C_L}\right)^2\left[\zeta^2 - \left(\frac{C_0}{C_L}\right)^2 \alpha^2 (1-\nu^2)\right]$$
$$D_1 = \gamma^4 \zeta^4$$
$$D_2 = \alpha^2\left[1 + [1 + \frac{2(1+\nu)}{\kappa^2(1-\nu^2)}]\gamma^4\zeta^2\left(\frac{C_0}{C_L}\right)^2(1-\nu^2)\right]$$
$$D_3 = \frac{2(1+\nu)}{\kappa^2}\alpha^4\gamma^4\left(\frac{C_0}{C_L}\right)^4(1-\nu^2)$$
$$D_4 = Z_F \frac{\alpha_0 \alpha^2}{\gamma^2}$$
$$D_5 = \frac{Q}{\rho_v h}\left(\frac{\zeta^2}{C_0^2}\right)$$

3. Results and Discussions

Investigation of the problem has been undertaken to evaluate total radiated sound power for an inplane loaded Timoshenko-Mindlin beam. Accordingly equation (14) is numerically evaluated for a case wherein the beam is floating on water. Material of beam considered is steel with properties as $E = 20 \times 10^{10}\ N/m^2$, $\rho_v = 7800\ kg/m^3$ (i.e, $D = 560\ KNm$), $h = 2.54 \times 10^{-2}\ m$, $\nu = 0.3$, $\kappa^2 = 0.85$, $C_0 = 1481\ m/s$ and $\rho_0 = 1000\ kg/m$. External force (f_0) is assumed to be of unit magnitude. By varying values of parameters M and $K_0 L$, sound power is computed and then plotted versus wave number ratio (γ) or non-dimensional frequency. Compressive (*Q*) / Tensile (-*Q*) loading is taken as varying between *50 MN* to *200 MN*. Sound power has been calculated for $K_0L = 0.1$ and 2π in frequency range $0.01 < \gamma < 2.2$. All calculations have been undertaken using MATLAB.

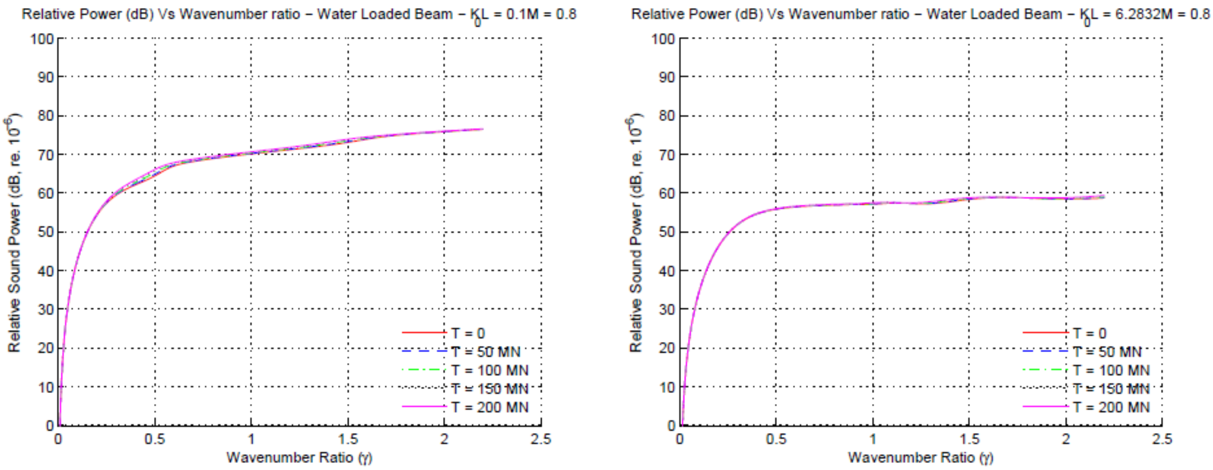


(a) $K_0L = 0.1$ (b) $K_0L = 2\pi$

Fig. 2: Relative sound power v/s wavenumber ratio under Compressive Load; M = 0.5

Sound power generated by moving load on a one dimensional Timoshenko-Mindlin beam model subjected to compressive inplane loading can be seen in Figs. 2(a) and 2(b) and that due to tensile inplane loading in Figs. 3(a) and 3(b). It is observed that for a compressive inplane load, with increased speed, there is a marginal increase in sound power generated, while an increased acoustic length K_0L reduces sound power level over the entire range of frequency range. This happens since total applied force strength remains a constant. Due to a denser medium, like water, wherein energy drain is faster, no pronounced peaks are noticed.

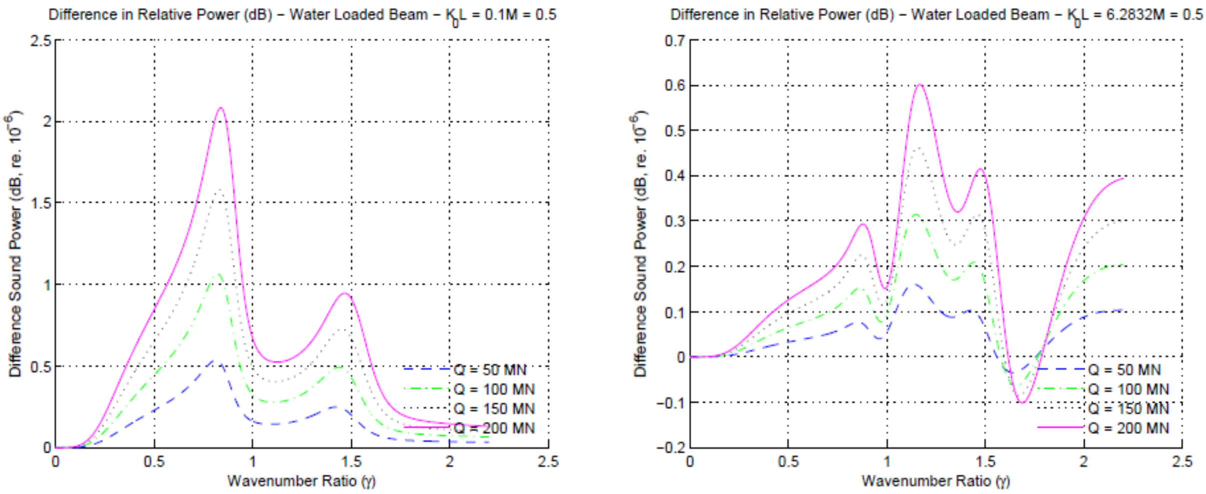
Since a tensile load is considered to be similar to a compressive load but with opposite direction, the effect of tensile load due to increased speed is a marginal decrease in sound power generated. However reduction in sound power level due to increased acoustic length K_0L is observed over the entire range of frequency range. Effect of compressive and tensile inplane loads on a beam can be better understood if one was to imagine a sheet of paper being pulled at the edges defining tensile loading. Sound generated from such a paper may be considered analogous to results observed herein.



(a) $K_0L = 0.1$ (b) $K_0L = 2\pi$

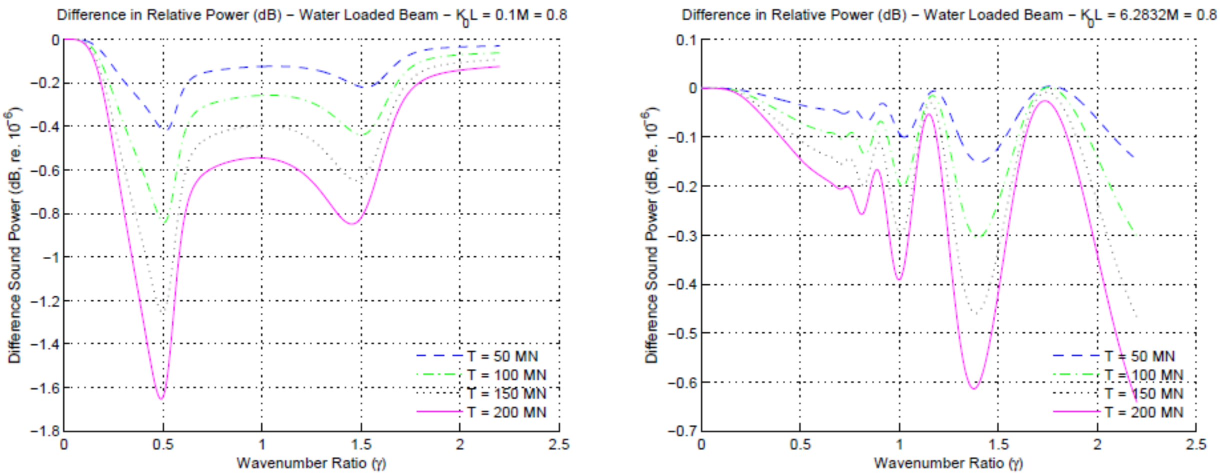
Fig. 3: Relative sound power v/s wave number ratio under Tensile Load; M = 0.8

Increase of acoustic power observed in Figs. 2 and 3 is however not very large over the entire range of frequency as seen in Figs. 4 and 5. However the magnitude of the change in sound power is of the order of 2dB which cannot be neglected since inplane loading is one of the components of the loads acting on the floating airport. What is interesting to note is the differences tend to converge for varying convective speed of loading at higher frequencies as noted by Keltie and Peng (1985).



(a) $K_0L = 0.1$ (b) $K_0L = 2\pi$

Fig. 4: Relative sound power v/s wave number ratio under Compressive Load; M=0.5



(a) $K_0L = 0.1$ (b) $K_0L = 2\pi$

Fig. 5: Relative sound power v/s wavenumber ratio under Tensile Load; M=0.8

4. Conclusion

The effect of inplane loading on total sound power generated by a floating runway due to landing / taking off of an airplane has been analysed. For such large structures physical modeling is not feasible and one has to rely on mathematical models. One such model has been proposed in this study. The following are concluded from this study

- Sound power decreases due to presence of compressive loading.
- However sound power increases due to presence of tensile loading.
- Change in sound power for inplane loading is limited to 2dB.

- Though magnitude of change is small, effect of inplane loading should not be neglected for floating airports subjected to landing / taking off of airplanes.
- Change in sound power tends to converge for varying convective speed at higher frequencies.

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