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DETERMINATION OF EFFECT OF ELLIPTIC NOTCHES AND GROOVES ON STRESS CONCENTRATION FACTORS ON NOTCHED BAR IN TENSION AND GROOVED SHAFT UNDER TORSION

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Abstract:

Stress concentration of structural members can be reduced considerably by the judicious choice of elliptic shaped stress raisers like notch and groove. Substantial effort has been given by numerous researchers to accurately measure the effect of such stress raisers, particularly of semicircular shaped notch and groove. An exhaustive bibliographical study proved that there is scope to investigate further and establish an alternative design criteria; concerning the elliptic geometry. Computational method, primarily the finite element method has been used to analyze the models under loading.

This paper suggests the use of a modified elliptic shape which gives less stress concentration when compared to semicircular notch and groove. The ratio of minor and major half axes of the ellipse should be between 0.3 and 0.4. The introduction of shoulder with elliptic notch and groove even reduces the stress concentration. The results obtained from FEM analysis propose optimal values of geometrical design parameters. The study represents not only a precise view of stress distribution, but also to develop charts that can be used by designer for practical purposes.

Keywords: Notch and groove, stress concentration factor, elliptic notch, finite element, Von-Mises stress.

NOMENCLATURE

K_t	stress concentration factor for normal stress (gross)
K_{ts}	stress concentration factor for shear stress (gross)
a, b, c	semi-axes of an ellipsoid
r	radius of semicircular notch or groove
t	shoulder thickness
W	width of flat bar in tension
D	diameter of round shaft under torsion
d	characteristic length of the least cross sectional area

Greek symbols

σ_{max}, σ_{nom}	maximum and nominal normal stress
τ_{max}, τ_{nom}	maximum and nominal shear stress
σ_y, τ_y	normal and shear yield strength
σ_{eq}, τ_{eq}	normal and shear Equivalent stress
σ_1, σ_2, σ_3	principle stresses
ν	Poisson's ratio

1. Introduction

Machine parts are only strong as its weakest point. In strictly engineering terms, the design of machine elements is focused reasonably on the regions where stress concentration occurs. Geometrical discontinuities, such as, notch, groove, keyhole, fillet etc. contribute to the concentration of stress to a considerable extent. Among these features, U-shaped or semicircular notches and grooves are found in numerous machine elements like, turbine rotor between blade rows, at seals; in variety of shafts such as, shoulder relieve grooves, retainer for spring washer etc. The semicircular design of notch and groove are most extensively used as it causes less stress concentration and are easier to generate compared to other traditional ones. Vigorous researches on other shapes are not encouraged as a complicated design may increase the cost and intricacy which may overthrow the advantage of reduced stress concentration. In this paper, a systematic study is presented in pursuit of a design that meets up both criteria; reduces stress concentration as well as reduced cost and complexity of manufacturing process. To achieve this, numerical analysis was carried out to establish an accurate and also a computationally efficient model.

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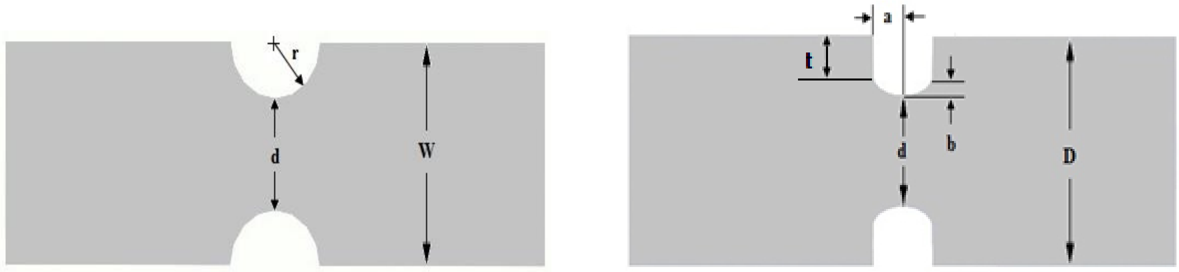
Designers often rely on circular or elliptical profiles, for which the theoretical stress concentration factors are provided for some standardized geometries Pilkey *et al* (2008) and Petersen (1974). Recently researchers have concentrated their focus on determining stress as well as strain concentration factors in 3D models using numerical schemes. Chiang (2011) numerically determined the stress concentration factor of an oblate ellipsoidal cavity by using equivalent inclusion method. Numerical results were presented for strongly oblate cavities and the range of validity of the cases was extended by developing a 2D model and established a simplified equation for stress concentration factor, K_t as a function of ratios of semi-axes of ellipsoid (b/a, c/b), Poisson's ratio (ν) and a dimensionless parameter defined as, $\Phi = \sqrt{1 + 1.464\left(b/a\right)^{1.65}}$. Chiang (2011) also assured the validity of the model for c/b less than 0.5.

Stress field and stress concentration factor in 3D models of elliptic holes in flat rectangular bars were also studied numerically by Yang (2009), Zhao et al (2008), Snowberger (2008), Pedersen et al (2007) etc. Yang (2009) showed that the plane stress condition of 3D models is approximately achieved by the 2D ones when the plate thickness tends to zero. Zhao et al (2008) is a summarized the 3D effect of stress concentrations at notches. They also concluded that for a plate with notch, the maximum 3D stress concentration factor through the thickness is always consumedly higher than the corresponding planer solution. Snowberger (2008) searched for the suitable mesh element size. Two element types were compared, which are the 4-noded quad and the 3-noded triangle. The results for 4-noded quad were found to be more accurate than results for 3-noded triangle. Pedersen et al (2007) used a 2D FE model and solved by using a different analysis tool (FORTRAN) to present the comparison between K_t value associated with geometric irregularities of semicircular and elliptic shape. They have drawn a conclusion that refers to a clear reduction in K_t for elliptic design. The present work utilizes findings of various researchers as mentioned above and aims to develop design charts which can be directly used for practical design purposes.

2. Specimen Geometries and Material Properties

We consider a steady two-dimensional laminar free convection flow of an electrically conducting, viscous and incompressible fluid along a vertical flat plate of length *l* and thickness *b*. It is assumed that the temperature at the outer surface of the plate is maintained at a constant temperature T_b, where $T_b > T_\infty$. Here T_∞ the temperature of the fluid outside the boundary layer. The coordinates system and the configuration are shown in Fig. 1



(a) (b)

Fig. 1: Geometric models; **(a)** flat bar with circular notch, **(b)** round shaft with elliptic groove

3. Mathematical Background

Stress concentration factor is defined as the ratio of maximum stress and nominal stress of an element under loading. The stress concentration factor for normal and shear stress can be thus expressed as,

$$K_t = \sigma_{max}/\sigma_{nom} \text{ ; for normal stress (tension and bending)} \quad (1)$$

$$K_{ts} = \tau_{max}/\tau_{nom} \text{ ; for shear stress (torsion)} \quad (2)$$

where, σ_{max} and τ_{max} represents maximum normal and shear stress developed under tension and torsion load; the nominal stresses σ_{nom} and τ_{nom} are the reference normal and shear stress. There exist a considerable number of theories to limit or determine the maximum stress σ_{max} in 2D or 3D stress systems. Petersen (1974) reported that yield tests for ductile materials proved the compatibility of Von-Mises criterion with the results of a variety of 2D conditions. The following equation is known as Von-Mises stress criterion of failure by yielding of a uniaxially loaded bar made of ductile material:

$$\sigma_y = \sqrt{\frac{\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_1-\sigma_3)^2\}}{2}} \qquad (3)$$

where, σ_y is the yield strength and $\sigma_{1,}$ $\sigma_{2,}$ σ_3 are the principle stresses. According to Pilkey (2008), equation (3) can also be used to determine *Equivalent stress,* σ_{eq}. For 2D cases with $\sigma_3 = 0$, equation (3) reduces to give the value of equivalent stress as below-

$$\sigma_y = \sigma_{eq} = \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2} \qquad (4)$$

For torsion cases with $\sigma_2 = -\sigma_1 = \tau_y$, Von-Mises stress equation becomes,

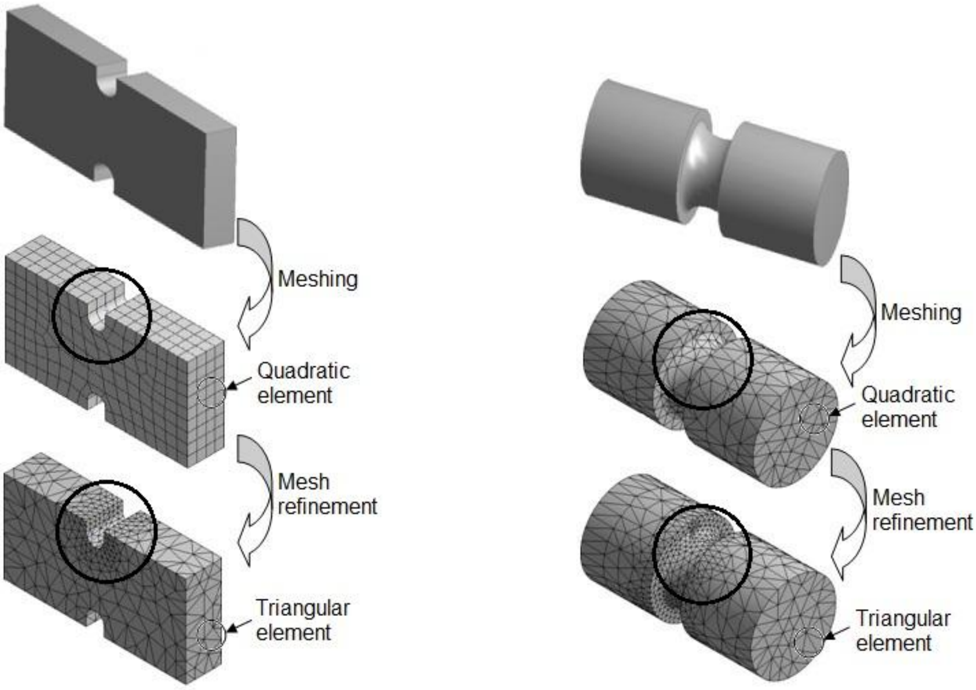
$$\tau_{eq} = \tau_y = 0.577\sigma_y = 0.577\sigma_{eq} \qquad (5)$$

By replacing the maximum stresses with Von-Mises equivalent stresses in equation (1) and (2), the stress concentration factors can be calculated using the following equations-

$$K_t = {}^{\sigma_{eq}}/_{\sigma_{nom}} \qquad (6)$$

$$K_{ts} = {}^{\tau_{eq}}/_{\tau_{nom}} \qquad (7)$$

The loads are applied after mesh generation followed by mesh refinement. The flat bar models are loaded with axial force of 1000 N and 100 N-m torque is applied on the round shafts are at both ends. The magnitude of loads in each case is determined such that the yielding remains within the elastic limits. To simplify the analysis, the loads are applied in static structural condition and in one step. In Figs. 2(a) and 2(b), example of meshing and mesh refinement is shown for two model geometries; flat bar with semicircular notch and round shaft with elliptic groove.



(a) (b)

Fig. 2: Sample meshing and mesh refinement of **(a)** a flat rectangular bar with opposite semicircular notches and, **(b)** a round shaft with circumferential elliptic groove

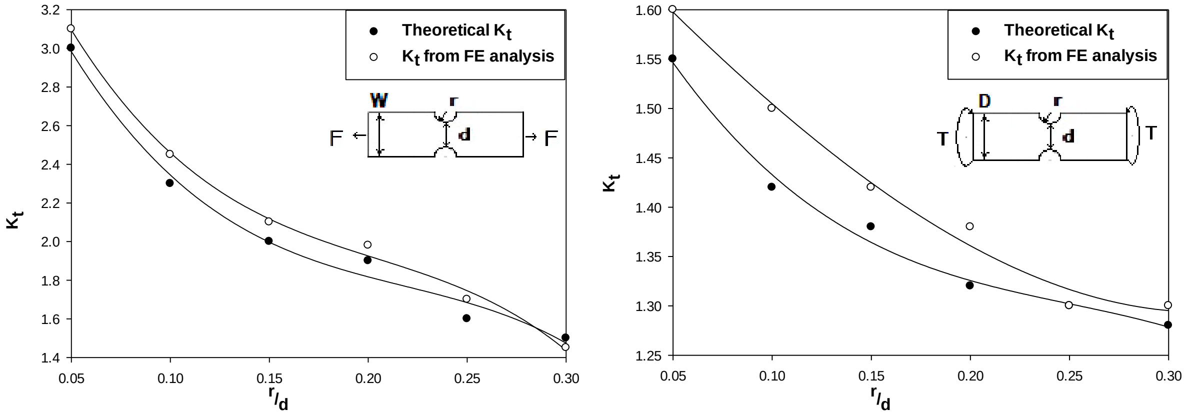
4. Results and Discussion

The result of this study as described in the previous section was reached by performing numerical analysis on finite element models. But before performing rigorous experiments, the models were verified against previously established results. The values of stress obtained by numerical method were separately analyzed to calculate the values of stress concentration factor for various geometries as the final outcome. In this section, the results including the verification of models used in the present study in presented in mostly in graphical form.

4.1. Verification of finite element model

Before performing rigorous numerical analysis, the finite element models required to be verified. The finite element models presented in this paper is justified by referring to the theoretical values of K_t extracted from works of Pilkey (2008), Petersen (1974) and conclusion offered by Yang (2009). According to Yang (2009), stress concentration factors for 2-D designs are in good agreement with 3-D designs as long as the thickness of the geometry in kept small enough. Based on this comment, the thickness and diameter of the finite element models analyzed in this paper were kept considerably small.

In Figs. 3(a) and 3(b), the K_t values obtained from finite element analysis are compared to the theoretical values of K_t [Petersen (1974)], for semicircular notch in flat bar in tension and semicircular groove in round shaft under torsion. In both cases the values of K_t is found to be in reasonable agreement with those of theoretical K_t.

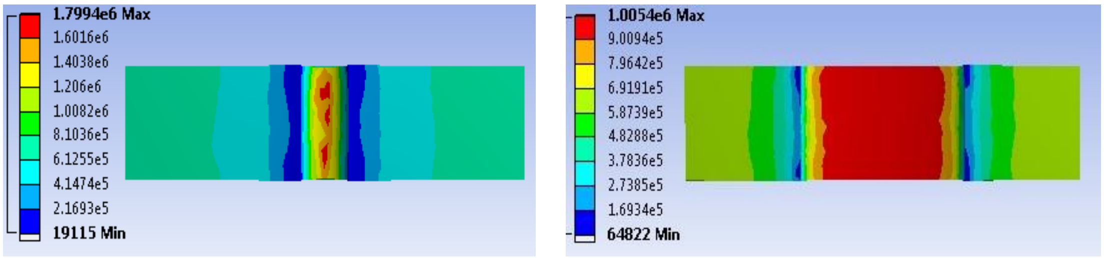


(a) (b)

Fig. 3: Verification of FE Model; **(a)** flat bar with semicircular notch in tension, **(b)** grooved round shaft in torsion

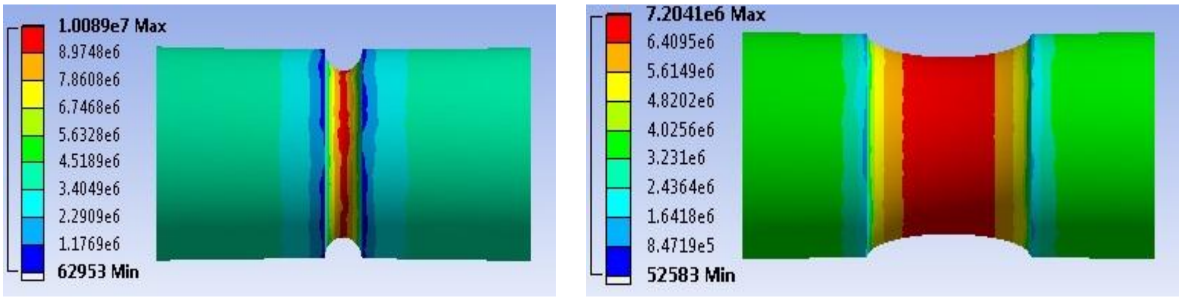
4.2. Comparison between semicircular and elliptical shapes

In this section, a comparison between the semicircular and elliptic shape of notch and groove is presented. The design parameters used for assessment are r/d and b/a; for round and elliptic shapes respectively. The comparison is justified by least cross sectional area, or the area of the cross section having minimum diameter. In both models, the least cross sectional area is constant. Fig. 4 and 5 shows stress distributions due to tension in a flat bar and torsion in a round shaft for both types of notch and grooves. For both loading conditions, the maximum stress in cases of elliptic design is found to be spread over a wider region, but has a considerably lower magnitude than that of semicircular design.



(a) (b)

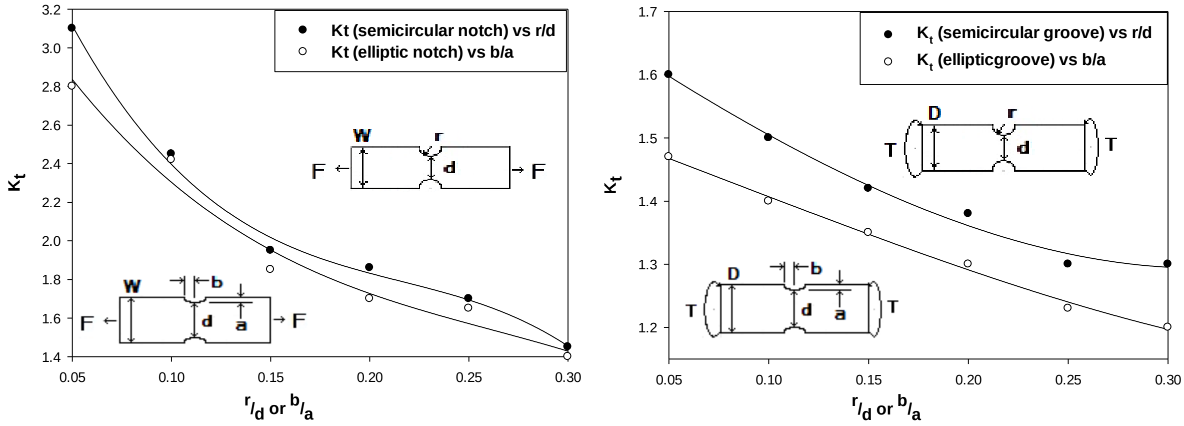
Fig. 4: Numerical simulation of stress distribution over **(a)** semicircular notch in flat bar, **(b)** elliptic notch in flat bar under tensile load of 1000N



(a) (b)

Fig. 5: Numerical simulation of stress distribution over **(a)** semicircular groove in round shaft, **(b)** elliptic groove in round shaft under torsional load of 100Nm

After repeating the same FE analysis for several times for different r/d and b/a ratio, the results were summed up in Fig. 6. With small r/d ratio the K_t in semicircular notch is significantly higher than the K_t in elliptic design. But at higher values of r/d or b/a, the difference decreases. In case of round shaft under torsion, elliptic shaped grooves provide reduced K_t. However, unlike the flat bar in tension difference in K_t values does not vary much with increasing r/d or b/a ratio.



(a) (b)

Fig. 6: Stress concentration factor K_t as function of relative notch size r/d for semi-circular notch or as ratio of ellipse half axes b/a for elliptic notch, in **(a)** flat bar in tension and **(b)** round shaft under torsion.

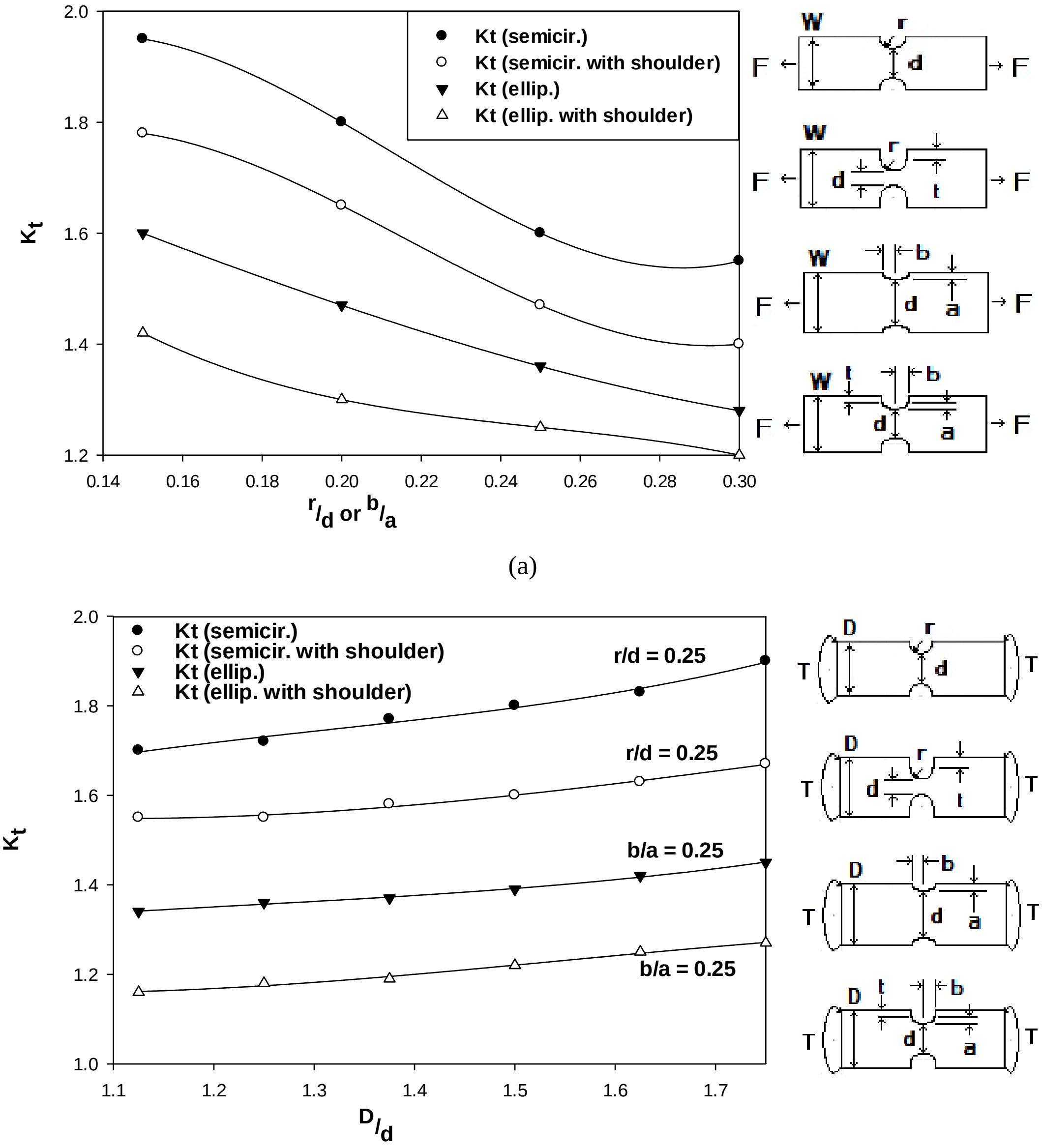
4.3. Effect of shoulder

In this paper, it is shown by using FE analysis that an elliptic groove combined with a shoulder gives the lowest value of K_t when compared with groove with no shoulder as illustrated in Fig. 7. The analysis was carried out for two design parameters of groove; r/d or b/a and D/d. Elliptic shaped grooves with shoulder give lowest values of K_t among the four cases presented in the figure.

For the cases of specific diameter ratio D/d, the relative groove size is maintained at r/d = 0.25. The results corresponding to D/d > 1.5 are for deep grooves and those for D/d < 1.5 are for shallow grooves. It should be noted that optimized stress concentration factor flatten out for shallow groove as the width of design is limited to that of circular design.

4.4. Design optimization

A parametric study is the practical approach to obtain the optimum result for a specific problem. In Fig. 8(a), values of K_t for elliptic notch in flat bar subjected to tensile load were plotted against a wide range of L/a, where, L/a = 1-b/a to fulfill the purpose. Fig. 8(b) was plotted for elliptic groove in round shaft under torsion. In both cases, the optimum value of b/a is recorded the one for which the K_t is minimum. The optimum value lies between 0.3 < b/a < 0.4.



(a)

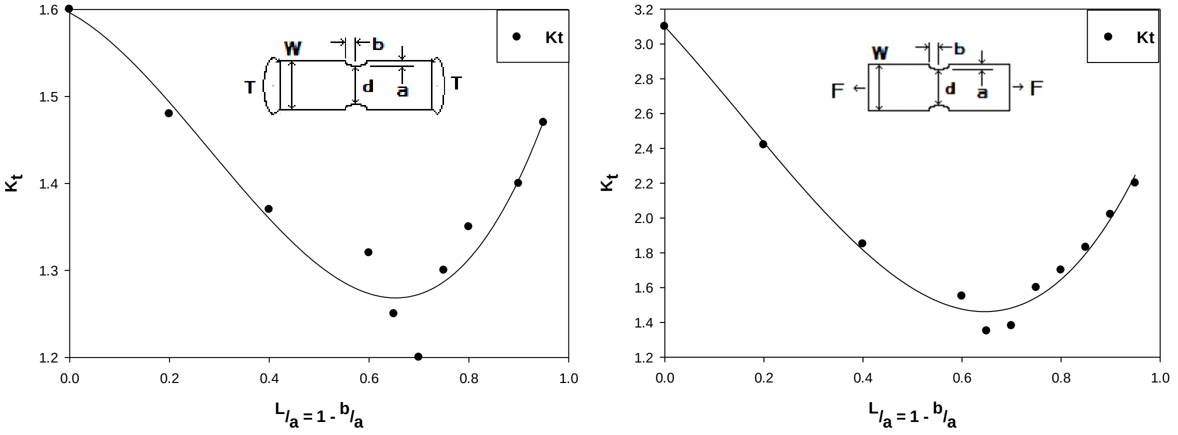
(b)

Fig. 7: Effect of shoulder in design of semicircular and elliptic groove in a round shaft under torsion; **(a)** stress concentration factor as a function of r/d or b/a, **(b)** stress concentration factor as a function of D/d

4.5. Proposed charts for practical design of elliptic shaped notches and grooves

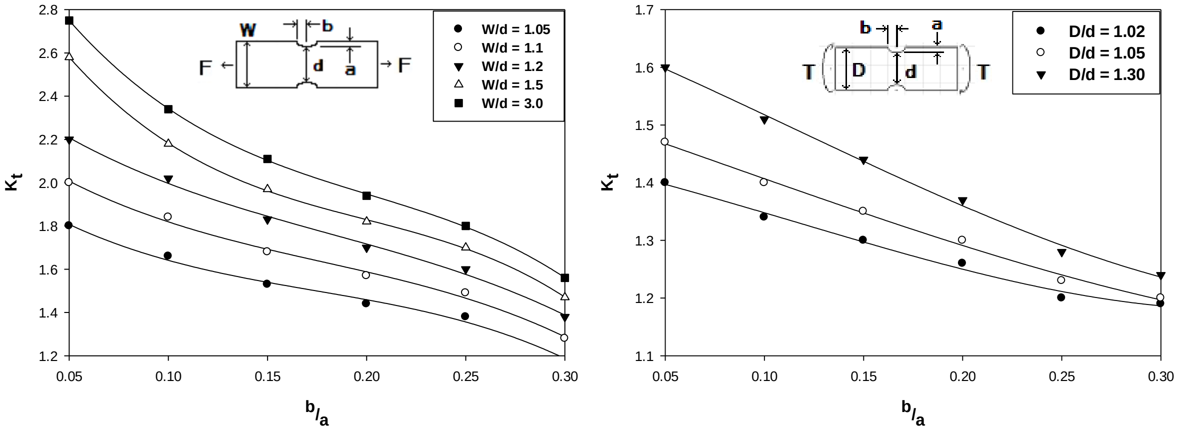
In all the loading conditions and model geometries analyzed in this paper, it is shown that stress concentration can be reduced by adopting elliptic notch and groove to design various machine elements. The final aim of the paper is to provide some practical rules that can be used as initial design optimization by designers without performing FE analysis. Figs. 9(a) and 9(b) are charts constructed to facilitate the purpose. In Fig. 9(a), stress concentration factors are plotted against b/a, for five different values of W/d for elliptic notch in flat bar in tension. Fig. 9(b) shows stress concentration factors vs. b/a, for three values of D/d for elliptic groove in round shaft in torsion.

From the design chart for elliptic shaped notch in flat bar in tension, it is observed that for constant geometric ratio W/d, stress concentration decreases with increase in the value of design parameter b/a ratio. Again as W/d increases, the value of K_t tends to increase.



(a) (b)

Fig. 8: Result from parametric study of ratio of ellipse half axes for **(a)** notch in flat bar in tension, **(b)** groove in round shaft under torsion. K_t is lowest for values within 0.3 < b/a < 0.4



(a) (b)

Fig. 9: Chart for designing elliptic shaped **(a)** notch in flat bar, **(b)** groove in round shaft

5. Conclusion

In summary, the FE analysis shows that elliptic notches and grooves produce reduced stress concentration as expected. The principle findings of the present study can be synthesized as written below:

- The FE analysis for all the cases presented in this paper proves the supremacy of elliptic design over conventional semicircular design on notches and grooves in term of reduced stress concentration at various b/a.
- Using parametric design technique, it is established that least stress concentration occurs within the range of 0.3 < b/a < 0.4.

Finally, charts are also constructed for elliptic notch and grooves which can be directly used for practical design purposes.

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