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SIMILARITY TRANSFORMATIONS OF HEAT AND MASS TRANSFER EFFECTS ON STEADY MHD FREE CONVECTION DISSIPATIVE FLUID FLOW PAST AN INCLINED POROUS SURFACE WITH CHEMICAL REACTION

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Abstract:

This paper concerns with a steady two-dimensional flow of an electrically conducting incompressible dissipating fluid over an inclined semi-infinite porous surface with heat and mass transfer in presence of chemical reaction. The flow is permeated by a uniform transverse magnetic field. A scaling group of transformation is applied to the governing equations. The system remains invariant due to some relations among the parameters of the transformations. After finding three absolute invariants, a third-order ordinary differential equation corresponding to the momentum equation, two second-order ordinary differential equations corresponding to energy and diffusion equations are derived. The coupled ordinary differential equations along with the boundary conditions are solved numerically by using Runge-Kutta method along with shooting technique. The velocity and concentration is found to decrease gradually as the chemical reaction is increased. The results of the study are of great interest because flows on a vertical stretching surface play a predominant role in applications of science and engineering, as well as in many transport processes in nature. The effects of various parameters on velocity, temperature and concentration fields as well as skin-friction, Nusselt number and Sherwood number are presented graphically and tabulated form discussed qualitatively.

***Keywords*:** Lie group analysis, MHD, radiation, viscous dissipation, chemical reaction, inclined porous surface

1. Introduction

The study of free convection flow for an incompressible viscous fluid past an inclined porous surface has attracted the interest of many researchers in view of its important applications to many engineering problems such as cooling of nuclear reactors, the boundary layer control in aerodynamics, crystal growth, food processing and cooling towers. Effect of porosity on the free convection flow along a vertical plate embedded in a porous medium was investigated by Beithou *et al*. (1998). Their results show that as the porosity is increased the temperature variation becomes steeper, that is, the heat transfer rate is increased. Chen (2004) studied the natural convection flow over a permeable inclined surface with variable wall temperature and concentration. The results show that the velocity is decreased in the presence of a magnetic field. Increasing the angle of inclination decreases the effect of buoyancy force. Heat transfer rate is increased when the Prandtl number is increased. Duwairi (2005), who investigated the effect of viscous and Joule heating on forced convection flow from radiative isothermal surfaces found that the heat transfer rate is decreased as the radiation parameter is increased. Radiative and magnetic effects on free convection and mass-transfer flow past a flat plate were studied by Ibrahim *et al*. (2005). They obtained similarity reductions and found analytical and numerical solutions using scaling symmetry. Kalpadides and Balassas (2004) also studied the free convective boundary layer problem using Lie group analysis.

Lie group analysis is a classical method discovered by Norwegian mathematician Sophus Lie for finding invariant and similarity solutions [(1989, 1995, 1989, 1982, 1999)]. Yurusoy and Pakdemirli (2001) presented exact solution of boundary layer equations of a special non-Newtonian fluid over a stretching sheet by the method of Lie group analysis. They extended their work to creeping flow of second-grade fluid (2006). Sivasankaran *et al* (2006) studied the problem of natural convection heat and mass transfer flow past an inclined plate for various parameters using Lie group analysis without and with heat generation. Kumari *et al* (2001) investigated the mixed convection flow over a vertical wedge embedded in a porous medium. They found that the heat transfer is increased with the Prandtl number and the effect of permeability on the heat transfer is very small. Ramana Reddy *et al* (2011) studied the problem of MHD flow over a vertical moving porous plate with

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heat generation by considering double diffusive convection, Lai and Kulacki (1994) studied the convection from horizontal impermeable surfaces in saturated porous medium. It is observed that the inertial term has a pronounced effect on the flow for higher values of parameter by the inertial effect on natural convection. Most recently, heat transfer problems for boundary layer flow concerning a convective boundary condition was investigated by Aziz (2009) for the Blasius flow. Similar analysis was applied to the Blasius and Sakiadis (2010) flow with radiation effects. Makinde (2010) studied the heat and mass transfer over a vertical plate with convective boundary conditions. Ishak (2010) studied the steady laminar boundary layer flow and heat transfer over a stationary permeable flat plate immersed in an uniform free stream with convective boundary condition. Kishan *et al* (2011) were investigated the effects of viscous dissipation on MHD flow with heat and mass transfer over a stretching surface with heat source, thermal stratification and chemical reaction. Ishak et al. (2011) studied the problem of steady laminar boundary layer flow and heat transfer over a moving flat surface in a parallel stream with convective boundary condition. Subhashini *et al*. (2011) investigated the simultaneous effects of thermal and concentration diffusions on a mixed convection boundary layer flow over a permeable surface under convective surface boundary condition. Recently, Hayat *et al*. (2012) studied the flow and heat transfer of Eyeing Powell fluid over a continuously moving surface in the presence of convective boundary conditions. Lie group analysis, also called symmetry analysis was developed by Sophius Lie to find point transformations which map a given differential equation to it. This method unifies almost all known exact integration techniques for both ordinary and partial differential equations Oberlack (1999). In the field of viscous fluids there are many papers dealing with aspect of group theory transformation [(1952, 1968, 1999, 2006, 2009, 2011, 2012)]. Shu and Pop (1997) numerically studied the natural convection from inclined wall plumes in a porous medium. The velocity is increased while the temperature is decreased with increasing the tilting angle. Parveen *et al* (2012) have studied the Joule heating effect on magnetohydrodynamic natural convection flow along a vertical wavy surface. Yurusoy and Pakdemirli (1997) studied the boundary layer equations for Newtonian / non-Newtonian fluids by using Lie group method. Gnaneswar Reddy (2013) was analyzed the scaling transformation for heat and mass transfer effects on steady MHD free convection dissipative flow past an inclined porous surface. So far no attempt has been made to study the heat and mass transfer in a porous medium using Lie groups, and hence we study the problem of MHD free convection dissipative fluid flow past an inclined porous surface with chemical reaction for various parameters using Lie groups.

In this paper, application of scaling group of transformation for chemical reaction, heat and mass transfer effects on steady free convection flow in an inclined porous plate in the presence of MHD and viscous dissipation has been employed. This reduces the system of nonlinear coupled partial differential equations governing the motion of fluid into a system of coupled ordinary differential equations by reducing the number of independent variables. The system remains invariant due to some relations among the parameters of the transformations. Three absolute invariants are obtained and used to derive a third-order ordinary differential equation corresponding to momentum equation and two second-order ordinary differential equations corresponding to energy and diffusion equations. With the use of Runge-Kutta fourth order along shooting method, the equations are solved. Finally, analysis has been made to investigate the effects of thermal and Solutal Grashof numbers, magnetic field parameter, Prandtl number, Viscous dissipation parameter, Schmidt number and chemical reaction on the motion of fluid using scaling group of transformations, viz., Lie group transformations.

2. Mathematical Analysis

Consider the heat and mass transfer of a steady two-dimensional hydromagnetic flow of a viscous, incompressible, electrically conducting and dissipating fluid past a semi-infinite inclined plate with an acute angle α to the vertical. The flow is assumed to be in the $x-$ direction, which is taken along the semi-infinite inclined porous plate and $y-$ axis normal to it. A magnetic field of uniform strength B_0 is introduced normal to the direction of the flow. In the analysis, we assume that the magnetic Reynolds number is much less than unity so that the induced magnetic field is neglected in comparison to the applied magnetic field. It is also assumed that all fluid properties are constant except that of the influence of the density variation with temperature and concentration in the body force term. The surface is maintained at a constant temperature T_w , which is higher than the constant temperature T_∞ of the surrounding fluid and the concentration C_w is greater than the constant concentration C_∞ . The level of concentration of foreign mass is assumed to be low, so that the Soret and Dufour effects are negligible. Then, under the usual Boussinesq's and boundary layer approximations, the governing equations are

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left[\nu\frac{\partial^2 u}{\partial y^2}+g\beta\left(T-T_\infty\right)\cos\alpha+g\beta^*\left(C-C_\infty\right)\cos\alpha-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{K'}u\right] \tag{2}$$

Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

Species equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_r'\left(C-C_\infty\right) \tag{4}$$

The boundary conditions for the velocity, temperature and concentration fields are

$$\begin{aligned}&u=v=0,\ T=T_w,\ C=C_w\ \text{ at }\ y=0\\&u\to 0,\ T\to T_\infty,\ C\to C_\infty\ \text{ as }\ y\to\infty\end{aligned} \tag{5}$$

On introducing the following non-dimensional quantities

$$\begin{aligned}&x^*=\frac{xU_\infty}{\nu},\mathrm{y}^*=\frac{yU_\infty}{\nu},u^*=\frac{u}{U_\infty},v^*=\frac{v}{U_\infty},M=\frac{\sigma B_0^2\nu}{U_\infty^3},K=\frac{K'U_\infty^3}{\nu^3},Gr=\frac{\nu g\beta\left(T_w-T_\infty\right)}{U_\infty^3},\\&Gm=\frac{\nu g\beta^*\left(C_w-C_\infty\right)}{U_\infty^3},\ \theta=\frac{T-T_\infty}{T_w-T_\infty},\phi=\frac{C-C_\infty}{C_w-C_\infty},\Pr=\frac{\nu}{\alpha},\ Ec=\frac{U_\infty^2}{c_p\left(T_w-T_\infty\right)},\\&Sc=\frac{\nu}{D},Kr=\frac{K_r'\nu}{V_0^2},\end{aligned} \tag{6}$$

Substituting Eq.(6) into Eqs. (1) - (4) and dropping bars, we obtain,

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left[\frac{\partial^2 u}{\partial y^2}+Gr\theta Cos\,\alpha+Gm\phi Cos\alpha-\left(M+\frac{1}{K}\right)u\right] \tag{8}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+Ec\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

$$u\frac{\partial\phi}{\partial x}+v\frac{\partial\phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}-K_r\phi \tag{10}$$

The corresponding boundary conditions take the form

$$\begin{aligned}&u=v=0,\ \theta=1,\ \phi=1\text{ at }\ y=0\\&u\to 0,\ \theta\to 0,\ \phi\to 0\text{ as }\ y\to\infty\end{aligned} \tag{11}$$

By using the stream function $u=\frac{\partial\psi}{\partial y},v=-\frac{\partial\psi}{\partial x}$ we have

$$\left(\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right)=\left[\frac{\partial^3\psi}{\partial y^3}+Gr\theta\ \mathrm{Cos}\,\alpha+Gm\phi\mathrm{Cos}\alpha-\left(M+\frac{1}{K}\right)\frac{\partial\psi}{\partial y}\right] \tag{12}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+Ec\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 \tag{13}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial\phi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\phi}{\partial y}=\frac{1}{Sc}\frac{\partial^2\phi}{\partial y^2}-K_r\phi \tag{14}$$

Finding the similarity solutions of Equations (12) - (14) is equivalent to determining the invariant solutions of these equations under a particular continuous one parameter group. One of the methods is to search for a transformation group from the elementary set of one parameter scaling transformation. We now introduce the simplified form of Lie-group transformations namely, the scaling group of transformations.

$$\Gamma:\; x^* = xe^{\varepsilon\alpha_1},\; y^* = ye^{\varepsilon\alpha_2},\; \psi^* = \psi e^{\varepsilon\alpha_3},\; u^* = ue^{\varepsilon\alpha_4}, v^* = ve^{\varepsilon\alpha_5}, \theta^* = \theta e^{\varepsilon\alpha_6}, \phi^* = \phi e^{\varepsilon\alpha_7}, \tag{15}$$

where $\alpha_1,\ \alpha_2,\ \alpha_3,\ \alpha_4,\ \alpha_5,\ \alpha_6$ and α_7 are transformation parameters and ε is a small parameter whose interrelationship will be determined by our analysis. Equation (15) may be considered as a point-transformation which transforms co-ordinates $(x, y, \psi, u, v, \theta, \phi)$ to the coordinates $(x*, y*, \psi*, u*, v*, \theta*, \phi*)$. Substituting transformations equation (15) in (12), (13) and (14), we get

$$e^{\varepsilon(\alpha_1+2\alpha_2-2\alpha_3)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial^2\psi^*}{\partial x^*\partial y^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial^2\psi^*}{\partial y^{2*}}\right) = \begin{bmatrix} e^{\varepsilon(3\alpha_2-\alpha_3)}\dfrac{\partial^3\psi}{\partial y^3}+e^{-\varepsilon\alpha_6}Gr\theta\,Cos\,\alpha \\ +e^{-\varepsilon\alpha_7}Gm\phi Cos\alpha - e^{\varepsilon(\alpha_2-\alpha_3)}\left(M+\dfrac{1}{K}\right)\dfrac{\partial\psi^*}{\partial y^*} \end{bmatrix} \tag{16}$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_6)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\theta^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\theta^*}{\partial y^*}\right) = e^{\varepsilon(2\alpha_2-\alpha_6)}\frac{1}{\Pr}\frac{\partial^2\theta^*}{\partial y^{*2}}+e^{\varepsilon(4\alpha_2-2\alpha_3)}Ec\left(\frac{\partial^2\psi}{\partial y^2}\right)^2 \tag{17}$$

$$e^{\varepsilon(\alpha_1+\alpha_2-\alpha_3-\alpha_7)}\left(\frac{\partial\psi^*}{\partial y^*}\frac{\partial\phi^*}{\partial x^*}-\frac{\partial\psi^*}{\partial x^*}\frac{\partial\phi^*}{\partial y^*}\right) = \frac{1}{Sc}e^{\varepsilon(2\alpha_2-\alpha_7)}\frac{\partial^2\phi^*}{\partial y^{2^*}}-K_r e^{-\varepsilon\alpha_7}\phi^* \tag{18}$$

The system will remain invariant under the group of transformations Γ, and we would have the following relations among the parameters, namely

$$\alpha_1 + 2\alpha_2 - 2\alpha_3 = 3\alpha_2 - \alpha_3 = -\alpha_6 = -\alpha_7 = \alpha_2 - \alpha_3$$
$$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_6 = 2\alpha_2 - \alpha_6 = 4\alpha_2 - 2\alpha_3$$
$$\alpha_1 + \alpha_2 - \alpha_3 - \alpha_7 = 2\alpha_2 - \alpha_7 = -\alpha_7$$

These relations gives

$$\alpha_2 = \frac{1}{4}\alpha_1 = \frac{1}{3}\alpha_3,\ \alpha_4 = \frac{1}{2}\alpha_1, \alpha_5 = -\frac{1}{4}\alpha_1,\ \alpha_6 = \alpha_7 = 0$$

Thus the set of transformations Γ reduces to one parameter group of transformations as

$$x^* = xe^{\varepsilon\alpha_1},\ y^* = ye^{\varepsilon\frac{\alpha_1}{4}},\ \psi^* = \psi e^{\varepsilon\frac{3\alpha_1}{4}}, u^* = ue^{\varepsilon\frac{\alpha_1}{2}}, v^* = ve^{-\varepsilon\frac{\alpha_1}{4}}, \theta^* = \theta, \phi^* = \phi,$$

Expanding by Taylors method in powers of ε and keeping terms up to the order ε we get

$$x^* - x = x\varepsilon\alpha_1,\ y^* - y = y\varepsilon\frac{\alpha_1}{4},\ \psi^* - \psi = \psi\frac{3\alpha_1}{4}, u^* - u = u\varepsilon\frac{\alpha_1}{2}, v^* - v = -v\varepsilon\frac{\alpha_1}{4}, \theta^* - \theta = 0, \phi^* - \phi = 0$$

$$\frac{dx}{x\alpha_1} = \frac{dy}{y\frac{\alpha_1}{4}} = \frac{d\psi}{\psi\frac{3\alpha_1}{4}} = \frac{du}{u\frac{\alpha_1}{4}} = \frac{dv}{-v\frac{\alpha_1}{4}} = \frac{d\theta}{0} = \frac{d\phi}{0} \tag{19}$$

Solving the above equations, we find the similarity transformations

$$\eta = x^{-\frac{1}{4}}y,\ \psi^* = x^{\frac{3}{4}}f(\eta),\ \theta^* = \theta(\eta),\ \phi^* = \phi(\eta) \tag{20}$$

Substituting these values in equations (16)-(18), we finally obtain the system of nonlinear ordinary differential equations

$$f''' + \frac{3}{4}ff'' - \frac{1}{2}f'^2 + Gr\theta\,\text{Cos}\alpha + Gm\phi\,\text{Cos}\alpha + \left(M + \frac{1}{K}\right)f' = 0 \tag{21}$$

$$\theta'' + \frac{3}{4}\Pr f\theta' + \Pr Ecf''^2 = 0 \tag{22}$$

$$\phi'' + \frac{3}{4}Scf\phi' - kr\phi = 0 \tag{23}$$

The corresponding boundary conditions are

$$f = 0, f' = 0, \theta = 1, \phi = 1\ at\ \eta = 0$$
$$f' \to 0, \theta \to 0, \phi \to 0\ as\ \eta \to \infty \tag{24}$$

3. Numerical Solution

The set of nonlinear ordinary differential equations (21) - (23) with boundary conditions equation (24) have been solved by using the Runge-Kutta method of fourth order along with shooting technique. First of all, higher order non-linear differential equations (21) - (23) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Jain *et al* [(1985)]. The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta\eta = 0.01$ is used to obtain the numerical solution with five decimal place accuracy as the criterion of convergence. In the next section, the results are discussed in detail.

4. Results and Discussion

To analyze the results, numerical computation has been carried out using the method described in the previous paragraph for various in governing parameters, namely, thermal Grashof number *Gr*, modified Grashof number *Gc*, magnetic field parameter *M*, permeability parameter *K*, Prandtl number *Pr*, Eckert number *Ec*, inclination angles *α*, Schmidt number *Sc*, chemical reaction parameter *Kr*. In the present study following default parameter values are adopted for computations: $Gr = 2.0$, $Gc = 2.0$, $M = 1.0$, $K = 1.0$, $Pr = 0.71$, $Ec = 0.01$, $\alpha = 30^0$, $Sc = 0.6$, $Kr = 0.5$. All graphs therefore correspond to these values unless specifically indicated on the appropriate graphs.

The influence of the thermal Grashof number on the velocity is presented in Fig. 1. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the boundary layer. As expected, it is observed that there is a rise in the velocity due to the enhancement of thermo buoyancy force. Here, the positive values of *Gr* correspond to cooling of the plate. Also, as *Gr* increases, the peak values of the velocity increases rapidly near the porous plate and then decays smoothly to the free stream velocity.
Fig. 2 presents typical velocity profiles in the boundary layer for various values of the solutal Grashof number *Gc*, while all other parameters are kept at some fixed values. The solutal Grashof number *Gc* defines the ratio of the species buoyancy force to the viscous hydrodynamic force. As expected, the fluid velocity increases and the peak value is more distinctive due to increase in the species buoyancy force. The velocity distribution attains a distinctive maximum value in the vicinity of the plate and then decreases properly to approach the free stream value.

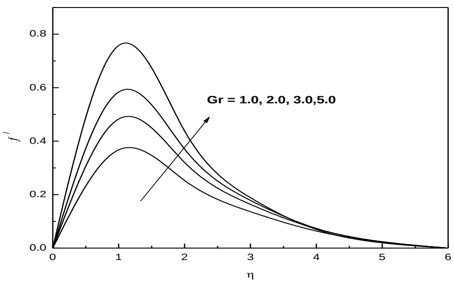

Fig. 1: Velocity profiles for different values of *Gr*

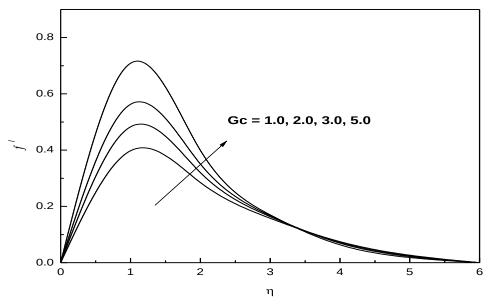

Fig. 2: Velocity profiles for different values of *Gc*

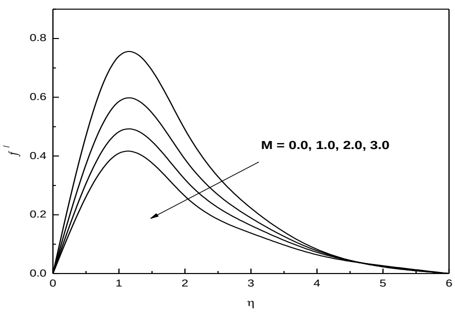

Fig. 3: Velocity profiles for different values of *M.*

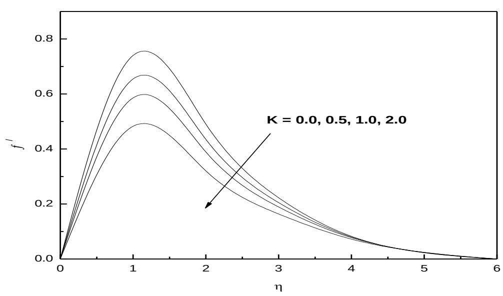

Fig. 4: Velocity profiles for different values of *K*

For various values of the magnetic parameter *M* and permeability parameter *K*, the velocity profiles are plotted in Figs. 3 and 4 respectively. It can be seen that as *M* and *K* are increases, the velocity profiles are decreases.

This result qualitatively agrees with the expectations, since the magnetic field exerts a retarding force on the free convection flow.

Figs. 5-9, 10-13, 14-15 show the velocity profiles, temperature profile and concentration profiles respectively for different parameters. Figs. 5 and 12 illustrate the velocity and temperature profiles for different values of the Prandtl number *Pr.* The Prandtl number defines the ratio of momentum *Pr* diffusivity to thermal diffusivity. The numerical results show that the effect of increasing values of Prandtl number results in a decreasing velocity (Fig. 5). From Fig. 12, it is observed that an increase in the Prandtl number results a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of .Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

The effect of the viscous dissipation parameter i.e., the Eckert number *Ec*on the velocity and temperature are shown in Figs 6 and 13 respectively. The Eckert number expresses the relationship between the kinetic energy in the flow and the enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. The positive Eckert number implies cooling of the plate i.e., loss of heat from the plate to the fluid. Hence, greater viscous dissipative heat causes a rise in the temperature as well as the velocity, which is evident from Figs 6 and13.

The influences of the Schmidt number *Sc* on the velocity and concentration profiles are plotted in Figs 7 and 14 respectively. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. As the Schmidt number increases the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. The reductions in the velocity and concentration profiles are accompanied by simultaneous reductions in the velocity and concentration boundary layers. These behaviors are clear from Figs 7 and 14. The influences of chemical reaction parameter *Kr* on the velocity and concentration profiles across the boundary layer are presented in Fig.8 and Fig. 15. We see that the velocity as well as concentration distribution across the boundary layer decreases with increasing of *Kr.*

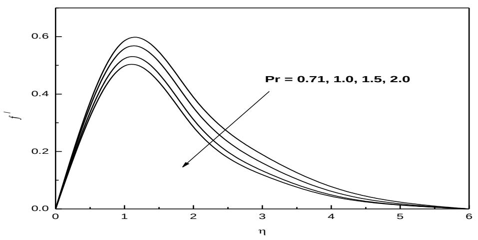

Fig. 5: Velocity profiles for different values of *Pr*

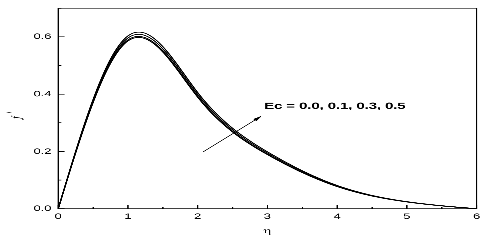

Fig. 6: Velocity profiles for different values of *Ec*

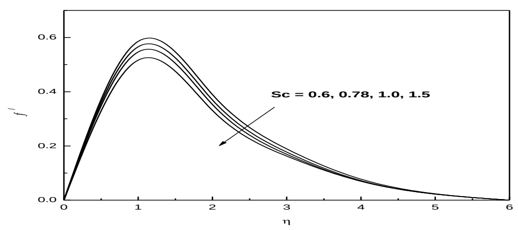

Fig. 7: Velocity profiles for different values of *Sc*

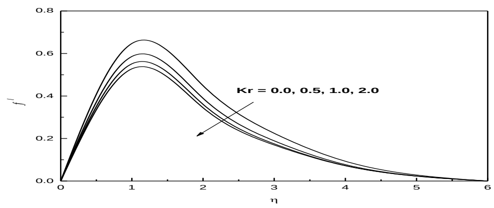

Fig. 8: Velocity profiles for different values of *Kr*

The effect of inclination of the surface for different parameters is depicted in Fig. 9. For fixed values of the all parameter the velocity is decreased with inclination angles as shown in Fig. 9. The fluid has higher velocity when the surface is vertical than when inclined because the buoyancy effect decreases due to gravity components $\cos\alpha$ as the plate is inclined. The fact is that as the angle of inclination increases the effect of the buoyancy force due to thermal diffusion decreases by a factor of $\cos\alpha$. Consequently the driving force to the fluid decreases as a result velocity profiles decreases. For different values of the magnetic field parameter *M* on the temperature profiles are plotted in Fig.10. It is observed that the magnetic parameter increases, the temperature also increases. Fig. 11 represents the effect of the porosity parameter on the temperature profiles.

There are very small changes that occur in both momentum and thermal boundary layers when changes are made in the porosity parameter. The temperature is increased with increase in the porosity parameter.

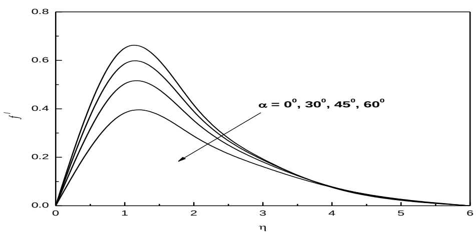

Fig. 9: Velocity profiles for different values of α

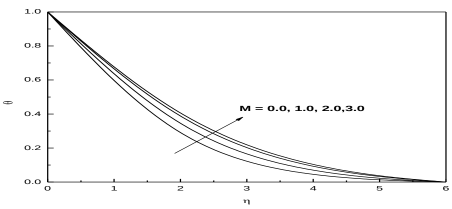

Fig. 10: Temperature profiles for different values of M

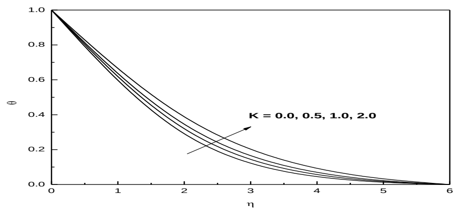

Fig. 11: Temperature profiles for different values of K

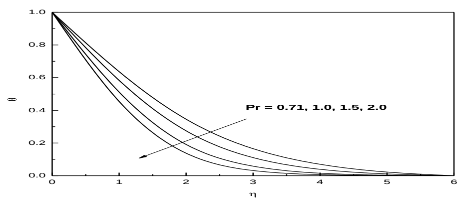

Fig. 12: Temperature profiles for different values of Pr

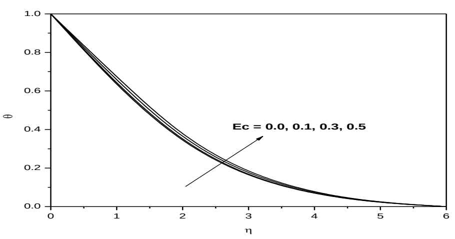

Fig. 13: Temperature profiles for different values of Ec

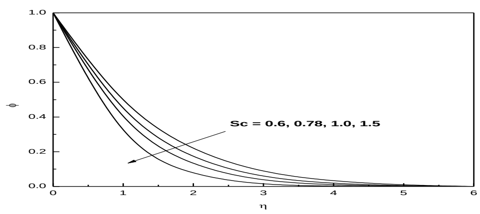

Fig. 14: Concentration profiles for different values of Sc

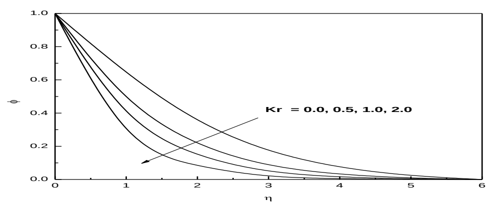

Fig. 15: Concentration profiles for different values of Kr.

Table 1: Computation showing $f''(0), -\theta'(0)$ and $-\phi'(0)$ for $Pr = 0.71,\ Ec = 0.01, Sc = 0.6,\ Kr = 0.5.$

Gr	Gc	M	K	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
2.0	2.0	1.0	1.0	1.67872	0.37536	0.632846
3.0	2.0	1.0	1.0	2.06767	0.404755	0.648809
4.0	2.0	1.0	1.0	2.44141	0.429506	0.66312
2.0	3.0	1.0	1.0	2.03459	0.399334	0.646135
2.0	4.0	1.0	1.0	2.38237	0.420826	0.658602
2.0	2.0	2.0	1.0	1.49761	0.348147	0.619813
2.0	2.0	3.0	1.0	1.36399	0.327933	0.610413
2.0	2.0	1.0	2.0	1.49761	0.348174	0.619813
2.0	2.0	1.0	3.0	1.36399	0.327933	0.610413

Table 1-3 indicate the values of skin-friction coefficient, the wall temperature gradient and the wall concentration gradient in terms of $f''(0), -\theta'(0)$ and $-\phi'(0)$ respectively for various values embedded flow parameter. From Table 1 - 3, we have noticed that skin-friction coefficient, Nusselt number and Sherwood number are increases with an increasing of Grashof number or mass Grashof number, as increasing values of magnetic field parameter *(M)* or porosity parameter a reduces in the skin-friction, Nusselt number and Sherwood. The Nusselt number reduces as increase the values of dissipation *Ec* or inclination angle, while it is increases for increasing value of Prandtl number *Pr*. It is also observed that the increase in Schmidt number *Sc* or chemical reaction parameter *Kr* parameter lead to the increase in the Sherwood number.

Table 2: Computation showing, $f''(0), -\theta'(0)$ and $\phi'(0)$ for $Gr = 2.0, Gc = 2.0, M = 1.0, K = 1.0, Sc = 0.60$.

Pr	Ec	α	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.71	0.01	30^0	1.67872	0.37536	0.632846
1.0	0.01	30^0	1.64902	0.429375	0.629509
2.0	0.01	30^0	1.57721	0.570278	0.622038
0.71	0.1	30^0	1.68425	0.337882	0.633256
0.71	0.2	30^0	1.69051	0.29549	0.633719
0.71	0.01	45^0	1.39653	0.352877	0.621139
0.71	0.01	60^0	1.01526	0.318826	0.60434

Table 3: Computation showing $f''(0), -\theta'(0)$ and $-\phi'(0)$ for *Gr* = 2.0, *Gc* =2.0, *M*= 1.0, *K* = 1.0, *Pr* = 0.71, *Ec* = 0.01, $\alpha = 30^0$.

Sc	Kr	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
0.6	0.5	1.67872	0.37536	0.632846
0.78	0.5	1.65106	0.369773	0.716312
1.0	0.5	1.62341	0.364526	0.805778
0.6	1.0	1.6253	0.36603	0.828406
0.6	2.0	1.55591	0.355195	1.12767

5. Conclusions

The present work helps us understanding numerically as well as physically free convection flow in an inclined porous plate in the presence of MHD and viscous dissipation has been employed. This reduces the system of nonlinear coupled partial differential equations governing the motion of fluid into a system of coupled ordinary differential equations by reducing the number of independent variables. The similarity solutions are obtained using scaling transformations. The set of governing equations and the boundary condition are reduced to ordinary differential equations with appropriate boundary conditions. Furthermore the similarity equations are solved numerically by using Runge–Kutta fourth order method along shooting technique. A comparison with previously published work is performed and the results are found to be in good agreement. Based on the obtained results, the following conclusions may be drawn.

From the numerical results, it is seen that the effect of increasing thermal Grashof number or solutalGrashof number is manifested as an increase in flow velocity. It is interesting to note that the temperature decreases much faster than the air temperature. In the presence of a magnetic field parameter, the permeability of porous medium, viscous dissipation is demonstrated to exert a more significant effect on the flow field and, thus, on the heat transfer from the plate to the fluid. The velocity and concentration is found to decrease gradually as the Schmidt number is increased. The velocity and concentration is found to decrease gradually as the chemical reaction is increased. The results of the study are of great interest because flows on a vertical stretching surface play a predominant role in applications of science and engineering, as well as in many transport processes in nature.

References

Afify, A. A.(2009): Similarity Solution in MHD Effects of Thermal Diffusion and Diffusion Thermo on Free Convective Heat and Mass Transfer over a Stretching Sheet Considering Suction or Injection, Communications in Nonlinear Science and Numerical Simulation, Vol. 14, pp. 2202–2214. http://dx.doi.org/10.1016/j.cnsns.2008.07.001

Aziz, A. (2009): A Similarity Solution for Laminar Thermal Boundary Layer over a Flat Plate with a Convective Surface Boundary Condition, Communications in Nonlinear Science and Numerical Simulation, Vol.14,No.4, pp. 1064–1068. http://dx.doi.org/10.1016/j.cnsns.2008.05.003

Bataller, R. C. (2008): Radiation Effects for the Blasius and Sakiadis Flows with a Convective Surface Boundary Condition, Applied Mathematics and Computation, Vol. 206, pp. 832–840. http://dx.doi.org/10.1016/j.amc.2008.10.001

Beithou, N. Albayrak, K. and Abdulmajeed, A. (1998): Effects of Porosity on the Free Convection Flow of Non- Newtonian Fluids along a Vertical Plate Embedded in a Porous Medium, Turkish Journal of Engineering and Environmental Sciences, Vol.22, pp. 203-209.

Bluman, G. W. and Kumei, S. (1989): Symmetries and Differential Equations, Applied Mathematical Sciences,Vol.81, Springer, New York, USA. http://dx.doi.org/10.1007/978-1-4757-4307-4

Chen C. H. (2004): Heat and Mass Transfer in MHD Flow by Natural Convection from a Permeable, Inclined Surface with Variable Wall Temperature and Concentration, Acta Mechanica, Vol.172, pp. 219-235. http://dx.doi.org/10.1007/s00707-004-0155-5

Duwairi, H. M. (2005): Viscous and Joule Heating Effects on Forced Convection Flow from Radiate Isothermal Porous Surfaces, International Journal of Numerical Methods Heat Fluid Flow, Vol.15, No.5, pp. 429-440. http://dx.doi.org/10.1108/09615530510593620

Fan, J.R., Shi, J.M., Xu, X.Z. (1999): Similarity Solution of Free Convective Boundary-Layer Behavior at A Stretching Surface, Journal Heat and Mass Transfer, Vol. 35, pp. 191–196. http://dx.doi.org/10.1007/s002310050313

Gnaneswara Reddy, M. (2013): Scaling Transformation for Heat and Mass Transfer Effects on Steady MHD Free Convection Dissipative Flow Past an Inclined Porous Surface, International Journal of Applied Mathematics and Mechanics. Vol.9, No.10, pp.1-18.

Hamad, M.A.A. and Ferdows, M. (2012): Similarity Solution of Boundary Layer Stagnation Point Flow Towards a Heated Porous Stretching Sheet Saturated with a Nanofluid with Heat Absorption/Generation and Suction/Blowing: A Lie Group Analysis, Communications in Nonlinear Science and Numerical Simulation, Vol. 17, pp.132–140. http://dx.doi.org/10.1016/j.cnsns.2011.02.024

Hayat, T., Iqbal, Z., Qasim, M. and Obaidat, S. (2012): Steady Flow of an Eyring Powell Fluid over a Moving Surface with Convective Boundary Conditions, International Journal of Heat and Mass Transfer, Vol. 55, pp. 1817–1822. http://dx.doi.org/10.1016/j.ijheatmasstransfer.2011.10.046

Ibragimov, N. H.(1995): A Handbook of Lie Group Analysis of Differential Equations, Vol. 2, CRC Press, Boca Raton, Fla, USA.

Ibrahim, F. S. Mansour, M. A. and Hamad, M. A. (2005) : A Lie-group Analysis of Radiative and Magnetic Field Effects on Free Convection and Mass Transfer Flow Past a Semi-Infinite Vertical Flat Plate, Electronic Journal Differential Equations, Vol.39, pp.1-17.

Ibrahim, F.S. and Hamad, M.A.A. (2006): Group Method Analysis of Mixed Convection Boundary-Layer Flow of a Micro Polar Fluid Near a Stagnation Point on a Horizontal Cylinder, Acta Mech. Vol.18, pp.65–81. http://dx.doi.org/10.1007/s00707-005-0272-9

Ishak, A. (2010): Similarity Solutions for Flow and Heat Transfer over a Permeable Surface with Convective Boundary Condition, Applied Mathematics and Computation. Vol. 217, pp. 837–842. http://dx.doi.org/10.1016/j.amc.2010.06.026

Ishak, A. Yacob, N.A. and Bachok, N.(2011): Radiation Effects on the Thermal Boundary Layer Flow over a Moving Plate with Convective Boundary Condition, Meccanica, Vol.46, pp. 795–801. http://dx.doi.org/10.1007/s11012-010-9338-4

Jain, M.K. and Iyengar, S.R.K. and Jain, R.K. (1985): Numerical Methods for Scientific and Engineering Computation. Wiley Eastern Ltd., New Delhi, India.

Kalpakides, V. K. and Balassas, K. G. (2004): Symmetry Groups and Similarity Solutions for a Free Convective Boundary-Layer Problem, International Journal of Nonlinear Mechanics, Vol.39, pp. 1659-1670. http://dx.doi.org/10.1016/j.ijnonlinmec.2004.01.006

Kandasamy, R., Hayat, T. and Obaidat, S. (2011): Group Theory Transformation for Soret and Dufour Effects on Free Convective Heat and Mass Transfer with Thermophoresis and Chemical Reaction over a Porous Stretching Surface in the Presence of Heat Source/Sink, Nuclear Engineering and Design, Vol. 241, pp. 2155–2161. http://dx.doi.org/10.1016/j.nucengdes.2011.03.002

Kishan, N. and Amrutha, P. (2011): Effects of Viscous Dissipation on MHD Flow with Heat and Mass Transfer over a Stretching Surface with Heat Source, Thermal Stratification and Chemical Reaction, Journal Of Naval Architecture And Marine Engineering, Vol. 7, No. 1, pp. 11-18. http://dx.doi.org/10.3329/jname.v7i1.3254

Kumari, M., Takhar H. S. and Nath, G. (2001): Mixed Convection Flow over a Vertical Wedge Embedded in a Highly Porous Medium, Journal of Heat and Mass Transfer, Vol.37, pp. 139-146. http://dx.doi.org/10.1007/s002310000154

Lai, F. C. and Kulacki, F. A. (1994): Non-Darcy Convection from Horizontal Impermeable Surfaces in Saturated Porous Media, International Journal of Heat and Mass Transfer, Vol.30 No.10, pp. 2189-2192. http://dx.doi.org/10.1016/0017-9310(87)90096-2

Makinde, O.D. (2010): On MHD Heat and Mass Transfer over a Moving Vertical Plate with a Convective Surface Boundary Condition, Canadian Journal of Chemical Engineering, Vol.88, pp. 983–990. http://dx.doi.org/10.1002/cjce.20369

Makinde, O.D. (2010): Similarity Solution of Hydromagnetic Heat and Mass Transfer over a Vertical Plate with a Convective Surface Boundary Condition, International Journal of Physical Sciences, Vol.5, No.6, pp. 700–710.

Moran, A. J. A and Gaggioli, R.A. (1968): Reduction of the Number of Variables in Systems of Partial Differential Equations with Auxiliary Conditions, SIAM Journal of Applied Mathematics, Vol. 16, pp. 202–215. http://dx.doi.org/10.1137/0116018

Morgan, A.J.A. (1952): The Reduction by One of the Number of Independent Variables in Some Systems of Partial Differential Equations, Quarterly Journal of Mathematics, Vol. 3, pp. 250– 259. http://dx.doi.org/10.1093/qmath/3.1.250

Oberlack, M. (1999): Similarity in Non-Rotating and Rotating Turbulent Pipe Flows, Journal Fluid Mechanics. Vol. 379, pp.1–22. http://dx.doi.org/10.1017/S0022112098001542

Olver, P. J. (1989): Applications of Lie Groups to Differential Equations, Springer, New York, USA.

Parveen, N and Alim, M. A., (2012): Joule Heating Effect on Magnetohydrodynamic Natural Convection Flow along a Vertical Wavy Surface, Journal of Naval Architecture and Marine Engineering, Vol.9, No.1, pp.11-24.. http://dx.doi.org/10.3329/jname.v9i1.5954

Ramana Reddy, G. V., Ramana Murthy, and Ch.V. and Bhaskar Reddy, N. (2011): MHD Flow over a Vertical Moving Porous Plate with Heat Generation by Considering Double Diffusive Convection, International Journal of Applied Mathematics and Mechanics, Vol.7, No.4, pp. 53-69.

Shu, J. J. and Pop, I. (1997): Inclined Wall Plumes in Porous Media, Fluid Dynamics Research, Vol. 21, pp. 303-317. http://dx.doi.org/10.1016/S0169-5983(97)81115-X

Sivasankaran, S., Bhuvaneswari, M., Kandaswamy, P. and Ramasami, E. K. (2006) : Lie Group Analysis of Natural Convection Heat and Mass Transfer in an Inclined Porous Surface with Heat Generation, International Journal Applied Mathematics and Mechanics, Vol. 2, pp.34–40.

Sivasankaran, S., Bhuvaneswari, M., Kandaswamy, P. and Ramasami, E. K. (2006): Lie group Analysis of Natural Convection Heat and Mass Transfer in an Inclined Surface, Nonlinear Analysis: Modeling and Control, Vol.11, No.2, pp. 201–212.

Subhashini, S.V., Samuel, N.and Pop, I. (2011): Double-diffusive Convection from a Permeable Vertical Surface under Convective Boundary Condition, International Communications in Heat and Mass Transfer, Vol. 38, pp.1183–1188. http://dx.doi.org/10.1016/j.icheatmasstransfer.2011.06.006

Vsiannikov, L. V. (1982): Group Analysis of Differential Equations, Academic Press, New York, USA.

Yurusoy, M. andPakdemirli, M. (1997): Symmetry Reductions of Unsteady Three-Dimensional Boundary Layers of Some Non-Newtonian Fluids, International Journal of Engineering Science, Vol.35, No.8, pp. 731-740. http://dx.doi.org/10.1016/S0020-7225(96)00115-2

Yurusoy, M. and Pakdemirli, M. (1999): Exact Solutions of Boundary Layer Equations of a Special Non-Newtonian Fluid over a Stretching Sheet, Mechanics Research Communications, Vol. 26, No. 2, pp. 171–175. http://dx.doi.org/10.1016/S0093-6413(99)00009-9

Yurusoy, M., Pakdemirli,M. and Noyan, O. F. (2001): Lie Group Analysis of Creeping Flow of a Second Grade Fluid, International Journal of Non-Linear Mechanics, Vol. 36, No. 6, pp. 955–960. http://dx.doi.org/10.1016/S0020-7462(00)00060-3