Journal of Naval Architecture and Marine Engineering
June, 2008
DOI: 10.3329/jname.v5i1.1868 http://jname.8m.net

SEPARATION POINTS OF MAGNETO-HYDRODYNAMIC BOUNDARY LAYER FLOW ALONG A VERTICAL PLATE WITH EXPONENTIALLY DECREASING FREE STREAM VELOCITY

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Abstract:

The points of separation of magneto-hydrodynamic mixed convection boundary layer flow along a vertical plate have been investigated. The free stream velocity is considered decreasing exponentially in the stream wise direction. The governing boundary layer equations are transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations are reduced to local non-similar boundary layer equations, which are solved numerically by implicit finite difference method known as Keller box scheme. Here we have focused our attention to find the effects of suction, magnetic field and other relevant physical parameters on the position of boundary layer separation. The numerical results are expressed in terms of local shear stress showing the effects of suction, buoyancy, Prandlt number and magnetic field on the shear stress as well as on the points of separation.

Keywords: Separation points, magneto-hydrodynamic, mixed convection, boundary layer, suction, finite difference method, Keller box scheme.

NOMENCLATURE

B_0	magnetic field strength
g	acceleration due to gravity
k	coefficient of thermal diffusivity
M	Magneto-hydrodynamic parameter
Pr	Prandtl number
Re	Reynolds number
T	temperature of the fluid
T_w	temperature of the heated surface
T_∞	temperature of the ambient fluid
u,v	velocity along the x & y direction
x	stream wise coordinate measuring distance along the surface
y	stream wise coordinate measuring distance normal to the surface

Greek symbols

α	suction parameter
β	volumetric coefficient of thermal expansion
γ_0	transpiration (suction) velocity
η	similarity variable
θ	dimensionless temperature function
ν	coefficient of viscosity
ξ	a scaled stream wise coordinate
ρ	density of the fluid
σ	electric conductivity
ψ	stream function

1. Introduction

The phenomenon of separation is one of the most interesting features of the motion of an incompressible fluid past a solid body at high Reynolds number. In the presence of an adverse pressure gradient, the thin boundary layer grows in thickness and eventually breaks away from the solid surface. The point, at which separation of boundary layer occurs, for steady flow over a stationary surface, is generally taken as coinciding with or very

 Received on: 3 March 2008

near the point at which the skin friction vanishes. The determination of the separation point in boundary layer flow has been the subject of many investigations over the past few decades. The usual procedure is to apply numerical methods to the governing partial differential equation, compute the full-field solution, and thereby obtain the stream wise station at which the wall shear stress becomes zero. Brown & Stewartson (1969) and Laura et al. (1990) investigated the points of separation in their works. Curle (1981) investigated the development of a steady two-dimensional incompressible laminar boundary layer when the external flow velocity is given by $u_e = u_0(1-\varepsilon e^{\xi})$, $0 < \varepsilon < 1$. When ε is very small and ξ is not too large then εe^{ξ} is also small and u_e is approximately constant and hence the flow is just a perturbation of the Blasius flow. However, as ξ increases the effect of εe^{ξ} is felt more and more. As ξ approaches $\log(\varepsilon^{-1})$, u_e falls rapidly causing the boundary layer to separate. This type of problem was analyzed by Curle (1981) and he used the equation $F_{\eta\eta\eta} + FF_{\eta\eta} + 2\xi\left(F_{\xi}F_{\eta\eta} - F_{\eta}F_{\xi\eta}\right) = 2\varepsilon\xi e^{\xi}(1-\varepsilon e^{\xi})$. This is obtained from two-dimensional boundary layer equation after introducing the variable $\eta = \left[\dfrac{u_0}{2\nu x}\right]^{1/2} y$ and the stream function $\psi = (2u_0 \nu x)^{1/2} F(\xi,\eta)$

Curle solved the above equation by writing $F(\xi,\eta) = F_0(\eta) + \varepsilon e^{\xi} F_1(\xi,\eta) + \varepsilon^2 e^{2\xi} F_1(\xi,\eta) + \cdots$
where $F_0(\eta)$ satisfies the Blasius equation. For small ξ, $F_1(\xi,\eta)$ is further expanded as
$F_1(\xi,\eta) = f_0(\eta) + \xi f_1(\eta) + \xi^2 f_1(\eta) + \cdots$

and the results were used to calculate the skin friction and displacement thickness. For large ξ inner and outer asymptotic expansions were determined and matched. The skin friction is expressed in a power series as: $X = \varepsilon\xi^{2/3} e^{\xi} = 0.1110187 + 0.00983\xi^{-2/3} + \cdots$

Chiam (1998) numerically solved quite similar type of problem for no suction and for uniform suction at the plate.

A study of the flow of electrically conducting fluid in presence of magnetic field is also important from the technical point of view and such types of problems have received much attention by many researchers. The specific problem selected for study is the flow and heat transfer in an electrically conducting fluid adjacent to the surface. The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force, which tends to oppose the fluid motion. And near the leading edge the velocity is very small so that the magnetic force, which is proportional to the magnitude of the longitudinal velocity and acts in the opposite direction, is also very small. Consequently, the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from the free stream pressure gradient. Magneto-hydrodynamic was originally applied to astrophysical and geophysical problems, where it is still very important, but more recently to the problem of fusion power, where the application is the creation and containment of hot plasmas by electromagnetic forces, since material walls would be destroyed. Astrophysical problems include solar structure, especially in the outer layers, the solar wind bathing the earth and other planets, and interstellar magnetic fields. The primary geophysical problem is planetary magnetism, produced by currents deep in the planet, a problem that has not been solved to any degree of satisfaction.

The hydrodynamic behavior of boundary layers along a flat plate in the presence of a constant transverse magnetic field was first analyzed by Rossow (1958), who assumed that magnetic Reynolds number was so small that the induced magnetic field could be ignored. MHD free convection flow of visco-elastic fluid past an infinite porous plate was investigated by Chowdhury and Islam (2000). Raptis and Kafoussias (1982) investigated the problem of magneto-hydrodynamic free convection flow and mass transfer through a porous medium bounded by an infinite vertical porous plate with constant heat flux. Moreover, Hossain et al. (1997) discussed both forced and free convection boundary layer flow of an electrically conducting fluid in presence of magnetic field. The magneto-hydrodynamic boundary layer flow and heat transfer on a continuous moving wavy surface was investigated by Hossain & Pop (1996).

The present work considered the magneto-hydrodynamic boundary layer flow along a vertical plate, with exponentially decreasing free stream velocity and has solved the problem numerically using the implicit finite difference method together with the Keller-Box scheme (1978). This method is described in details by Cebeci & Bradshaw (1984) and used by Hossain et al. (1997, 1998 and 1999). The purpose of this paper is to study the MHD boundary layer flow and to show the effect of MHD on the points of separation. The numerical results are expressed in terms of local shear stress showing the effects of suction, buoyancy, Prandlt number and magnetic field on the shear stress as well as on the points of separation and some results are compared with those of Chiam (1998).

2. Formulation of the problem

A steady two-dimensional incompressible boundary layer flow along a vertical plate with external velocity is considered. It is assumed that the surface temperature of the plate is T_w and the temperature of the ambient fluid is T_∞, where $T_w > T_\infty$. The physical coordinates (x, y) are chosen such that x is measured from the leading edge in the stream wise direction and y is measured normal to the surface of the plate. The flow configuration and the coordinate system are shown in Fig.1.

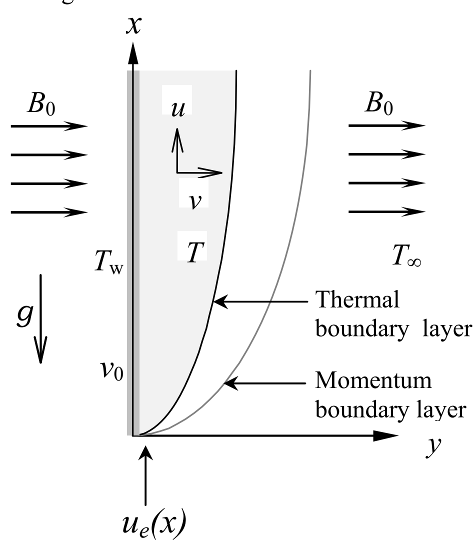

Fig. 1: The co-ordinate system and the physical model

The governing equations for mass continuity, momentum and energy take the following forms:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2} + \rho g\beta(T - T_\infty) - \frac{\sigma B_0^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

where, u and v are velocity components in the x and y directions respectively, ν is the coefficient of viscosity, ρ is the density of ambient fluid, g is the acceleration due to gravity, σ is the electric conductivity, B_0 is the magnetic field strength, β is the volumetric coefficient of thermal expansion, κ is the coefficient of thermal conductivity and T is the temperature of the fluid. The boundary conditions for the present problem are:

$$\begin{aligned} &y = 0: \quad u = 0,\ v = v_0,\quad T = T_w \\ &y \to \infty: u \to u_e(x)\ ,\quad T \to T_\infty \end{aligned} \quad \text{and} \quad u_e = u_0(1 - \varepsilon e^{\xi}),\ 0 < \varepsilon < 1 \tag{4}$$

In equation (4), v_0 represents the suction velocity of fluid through the surface of the plate. Here, for suction or withdrawal of fluid the transpiration velocity v_0 is negative whereas for blowing or injection of fluid $v_0 > 0$. Near the leading edge, the boundary layer is very much like that of the free convection boundary layer in the absence of suction. Therefore the following group of transformations may be introduced:

$$\eta = y\left(\frac{u_e}{\nu x}\right)^{1/2} = \mathrm{Re}_x^{1/2}\frac{y}{x},\ \ \xi = \frac{x}{L} \tag{5}$$

$$\psi(x,y) = \left(u_e \nu x\right)^{1/2} f(\xi,\eta) \tag{6}$$

$$\theta(\xi,\eta)=\frac{T(x,y)-T_\infty}{T_w - T_\infty} \tag{7}$$

where the stream function $\psi(x, y)$ satisfies the mass conservation equation with

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \tag{8}$$

Equations (2) and (3) can be transformed into

$$f_{\eta\eta\eta} + \frac{m+1}{2} ff_{\eta\eta} + m\left(1 - f_\eta^{\ 2}\right) + \frac{Gr}{\mathrm{Re}^2}\xi\theta - \xi M f' = \xi\left(f_\eta \frac{\partial f_\eta}{\partial \xi} - f_{\eta\eta}\frac{\partial f}{\partial \xi}\right) \tag{9}$$

$$\frac{1}{\mathrm{Pr}}\theta_{\eta\eta} + \frac{m+1}{2} f\theta_\eta = \xi\left(f_\eta \frac{\partial \theta}{\partial \xi} - \theta_\eta \frac{\partial f}{\partial \xi}\right) \tag{10}$$

where m is a dimensionless pressure gradient parameter and M is magnetic parameter defined respectively by

$$m = \frac{x}{u_e}\frac{du_e}{dx} \quad \text{and} \quad M = \frac{\nu\sigma B_0 L}{\rho u_e} \tag{11}$$

And Pr = ν/κ is the Prandtl number. Equations (9)-(11) are the local non-similarity equations governing the flow under consideration. The boundary condition (4) takes the form

$$\begin{aligned} &f_\eta = 0,\ \theta = 1 \ \text{ at } \ \eta = 0 \\ &f_\eta = 1,\ \theta = 0 \ \text{ as } \ \eta \to \infty \end{aligned} \tag{12}$$

which leads to

$$f(\xi,0) = \alpha \frac{(1-\varepsilon \ \ e^{\xi})^{1/2}}{1+(1+\xi)\varepsilon \ \ e^{\xi}}\sqrt{\xi} = \alpha\,\Gamma\sqrt{\xi} \tag{13}$$

where the suction parameter $\alpha = -2v_0\sqrt{\dfrac{L}{u_0 \nu}}$

Chiam (1998) assumed

$$\Gamma = \frac{(1-\varepsilon e^{\xi})^{1/2}}{1+(1+\xi)\varepsilon e^{\xi}} = 1 \tag{14}$$

and obtained the solution numerically.

The solutions of Equations (9)-(11) enable us to compute the local shear stress $f_{\eta\eta}(\xi,0)$ and the local rate of heat transfer $\theta_\eta(\xi,0)$ from the wall values of

$$f_{\eta\eta}(\xi,\eta) \ \text{ and } \ \theta_\eta(\xi,\eta) \tag{15}$$

In this paper only the effect of Γ on shear stress as well as separation point is considered.

3. Method of solution

The numerical method used here is finite difference method known as Keller box Scheme (1978) which has been described in details by Cebeci and Bradshow (1984). The method has been used successfully by Hossain & Alim (1997) and Hossain et al. (1998, 1999) and it has also been used by many authors in a wide variety of boundary layer problems. To employ the finite difference method, the system of partial differential equations (9)-(10) are first converted into a system of first order differential equations. The discretization of momentum and energy equations carried out with respect to non-dimensional coordinates ξ and η to express the equations in finite difference form by approximating the functions and their derivatives in terms of the central differences in both coordinate directions. Denoting the mesh points in the ξ,η-plane by ξ_i and η_j where i = 1, 2, . . . , M and j = 1, 2, . . . , N, central difference approximations are made, such that those equations involving ξ explicitly are centered at ($\xi_{i-1/2}$, $\eta_{j-1/2}$) and the remainder at (ξ_i , $\eta_{j-1/2}$), where $\eta_{j-1/2} = \frac{1}{2}(\eta_j + \eta_{j-1})$ etc. The above central difference approximations reduces the system of first order differential equations to a set of non-linear difference equations for the unknown at ξ_i in terms of their values at ξ_{i-1}. The resulting set of nonlinear difference equations are solved by using the Newton's quasi-linearization method. The Jacobian matrix has a block-tridiagonal structure and the difference equations are solved using a block-matrix version of the Thomas algorithm; the details of the computational procedure have been discussed further in the book by Cebecci and Bradshow (1984).

4. Results and Discussion

Equations (9 - 11) subject to the boundary conditions (12) are solved numerically using implicit finite difference method of Keller (1978), which is described by Cebeci and Bradshow (1984). The numerical results of $f_{\eta\eta}(\xi,\eta)$ for η = 0 denoted by $f_{\eta\eta}(\xi,0)$ are obtained for representative values of the suction parameter $\alpha = -2v_0\sqrt{L/u_0\nu}$ between 0.0 and 1.0 for different values of Prandtl number Pr, the buoyancy parameter Gr/Re^2, magneto-hydrodynamic parameter M and for several values of ε. Fig.2 shows the results of separation points for Pr=1.0, M = 0.0, ε =0.1 and Gr/Re^2 = 0.0 considering the effect of Γ (Eq^n 14) and compares with the results of Chiam (1998). The effect of Γ is clearly demonstrated in the figure. At α = 0.0 the effect of Γ is insignificant thus the present solution gives the identical result as Chiam (1998). As the value of α increases, the effect of Γ becomes prominent as shown in the figure. It is well known that suction tends to delay separation. This can readily be seen from Table 1 where ξ_s is shown for three representative values of the suction parameter α for each ε. The percentage shift of ξ_s to higher values is more pronounced for larger values of ε.

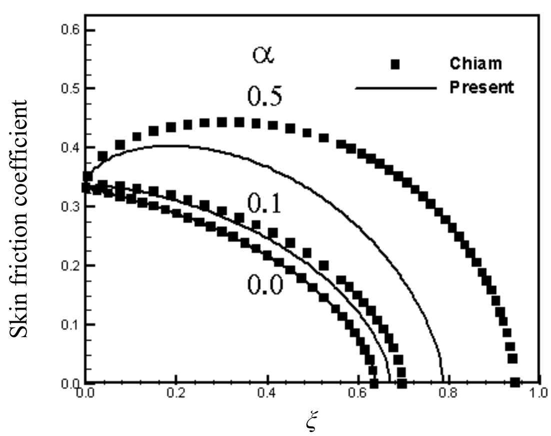

Fig. 2: Plot of the wall values of $f_{\eta\eta}(\xi,\eta)$ Versus ξ for different values of the suction parameter α when Pr=1.0, Gr/Re^2=0.0, M=0.0 and ε=0.1

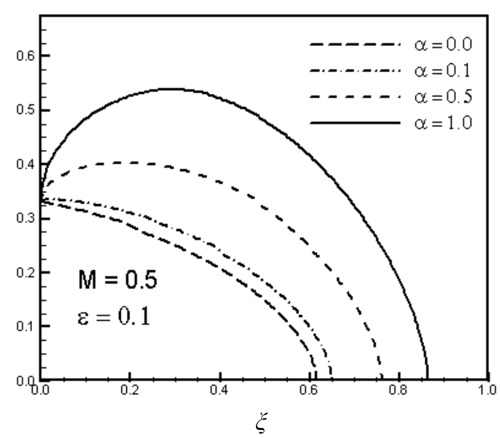

Fig. 3: Plot of the wall values of $f_{\eta\eta}(\xi,\eta)$ Versus ξ for different values of the suction parameter α when Pr= 1.0, Gr/Re^2 =0.0, M=0.5, and ε=0.1

Figs. 3-5 show plots of the wall values of $f_{\eta\eta}(\xi,\eta)$ against ξ, the stream wise coordinate. The behavior of the curves can be understood on the basis of the interplay of two effects: the adverse pressure gradient tending to bring about separation and suction tending to delay it. Fig. 3 show the effect of the suction parameter α for Pr=1.0, M=0.5, Gr/Re^2 =0.0 when ε = 0.1. The effect of α is also shown for ε = 0.05 and 0.01 for the same parameter in Fig. 4 and Fig. 5 respectively. For all values of ε, the distance of separation point from the leading edge increases with increase in α. On the other hand, the distance of separation point from leading edge increases when the value of ε decrease for a particular value of α. Figs. 6-7 describe the effect of buoyancy parameter Gr/Re^2 on skin friction and on the points of separation. We observe that the points of separation occur for Gr/Re^2 less than 0.5 and at above this value there is no point of separation. Fig. 8 shows the effect of Prandtl number Pr on $f_{\eta\eta}(\xi, \eta)$ as well as that on the points of separation for the suction parameter $\alpha = 0.2$ when $Gr/Re^2 = 0.0$, $M = 0.5$, and $\varepsilon = 0.1$. It is seen that, the lengths of separation points from the leading edge decrease when the Prandalt number increases.

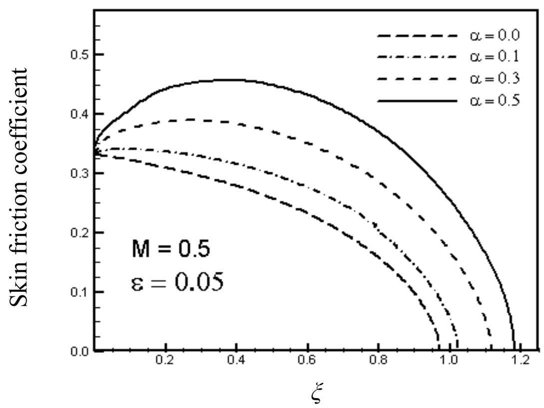

Fig. 4: Plot of the wall values of $f_{\eta\eta}(\xi, \eta)$ Versus ξ for different values of the suction parameter α when $\varepsilon = 0.05$, Pr= 1.0, M=0.5 and $Gr/Re^2 = 0.0$

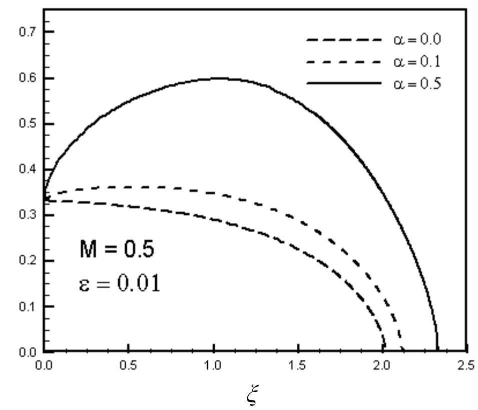

Fig. 5: Plot of the wall values of $f_{\eta\eta}(\xi, \eta)$ Versus ξ for different values of the suction parameter α when $\varepsilon = 0.01$, M=0.5, Pr= 1.0, $Gr/Re^2 = 0.0$

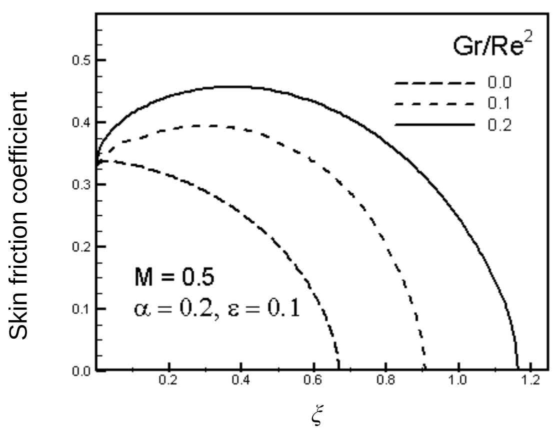

Fig. 6: Plot of the wall values of $f_{\eta\eta}(\xi, \eta)$ Versus ξ for different values of the buoyancy parameter Gr/Re^2 when $\varepsilon = 0.1$, M=0.5 and α=0.2

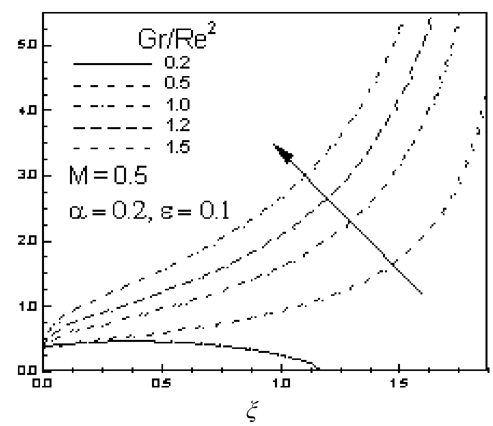

Fig. 7: Plot of the wall values of $f_{\eta\eta}(\xi, \eta)$ Versus ξ for different values of the buoyancy parameter Gr/Re^2 when $\varepsilon = 0.1$, M=0.5 and α=0.2

Finally, Fig. 9 exhibiting the effect of magnetic parameter M on $f_{\eta\eta}(\xi, \eta)$ as well as that on the points of separation for $\alpha = 0.2$ when $Gr/Re^2 = 0.0$, Pr = 0.72 and $\varepsilon = 0.1$. From this figure, it can easily be seen that an increase in the magnetic parameter M leads to decrease in the local skin friction coefficient. This phenomenon can easily be understood from the fact that when the magnetic parameter M increases the temperature of the fluid rises and the thickness of the velocity boundary layer grows i.e. the thermal boundary layer becomes thinner than the velocity boundary layer. Therefore the skin friction coefficient decreases. As a result, the separation becomes faster with the increase of M. Table-1 depicts the comparisons of the present numerical results of the separation points for different values of ε and α with those obtained by Chiam (1998). The present results agreed well with the solutions of Chiam (1998) in absence of Magnetic field, i.e., for $M = 0.0$.

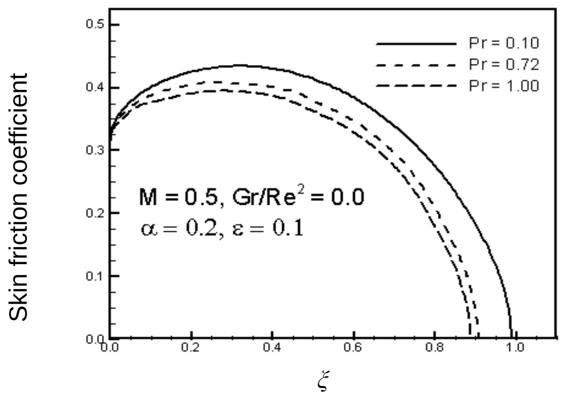

Fig. 8: Plot of the wall values of $f_{\eta\eta}(\xi,\eta)$ Versus ξ for different values of the Prandtl number Pr when ε =0.1, α=0.2, M=0.5 and Gr/Re^2 = 0.0

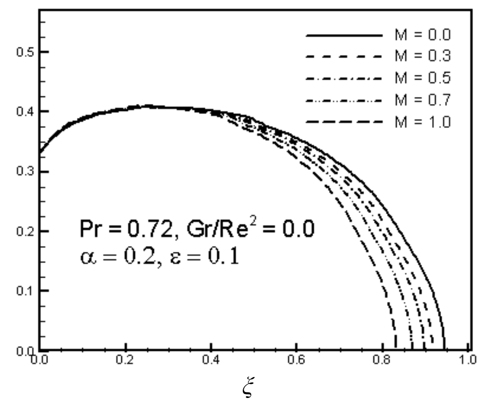

Fig. 9: Plot of the wall values of $f_{\eta\eta}(\xi,\eta)$ Versus ξ for different values of M when ε =0.1, α=0.2, Pr=0.72 and Gr/Re^2 = 0.0

Table 1: Effect of suction on the position of separation ξ_s [$f_{\eta\eta}(\xi_s, 0)$ less than 0.0001)] for Pr= 1.0, M=0.0 and Gr/Re^2 = 0.0

ε	Suction parameter α	Position of separation ξ_s	
		Present result	Chiam's result
0.1	0.0	0.6352	
	0.1	0.6705	0.6930
	0.5	0.7865	0.9475
0.05	0.0	0.9986	
	0.1	1.0477	1.0916
	0.5	1.2132	1.4565
0.01	0.0	2.0458	
	0.1	2.1453	2.2380
	0.5	2.3613	2.8203
0.001	0.0	3.8750	
	0.1	3.9950	4.1796
	0.5	4.1627	4.9418

5. Conclusion

The numerical results of the skin friction coefficients for different values of the pertinent parameter considering the effect of Γ have been shown. The results on the position of separation have been presented for the boundary layer flow in the presence of buoyancy with exponentially decreasing free stream velocity. The effects of suction parameter, Prandtl number, buoyancy parameters and magnetic parameter on the separation point are described and the results are compared with those of Chiam (1998). From this investigation, following conclusions can be drawn:

i) At α = 0.0, the effect of Γ is insignificant thus the present solution gives the identical result as Chiam (1998).

ii) As the magnitude of α increases, the effect of Γ becomes prominent.

iii) For all values of ε, the distance of separation point from the leading edge increases with increase in α.

iv) For a particular value of α, the distance of separation point from leading edge increases for lower values of ε.

v) As M increases, the point of separation becomes closer to the leading edge, i.e., separation becomes faster with the increase in M.

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