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NATURAL CONVECTION FLOW FROM AN ISOTHERMAL SPHERE WITH TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY

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Abstract

Laminar free convection flow from an isothermal sphere immersed in a fluid with thermal conductivity proportional to linear function of temperature has been studied. The governing boundary layer equations are transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations is reduced to local non-similarity equations, which are solved numerically by very efficient implicit finite difference method together with Keller box scheme. Numerical results are presented by velocity and temperature distribution of the fluid as well as heat transfer characteristics, namely the heat transfer rate and the skin-friction coefficients for a wide range of thermal conductivity parameter γ (= 0.0, 0.5, 1.0, 2.0, 3.0, 5.0) and the Prandtl number Pr (= 0.7, 1.0, 3.0, 5.0, 7.0).

Keywords: Natural convection, temperature dependent thermal conductivity, isothermal sphere.

NOMENCLATURE:

a	Radius of the sphere
C_p	Specific heat at constant pressure
C_f	Skin-friction coefficient
f	Dimensionless stream function
g	Acceleration due to gravity
Gr	Grashof number
$k(T)$	Thermal conductivity of the fluid
Nu	Nusselt number
Pr	Prandtl number
q_w	Heat flux at the surface
r	Local radius of the sphere
T	Temperature of the fluid in the boundary layer
T_∞	Temperature of the ambient fluid
T_w	Temperature at the surface
u, v	The dimensionless x and y- component of the velocity
$\hat{u}, \hat{v}$	The dimensional $\hat{x}$ and $\hat{y}$ component of the velocity
x,y	Axis in the direction along and normal to the surface

Greek symbols

β	Volumetric coefficient of thermal expansion
ψ	Stream function
τ_w	Shearing stress
γ	Conductivity-variation parameter
γ^*	Constant
ρ	Density of the fluid
ν	Reference kinematic viscosity
μ	Viscosity of the fluid
θ	Dimensionless temperature function

Subscript

w	Wall conditions
f	Film temperature of the fluid
∞	Ambient temperature

Superscript

$'$	Differentiation with respect to y

1. Introduction:

Natural convection flow of viscous incompressible fluid from an isothermal sphere represents an important problem, which is related to numerous engineering applications. Conjugate effect heat and mass transfer in natural convection flow from an isothermal sphere with chemical reaction has been investigated by Molla et al. (2004). The natural convection flow from an isothermal horizontal circular cylinder and sphere with temperature dependent viscosity has been investigated by Molla et al. (2005). Nazar et al. (2002) have considered the problem of natural convection flow from lower stagnation point to upper stagnation point of a horizontal circular cylinder and an isothermal sphere immersed in a micropolar fluid. Chiang et al. (1964) investigated the laminar free convection from a sphere by considering prescribed surface temperature and surface heat flux. Natural convection from a sphere with blowing and suction studied by Huang and Chen (1987). Analysis of mixed forced and free convection about a sphere studied by Chen and Mucoglu (1977).

Sparrow and Lee (1976), looked at the problem of vertical stream over a heated horizontal circular cylinder. They obtain a solution by expanding velocity and temperature profiles in powers of x, the co-ordinate measuring distance from the lowest point on the cylinder. The exact solution is still out of reach due to the non-linearity in the Navier-Stokes equations. It appears that Merkin (1977), was the first who presented a complete solution of this problem using Blasius and Gortler series expansion method along with an integral method and a finite-difference scheme. Also the problem of free convection boundary layer flow on cylinder of elliptic cross-section was studied by Merkin (1977). Ingham (1978) investigated the boundary layer flow on an isothermal horizontal cylinder. Hossain and Alim (1997) have investigated natural convection-radiation interaction on boundary layer flow along a vertical thin cylinder. Hossain et al. (1999), have studied radiation-conduction interaction on mixed convection from a horizontal circular cylinder.

All the above studies were confined to the fluid with constant thermal conductivity. However, it is known that this physical property may change significantly with temperature. To predict accurately the flow behavior, it is necessary to take into account this variation of thermal conductivity. A semi-empirical formula for the variation of the thermal conductivity with temperature was used by Arunachalam and Rajappa (1978). On assuming that the viscosity and thermal conductivity of the fluid are linear functions of temperature, two semi-empirical formulae were proposed by Charraudeau (1975). Following him Hossain et al. (2000) investigated the natural convection flow past a permeable wedge, a flat plate and a wavy surface for the fluid having temperature dependent viscosity and thermal conductivity.

In the present study it is proposed to investigate the natural convection flow of a viscous incompressible fluid having thermal conductivity $k(T)$ depending on temperature from an isothermal sphere. The surface temperature T_w of the sphere is higher than that of the ambient fluid temperature T_∞. In formulating the equations governing the flow the conductivity of the fluid has been assumed to be proportional to a linear function of temperature, a semi-empirical formula for the conductivity $k(T)$, as Charrudeau (1975). The governing partial differential equations are reduced to locally non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations are solved numerically using very efficient finite-difference scheme known as Keller box technique (1978). Effect of conductivity-variation parameter γ , on the velocity and temperature distribution of the fluid as well as on the rate of heat transfer in terms of the Nusselt number and the skin-friction are shown graphically for fluids having Prandtl number Pr ranging from 0.7 to 7.0.

2. Formulation of problem

A steady two-dimensional laminar free convective flow from a uniformly heated sphere of radius a, which is immersed in a viscous and incompressible fluid having temperature dependent thermal conductivity, is considered. It is assumed that the surface temperature of the cylinder is T_w, where $T_w > T_\infty$. Here T_∞ is the ambient temperature of the fluid, the configuration considered is as shown in Figure 1.

The equations governing the flow are

$$\frac{\partial(\hat{r}\hat{u})}{\partial\hat{x}} + \frac{\partial(\hat{r}\hat{v})}{\partial\hat{y}} = 0 \tag{1}$$

$$\rho\left(\hat{u}\frac{\partial\hat{u}}{\partial\hat{x}} + \hat{v}\frac{\partial\hat{u}}{\partial\hat{y}}\right) = \mu\frac{\partial^2\hat{u}}{\partial\hat{y}^2} + \rho g\beta\left(T - T_\infty\right)\sin\left(\frac{\hat{x}}{a}\right) \tag{2}$$

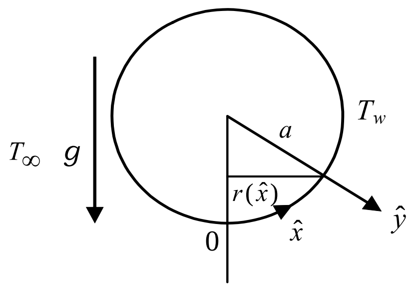

Figure 1: Physical model and coordinate system.

$$\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}} = \frac{1}{\rho C_p}\frac{\partial}{\partial \hat{y}}\left(k\frac{\partial T}{\partial \hat{y}}\right) \tag{3}$$

The boundary conditions of equation (1) to (3) are

$$\hat{u} = \hat{v} = 0, \quad T = T_w, \text{ at } \hat{y} = 0 \tag{4a}$$

$$\hat{u} \to 0, \quad T \to T_\infty \quad \text{as} \quad \hat{y} \to \infty \tag{4b}$$

where $(\hat{u}, \hat{v})$ are velocity components along the $(\hat{x}, \hat{y})$ axes, g is the acceleration due to gravity, ρ is the density, μ is the viscosity of the fluid, β is the coefficient of thermal expansion, C_p is the specific heat at constant pressure, k (T) is the thermal conductivity of the fluid depending on the fluid temperature T. Here $r(\hat{x}) = a\sin(\hat{x}/a)$

Here we will consider the form of the temperature dependent thermal conductivity which is proposed by Charraudeau (1975), as follows

$$k = k_\infty\left[1 + \gamma^*(T - T_\infty)\right] \tag{5a}$$

where k_∞ is the thermal conductivity of the ambient fluid and γ^* is defined as follows

$$\gamma^* = \frac{1}{k_f}\left(\frac{\partial k}{\partial T}\right)_f \tag{5b}$$

We now introduce the following non-dimensional variables:

$$x = \frac{\hat{x}}{a}, \; y = Gr^{1/4}\left(\frac{\hat{y}}{a}\right), \; u = \frac{\rho a}{\mu}Gr^{-1/2}\hat{u}$$

$$v = \frac{\rho a}{\mu}Gr^{-1/4}\hat{v}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad Gr = \frac{g\beta(T_w - T_\infty)a^3}{\nu^2} \tag{6}$$

where ν (=μ/ρ) is the reference kinematic viscosity and Gr is the Grashof number and θ is the non-dimensional temperature.

Substituting variables (6) into equations (1)-(3) leads to the following non-dimensional equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \theta\sin x \tag{8}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}(1 + \gamma\theta)\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{\Pr}\gamma\left(\frac{\partial \theta}{\partial y}\right)^2 \tag{9}$$

With the boundary conditions (4) become

$$u = v = 0, \quad \theta = 1 \text{ at } \quad x = 0, \quad \text{for any} \quad y \tag{10a}$$

$$u = v = 0, \quad \theta = 1, \quad \text{at} \quad y = 0, \quad x > 0 \tag{10b}$$

$$u \to 0, \quad \theta \to 0, \quad \text{as} \quad y \to \infty, \quad x > 0 \tag{10c}$$

In equation (9) the conductivity-variation parameter γ and Prandtl number Pr are defined as

$$\gamma = \frac{1}{k_f}\left(\frac{\partial k}{\partial T}\right)_f \left(T_w - T_\infty\right) \quad \text{and} \quad \Pr = \frac{\mu C_p}{k_\infty} \tag{11}$$

To solve equations (7)-(9), subject to the boundary conditions (10), we assume the following functions

$$\psi = xrf(x, y), \quad \theta = \theta(x, y) \tag{12}$$

where ψ is the non-dimensional stream function defined in the usual way as

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{r}\frac{\partial \psi}{\partial x} \tag{13}$$

Substituting (12) into equations (8)-(9) we get, after some algebra, the following transformed equations

$$\frac{\partial^3 f}{\partial y^3} + \left(1 + \frac{x}{\sin x}\cos x\right) f \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{\theta \sin x}{x} = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right) \tag{14}$$

$$\frac{1}{\Pr}(1 + \gamma\theta)\frac{\partial^2 \theta}{\partial y^2} + \frac{1}{\Pr}\gamma\left(\frac{\partial \theta}{\partial y}\right)^2 + \left(1 + \frac{x}{\sin x}\cos x\right) f \frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \theta}{\partial y}\frac{\partial f}{\partial x}\right) \tag{15}$$

Along with boundary conditions

$$f = \frac{\partial f}{\partial y} = 0, \quad \theta = 1 \text{ at } \quad x = 0 \quad \text{any} \quad y \tag{16a}$$

$$f = \frac{\partial f}{\partial y} = 0, \quad \theta = 1, \quad \text{at} \quad y = 0, \quad x > 0 \tag{16b}$$

$$\frac{\partial f}{\partial y} \to 0, \quad \theta \to 0, \quad \text{as} \quad y \to \infty, \quad x > 0 \tag{16c}$$

Now we calculate the important quantities for the experimentalists are the shearing stress in terms of the skin-friction coefficient and the rate heat transfer in terms of the Nusselt number, which can be written, in non-dimensional form as

$$C_f = \frac{Gr^{-3/4} a^2}{\mu \nu}\tau_w, \quad Nu = \frac{a Gr^{-1/4}}{k\left(T_w - T_\infty\right)} q_w \tag{17}$$

$$\text{where} \quad \tau_w = \mu\left(\frac{\partial \hat{u}}{\partial \hat{y}}\right)_{\hat{y}=0}, \quad q_w = -k\left(\frac{\partial T}{\partial \hat{y}}\right)_{\hat{y}=0} \tag{18}$$

Using the variables (6),(12) and the boundary condition (16b), get

$$C_f = x\frac{\partial^2 f(x,0)}{\partial y^2} \tag{19}$$

$$Nu = -\frac{\partial \theta(x,0)}{\partial y} \tag{20}$$

3. Solution Methodology

In the present investigation we have integrated the equations (14) to (15) for all x by implicit finite difference method.

Implicit finite difference method (IFDM)

In the present analysis, we shall employ a very efficient solution method, known as implicit finite difference method, which was first introduced by Keller (1978) and elaborately describe by Cebeci and Bradshaw (1984). An elaborate discussion on the development of algorithm of implicit finite difference method together with Keller-box elimination scheme is given below.

To apply the aforementioned method, we first convert the equations (14) and (15) into the following system of first order equations with dependent variables, $U(x, y)$, $V(x, y)$ and $p(x, y)$ as

$$\frac{\partial f}{\partial y} = U, \quad \frac{\partial U}{\partial y} = V, \quad \frac{\partial \theta}{\partial y} = p \tag{21a}$$

$$V' + P_1 fV - P_2 U^2 + P_3 \theta = x\left(U \frac{\partial U}{\partial x} - V \frac{\partial f}{\partial x} \right) \tag{21b}$$

$$\frac{1}{\Pr} P_4 p' + \frac{1}{\Pr} P_5 p^2 + P_1 fp = x\left(U \frac{\partial \theta}{\partial x} - p \frac{\partial f}{\partial x} \right) \tag{21c}$$

where

$$P_1 = 1 + \frac{x}{\sin x}\cos x, \quad P_2 = 1,$$
$$P_3 = \frac{\sin x}{x}, \quad P_4 = 1 + \gamma\theta, \; P_5 = \gamma \tag{21d}$$

And the boundary conditions (16) reduce to

$$f(x,0) = 0, \; U(x,0) = 0, \; \theta(x,0) = 1.0$$
$$U(x,\infty) \to 0, \quad \theta(x,\infty) \to 0 \tag{22}$$

We consider the net rectangle on the (x, y) plane and denoted by the net points

$$y_0 = 0, \; y_j = y_{j-1} + h_j, \; j = 1,2,\cdots,J$$
$$x^0 = 0, \; x^n = x^{n-1} + k_n, \; n = 1,2,\cdots,N \tag{23}$$

Here n and j are just sequence of numbers on the (x, y) plane, k_n and h_j be the variable mesh widths.

We approximate the quantities (f, U, V, θ) at the point (x^n, y_j) of the net by $(f_j^n, U_j^n, V_j^n, \theta_j^n)$which we call net function. We also employed the notation m_j^n for the quantities midway between net points and for any net function as

$$x^{n-1/2} = \frac{1}{2}(x^n + x^{n-1}), \; y_{j-1/2} = \frac{1}{2}(y_j + y_{j-1}) \tag{24a}$$

$$m_j^{n-1/2} = \frac{1}{2}(m_j^n + m_j^{n-1}), \quad m_{j-1/2}^{\;n} = \frac{1}{2}(m_j^n + m_{j-1}^{\;n-1}) \tag{24b}$$

Now we write the difference equations that are to approximate equations (21a)-(21d) by considering one mesh rectangle for the mid point $(x^n, m_{j-1/2})$ to obtain

$$\frac{f_j^n - f_{j-1}^n}{h_j} = U_{j-1/2}^n, \quad \frac{U_j^n - U_{j-1}^n}{h_j} = V_{j-1/2}^n, \quad \frac{\theta_j^n - \theta_{j-1}^n}{h_j} = p_{j-1/2}^n \tag{25}$$

Similarly equations (21c)-(21d) are approximated by centering about the mid points $\left(x^{n-1/2}, y_{j-1/2}\right)$. Centering the equations (21c) and (21d) about the point $\left(x^{n-1/2}, y\right)$ without specifying y to obtain the algebraic equations. The difference approximation to equations (21c)- (21d) become

$$\frac{1}{2}(L^n + L^{n-1}) = x^{n-\frac{1}{2}}\left[U^{n-\frac{1}{2}}\left(\frac{U^n - U^{n-1}}{k_n}\right) - V^{n-\frac{1}{2}}\left(\frac{f^n - f^{n-1}}{k_n}\right)\right] \tag{26}$$

$$\frac{1}{2}(M^n + M^{n-1}) = x^{n-\frac{1}{2}}\left[U^{n-\frac{1}{2}}\left(\frac{\theta^n - \theta^{n-1}}{k_n}\right) - p^{n-\frac{1}{2}}\left(\frac{f^n - f^{n-1}}{k_n}\right)\right] \tag{27}$$

Rearranging these equations and using equation (24), we can write

$$(V')^n + \alpha_1 (fV)^n - \alpha_2 (U^2)^n + P_3\theta^n + \alpha\left(V^{n-1}f^n - V^n f^{n-1}\right) = R^{n-1} \tag{28}$$

$$\frac{1}{\Pr}(P_4 p')^n + \frac{1}{\Pr}(P_5 p^2)^n + \alpha_1 (fp)^n - \alpha\left(U^{n-1}\theta^n - U^n\theta^{n-1} + f^{n-1}p^n - f^n p^{n-1}\right) = T^{n-1} \tag{29}$$

Where

$$\alpha_0 = \frac{x^{n-\frac{1}{2}}}{k_n}, \quad \alpha_1 = P_1 + \alpha_0, \quad \alpha_2 = P_2 + \alpha_0 \tag{30a}$$

$$R^{n-1} = -L^{n-1} + \alpha_0\left[(fV)^{n-1} - (U^2)^{n-1}\right] \tag{30b}$$

$$L^{n-1} = \left[V' + P_1 fV - P_2 U^2 + P_3\theta\right]^{n-1} \tag{30c}$$

$$T^{n-1} = -M^{n-1} + \alpha_0\left[(fp)^{n-1} - (U\theta)^{n-1}\right] \tag{30d}$$

$$M^{n-1} = \left[\frac{1}{\Pr}P_4 p' + \frac{1}{\Pr}P_5 p^2 + P_1 fp\right]^{n-1} \tag{30e}$$

The corresponding boundary conditions (22) become

$$f_0^n = 0, \quad U_0^n = 0, \quad \theta_0^n = 1$$
$$U_J^n = \theta_J^n = 0 \tag{31}$$

If we assume $f_j^{n-1}, U_j^{n-1}, V_j^{n-1}, \theta_j^{n-1}$ and p_j^{n-1} to be known for $0 \le j \le J$, equations (30) are a system of 5*J*+5 equations for the solutions of 5*J*+5 unknowns $(f_j^n, U_j^n, V_j^n, \theta_j^n, p_j^n)$, *j*=0,1,2,…,*J*. These nonlinear systems of algebraic equations are then linearized by Newton's quasi-linearization method.

We define the iterations $(f_j^{(i)}, U_j^{(i)}, V_j^{(i)}, \theta_j^{(i)}, p_j^{(i)})$, *i*=0,1,2,…,IMAX with initial values equal to those at the previous τ station (which is usually the best initial guess available). For higher iterates we get

$$f_j^{(i+1)} = f_j^{(i)} + \delta f_j^{(i)} \tag{32a}$$

$$U_j^{(i+1)} = U_j^{(i)} + \delta U_j^{(i)} \tag{32b}$$

$$V_j^{(i+1)} = V_j^{(i)} + \delta V_j^{(i)} \tag{32c}$$

$$\theta_j^{(i+1)} = \theta_j^{(i)} + \delta\theta_j^{(i)} \tag{32d}$$

$$p_j^{(i+1)} = p_j^{(i)} + \delta p_j^{(i)} \tag{32e}$$

We then insert the right hand side of the expressions (32) in place of f_j, U_j, V_j, θ_j and p_j in equations (26)-(30) dropping the terms that are quadratic in $\delta f_j^{(i)}, \delta U_j^{(i)}, \delta V_j^{(i)}, \delta\theta_j^{(i)}$ and $\delta p_j^{(i)}$. This procedure yields the following linear system of algebraic equations:

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2}\left(\delta U_j - \delta U_{j-1}\right) = (r_1)_j \tag{33a}$$

$$\delta U_j - \delta U_{j-1} - \frac{h_j}{2}\left(\delta V_j - \delta V_{j-1}\right) = (r_4)_{j-1} \tag{33b}$$

$$\delta \theta_j - \delta \theta_{j-1} - \frac{h_j}{2}\left(\delta p_j - \delta p_{j-1}\right) = (r_5)_{j-1} \tag{33c}$$

$$\begin{aligned}(s_1)\delta V_j + (s_2)\delta V_{j-1} + (s_3)\delta f_j + (s_4)\delta f_{j-1} + (s_5)\delta u_j + (s_6)\delta U_{j-1} \\ + (s_7)\delta\theta_j + (s_8)\delta\theta_{j-1} = (r_2)_j\end{aligned} \tag{33d}$$

$$\begin{aligned}(t_1)\delta p_j + (t_2)\delta p_{j-1} + (t_3)\delta f_j + (t_4)\delta f_{j-1} + (t_5)\delta U_j + (t_6)\delta U_{j-1} \\ + (t_7)\delta V_j + (t_8)\delta V_{j-1} + (t_9)\delta\theta_j + (t_{10})\delta\theta_{j-1} = (r_3)_j\end{aligned} \tag{33e}$$

$$(r_1)_j = f_{j-1}^{(i)} - f_j^{(i)} + h_j U_{j-1/2}^{(i)} \tag{33f}$$

$$(r_4)_{j-1} = U_{j-1}^{(i)} - U_j^{(i)} + h_j U_{j-1/2}^{(i)} \tag{33g}$$

$$(r_5)_{j-1} = \theta_{j-1}^{(i)} - \theta_j^{(i)} + h_j p_{j-1/2}^{(i)} \tag{33h}$$

$$\begin{aligned}(r_2)_j = R_{j-1/2}^{n-1} - [h_j^{-1}\left(V_j^{(i)} - V_{j-1}^{(i)}\right) + \alpha_1 (fV)_{j-1/2}^{(i)} - \alpha_2 \left(U^2\right)_{j-1/2}^{(i)} \\ + p_3(\theta)_{j-1/2}^{(i)} + \alpha_0 (V_{j-1/2}^{n-1} f_{j-1/2}^{(i)} - f_{j-1/2}^{n-1} V_{j-1/2}^{(i)})]\end{aligned} \tag{33i}$$

$$\begin{aligned}(r_3)_j = T_{j-1/2}^{n-1} - [\Pr^{-1} P_4 h_j^{-1}\left(p_j^{(i)} - p_{j-1}^{(i)}\right) + P_5\left(p^2\right)_{j-1/2}^{(i)} + \alpha_1 (fp)_{j-1/2}^{(i)} \\ + \alpha_0 (\theta_{j-1/2}^{n-1} U_{j-1/2}^{(i)} - U_{j-1/2}^{n-1}\theta_{j-1/2}^{(i)} + p_{j-1/2}^{n-1} f_{j-1/2}^{(i)} - f_{j-1/2}^{n-1} p_{j-1/2}^{(i)})]\end{aligned} \tag{33j}$$

The coefficients of momentum equation are

$$(s_1)_j = h_j^{-1} + \frac{1}{2}\left[\alpha_1 f_j^{(i)} - \alpha_0 f_{j-1/2}^{n-1}\right] \tag{34a}$$

$$(s_2)_j = -h_j^{-1} + \frac{1}{2}\left[\alpha_1 f_{j-1}^{(i)} - \alpha_0 f_{j-1/2}^{n-1}\right] \tag{34b}$$

$$(s_3)_j = \frac{1}{2}\left[\alpha_1 V_j^{(i)} + \alpha_0 V_{j-1/2}^{n-1}\right] \tag{34c}$$

$$(s_4)_j = \frac{1}{2}\left[\alpha_1 V_{j-1}^{(i)} + \alpha_0 V_{j-1/2}^{n-1}\right] \tag{34d}$$

$$(s_5)_j = -\alpha_2 U_j^{(i)} \tag{34e}$$

$$(s_6)_j = -\alpha_2 U_{j-1}^{(i)} \tag{34f}$$

$$(s_7) = 0 \tag{34g}$$

$$(s_8) = 0 \tag{34h}$$

The coefficients of energy equation are

$$(t_1)_j = \frac{P_4}{\Pr} h_j^{-1} + \frac{P_5}{\Pr} p_j^{(i)} + \frac{1}{2}\left[\alpha_1 f_j^{(i)} - \alpha_0 f_{j-1/2}^{n-1}\right] \tag{35a}$$

$$(t_2)_j = -\frac{P_4}{\Pr} h_j^{-1} + \frac{P_5}{\Pr} p_{j-1}^{(i)} + \frac{1}{2}\left[\alpha_1 F_{j-1}^{(i)} - \alpha_0 F_{j-1/2}^{n-1}\right] \quad (35b)$$

$$(t_3)_j = \frac{1}{2}\left[\alpha_1 p_j^{(i)} + \alpha_0 p_{j-1/2}^{n-1}\right] \quad (35c)$$

$$(t_4)_j = \frac{1}{2}\left[\alpha_1 p_{j-1}^{(i)} + \alpha_0 p_{j-1/2}^{n-1}\right] \quad (35d)$$

$$(t_5)_j = -\frac{1}{2}\alpha_0[\theta_j^{(i)} - \theta_{j-1/2}^{n-1}] \quad (35e)$$

$$(t_6)_j = -\frac{1}{2}\alpha_0[\theta_{j-1}^{(i)} - \theta_{j-1/2}^{n-1}] \quad (35f)$$

$$(t_7) = 0 \quad (35g)$$

$$(t_8) = 0 \quad (35h)$$

$$(t_9)_j = -\frac{1}{2}\alpha_0[U_j^{(i)} + U_{j-1/2}^{n-1}] + \frac{1}{2}P_5 h_j^{-1}\left(p_j^{(i)} - p_{j-1}^{(i)}\right) \quad (35i)$$

$$(t_{10})_j = -\frac{1}{2}\alpha_0[U_{j-1}^{(i)} + U_{j-1/2}^{n-1}] + \frac{1}{2}P_5 h_j^{-1}\left(p_j^{(i)} - p_{j-1}^{(i)}\right) \quad (35j)$$

The boundary conditions (31) become

$$\delta f_0 = 0, \quad \delta U_0 = 0, \quad \delta\theta_0 = 1, \quad \delta U_j = 0, \quad \delta\theta_j = 0 \quad (36)$$

which just express the requirement for the boundary conditions to remain during the iteration process. Now the system of nonlinear equations (33)-(35) together with the boundary conditions (36) can be written in matrix/vector form where the coefficient matrix has a block tri-diagonal structure. Such a system is solved using a block-matrix version of well known Thomas or tri-diagonal matrix algorithm. The whole procedure, namely reduction to first order form followed by central difference approximations, Newton's quasi-linearization method and the block Thomas algorithm, is well known as the Keller-box method. To initiate the process at the leading edge $x = 0.0$ we first prescribed guess profiles for the functions f and θ and their derivatives from the exact solutions of the following equations:

$$f''' + 2ff'' - f'^2 + \theta = 0 \quad (37)$$

$$\frac{1}{\Pr}(1+\gamma\theta)\theta'' + \frac{1}{\Pr}\gamma\theta'^2 + 2f\theta' = 0 \quad (38)$$

Satisfying the boundary conditions

$$f(0) = f'(0) = 0,\ \theta(0) = 1$$
$$f'(\infty) = \theta(\infty) = 0 \quad (39)$$

These solutions are then employed in the Keller-box scheme with second order accuracy to march step by step along the boundary layer. For a given value of x, the iterative procedure is stopped when the maximum change between successive iterates is less than 10^{-5}. A uniform grid of 2001 points are used in the x-direction with the step size $\Delta x = 0.01$ and another non-uniform grid in the y-direction has been incorporated, considering $y_j = \sinh\{(j\text{-}1)/a\}$ where j = 1, 2, 3, …, N with N = 301 and a = 100 to get quick convergence and thus save computational time and memory space.

4. Results and Discussion:

In this paper we have investigated in the problem of laminar natural convection flow and heat transfer from an isothermal sphere with temperature dependent thermal conductivity. Here we have considered the thermal conductivity of the fluid is proportional to the linear function of temperature that means if the temperature of the fluid increase then the conductivity of the fluid increases. The thermal conductivity of air is 0.009246 $W.m^{-1}.^0K^{-1}$, 0.013735 $W.m^{-1}.^0K^{-1}$, 0.03003 $W.m^{-1}.^0K^{-1}$ and 0.05779 $W.m^{-1}.^0K^{-1}$ at 100^0K, 200^0K, 400^0K and 800^0K temperature respectively. (See Cebeci and Bradshaw (1984)).

Table 1. Compares the present numerical values of *Nu* for the values of Prandtl number Pr (= 0.7, 7.0) without effect of conductivity variation parameter with those obtained by Nazar et al. (2002) and Huang and Chen (1987).

x in degree	Pr = 0.7			Pr = 7.0		
	Nazar et al. [5]	Huang and Chen [7]	Present results	Nazar et al. [7]	Huang and Chen [5]	Present results
0	0.4576	0.4574	0.4576	0.9595	0.9581	0.9582
10	0.4565	0.4563	0.4564	0.9572	0.9559	0.9558
20	0.4533	0.4532	0.4532	0.9506	0.9496	0.9492
30	0.4480	0.4480	0.4479	0.9397	0.9389	0.9383
40	0.4405	0.4407	0.4404	0.9239	0.9239	0.9231
50	0.4308	0.4312	0.4307	0.9045	0.9045	0.9034
60	0.4189	0.4194	0.4188	0.8801	0.8805	0.8791
70	0.4046	0.4053	0.4045	0.8510	0.8518	0.8501
80	0.3879	0.3886	0.3877	0.8168	0.8182	0.8161
90	0.3684	0.3694	0.3683	0.7774	0.7792	0.7768

Equations (14)-(15) subject to the boundary conditions (16) are solved numerically using a very efficient implicit finite finite-difference together with Keller box, which is described in above section. The numerical solutions start at the lower stagnation point of the sphere, $x \approx 0$, with initial profiles as given by equations (37)-(38) along with the boundary conditions (39) and proceed round the sphere up to the upper point, $x \approx \pi/2$. Solutions are obtained for fluids having Prandtl number Pr (= 0.7, 1.0, 3.0, 5.0, 7.0) and for a wide range of values of the variable conductivity parameter γ (= 0.0, 0.5, 1.0, 2.0, 3.0, 5.0.). Since the values of $f''(x,0)$ and $\theta'(x,0)$ are known from the solutions of the coupled equations (14) and (15), numerical values of the shearing stress in terms the skin-friction coefficients C_f from (19) and the heat transfer rate in terms of Nusselt number Nu from (20) are calculated from lower stagnation point to upper stagnation point of the circular cylinder. Numerical values of C_f and Nu are entered in Table 1 and depicted by figures 2 and 3**.** It should be noted that for constant thermal conductivity we recover the problem that discussed by Nazar et al. (2002) in absence of micro polar parameter and Huang and Chen (1987) in absence of suction and blowing.

The numerical values of the skin-friction coefficient C_f and the local Nusselt number Nu, against the curvature parameter x for different values of conductivity variation parameter γ (= 0.0, 0.5, 1.0, 2.0, 5.0) while Pr = 0.7 (air at 20^0C and 1 atm. pressure) are depicted in Fig. 2(a)-(b). With the increasing values of the conductivity-variation parameterγ, it can be observed that the values of skin-friction coefficient C_f increases and the Nusselt number Nu decreases. For increasing values of γ, the temperature of the fluid within the boundary layer increases and hence the viscosity of the air increases and the corresponding skin-friction coefficient C_f increases. Since the temperature of the fluid increases and hence the corresponding temperature difference between the surface and the fluid enhances. Due to higher temperature of the fluid, the rate of heat transfer that means the Nusselt number Nu decreases. It can also be calculated at $x = \pi/4$, the values of skin-friction coefficient C_f increases by 25.32% and the Nusselt number Nu decrease by 59.20% respectively for increasing values of γ from 0.0 to 5.0.

The effect of Prandtl number Pr (= 0.7, 1.0, 3.0, 5.0, 7.0) on the skin-friction coefficient C_f and the Nusselt number Nu against the curvature parameter $x \in [0, \pi/2]$ for $\gamma = 2.0$ is shown in Fig. 3(a)-(b). It is found that values of the skin-friction coefficient C_f decreases and the Nusselt number Nu increases for increasing values of the Prandtl number Pr. For example, at $x = \pi/4$, the values of the skin-friction C_f decreases by 31.88% and the Nusselt number increases by 120.17% while Pr increasing from 0.7 to 7.0. According to definition of Pr, for increasing values of Pr the thermal conductivity of the fluid decreases and the viscosity of the fluid increases. Then in any one point on the surface, the shearing stress that means the skin-friction coefficient is larger and the heat is not able to conduct easily into the fluid as Pr increases and therefore the thermal boundary layer becomes thinner, hence the corresponding temperature gradients are larger and the surface rate of heat transfer increases.

Attention is now given to the effect of pertinent parameter on the dimensionless velocity $f'(x, y) = u/x$ and the dimensionless temperature distribution $\theta(x,y)$ in the flow field are shown graphically in Fig. 4. Fig. 4(a)-(b) illustrate the velocity and temperature distribution against the variable y for different values of the conductivity-variation parameter γ (= 0.0, 0.5, 1.0, 2.0, 3.0, 5.0) at $x = \pi/3$ while Pr = 0.7. It can be observed that the velocity and temperature distribution increases with the increasing values of the conductivity-variation parameter, γ. It should be noted that at each value of the conductivity-variation-parameter γ, the velocity profile has a local maximum value within the boundary layer. The maximum values of the velocity are 0.32806, 0.35586, at y = 1.1144 for γ = 0.0, 0.5 respectively and for γ = 1.0, 2.0 the maximum values of the velocity are 0.37855, 0.41383 respectively at y = 1.1752. And the corresponding temperature at that location are 0.53497, 0.59759, 0.62240, 0.68437, 0.70813 and 0.75963 for γ = 0.0, 0.5, 1.0, 2.0, 3.0, 5.0 respectively. Here it can be seen that for large values of γ, the location of the maximum values of the velocity are shifted. The maximum velocity and temperature increase by 26.14% and 41.99% respectively as γ increases from 0.0 to 5.0. It also be concluded that the velocity boundary layer and the thermal boundary layer thickness increase for large values of γ.

5. Conclusions:

The effect of temperature-dependent thermal conductivity on the natural convection boundary layer flow from an isothermal sphere has been investigated theoretically. Numerical solutions of the equations governing the flow are obtained by using the very efficient implicit finite difference method together with Keller box scheme. From the present investigation the following conclusions may be drawn:

- Increasing the value of the thermal conductivity- variation parameter γ leads to increase the local skin-friction coefficient C_f and a decrease the local Nusselt number *Nu.*
- The velocity distribution and the temperature distribution increase for increasing value of thermal conductivity- variation parameter γ.
- It is seen that the skin-friction coefficient C_f decrease as well as the rate of heat transfer increase with the increasing values of Prandtl number Pr.
- The results have demonstrated that the assumption of constant fluid properties may introduce severe errors in the prediction of the surface shearing stress and the rate of heat transfer.

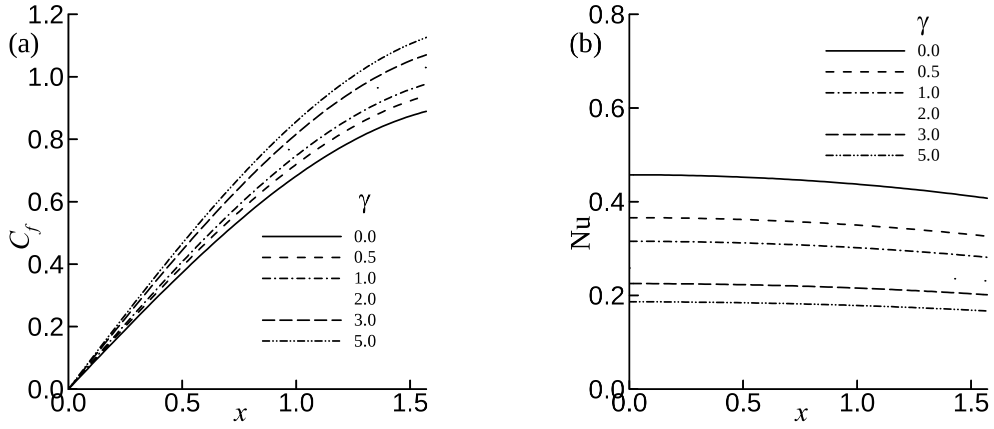

Figure 2: (a) Skin-friction coefficient (b) Rate of heat transfer for different values of γ while Pr = 0.7

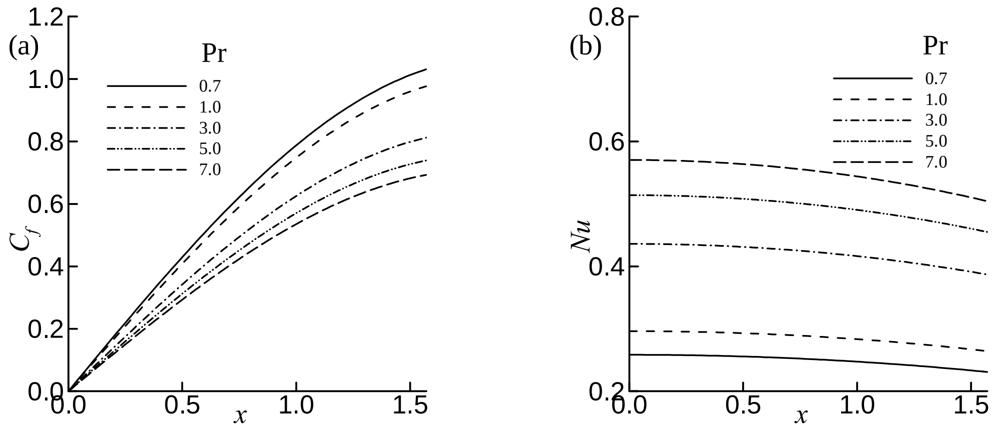

Figure 3: (a) Skin-friction coefficient (b) Rate of heat transfer for different values of Pr while γ = 2.0.

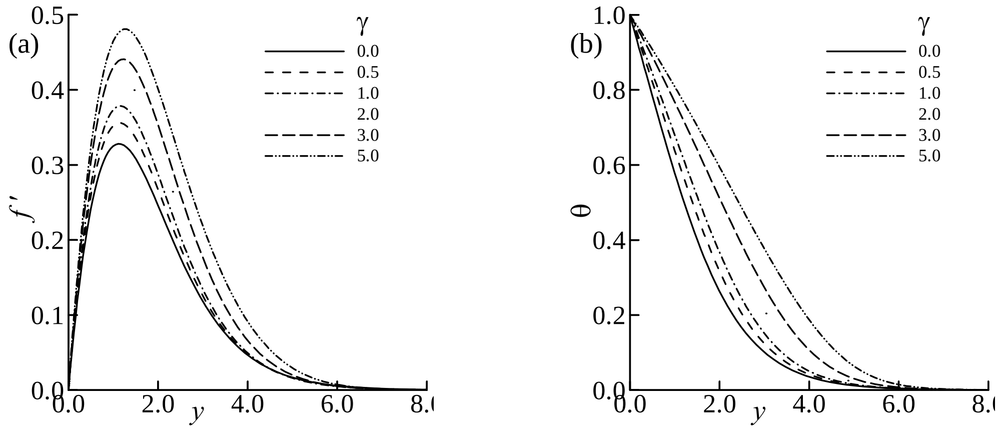

Figure 4: (a) Velocity and (b) Temperature distribution for different values of γ while Pr = 0.7 at $x = \pi/3$.

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