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STRUCTURAL RESPONSE OF A FLOATING RUNWAY EXCITED BY THE TAKING OFF OF AN AIRPLANE

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Abstract:

When designing a floating airport we need to address the structural response both by ocean waves and dynamic loads such as the landing / take off of an airplane. Since such problems are not conducive to physical modeling and experimental validation due to their size and speeds involved, numerical analysis is an accepted norm. However conventional means to study structural responses using a three dimensional runway with time varying dynamic loads is numerically difficult and time consuming. The analysis is made simpler by assuming the airport to be a simple, infinitely long beam, given by a one dimensional Timoshenko-Mindlin plate equation, in contact with the water surface. In developing this expression, a Fourier transformation in space in wave number domain is utilized rather than using the wave propagation method to reduce the analysis to a substructure. On analyzing, the structural response is seen as local peaks emanating from the point of load application which moves in a curvilinear path with increasing speed of the airplane. The location of these peaks a priori is however not feasible.

Keywords: Moving Load, Timoshenko-mindlin plate, floating runway, in-plane loading

NOMENCLATURE

ξ	Wave number variable	E	Elastic modulus, N / m^2
γ	Wave number ratio	h	Height of the beam, m
ν	Poisson's ratio	$H(x)$	Heavyside step function
ρ_v	Mass density of the material, kg / m^3	$G=\dfrac{E}{2(1+\nu)}$	Complex shear modulus
ρ_0	Mass density of the acoustic medium, kg / m^3	$I=\dfrac{h^3}{12}$	The cross sectional moment of inertia per unit width
$\kappa^2=\pi^2/12$	Cross sectional shape factor or the shear correction factor	$K_0=\omega/C_0$	Acoustic wave number
ζ	Non dimensional wave number variable	K_B	Free bending wave number
α_0	Fluid loading parameter	$M=V/C_0$	Mach number, non-dimensional number
$\delta(x-Vt)$	Delta function	P	Sound pressure on the beam surface
$p(x,y=0,t)$	Acoustic pressure acting on the beam's surface	$u(x,t)$	Transverse displacement of the beam, m
C_L	Longitudinal wave speed, m / s	V	Subsonic speed of moving force of length 2L, m / s
C_0	Sound speed in the acoustic medium, m / s	f_0	Strength of external force per unit width, Nm / s
$D=\dfrac{Eh^3}{12(1-\nu^2)}$	Flexural rigidity of the plate		

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1. Introduction

Many developed countries with long coastlines for want of space have successfully reclaimed land from sea. Due to negative ecological effects of land reclamation, researchers and engineers proposed the construction of *Very Large Floating Structures* (VLFS). The relatively simple construction and ease of maintenance of a VLFS thus makes it a promising design candidate for floating airports or runways. Unlike conventional floating structures, the VLFS has a huge horizontal dimension when compared to its height, thus making it flexible. This implies that when the VLFS is subjected to a landing / takeoff load of an airplane it is likely to exhibit larger elastic deformation. It is thus essential that the dynamic analysis of a VLFS when subjected to such transient loads is done carefully.

Dynamic analysis of elastic structures has attracted much attention from researchers for many years. The problem arises from the observation that a structure subjected to moving load can exhibit higher deflection and stresses than those for static load. Today, the analysis of moving load problem is applicable for various engineering applications such as high speed drilling, turning, work piece transportation, fluid flow induced vibrations to name a few. For an elastic structure such as a VLFS subjected to a landing / taking off load of an airplane, this analysis has its importance because a floating runway is flexible and receives buoyant support from the water. This makes the runway to deflect due to its weight forming a dish-like "dent"" around the aircraft when the aircraft is static. However this dent moves and progresses like a wave down the runway when the airplane takes off. This moving dent in return causes an increased drag on the aircraft resulting into increased time, distance of take-off and fuel consumed thus leading to increased operating cost. The designer is hence required to address the transient dynamics problem due to the impulsive and moving load excited by the landing / taking off of an airplane on these structures.

The difficulty in solving this transient dynamic problem has resulted in only a few simplified studies to have been reported to date. Using a finite element (FE) program, Watanabe and Utsunomiya (1996) presented the numerical results for elastic responses due to prescribed impulsive loading on a circular VLFS excited by impulsive loading. Kim and Webster (1998) and Yeung and Kim (2000) studied transient phenomena of an infinite elastic runway using a double Fourier transform approach. The former studied the added drag caused by the flexibility of the runway while the latter focused on the resonance phenomenon caused by the accumulation of energy near the moving load. Ohmatsu (1998) developed another kind of time-domain analysis method in which the structural response is obtained from the convolution integral of the frequency response function and impulse response function. Endo and Yago (1998) adopted a FE scheme and Wilson-θ method to investigate the transient behavior of an airplane taking off from and landing on a VLFS in rough sea conditions using a triangle time impulse load applied at the nodes of the structure to represent the loads introduced by the weight of the airplane. Endo (2000) calculated the behaviour of a VLFS and airplane during takeoff / landing run in wave condition allowing for the effects of hydroelasticity. Lee and Choi (2003) developed a FE-BE hybrid method to analyze transient hydroelastic response of VLFS. Kashiwagi (2004) presented the transient elastic deformation of a pontoon-type VLFS caused by the landing and takeoff of an airplane based on the mode superposition method using realistic numerical data from a Boeing 747-400 Jumbo jet. Fleischer and Park (2004) used the modal analysis with Fourier series to solve the plane hydroelastics of a beam due to uniformly moving one-axle vehicle. Kyoung et al. (2006) developed a finite element method for the time-domain analysis of the hydroelastic deformation of a pontoon-type VLFS with fully nonlinear free-surface conditions. Jin and Xing (2007) proposed a mixed mode function-boundary element method to solve the dynamic responses of a floating beam excited by landing loads. Qiu and Liu (2007) proposed a time-domain finite element procedure to analyze the transient hydroelastic responses of VLFS subjected to dynamic loads. Qiu (2007) proposed a time-domain finite element model to analyze the fluid-structure interactive dynamical system. He validated the proposed approach by comparing the existing experimental and numerical results with the results obtained.

In the present paper, an expression for the structural response of a floating airport due to an airplane landing / taking off modeled as a moving load is proposed. In developing this expression, a Fourier transformation in space in wave number domain is utilized rather than using the wave propagation method to reduce the analysis to a substructure. The procedure employed is similar to that demonstrated by Cray (1994) for stiffened plates and Cheng and Chui (1999) and Cheng et al. (2000; 2001) for calculating the transverse response and acoustic radiation of a periodically supported beam. The advantage of expressing the response in terms of a wave number arises from the fact that the periodic boundary conditions and the phase relation between two adjacent substructures will not be required to be used.

2. Mathematical Formulation

To study structural response of a floating airport, dynamic analysis of a three-dimensional runway with time varying loading during take-off would be exceeding difficult. This analysis is made simpler by making following assumptions

- The floating airport behaves as a simple, infinitely long beam in contact with water surface. To eliminate boundary effect of the finite length of the airport, we assume modeled beam to extend to infinity.
- The geometry and material properties are assumed to be linearly elastic.
- The structural damping is ignored since there is no apparent resonant mechanism in this problem.
- The water is assumed to be inviscid, and flow resulting from the airplane take-off is irrotational.
- Because the floating runway is very narrow compared to its length, as a simplification, we assume that deformation and loading do not vary across the runway.

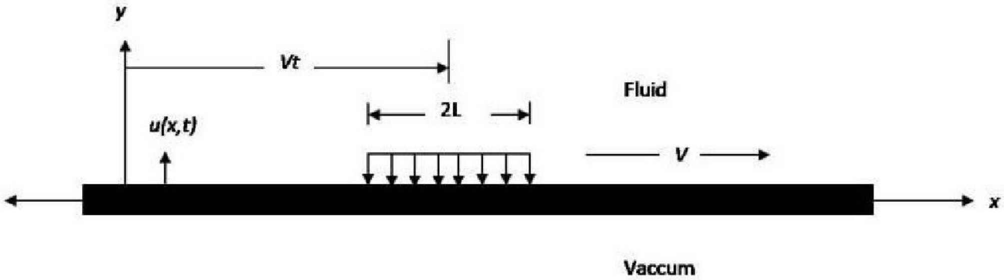

Fig.1: Schematic representation of the problem geometry

Accordingly, the structure is assumed to behave like a beam, described by the Timoshenko-Mindlin beam equation. The x-axis is aligned with the length of the runway and the y-axis is directed vertically upwards, as seen in Fig. 1. An excitation force of length 2L moving at a subsonic speed V is assumed to be acting on the runway. The space y > 0 is filled with a medium such as water. The other side of the plate is assumed to be vacuum. The vibration equation for a one dimensional elastic plate, including rotational inertia, transverse shear effects, given by the Timoshenko-Mindlin plate equation as discussed by Junger and Feit (1986) is:

$$D\frac{\partial^4 u(x,t)}{\partial x^4}-\rho_v I\left(1+\frac{D\rho_v}{\kappa^2 G}\right)\frac{\partial^4 u(x,t)}{\partial x^2\partial t^2}+\rho_v I\frac{\rho_v}{\kappa^2 G}\frac{\partial^4 u(x,t)}{\partial t^4}=\left(1-\frac{D}{\kappa^2 Gh}\frac{\partial^2}{\partial x^2}+\frac{\rho_v h^2}{12\kappa^2 G}\frac{\partial^2}{\partial t^2}\right)\left[f(x,t)-p(x,y=0,t)\right] \tag{1}$$

To model landing / taking off of an airplane a uniform distributed moving load on the beam is used. We use either a line load or a point load as the moving force since the distributed load is a general representation of the extreme case of a point load. These are given by

$$f(x,t)=f_0/2L[H(x-Vt+L)-H(x-Vt-L)]e^{j\omega t} \quad \text{for harmonic line force} \tag{2a}$$

$$f(x,t)=f_0 e^{j\omega t}\delta(x-Vt) \quad \text{for a point force} \tag{2b}$$

The pressure distribution induced by the vibrating plate in the acoustic medium is denoted by $p(x,y,t)$ and satisfies the wave equation in two-dimensional space, given by

$$\left(\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}-\frac{1}{C_0^2}\frac{\partial^2}{\partial t^2}\right)p(x,y,t)=0 \tag{3}$$

If ρ_0 is the mass density of the acoustic medium, the boundary condition at $y=0$ is given by

$$\rho_0\frac{\partial^2 u}{\partial t^2}=-\frac{\partial p}{\partial y}\Big|_{y=0} \tag{4}$$

By applying spatial Fourier transformation $FT() = \int_{-\infty}^{\infty} ()e^{i\xi x}dx$, using ξ as the wave number variable, the force functions in wave number domain may be written as

$$\tilde{F}(\xi,t) = f_0(\frac{\sin\xi L}{\xi L})e^{j(\omega+\xi V)t} \quad \text{for harmonic line force} \tag{5a}$$

$$\tilde{F}(\xi,t) = f_0 e^{j(\omega+\xi V)t} \quad \text{for a point force} \tag{5b}$$

the transformed displacement as

$$\tilde{U}_s(\xi,t) = U_s(\xi)e^{j(\omega+\xi V)t} \tag{6}$$

and the transformed pressure as

$$\tilde{P}(\xi,y,t) = P(\xi,y)e^{j(\omega+\xi V)t} \tag{7}$$

substitution of Eq. (5), (6) and (7) in the transformed Eq. (1) and the transformed combination of Eq. (3) and (4), we get

$$U_S(\xi) = \frac{Z_F F(\xi)}{Z_m + Z_F Z_a} \tag{8}$$

where the acoustic impedance operator (Z_a) is given by

$$Z_a = \frac{j\rho_0(\omega+\xi V)^2}{K_y} \tag{9}$$

the beam impedance operator (Z_m) as

$$Z_m = D\xi^4 - \rho_v h(\omega+\xi V)^2 - \xi^2\left(\rho_v I + \frac{D\rho_v}{\kappa^2 G}\right)(\omega+\xi V)^2 + \rho_v I\frac{\rho_v}{\kappa^2 G}(\omega+\xi V)^4 \tag{10}$$

The Z_F by

$$Z_F = 1 + \frac{D}{\kappa^2 G h}\xi^2 - \frac{\rho_v h^2}{12\kappa^2 G}(\omega+\xi V)^2 \tag{11}$$

and K_y is given by

$$K_y = \begin{cases} -j\sqrt{\xi^2 - (K_0 + M\xi)^2} & for\ \xi^2 > (K_0 + M\xi)^2 \\ \sqrt{(K_0 + M\xi)^2 - \xi^2} & for\ \xi^2 < (K_0 + M\xi)^2 \end{cases} \tag{12}$$

Nondimensionalization

In order to present numerical results, concept of non-dimensional parameters is used. Hence following non-dimensional parameters are defined

$$\text{Wavenumber variable } [\zeta] = \frac{\text{Wavenumber variable } (\xi)}{\text{Acoustic wavenumber } (K_0)} \tag{13a}$$

$$\text{Free bending wavenumber } [K_B] = \left[\frac{\rho_v h\omega^2}{D}\right]^{\frac{1}{4}} \tag{13b}$$

$$\text{Wavenumber ratio } [\gamma] = \frac{K_0}{K_B} \tag{13c}$$

$$\text{Longitudinal wavenumber } [C_L] = \sqrt{\frac{E}{\rho_v}} \tag{13d}$$

$$\text{Fluid loading parameter } [\alpha_0] = \frac{\rho_0 C_L}{\sqrt{12}\rho_v C_0} \tag{13e}$$

$$\text{Displacement of plate } [U(\zeta)] = \frac{E}{f_0 h}U(\xi) \tag{13f}$$

$$\text{Strength of external force } [F(\zeta)] = \frac{F(\xi)}{f_0} \tag{13g}$$

$$K_0 h = \left[\frac{K_0}{K_B}\right]^2 \left[\frac{12\rho_v C_0^2}{E}\right]^{\frac{1}{2}} \tag{13h}$$

Using Eq. (8) and the above mentioned non dimensional variables, we get an expression for displacement as

$$U(\zeta) = \frac{E}{h}\frac{F(\zeta)}{f_0}\frac{Z_F}{(Z_m + Z_F Z_a)} = \frac{F(\zeta)A(\zeta)}{B(\zeta)+A(\zeta)C(\zeta)} \tag{14}$$

where

$$F(\zeta) = f_0 \frac{\sin(K_0\zeta L)}{K_0\zeta L} = f_0 \frac{\sin[(K_0 h)\zeta(L/h)]}{(K_0 h)\zeta(L/h)} \qquad \text{for harmonic line force}$$

$$F(\zeta) = f_0 \qquad \text{for a point force}$$

$$A(\zeta) = 1 + \left(\frac{1+\nu}{6\kappa^2}\right)(K_0 h)^2 \zeta^2 - \left(\frac{1+\nu}{6\kappa^2}\right)\left(\frac{C_0}{C_L}\right)^2 (K_0 h)^2 H(\zeta)$$

$$B(\zeta) = \frac{Z_m}{E/h} = \frac{1}{12}(K_0 h)^4 \zeta^4 - H(\zeta)\left(\frac{C_0}{C_L}\right)^2 (K_0 h)^2 \left[1 + \frac{(K_0 h)^2}{12}(1 + \frac{2(1+\nu)}{\kappa^2})\zeta^2\right] + \frac{(1+\nu)}{6\kappa^2}\left(\frac{C_0}{C_L}\right)^4 (K_0 h)^4 H(\zeta)^2$$

$$C(\zeta) = \frac{Z_a}{E/h} = j\frac{H(\zeta)}{\sqrt{H(\zeta)-\zeta^2}}\left(\frac{C_0}{C_L}\right)^2 \frac{\rho_0}{\rho_v}(K_0 h)$$

$$H(\zeta) = (\zeta M + 1)^2$$

3. Results and Discussion

Material chosen for the floating airport is low carbon steel with properties as $E = 20\,X\,10^{10}\,N/m^2$, $\rho_v = 7800\,kg\,/\,m^3$ (i.e $D = 560\,KNm$), $h = 2.54X10^{-2}m$, $\nu = 0.3$, $\kappa^2 = 0.85$. The acoustic medium considered is water with $C_0 = 1481 m/s$ and $\rho_0 = 1000 kg/m^3$. The numerical model in analyzed in the frequency range $0.01 < \gamma < 2.2$ to understand the structural response of a floating airport due to landing / take off of an airplane. The external force (f_0) is assumed to be of unit magnitude. The structural response of the floating airport is numerically obtained by evaluating Eq. (14). By varying the parameter *M*, between 0.1 and 0.9 the three dimensional plot of the normalized amplitude versus the wave number ratio (γ) or non-dimensional frequency obtained for a distributed load and a point load is seen in Figs. 2 and 3 respectively. All calculations have been undertaken using MATLAB.

In both the Figs. 2 and 3, large spectral responses of the fluid loaded flexural wave number are visible. These are seen as local peaks emanating from the point of load application, indicated by $\zeta = 0$ from the M = 0.1 to 0.9. The presence of peak in the positive wave number is noticed, which represents the flexural wave that propagates in the same direction as the convected loading. With increased convected loading speed, the peak moves towards a larger value of wave number due to the Doppler effect. Similarly the negative peak represents the flexural wave for which the propagating direction is opposite to that of the convected loading. The wave number for this peak is found to be decreasing with increased convected loading thus indicating that the negative going wave may be supersonic.

These sets of propagating positive going and negative going flexural waves can be noted in both the loading cases. The identification or pre-definition of the location of these peaks precisely can however not be done.
Looking closely at Fig. 2 and Fig. 3, one notices that the local peak, which is related to the maximum structural deflection of the platform caused by the relative movement of the airplane and the local deformation of the platform, moves in a curvilinear path with increasing speed of the airplane (indicated by the increasing *M*). Results obtained by this methodology are comparable in trend to those published by Kashiwagi (2004) (Fig. 4 for landing airplane and Fig. 8 for takeoff) and Kim and Webster (1998) (Fig.4 and Fig. 5 for varying bending rigidity). Since the dense plot seen in Fig. 2 and Fig. 3 above do not give a clear appreciation of this

understanding, inorder to improve visualization of these three dimensional plots, slice plots are presented in Fig. 4 and Fig. 5 for distributed loading and line loading respectively.

It may be observed that for lower convective loading the structural response peaks are approximately symmetrical and are identified as propagating flexural waves. With increased convective loading speed, the spectral response changes as seen in Fig. 4 and Fig. 5.

In the present study the problem has been made time independent by taking the Fourier transformation. This disallows time histories to be plotted explicitly for comparison of the obtained results with those of other researchers. However since the plots obtained are those at varying speeds of the same airplane, one can assume the plots at Fig. 4 and Fig. 5, for distributed loading and point loading respectively, to be equivalent to time histories wherein the maximum structural deflection of the platform is seen to be following a curvilinear path. This is in concurrence with results obtained by Kim and Webster (1998) and Kashiwagi (2004). One may consider the landing of the airplane to be analogous to moving from a higher Mach number to a lower Mach number while the take off to be analogous to moving from a lower Mach number to a higher Mach number.

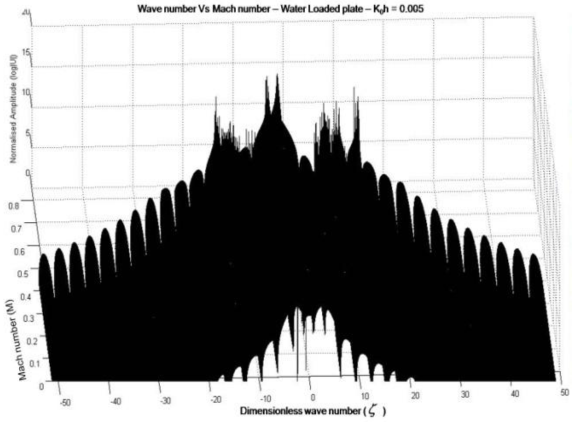

Fig. 2: Three Dimensional plot for wave number response V/s Dimensionless Frequency – Distributed Loading

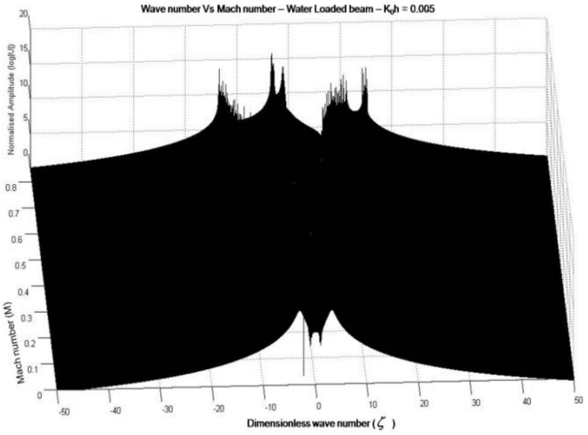

Fig. 3: Three Dimensional plot for wave number response V/s Dimensionless Frequency- Point Loading

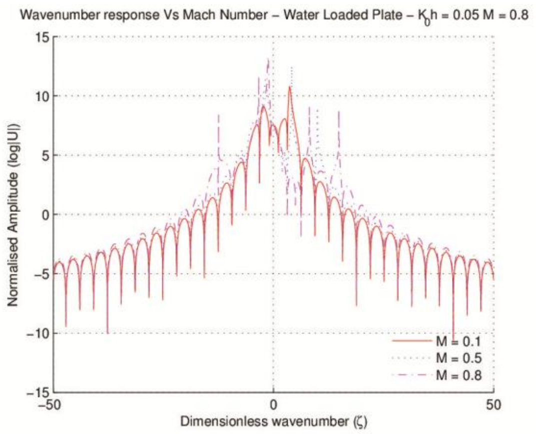

Fig. 4: Two Dimensional plot for wave number response V/s Dimensionless Frequency – Distributed Loading

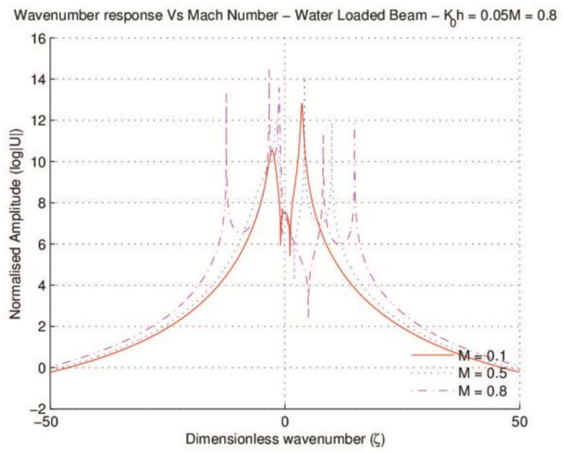

Fig. 5: Two Dimensional plot for wave number response V/s Dimensionless Frequency - Point Loading

The study by Kim and Webster (1998), is based on using the Fourier Transformation for the governing equations and then taking the inverse transform to get the shape of the runway as a function of time. On the other hand, results as obtained by Kashiwagi (2004) are based on the time domain mode expansion method, taking account of memory effects in the hydrodynamic forces, which can be applied to any problem when the external force term on the right hand side of the differential equation is modified in accordance with the problem being considered. In the present methodology the structural response is expressed in terms of a wave number which does not require the periodic boundary condition and the phase relation between the adjacent substructures to be used, further by using a Fourier Transformation in space in wave domain, one is able to reduce the analysis to a substructure thus making the entire formulation simpler and faster to calculate.

4. Conclusion

The present paper analyzes the structural response of a floating airport subjected to landing / taking off of an airplane by using a Fourier Transformation in space in wave number domain rather than using the wave propagation method to reduce the analysis to a substructure. Similar structural response of floating airports has been undertaken by Kashiwagi (2004), by using a time domain mode expansion method and Kim and Webster (1998) by using the Fourier transform and inverse method to obtain the shape of the runway as a function of time. The method used in this paper converts the transient dynamic problem to a time independent problem thus making analysis simpler. By doing so, the structural response of a floating airport subjected to landing / taking off of an airplane has been analyzed. Since such problems are not conducive to physical modeling and experimental validation due to their size and speeds involved, numerical analysis is an excepted norm. However conventional means of using a three dimensional runway with time varying loads is extremely difficult and time consuming. The problem has been simplified by using a Timoshenko-Mindlin beam model. One notices that

- A number of large spectral responses are visible when the wave number is plotted against the increasing speed of the airplane.
- These large spectral responses are seen as local peaks emanating from the point of load application and represent the flexural wave propagating in the same direction as the convected loading due to the Doppler Effect.
- Defining the location of these peaks precisely a priori is however not feasible.
- The local peak moves in a curvilinear path with increasing speed of the airplane.
- Path traced by local peaks with increasing speed (increasing time) is similar to that as reported by other researchers (Kim and Webster, 1998 and Kashiwagi, 2004) using time histories.

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