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RANDOM POLAR SAMPLING TECHNIQUE FOR THE RELIABILITY ANALYSIS OF SUBMARINE PRESSURE HULL

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Abstract

Reliability analysis is essential to reduce the structural system failures to an acceptance level. In this paper, a quasi-Monte Carlo Simulation method called 'Random Polar Sampling Technique' (RPST) has been described for the reliability analysis of ring stiffened shell structure, in which the combinations of variates are obtained using a polar sampling of Latin hypercube sampled values. An efficient computer program coded in C++ has been developed for use with the RPST. Reliability analysis of internally ring stiffened submarine pressure hull has been described as an illustration and the results are compared with those given by First Order Second Moment method (FOSM), Advanced First Order Second Moment method (AFOSM) and Monte-Carlo Simulation methods.

Keywords: Random Polar Sampling Technique, Reliability, Ring stiffened shells, Submarine pressure hull, Latin Hypercube Samples.

NOMENCLATURE:

b	Effective width of shell
d_W	Depth of web of stiffener
E	Young's modulus
f	Flange width of stiffener
FLH[ ]	Final LHS matrix
I_{XX}	Moment of inertia of the section about X-axis
ILH[ ]	Initial LHS matrix
K	Number of variables
L_S	Spacing of the stiffeners
N	Number of samples
p	External pressure.
p_f	Probability of failure
p_{RC}	Ring collapse pressure.
R	Resistance
R_e	Reliability of structure
R'	Mean radius of circular cylindrical shell.
S	Load
t	Shell thickness
t_f	Flange thickness of stiffener
t_W	Web thickness of stiffener
β	Reliability index

1. Introduction:

In all engineering structural system design, uncertainties are unavoidable due to stochastic nature of materials and loads, and imperfect nature of mathematical model. These uncertainties can be accounted only through a reliability analysis and for stiffened shell structures, it can be analysed through the failure probability distribution of strength and loads. The common methods used for structural reliability assessment are First-Order Second-Moment (FOSM) method, Advanced FOSM (AFOSM) method, Second Order Reliability Method (SORM) etc., and these require the evaluation of derivatives of the failure equation, which is difficult in complex situations.

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The most general method is the Monte-Carlo Simulation (MCS) technique. It consists of obtaining cumulative distribution functions for each and every random variable and simulating the ultimate strength of stiffened shells for combinations of random variable values. It uses randomly generated samples of the input variables for each deterministic analysis, and estimates reliability after numerous repetitions of the deterministic analysis as given by Harr (1987). However for MCS to be successful, the sample size should be very large.

Hence methods have been proposed to reduce the sample size without however sacrificing any accuracy on reliability. 'Point Estimation Method' (PEM), 'Response Surface Technique' (RST), 'Importance Sampling Procedure Using Design points' (ISPUD), 'Latin Hypercube Sampling' (LHS) etc., are some of these methods. In this paper, a method called 'Random Polar Sampling Technique' (RPST) is proposed for efficient reliability estimation and a computer program coded in C++ for use with RPST for the reliability analysis of ring stiffened shell is also described. Since the RPST is used with the Latin Hypercube Sampling (LHS), the number of simulation cycles required for the analysis can be much reduced. A typical submarine pressure hull has been taken for the illustration of the proposed method.

2. Concepts and Evaluation of Structural Reliability:

In reliability analysis, random variables occur in modelling loads (S) and resistances (R). Their randomness is characterized by means, μ_S and μ_R; standard deviations, σ_S and σ_R; and corresponding probability density functions, $f_S(s)$ and $f_R(r)$.

The probability of failure, p_f is given by

$$p_f = \int_0^\infty F_R(s)\, f_S(s)\, ds \tag{1}$$

where $F_R(s)$ is the Cumulative Distribution Function (CDF) of R evaluated at s.

The performance function can be described as

$$\begin{aligned} Z &= R - S \\ &= g\,(X_1, X_2, \ldots, X_n) \end{aligned} \tag{2}$$

where X_i are the basic variables.

The failure surface or limit state of interest is given as Z = 0. The transformation of original limit state, $g(X) = 0$ to the reduced limit state, $g(X') = 0$ is given by

$$X_i' = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}}; \qquad (\text{ i} = 1, 2, \ldots, \text{n}) \tag{3}$$

where X_i' is the standard normal random variable with zero mean and unit standard deviation.

For two random variables, **Fig. 1** shows the limit state concept in which β is called the reliability index, which is the minimum distance from the origin of axes in the reduced coordinate system to the failure surface as given by Haldar et al. (2000).

The reliability index β is calculated by

$$\beta = \frac{\mu_Z}{\sigma_Z} \tag{4}$$

where,

Mean value, $$\mu_Z = \frac{1}{N}\sum_{i=1}^{N} z_i \tag{5}$$

Standard deviation, $$\sigma_Z = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(z_i - \mu_Z)^2} \tag{6}$$

where z_i is the value of performance function for each set of variables.

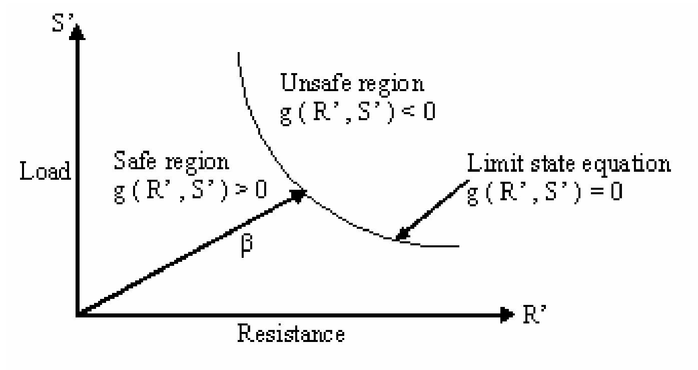

Fig. 1: Limit state concept.

The probability of failure is given by

$$p_f = \Phi(-\beta) \tag{7}$$

where Φ is the cumulative standard normal distribution function.

The common methods such as FOSM, AFOSM, etc., with various degrees of complexity can be used to estimate the reliability or safety index of the probability of failure. In FOSM, the reliability index and the probability of failure are calculated by Eqs. (4) and (7). These methods require the evaluation of a number of derivatives and also in any case, the estimation of reliability using these techniques requires a good background in probability and statistics.

The commonly used MCS method is robust, simple to use and generally faster than full probabilistic approaches. The generation of random variates according to specified distributions is the core of MCS. But this sampling scheme has a major drawback that many samples are required for good accuracy and repeatability. Also it is a crude and time-consuming process. Other methods such as Point Estimation Method (Rajagopalan, 1993), Variance Reduction Techniques (Ayyub et al. 1991), Importance Sampling method (Melchers, 1987), Latin Hypercube sampling (Nowak, 2000), etc., have been proposed to reduce the sample sizes required for efficiency.

Very few attempts have been done to use LHS in reliability analysis. Florian (1992) has introduced an updated LHS to compare the estimates of some statistical parameters obtained by different sampling schemes. He used specially modified tables of independent random permutations of rank numbers which form the strategy of generating input samples for simulation procedures. Huntington et al. (1998) have proposed two improvements to the performances of LHS. Olosson et al. (2003) suggested the LHS as a tool to improve the efficiency of different importance sampling methods for reliability analysis. A method to improve the reliability using optimal LHS has been proposed by Stocki (2003).

3. Random Polar Sampling Technique:

The Random Polar Sampling Technique is a method used as a modification of LHS. In RPST, the division of range of possible values of each random input variable depends on the number of samples required. The cumulative distribution function of each random variable is divided into equal probabilities (say) 1/N, where N is the number of required samples. Within this range, a value for the variable can be selected by either taking the mid value of the range or by generating a random number from 0 to 1 and picking a value in this range. These values will form the columns of initial LHS matrix, ILH. It must be ensured that same numbers of samples are taken for all the variables. Initially the sample values of each variable will be in an ascending order. In

RPST the column wise shuffling of the matrix to get the best initial LHS matrix has been adopted.

After obtaining the ILH, next step is the generation of arms. Depending upon the number of variables, the angle between each arm (θ) can be calculated as

$$\theta = \frac{2\Pi}{K} \quad (8)$$

where K is the number of variables in a problem. Each arm will have N sample values of corresponding variable.

In general the initial LHS matrix can be written as

$$ILH = \begin{bmatrix} x_{11} \; x_{12} \; \ldots\ldots\ldots \; x_{1K} \\ x_{21} \; x_{22} \; \ldots\ldots\ldots \; x_{2K} \\ x_{31} \; x_{32} \; \ldots\ldots\ldots \; x_{3K} \\ \vdots \quad \vdots \quad \ldots\ldots\ldots \quad \vdots \\ x_{N1} \; x_{N2} \; \ldots\ldots \; x_{NK} \end{bmatrix}$$

and the corresponding polar diagram is shown in **Fig. 2**.

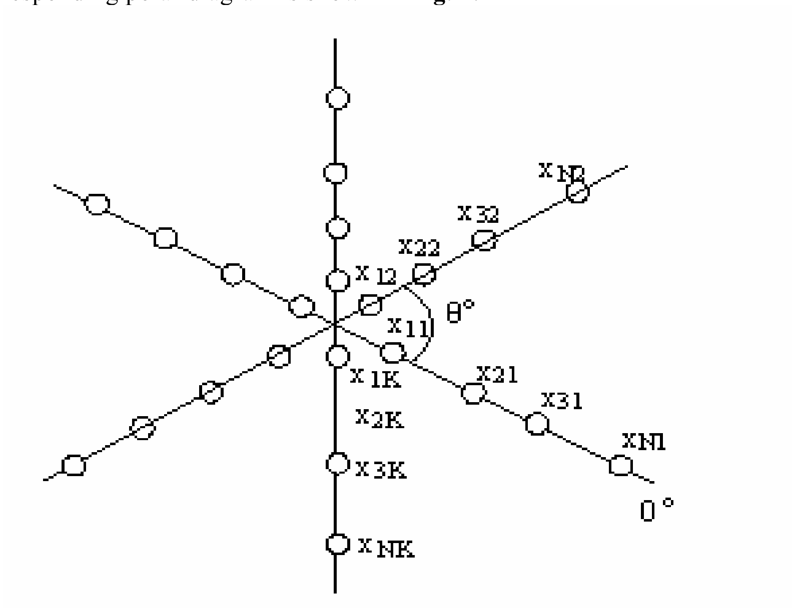

Fig. 2. Typical Polar Diagram for K variables and N samples.

By taking a U (0,1) random number, an arm can be randomly selected. The value of the variable lying on this arm can be selected by taking another random number. This process is repeated for a full cycle till one value from each of the variables gets selected. To accomplish this, each of the random numbers U_m is scaled to obtain a corresponding cumulative probability, P_m, so that each P_m lies within the m^{th} interval. Then all the arms of the polar diagram will be shrunk with samples of size (N – 1). The cycle evaluation described in the forgoing has to be continued till all the required samples are selected. The modified LHS matrix, FLH, thus obtained is one of the best forms of the LHS matrix.

In this paper by performing the RPST, the mean and variance of the strength function R is evaluated. Then the reliability index can be evaluated by using Eqs. (2) and (4) and the probability of failure by Eq. (7) and the reliability of system is given by

$$R_e = 1 - p_f \tag{9}$$

4. Reliability Analysis of Ring Stiffened Shells

Stiffened shell structures are widely used in ocean engineering in the form of submersibles, submarine pressure hulls, storage tanks in ports and harbours, legs of offshore platforms, etc. Several attempts have been made to use reliability concepts for the design of stiffened cylindrical shells. The method for the analysis of submarine structures up to the point of collapse has been discussed by Kendrick (1986). White et al. (1987) described the reliability based design format for marine structures and proposed the 'reliability-conditioned method'. Faulkner (1991) discussed the application of reliability theory in submarine design. Frieze (1991) described the basic requirements for physical modelling, probabilistic modelling and reliability evaluation. Das et al. (2001) have put an effort to establish a set of design equations for buckling strength assessment of ring and stringer stiffened shells related to marine structures such as TLPs and Spars. Das et al. (2003) described methods for the buckling and ultimate strength assessment of ring stiffened shells and ring and stringer stiffened shells involving various modes of buckling under various loading for reliability analysis.

In this paper, an internally ring stiffened pressure hull has been considered for illustrating the proposed method. The operating depth of submarine considered is 200 m and hence the external pressure (p) acting on it is 2 MPa. The distance between the bulkheads is taken as 21.336m. The arrangement of internal ring stiffeners used in the submarine pressure hull is shown in **Fig.3**.

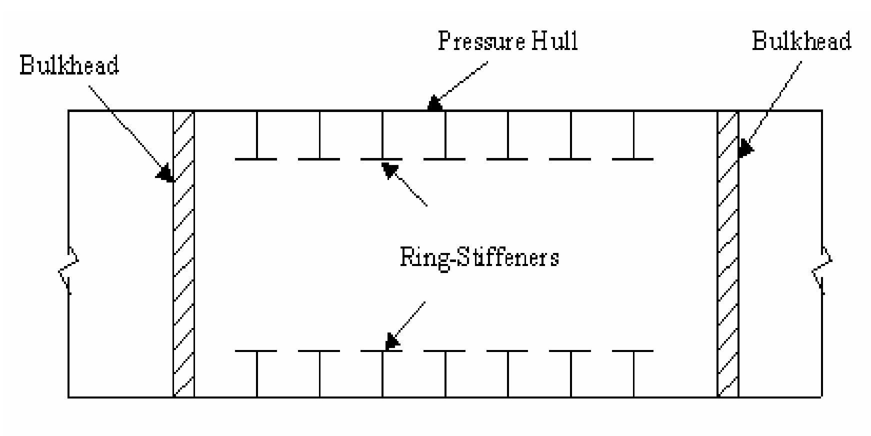

Fig. 3. Pressure hull with internal ring stiffeners

The shell and stiffener geometry considered here is given in **Fig. 4**.

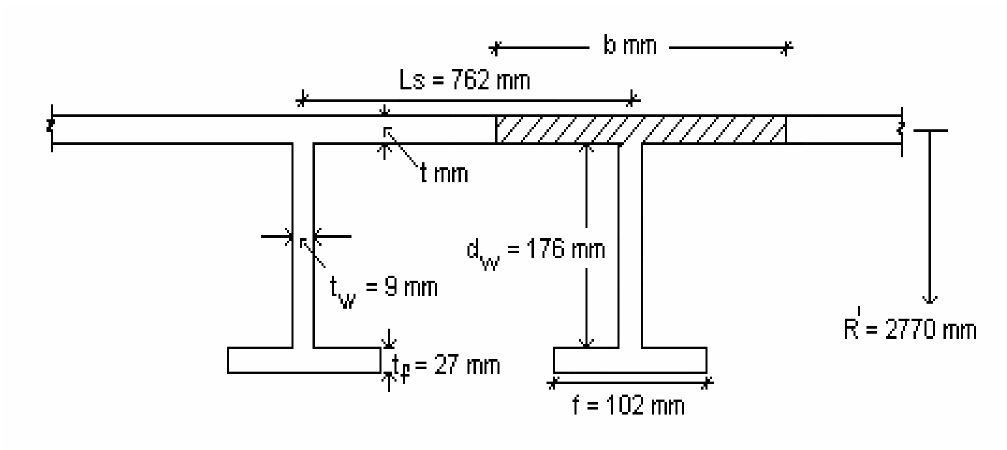

Fig. 4. Shell and Stiffener geometry

The stiffened shell structures can fail by elastic buckling in different modes. The collapse pressures obtained for ring buckling, interstiffener buckling and overall buckling, at the mean values of the random variables considered, are 4.53185 MPa, 5.81423 MPa and 5.925 MPa respectively. Hence in this study, the failure mode considered is the ring buckling and the corresponding expression for the collapse pressure is given by the Bresse's equation [Ross, (1990)] as

$$p_{RC} = \frac{3\ E\ I_{XX}}{R'^{\,3}\ L_S} \tag{10}$$

5. Program Description:

For rapid estimation of reliability of complex systems using RPST a computer program is needed. In this study, an effective computer program coded in C++ has been developed for the reliability analysis of submarine pressure hull.

The variables considered in the problem are E, t, b and p. The performance function thus formulated is

$$Z = p_{RC}\left(E, t, b\right) - p \tag{11}$$

The reliability index for the problem is given by

$$\beta = \frac{\mu_{p_{RC}} - \mu_p}{\sqrt{\left(\sigma_{p_{RC}}\right)^2 + \left(\sigma_p\right)^2}} \tag{12}$$

In order to calculate the p_{RC} using RPST program, the random variables considered are E, t and b. The mean values of E, t and b are taken as 200GPa, 24 mm and 510 mm respectively. A 5% variation in E, 10% variation in t and 2% variation in b are considered. Hence E values will change from 190 GPa to 210 GPa, t will vary from 21.6 mm to 26.4 mm and that of b will vary from 499.8 mm to 520.2 mm. The mean value of pressure p is 2 MPa and the variation considered is 10%. The number of samples required for each variable has been taken as 10 for illustration.

It is assumed that all the variables have a continuous uniform distribution. The CDF of these variables are divided into a number of bins with equal probability of 0.1. The initial LHS matrix is obtained by using the random number generation between 0 and 1 and picking the variables within this range. From the RPST program, the initial LHS matrix, ILH [ ], for a particular combination of random numbers is obtained as

$$ILH = \begin{bmatrix} 190394 & 21.6665 & 499.924 \\ 193689 & 22.1646 & 502.190 \\ 194016 & 22.8122 & 505.137 \\ 196218 & 23.0761 & 506.901 \\ 199603 & 23.6072 & 508.390 \\ 201845 & 24.2159 & 511.611 \\ 203809 & 24.8722 & 512.088 \\ 205216 & 25.3894 & 514.252 \\ 207995 & 25.6993 & 516.299 \\ 208959 & 26.0266 & 519.381 \end{bmatrix}$$

The next important step in RPST is the arm generation. Since the number of variables is 3, the angle between the arms obtained using Eqn. (8) is 120° and each arm is having 10 sample values. A typical polar diagram for the problem is shown in **Fig.5**.

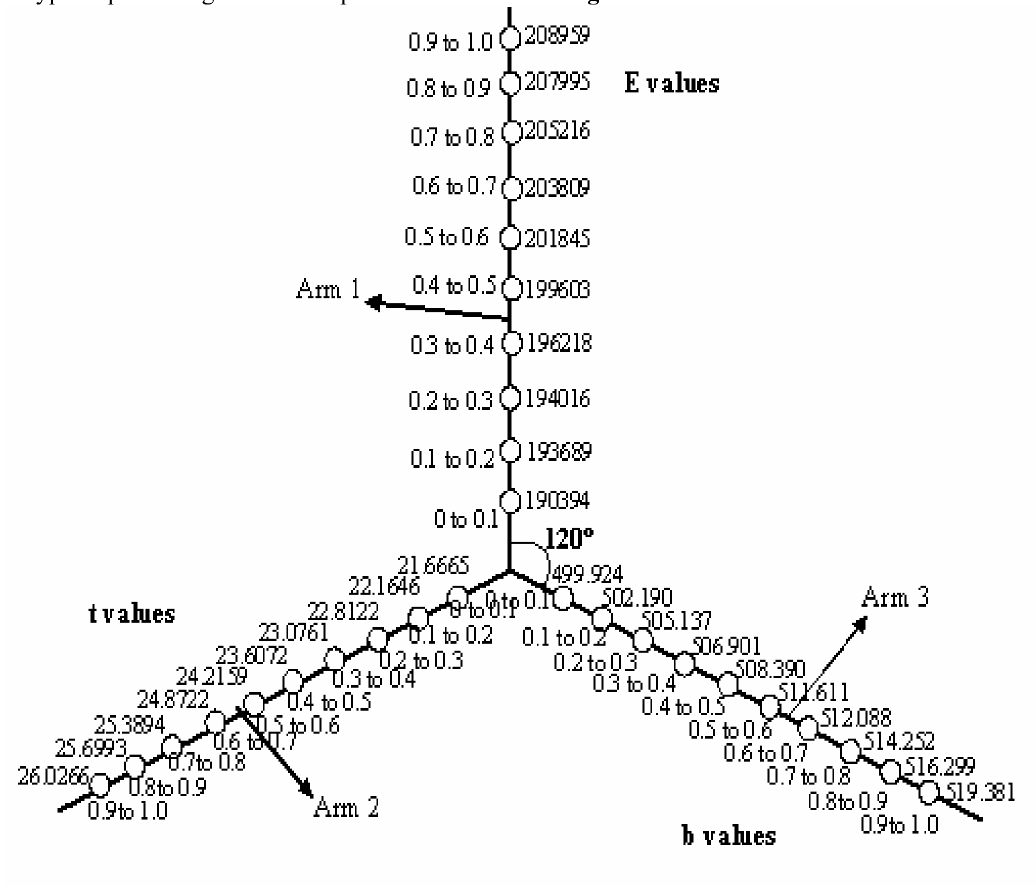

Fig. 5. Typical polar diagram for 3 variables, 10 samples problem

RPST is performed for selecting randomly one sample value from each arm. After performing one cycle of RPST, the arm will shrink to have only 9 sample values. Repeating the simulation cycles, all the samples are selected and this will form the columns of modified LHS matrix.

From the program, the modified LHS matrix, FLH [ ], is obtained as

$$FLH = \begin{bmatrix} 196218 & 22.8122 & 505.137 \\ 208959 & 25.6993 & 502.190 \\ 203809 & 21.6665 & 506.901 \\ 207995 & 25.3894 & 516.299 \\ 190394 & 24.2159 & 499.924 \\ 205216 & 22.1646 & 519.381 \\ 193689 & 23.6072 & 514.252 \\ 194016 & 24.8722 & 508.390 \\ 199603 & 26.0266 & 512.088 \\ 201845 & 23.0761 & 511.611 \end{bmatrix}$$

which is one of the matrices having the best pairs of variables.

From the C++ program, the following ring buckling collapse pressures in MPa, for each row values of FLH matrix are obtained.

Prc1 = 4.35194
Prc2 = 4.84273
Prc3 = 4.43685
Prc4 = 4.02756
Prc5 = 4.30911
Prc6 = 4.21322
Prc7 = 4.36992
Prc8 = 4.4529
Prc9 = 4.6682
Prc10 = 4.50971

Using these ring buckling collapse pressures, the mean value and standard deviation of the performance function are calculated and these are 2.418214 and 0.325082 respectively. The reliability index β for the problem, by assuming the variables as uncorrelated, is calculated using Eqn. (12). The probability of failure is obtained from Eqn. (7) and the corresponding reliability is calculated using Eqn. (9).

The methods like PEM, RST, ISPUD etc., are comparable with FOSM, AFOSM and MCS methods. In line with these methods, the values from RPST are compared with MCS method for 10000 samples, FOSM and AFOSM methods. The reliability values are also calculated for different mean pressure values and the results are shown in **Table. 1.**

Table 1: Reliability values from RPST, MCS, FOSM and AFOSM methods.

Mean Pressure (MPa)	**Methods**	**Reliability Index (β)**	**Probability of failure (p_f)**	**Reliability (R)**
2.0	RPST	7.438776	5.0404 E-14	0.9999999999999496
	MCS	7.492777	3.2196 E-14	0.999999999999967804
	FOSM	6.87531	2.6161 E-12	0.9999999999978329
	AFOSM	8.02868	1.4433 E-15	0.999999999999998557
2.5	RPST	5.263685	7.8928 E-08	0.999999921072
	MCS	5.281628	3.3396 E-08	0.99999996604
	FOSM	4.82337	7.9435 E-07	0.99999920565
	AFOSM	6.31852	1.4947 E-10	0.99999999985253
3.0	RPST	3.429697	3.0 E-04	0.99970
	MCS	3.460938	2.7 E-04	0.99973
	FOSM	2.91634	0.00175	0.99825
	AFOSM	4.26531	1.0696 E-05	0.999989304
3.5	RPST	1.915886	0.02743	0.97257
	MCS	1.966465	0.02442	0.97558
	FOSM	1.18715	0.11702	0.88298
	AFOSM	2.76253	0.00289	0.99711

From the Table, it is clear that the values obtained by using RPST are comparable to those by FOSM, AFOSM and MCS methods.

6. Conclusions:

This paper mainly concentrates on introducing a new method called Random Polar Sampling Technique (RPST) for the reliability analysis of internally ring stiffened submarine pressure hull structure. For computationally expensive problems, a computer code is needed for rapid reliability analysis. In this study, a computer program coded in C++, based on RPST, for the collapse pressure calculation of ring stiffened shell is also described The variables considered in the present program are having a continuous uniform distribution. By using RPST, which is a modification of LHS, the sample sizes can be much reduced than those used in MCS.

Another advantage of RPST is that it does not require the evaluation of derivatives of the failure equation as in FOSM and AFOSM, and also the proposed method can be used for large number of samples and variables. In the present work, a typical submarine pressure hull has been taken for the illustration purpose and the values from RPST have been used to compare with FOSM, AFOSM and MCS methods. The accuracy of result is not much affected by reducing the sample size from 10000 in MCS to 10 in RPST, as given in the illustration. Furthermore, this program can also be easily extended to variables having other probability distributions.

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