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INTRODUCING A PARTICULAR MATHEMATICALMODEL FOR PREDICTING THE RESISTANCE AND PERFORMANCE OF PRISMATIC PLANING HULLS IN CALM WATER BY MEANS OF TOTAL PRESSURE DISTRIBUTION

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Abstract:

Mathematical modeling of planing hulls and determination of their characteristics are the most important subjects in hydrodynamic study of planing vessels. In this paper, a new mathematical model has been developed based on pressure distribution. This model has been provided for two different situations: (1) for a situation in which all forces pass through the center of gravity and (2) for a situation in which forces don not necessarily pass through the center of gravity. Two algorithms have been designed for the governing equations. Computational results have been presented in the form of trim angle, total pressure, hydrodynamic and hydrostatic lift coefficients, spray apex and total resistance which includes frictional, spray and induced resistances. Accuracy of the model has been verified by comparing the numerical findings against the results of Savitsky's method and available experimental data. Good accuracy is displayed. Furthermore, effects of deadrise angle on trim angle of the craft, position of spray apex and resistance have been investigated.

Keywords: Planing hulls, mathematical modeling, resistance and performance, hydrodynamic characteristics, spray apex

1. Introduction

Determination of the resistance and effective power and prediction of the running trim angle, lift coefficient, spray apex and other characteristics of the planing crafts are very important in their design. Many researchers have studied hydrodynamics of the planing hulls and have tried to predict these parameters.

Hydrodynamics of the planing hulls was first studied through experimental methods. Through these efforts, many parametric studies were performed and effects of each of the intended parameter were examined, while other parameters were kept fixed. One of the most important results of these experiments was produced by Sottorf (1934) who tried to investigate the pressure distribution. In all likelihood, it was him who analyzed the pressure distribution and the effect of some parameters on it, for the first time. Smiley (1951) also made an important contribution in this regard. Important data resulted from his work which represented three dimensional pressure distributions.

Pressure distribution on the planing hulls was also investigated by Pierson and Leshnover (1948) and Kapryan and Boyd (1955). Experimental studies were not limited to the analysis of pressure distribution and also included the lift force. Experiments on lift force by Korvin-Kroukovsky et al. (1949) resulted in an empirical equation for determining the lift force of the planing surfaces. On the other hand, computing the wetted area and the center of pressure were other important issues in this regard. Other experiments conducted by Korvin-Kroukovsky et al. (1950) led to some new empirical equations for these important factors. Furthermore, Pierson and Leshnover (1950) tried to explore the phenomena related to the spray root. As a result of their experimental study, the bottom of a planing surface was divided into two areas of spray and pressure. Some important experimental data can also be found in the work by Lock (1933). However, prediction of performance of a planing hull was an issue which hadn’t been solved yet. As a result of Lock’s effort (Lock, 1948), it was found that there is an inception point in the planing surfaces, i.e. when these crafts reach a specific speed coefficient, planing reign begins, and trim angle varies with speed coefficients. Consequently, trim angle, wetted length, resistance, and some other hydrodynamic characteristics were unknown. In all of the mentioned studies, trim angle, wetted length, speed coefficient among other factors were assumed known, in advance. However, these assumptions could not be considered practical for a naval architect in designing a planing hull, because accurate prediction of resistance and performance is not possible without the trim angle and mean wetted length of the planing surface. In order to overcome this difficulty, a new method was developed by Savitsky (1964) which

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was based on empirical equations. A computational procedure was presented and the trim angle of a planing hull was obtained. Subsequently, the resistance and effective power were calculated. Savitsky and Brown (1976) made some efforts to further enhance the developed method by Savitsky (1964) in order to consider the warped hull shape and trim tab effects in their computations. In the same year, Blount and Fox initiated an empirical method for power prediction of planing craft. Latorre, on the other hand, focused on the flow around a planing boat. Latorre (1982) focused on wave pattern and its resistance component in planing motion. Later, Latorre (1983) investigated the spray of planning boats and frictional resistance acting on the bottom of a planing vessel. In another research by Latorre (1993), a parameter was introduced in order to identify whether or not spray blister is broken down into droplets.

Some years later, Zarnick (1978) established a new method for modeling the prismatic planing hulls in waves based on the added mass and strip theories. The origin of his work can be found in a study conducted by Von Karman (1929) who tried to find the hydrodynamic force of the water impact using virtual mass. Payne (1982) was another researcher in this field who made progress in theories related to the added mass theory in planing condition. His works started by computing the hydrodynamic load. However, it could also be used to predict the performance of the planing hulls. In addition to empirical methods and strip theory, numerical methods reached an important level in predicting the performance of the planing hulls. Zhao and Faltinsen (1993) numerically investigated the water entry of wedge shaped bodies using boundary element method (BEM) to find pressure distribution. Later, Zhao et al. (1996) using the obtained pressure distribution and by applying 2d+t theory calculated the performance of planing hulls in calm water. Finally, in the last decade, some important equations were developed by different researchers to determine spray resistance, pressure distribution, and spray apex. Van Deyzen (2008) used added mass planing theory and satisfied surge motion equation of a planing boat in order to compute the planing hull motion in calm water. Kim et al. (2013) also proposed a new method based on the added mass planing theory and pressure acting on a planing plate to compute the trim and resistance of the round bilge semi planing hulls. Recent advances in this field (2d+t theory)have led to new progress in motion prediction of planing hulls such as efforts by Ghadimi el al. (2013a) for predicting six degrees of freedom motion, research by Morabito (2015) for yaw force prediction, Tavakoli et al. (2015) and Ghadimi et al. (2015) for roll motion prediction. It should be noted by the recent advances in water entry problem, today, some other alternative methods for determining forces acting on a section exists such as FVM (Ghadimi et al., 2014) or SPH (Farsi and Ghadimi, 2014).

On the other hand, Savitsky et al. (2007) offered an empirical relation for predicting the spray resistance. Later, Saymsundar and Datla (2008) proposed a modification of Savitsky's method in order to consider the interceptors effect in performance prediction. Afterward, Bertorello and Oliviero (2007) further developed Savitsky method to predict the performance of a warped planning hullform. Savitsky (2012) also offered an empirical model in order to modify his 1964 model for modeling the warped planing boats.

In the recent years, Morabito (2010) presented a comprehensive discussion on the spray and pressure of the planing vessels. On the other hand, Savitsky and Morabito (2011) derived an equation for calculation of the position of spray apex of the planing hulls. Ghadimi et al. (2014) also performed a detailed study of the spray generated by planing hulls and investigated the effects of geometrical and physical parameters on the spray. Recently, Tavakoli et al. (2013) used Morabito's (2010) method for determination of the longitudinal dynamic pressure. Subsequently, Ghadimi et al. (2013b) introduced a procedure for computing the three-dimensional hydrodynamic and hydrostatic pressure distribution on the planing hulls.

In this paper, the derived equations by Morabito (2010) and the developed computational method by Ghadimi et al. (2013b) are utilized for determination of the performance of planing hulls in two different practical situations including one in which all forces pass through the center of gravity (CG) and the other in which not all forces pass through the CG. First, an iterative method is introduced for computing the running trim angle and the mean wetted length for both of these cases. Subsequently, these computed parameters are used along with empirical equations of the frictional and spray resistance to calculate the total resistance of a planing hull. On the other hand, the mentioned analytical relation of the spray apex developed by Savitsky and Moabito (2011) is included in the procedure and computed. Accuracy of the model is validated by comparing the current results against Savitsky's method and available experimental data. Effects of deadrise angle on the running trim angle, spray apex, and resistance are investigated and variations of these characteristics are examined based on the changes in speed coefficient. The proposed model may be considered as the initial point for modeling the motion of the planing boats in waves.

2. Pressure Distribution on Planing Hulls

2.1 Hydrodynamic pressure

As a planing plate moves forward, a stagnation point is generated and a maximum pressure occurs at the stagnation point. Subsequently, dynamic pressures of the points behind the stagnation point decrease as their distance from the stagnation point increases and the pressure becomes zero at the stern of the plate, as shown in Fig. 1. Pressure distribution on a planing hull is similar to the plot shown in Fig. 1, but slightly different. In these crafts, there is a stagnation line at the bottom of the hull as discussed by Pierson and Leshnover (1950), and if the longitudinal sections are used, there is a stagnation point and a maximum value for the dynamic pressure in every section. Dynamic pressure distribution in every section can be calculated using equations presented by Morabito (2010) and the computational procedure developed by Ghadimi et al. (2014) as follows:

$$\frac{P_D}{\frac{1}{2}\rho V^2} = P_T \frac{CX^{\frac{1}{3}}}{X+K} \tag{1}$$

Different parameters and variables in Equation (1) can be determined using Equations (2) through (9).

$$C = 0.006 P_Y \tau^{\frac{1}{3}} \tag{2}$$

$$K = \frac{C^{1.5}}{2.58\left(\frac{P_{MAX}}{\frac{1}{2}\rho V^2}\right)^{1.5}} \tag{3}$$

$$P_T = \frac{\left(\lambda_y - X\right)^{1.4}}{\left(\lambda_y - X\right)^{1.4} + 0.05} \tag{4}$$

$$P_Y = \left[1.02 - 0.05(\beta+5)Y^{1.4}\right]\frac{0.5-Y}{0.51-Y} \tag{5}$$

$$\frac{P_{MAX}}{\frac{1}{2}\rho V^2} = \frac{P_{YSTAG}}{P_N}\sin^2\alpha \tag{6}$$

$$\frac{P_{YSTAG}}{P_N} = [1.02 - 0.25Y^{1.4}]\frac{0.5-Y}{0.51-Y} \tag{7}$$

$$\lambda_y = \lambda - \frac{(Y-0.25)}{\tan\alpha} \tag{8}$$

$$\alpha = \tan^{-1}\left(\frac{\pi}{2}\frac{\tan\tau}{\tan\beta}\right) \tag{9}$$

Here, C and K are two parameters which are constant in each longitudinal section. P_Y is a parameter which causes the transverse decrease of the pressure and (P_{MAX}/q) is the maximum pressure in each longitudinal strip. α is the angle between the stagnation line and the keel, while λ_y is the non-dimensional distance from the transom stern. PT is the parameter which expresses the decreasing pressure in the longitudinal strips and causes the pressure at the transom to diminish. Finally, P_{YSTAG} / P_N represent pressure distribution on the stagnation line. All these parameters are explained in detail by Morabito (2014). Regarding the range of applicability of these equations, the following range has been previously used by Morabito (2010) in his studies which may be considered as a suitable range of applicability.

$C_V > 2$
$\beta < 45$
$2 < \tau < 30$

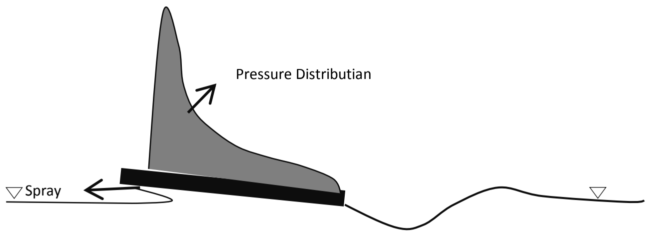

Fig. 1: Longitudinal dynamic pressure distribution on planing plate (Sottorf, 1934)

However, there is no limitation for the mean wetted length. In Equations (1) through (9), X is the non-dimensional distance from the stagnation point at each longitudinal section and Y is the non-dimensional distance from the keel which is illustrated in Fig. 2 and is obtained using Equation (10). Determination of X and Y require the descretization of the bottom of the hull which requires a finer mesh resolution for better accuracy. Errors many emanate from this step if size of the involved mesh is not fine enough. Part of the inevitable errors involved in the proposed mathematical model may also be attributed to the errors of numerical integration for determining the lift coefficient that will be explained in the next section.

$$X = x / B \qquad Y = y / B \tag{10}$$

By using the longitudinal sections and Equations (1) through (9), the dynamic pressure acting on the bottom of a planing hull with specified trim and deadrise angles as well as mean wetted length can be predicted.

2.2 Hydrostatic pressure

The hydrostatic force acting on the bottom of a planing surface is not equal to the buoyancy force, since the hydrostatic pressure is affected by the transom stern and the chine as dynamics pressure is affected, too. Hydrostatic pressure can be computed using a relation proposed by Morabito (2010) as in

$$\frac{P_B}{\frac{1}{2}\rho V^2} = \frac{2 P_T P_Y \sin\tau}{C_V^2}\left(X + Y\left(\frac{1}{\tan\alpha} - \frac{1}{\tan\alpha_w}\right)\right) \tag{11}$$

where α_w is the angle between calm water and the keel that can be written in the form of

$$\alpha_W = \tan^{-1}\left(\frac{\tan\tau}{\tan\beta}\right) \tag{12}$$

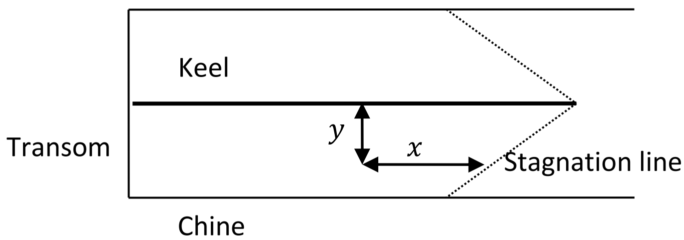

Fig. 2: Distances form stagnation line and center line for a point in the bottom of a planing hull

Numerical treatment of fluid flow for maritime crafts are much more complex than other types of vehicle because of special environmental treatments associated with its operation for example sailing at a free surface. Academic and research organizations have devoted long potential on numerical simulations in the maritime field accompanying traditional model testing.

Finally, total pressure acting on the bottom of a planing hull can be computed by summation of the hydrostatic and hydrodynamic pressures.

$$\frac{P_{TOTAL}}{\frac{1}{2}\rho V^2} = \frac{P_D}{\frac{1}{2}\rho V^2} + \frac{P_B}{\frac{1}{2}\rho V^2} \tag{13}$$

3. Lift and Center of Pressure

Lift force of a planing hull can be calculated by integration of the dynamic and hydrostatic pressures. Lift coefficient includes two terms; hydrodynamic and hydrostatic lifts. Lift coefficient is given in Equation (14), where the first term represents the hydrodynamic lift coefficient and the second represents the hydrostatic lift coefficient (Morabito, 2010).

$$C_L = \int_{-0.5}^{0.5}\int_{0}^{\lambda_y} \frac{P_D}{\frac{1}{2}\rho V^2} dXdY + \int_{-0.5}^{0.5}\int_{0}^{\lambda_y} \frac{P_B}{\frac{1}{2}\rho V^2} dXdY$$

(14)

Non-dimensional position of the center of pressure from the stern can also be obtained by using equation

$$\frac{l_P}{B} = \int_{-0.5}^{0.5}\int_{0}^{\lambda_y} \frac{(P_D + P_B)(\lambda_y - X)}{\frac{1}{2}\rho V^2} dXdY \tag{15}$$

which is introduced by Morabito (2010).

4. Resistance of Planing Hulls

Resistance of a planing hull can be obtained by applying the Savitsky's method (1964) which is written in the form of

$$R = \Delta \tan \tau + \frac{D_f}{\cos\tau} \tag{16}$$

here, Δ is the weight of the boat and D_f is the frictional drag and is evaluated from Equation (16) to be

$$D_f = \frac{\rho V^2 C_F \lambda b^2}{2\cos\beta} \tag{17}$$

where

$$C_F = \frac{0.0075}{(\log Rn - 2)} + 0.0004 \tag{18}$$

$$Rn = V\,\frac{\lambda b}{\upsilon} \tag{19}$$

Savitsky et al. (2007) derived an equation in order to compute the viscous force of the spray on a planing hull as in

$$R_S = \frac{1}{2}\rho V^2 \,\frac{B^2 \cos\Theta}{2\sin 2\alpha \cos\beta} \tag{20}$$

Where Θ is the angle between the forward edge of the whisker spray and the keel line in a plane that passes through the keel and is perpendicular to the hull center plane. Θ is written as

$$\Theta = \frac{\theta}{\cos\beta} \tag{21}$$

$$\theta = 2\alpha \tag{22}$$

In the proposed mathematical model, Equations (16) and (20) are used for calculation of the total resistance. On the other hand, the effective power of the planing hull can be written as:

$$P_E = R_T V \tag{23}$$

where,

$$R_T = R_f + R_S + R_i \tag{24}$$

In Equation (24), R_f is the frictional resistance which is $D_f/\cos\tau$ and R_i is the induced resistance being $\Delta \tan\tau$.

5. Spray Apex

Spray apex may be also considered as another important parameter in planing hull motion and has been highlighted by some researchers. For instance, Latorre and Tamiya (1975) developed an experimental method to measure spray around the planing boats. Latorre (1983) also developed a model for studying the spray apex. Recently, Savitsky and Morabito (2011) developed a new method for computing the spray apex position based on the swept wing theory. In the current paper, their method is utilized in order to determine the spray apex along with resistance, trim angle, and wetted length. In the developed by Savitsky and Morabito (2011), two components are considered for the velocity vector: 1) velocity component along the stagnation line shown as (V_s) and 2) velocity component normal to the stagnation line depicted as (V_n). In order to compute the spray apex, vertical velocity of the spray (V_V) should be calculated first and subsequently the spray apex is determined using the projectile principle. Vertical velocity of the spray is given by

$$V_V = V_n + V_{SV} = V \sin\alpha + V\cos\alpha \sin\alpha \sin\beta_E \tag{25}$$

In this equation, β_E is the angle between the stagnation line and the horizon in transverse plane and can be obtained using Equation (26), while V_{SV} is the vertical component of the velocity along the stagnation line in the transverse plane. The height of the spray apex is found using Equation (27) and the longitudinal and lateral positions of the spray apex can be determined using Equations (28) through (30).

$$\beta_E = \tan^{-1}\left[\left(1-\frac{2}{\pi}\right)\tan\beta\right] \tag{26}$$

$$Z' = \frac{V_V^2}{2g} \tag{27}$$

$$L_H = V_s\sqrt{\frac{2z}{g}} \tag{28}$$

$$X' = L_H \cos\alpha \tag{29}$$

$$Y' = L_H \sin\alpha \tag{30}$$

Here, Z' is height of the spray apex, L_H is the horizontal distance that spray travels before reaching apex, and X' and Y' are the longitudinal and lateral positions of the spray apex, respectively.

6. Prediction of Trim Angle and Performance

Savitsky (1964) developed his computational methods for two general conditions. In the first condition, all forces pass through the center of gravity (CG), while in the second condition, not all forces pass through the CG. The presented equations are hereby used to predict the trim angle of the planing hulls in both conditions. Subsequently, resistance and spray apex can be computed.

6.1 Case 1: All the forces pass through CG

Schematic of the forces in this case has been displayed in Fig. 3. As seen in this figure, thrust, frictional drag, lift and weight of the boat pass through the CG.

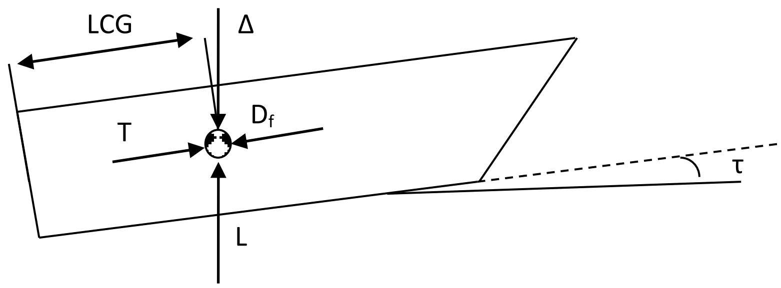

Fig. 3: Schematic of case 1: All forces pass through the CG (Savitsky, 1964)

The proposed computational procedure starts with determination of the lift coefficient of the planing boat (C_{lBoat}). This coefficient is calculated by

$$C_{lBoat} = \frac{\Delta}{\frac{1}{2}\rho B^2 V^2} \tag{31}$$

The trim angle of a planing hull can be obtained through logical steps of a computational process. In this section, a computational procedure is presented and steps are explained. First, a running trim angle is guessed for the planing hull. Next, the non-dimensional mean wetted length of the planing hull is also guessed. By using the assumed values of the trim angle and mean wetted length and having the geometry of the planing hull, total pressure distribution can be determined. Later, the computed pressure distribution is integrated to find the lift coefficient. Subsequently, lift force should be compared against lift coefficient of the planing hull (C_{lBoat}). There exist two possibilities: (1) the computed lift coefficient is greater than C_{lBoat} and (2) the computed lift coefficient is smaller than C_{lBoat}. If the computed lift is greater, a reduced mean wetted length should be predicted again. Otherwise, a greater mean wetted length should be estimated. The new mean wetted length and the estimated trim angle will then be used once again in order to estimate hydrodynamic and hydrostatic pressure distributions. The lift coefficient is then computed by integration of the pressure values. This coefficient is to be compared with C_{lBoat} again, and the iterative method continues until value of C_{lBoat} becomes approximately equal to C_L. When C_{lBoat} and C_L are nearly equal, the mean wetted length in which weight is supported by the lift, is determined. Subsequently, the correct value of the trim angle is computed. In this framework, the center of pressure is computed by using Equation (15). In the next step, the difference between determined center of pressure and LCG is evaluated. If LCG and l_p are located at the same longitudinal position, the estimated trim angle is a correct presumption. If these conditions are not correct, then the trim angle must be assessed again. Consequently, the position of the center of pressure is utilized for the second deduction. If the center of pressure is closer to the transom than the LCG, a new larger trim angle should be adopted. Otherwise, the new estimate for the trim angle will need to be a smaller value. For this new trim angle, a new mean wetted length in which ClBoat and CL are equal, should be determined again. Likewise, longitudinal position of the center of pressure is also computed and compared to LCG. This computational procedure will be performed until same longitudinal location for lp and LCG occurs. In the final step, resistance is determined by using Equation (23) and position of the spray apex is also calculated using Equations (26) through (30). Computational algorithm for this case is illustrated in Fig. 4.

6.2 Case 2: Not all the forces pass through CG

Schematic of the case in which not all forces pass through the center of gravity is shown in Fig. 5. In this figure, N is the normal force due to the hydrodynamic and hydrostatic pressures and all forces do not pass through the CG.

In order to predict the performance of planing hulls in this case, the equilibrium relation in Equation (32) should be satisfied. This equation was proposed by Savitsky (1964) and requires the exact position of the thrust.

$$\Delta\left[\frac{c}{\cos\tau}(1-\sin(\tau+\varepsilon)) - f\sin\tau\right] + D_f(a-f) = 0 \tag{32}$$

where,

c = Distance between N and CG

ϵ = Inclination of thrust line relative to keel

f = Distance between T and CG
a = Distance between D_f and CG (it is assumed that viscous draft is parallel to the keel line)
c and a can be determined using Equations (33) and (34), respectively. f and ϵ are the inputs of the problem.

$$a = VCG - \frac{B}{4}\tan\beta \tag{33}$$

$$c = LCG - l_p \tag{34}$$

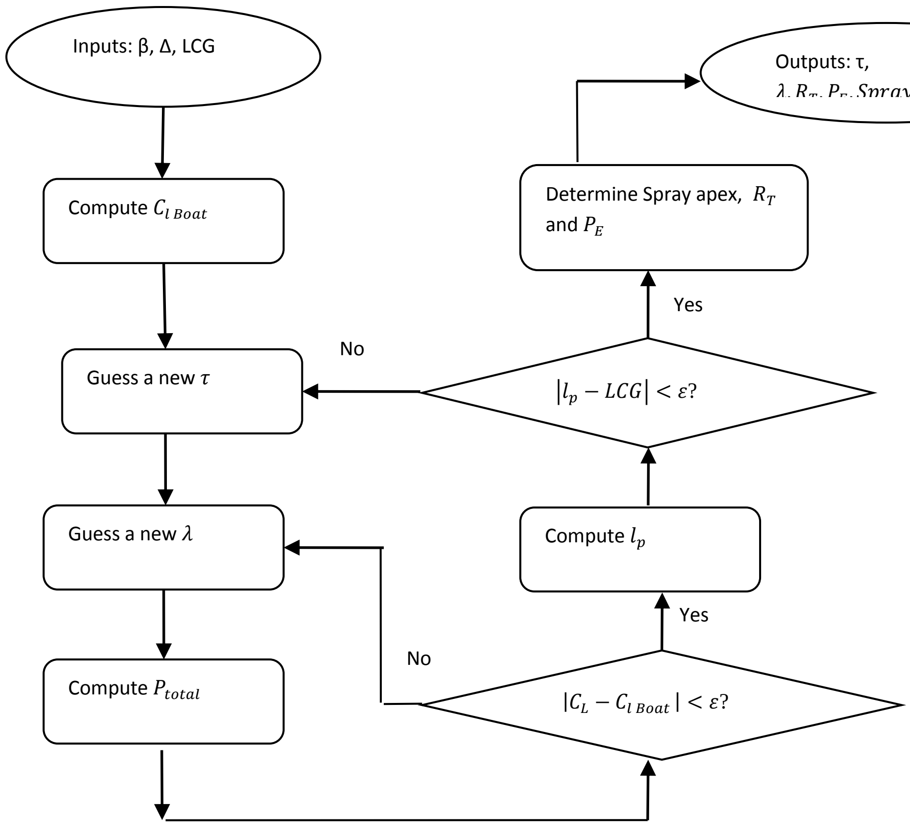

Fig. 4: Computational procedure for predicting trim angle, resistance, effective power and spray apex for the case in which all forces pass through the CG

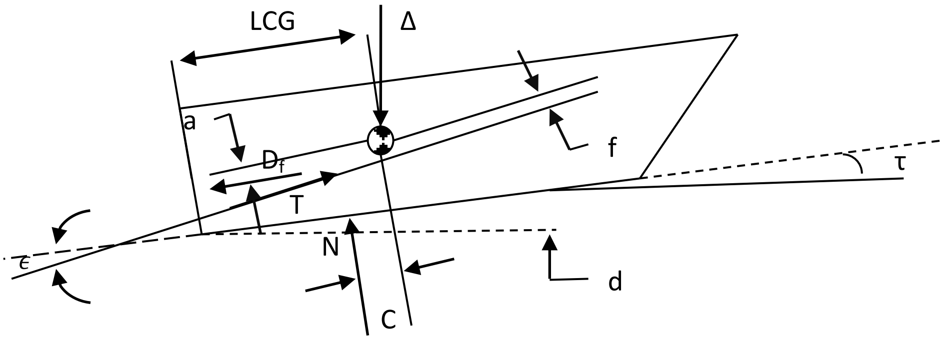

Fig. 5: Schematic of case 2: Not all forces pass through CG (Savitsky, 1964)

In Equation (33), VCG is the vertical position of the center of gravity. Computational procedure for this case is similar to case 1. Initially, trim angle and non-dimensional mean wetted length are estimated, respectively. Similar as in case 1, the mean wetted length at which C_{lBoat} and C_L are approximately equal, will be determined. Later, left side of Equation (32) is computed by the determined mean wetted length and the guessed trim angle. If this value is approximately equal to zero, then the guessed trim angle is accurate. Otherwise, the trim angle should be predicted again. This presumption is a function of the computed value for the left side of Equation (32). If this calculated value is positive, the new trim angle should be greater. Otherwise, it should be smaller.

This computational procedure continues until relative satisfaction of Equation (32) is achieved. Finally, the predicted trim angle and mean wetted length are used for computing the resistance and spray apex. The computational algorithm used for this case is displayed in Fig. 6.

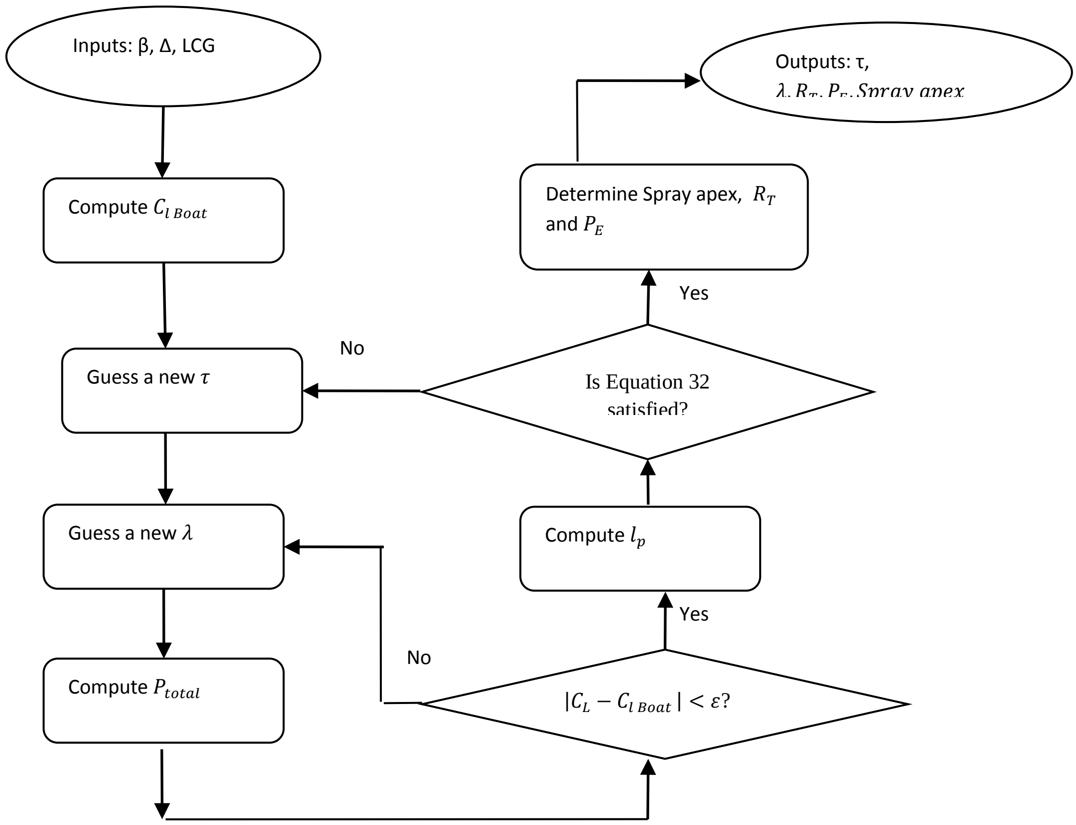

Fig. 6: Computational procedure for predicting trim angle, resistance, effective power and spray apex for the case in which all forces do not pass through CG

7. Validation

After presenting the computational procedure, it is necessary to investigate the accuracy of the algorithm. First, the computational procedure is compared against the well-known technique offered by Savitsky (1964). Primarily, the basic computation of the lift coefficient for a planing plate (β=0) is investigated. In this regard, values of $C_{L0}/\tau^{1.1}$ are calculated at four different speed coefficients of C_V =1, 2, 4 and 6 for $0.25\leq\lambda\leq4$ and compared against the empirical equation (35) which was introduced by Savitsky (1964). The hydrostatic lift and total lift are then compared against the results of empirical equation, separately.

$$C_{L0} = \tau^{1.1}\left(0.0120\sqrt{\lambda} + \frac{0.0055\lambda^{2.5}}{C_V^2}\right) \tag{35}$$

Figs. 7 and 8 demonstrate the comparison of the hydrostatic lift coefficients of the presented method and that of Equation (35). This comparison displays a fairly good agreement between the calculated hydrostatic term of $C_{L0}/\tau^{1.1}$ and the empirical equation of Savitsky (1964). There is only a slight difference between the presented method and empirical equations at large mean wetted lengths (λ>3). This error can probably be attributed to the numerical integration of the pressure. The value of $C_{L0}/\tau^{1.1}$ is determined by the proposed method and compared against the results of Equation (35).

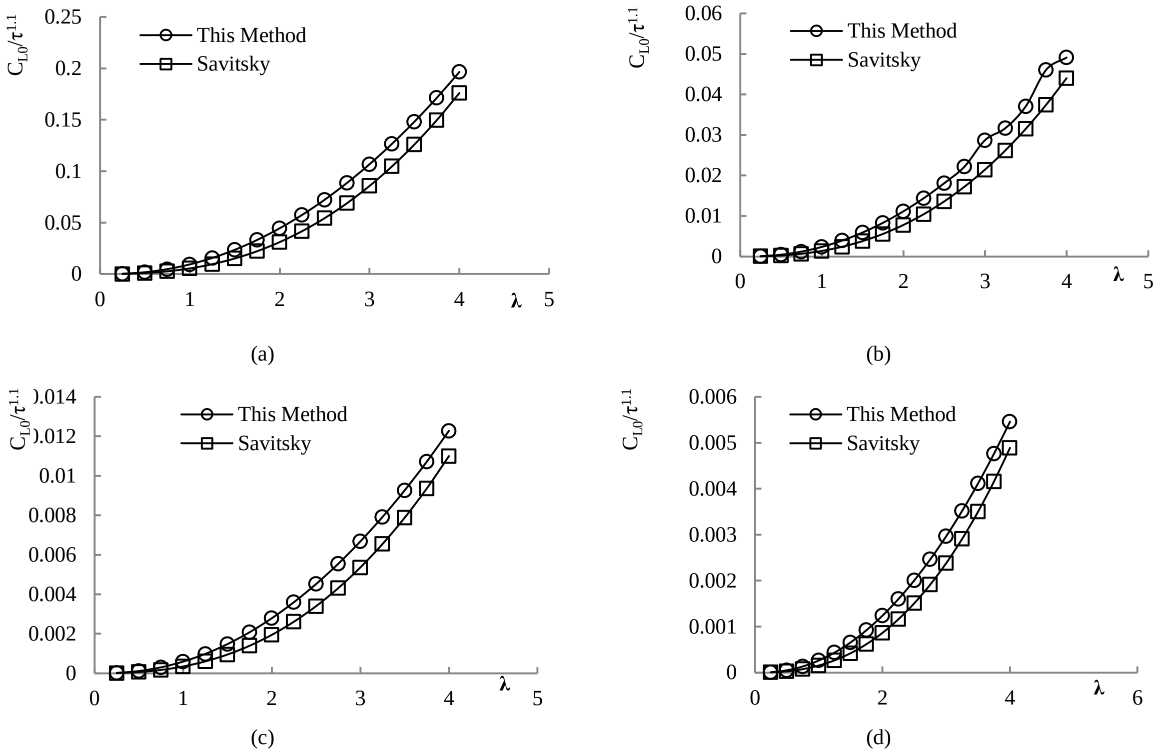

Fig. 7: Comparison of the determined hydrostatic lift coefficient using the proposed method and empirical equation (36) as presented by Savitsky (1964):(a) C_V =1 (b) C_V =2 (c) C_V =4 (d) C_V =6.

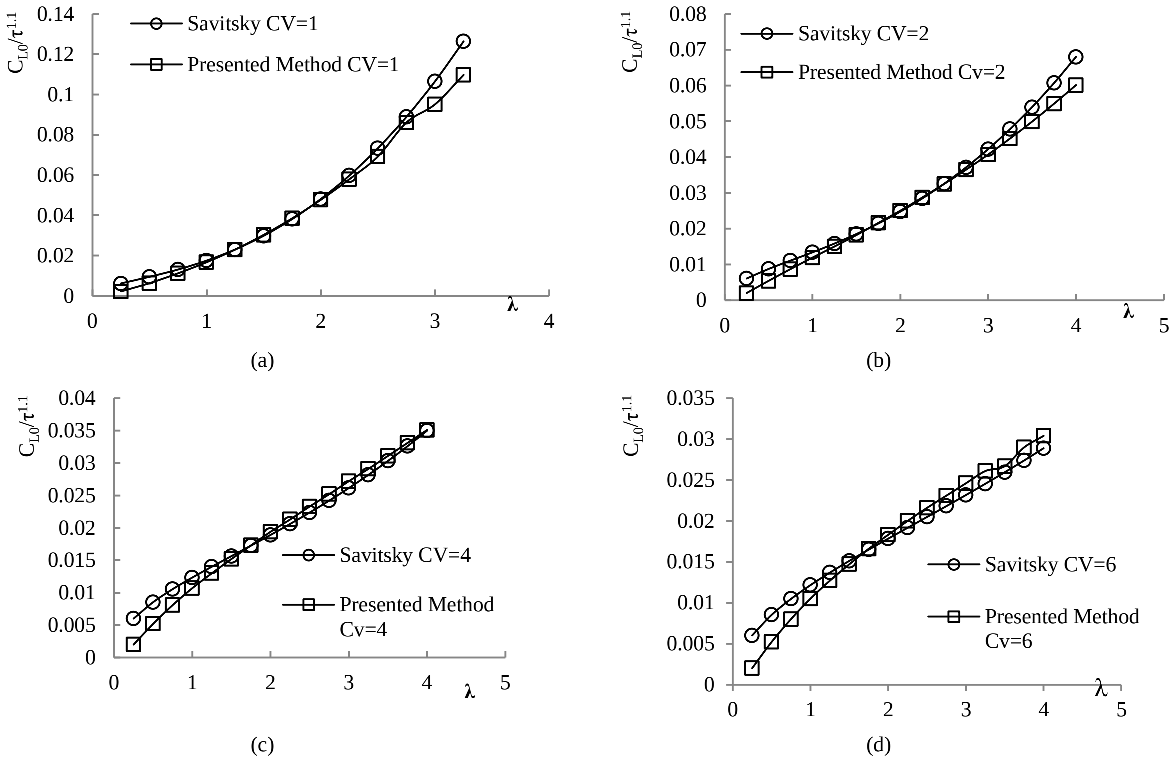

Fig. 8: Comparison of total lift coefficient using the proposed method and empirical equation (35) presented by Savitsky (1964):(a) C_V =1 (b) C_V=2 (c) C_V =4 (d) C_V =6.

Fig. 8 illustrates the stated comparison. There is also a satisfactory accuracy between the computed $C_{L0}/\tau^{1.1}$ and the result of empirical equation at four different speed coefficients. For small mean wetted lengths (λ<1), there is a difference between the presented method and Equation (36), which is created by the area over which the pressure is acting on, i.e. for the small mean wetted lengths, the area of maximum pressure is small and the resulting force is small, too. Accordingly, integration of the maximum pressure over this area does not yield a small value. Therefore, the results will not be close to the empirical equation of Savitsky (1964).

Fig. 10 further displays the comparison of the predicted trim angle by the proposed technique and Savitsky's method for a planing boat with the presented parameters in Table 1 and a body profile shown in Fig 9.

Table 1:Information of the planing boat for validation purposes

Parameter	Value
Mass	*27000 Kg*
LCG	*8.81m*
VCG	*0.61 m*
L/B	*5*
Beam (B)	*4.71 m*
Deadrise (β)	*10°*
f *(only for the condition that all the forces don't pass through CG)*	*0.1524 m*
ϵ *(only for the condition that all the forces don't pass through CG)*	*4°*
C_v	*2-6*

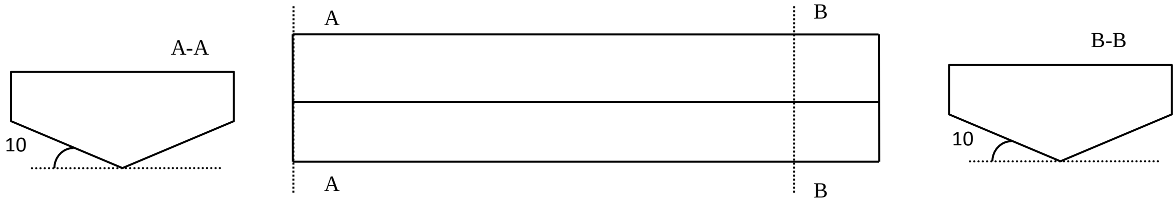

Fig. 9: Body profile of the planing hull of Table 1

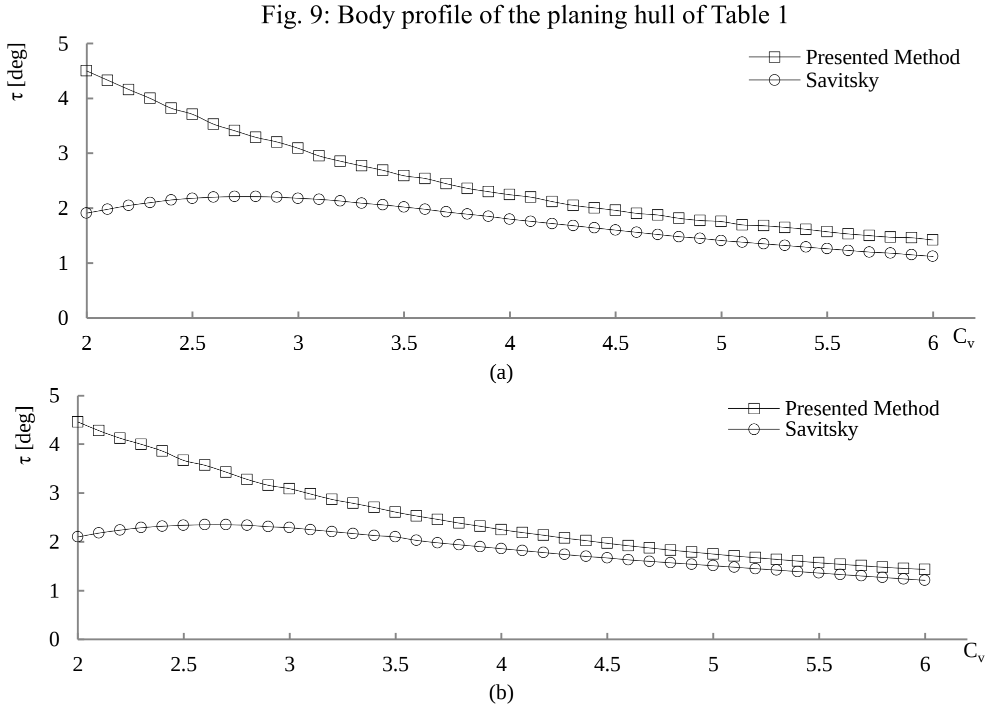

Fig. 10: Comparison of the predicted trim angle for the planing boat introduced in Table 1 by the presented method against Savitsky (1964); (a) Case 1: all the forces pass through CG, (b) Case 2: all the forces don't pass through CG

The trim angle is predicted at various speed coefficients ranging from 2 to 6. Both of these cases were utilized for this prediction. Fig. 10(a) shows the predicted trim angles for the case in which all the forces pass through the CG. Fig. 10(b) shows the results for the case in which not all the forces pass through the CG. This figure indicates that the computed trim angle resulting from both methods have converging results at speed coefficients greater than 3. The difference is obvious at smaller speed coefficients. This variance is caused by the manner in which the center of pressure is predicted. In other words, in the presented method, the center of pressure is calculated using the integration of hydrostatic and hydrodynamic pressures. At low speed coefficients, the computed hydrostatic pressure has greater values than the hydrodynamic pressure. Therefore, this force contributes significantly to the generation of lift coefficient and causes the center of pressure move towards the transom stern. This detail leads to the presented computational method for computing the trim angles which have larger values than that of Savitsky's model at this C_V. But at speed coefficients greater than 3, the hydrostatic pressure has a very small contribution. This effect vanishes and there is a small difference between the two methods.

The values of *R/*Δ are also computed by the proposed method and compared against those of Savitsky method. Accordingly, for the planing hull with presented information in Table 1 and only for the case in which not all the forces pass through the CG, the resistance is computed. The results of both methods are shown in Fig. 11.

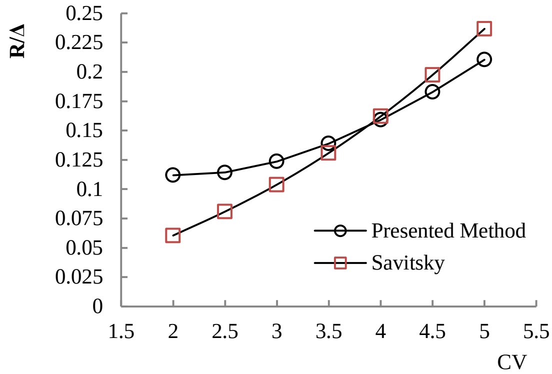

Fig. 11: Predicted R/Δ for the planing boat with information provided in Table 1

As evident for C_V>3, the results of the proposed method and those by Savitsky (1964) display favorable agreement. However, for C_V<3, there is a slight disagreement which is caused by the differences in the predicted trim angles for both methods (shown in Fig. 10 (a) and (b)). It should be noticed that both Savitsky's method and the suggested approach involve empirical approach which use simplified equations and also neglect the effects of some physical aspects of the problem. Therefore, the above comparison can only confirm the agreement between two simplified methods, but is not enough for validation. Alternatively, the method is validated by an experimental case in which the trim angle and resistance of a particular boat are measured. Accordingly, a prismatic monohull is considered and its specifications are given in Table 2 and its body profile is displayed in Fig. 12.

Table 2: Specifications of the considered monohull for validation purposes (Begovic and Bertorello, 2012)

Parameter	Value
Mass	*32.5*
LCG	*0.697m*
Length of Overall	*1.9 m*
Deadrise (*β)*	*16.7°*
L/B	*4.48*
C_v	*2.25-3.98*

Hydrodynamic characteristics of this hull have been reported by Begovic and Bertorello (2012).Comparison of the computed trim angle and resistance of the monohull against the experimental measurements reported by Begovic and Bertorello (2012) are displayed in Figs. 13 and 14, respectively. Fig. 13 shows that trim angle is well predicted at C_v>3 and mean of the error for this range of Cv is approximately 15%. However, if the range of C_v lowered to C_v>2, mean of the error becomes approximately 22.5%. This signifies the fact that applicability range of this method should be limited to C_v>3. It is noteworthy that same trend was observed when the current results were compared with that of Savitsky's method (1964).

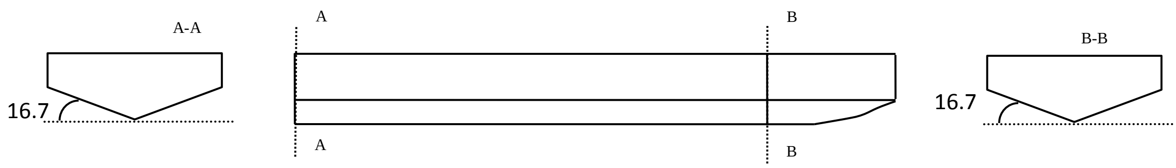

Fig.12: Body profile of the planing hull in Table 2 (Begovic and Bertorello (2012)).

Estimated resistance of the considered monohull is displayed in Fig. 14. This figure displays favorable accuracy for all values of C_v with the exception of C_v =3.98. Mean value of the error associated with the predicted resistance is approximately 10.8%. This error may have been caused by the residual resistance which is not considered in the formulations and the lower computed wetted surface area.

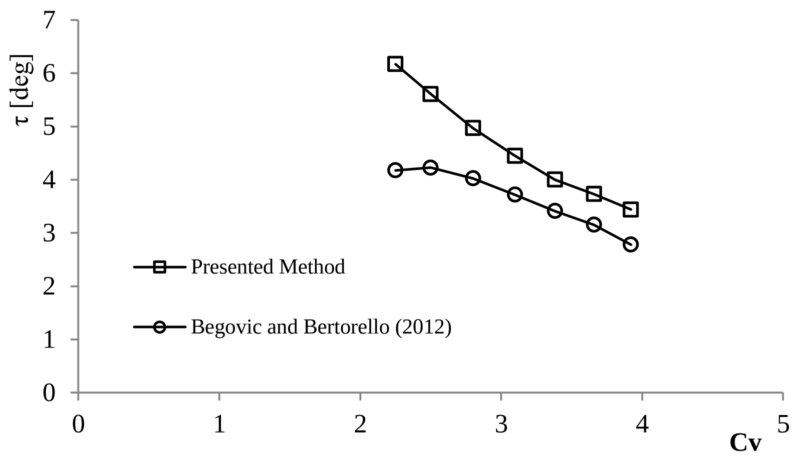

Fig. 13: Comparison of the predicted trim angle for the considered planing boat against the reported results of Begovic and Bertorello (2012)

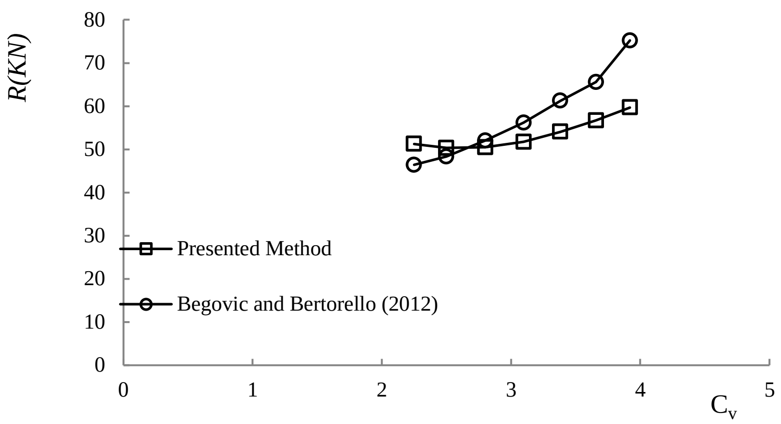

Fig. 14: Comparison of the predicted resistance for the considered planing boat against the reported results of Begovic and Bertorello (2012)

8. Results and Discussion

8.1 Trim angle and hydrodynamic lift coefficient

In this section, corresponding equations and mathematical models are utilized to study the trim angles of the planing boats. First, for a planing hull of M = 27000 Kg, LCG = 8.81 m, and B = 4.27 m at a constant speed coefficient C_V=6, various deadrise angles are considered and the trim angles are calculated with respect to the case in which forces pass through the CG. The result of this study is illustrated in Fig. 15. This figure shows that growth in deadrise with constant parameters causes an increase in the trim angle.

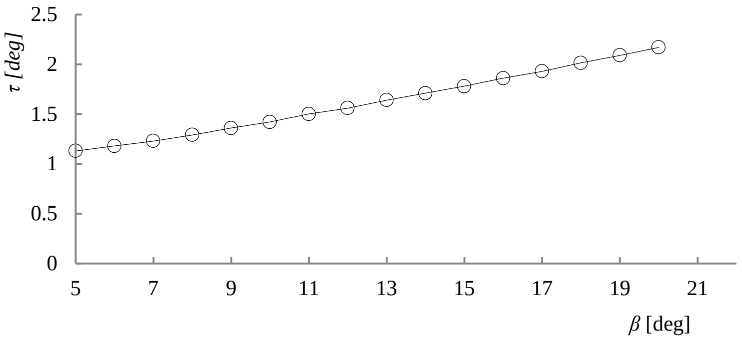

Fig. 15: Predicted trim angle of planing hulls of M=27000 kg, LCG=8.81 m and C_V =6 at various deadrise angles

This observation can be investigated more comprehensively by modeling two planing hulls with different deadrise angles. In this context, trim angles of two planing hulls with different deadrise angles *β*=10, 15 and same *M*=27000 Kg, *LCG*=8.81 m have been determined using case 1. The predicted trim angles are illustrated in Fig. 16. This figure indicates that trim angle of a planing hull with greater value of deadrise angle is larger, when all parameters are kept constant.

Differences between the trim angles which are evident in Fig. 16 can be attributed to the resolution of pressure distribution. Longitudinal pressure distribution on the center line has been computed for these crafts at a specific speed coefficient C_V=3. The computed pressure is illustrated in Fig. 17. This figure confirms that maximum pressure of the planing hull with β=15 is larger than the planing hull with β=10. On the other hand, values of pressure of the planing hull with smaller deadrise angles (*β*=10) are larger after the maximum pressure area (here, *x/b*>0.3). Conversely, planing hulls with *β*=15 should exhibit less pressure which occurs after the maximum pressure area, with a large maximum pressure. Maximum pressure has a direct relation with $\sin^2\alpha$. As a result of this, $\sin^2\alpha$ has a direct relation with the trim angle. Therefore, at the same condition, a planing hull with larger deadrise angle needs larger trim angle.

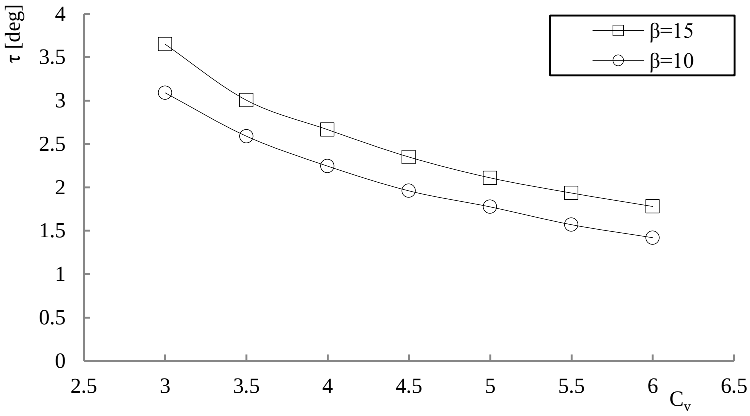

Fig. 16: Predicted Trim angles of two planing hulls with M=27000 kg, LCG=8.81 m at two different deadrise angles

Lift coefficient of the planing hulls with *β*=10, *M*=27000 kg, *B*=4.27 m, *LCG*=8.81 m, *VCG*=0.614 m, *f*=0.154 m and $\epsilon = 4°$ have been computed at different speed coefficients. Fig. 18 shows the computed lift coefficients. Total lift, hydrodynamic lift, and hydrostatic lift are illustrated separately and it is quite evident that they decrease as speed coefficient is gradually raised. Fig. 19 also shows the percentage of contribution of hydrodynamic lift at 3<C_V<6 for a planing hull with different center of gravity *LCG*=8.81 m and 10 m. Two notable points can be concluded from this figure. First, contribution of the hydrodynamic lift increases by an increase in the speed coefficient. Second, if *LCG* moves to the bow of the planing hull (*LCG*=10 m), the contribution of hydrodynamic lift decreases.

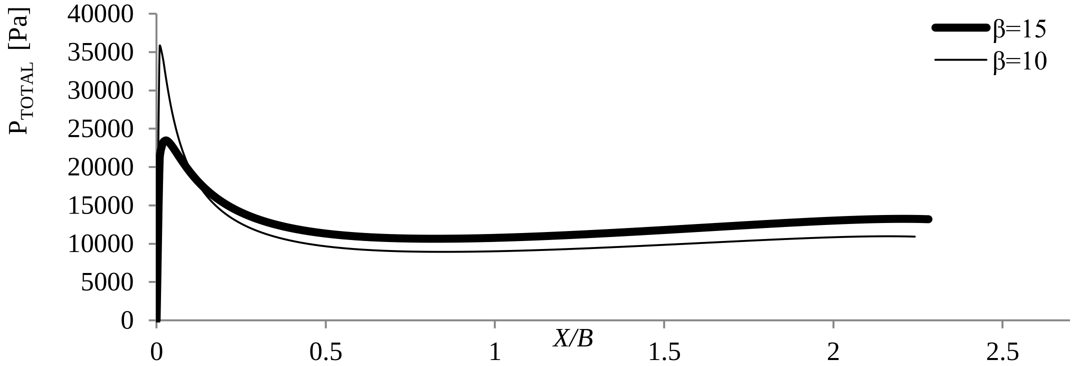

Fig. 17: Longitudinal pressure distribution over the centerline for two planing hulls with *β=10* and *β=15* at C_V=6

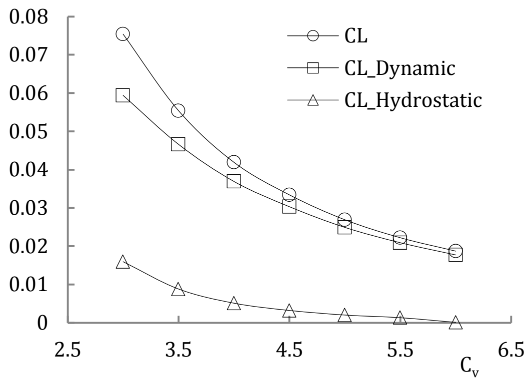

Fig. 18: Longitudinal pressure distribution over the centerline for two planing hulls with *β=10* and *β=15* at C_V=6

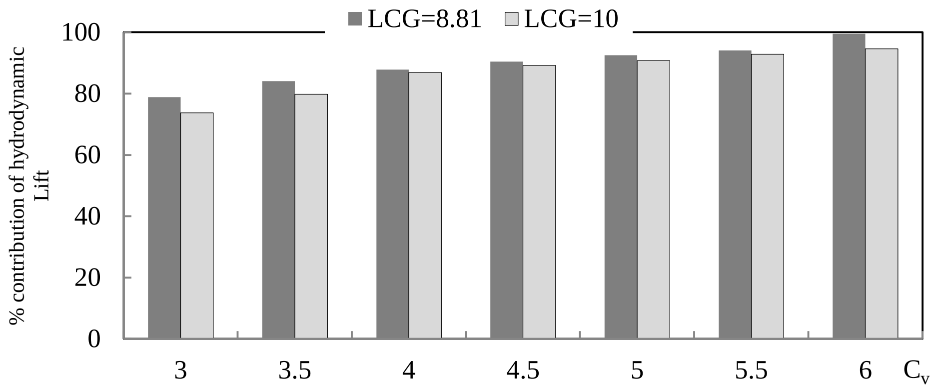

Fig. 19: Longitudinal pressure distribution over the centerline for two planing hulls with *β=10* and *β=15* at C_V=6

8.2 Spray Apex

In this section, varieties of spray apex positions as a function of speed coefficient have been studied. The position of spray apex is illustrated in Fig. 20. As discussed earlier, the spray apex is calculated by using Equations (27) through (30) after predicting the trim angle and the mean wetted length.

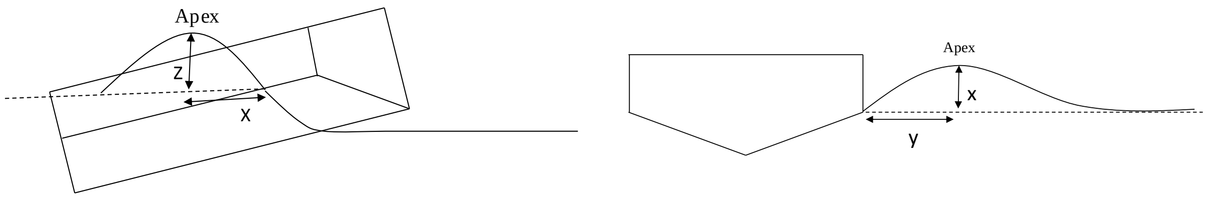

Fig. 20: Spray apex position with respect to chine, transverse section (right), and longitudinal section (left)

Spray apex height has been computed for two planing hulls with predicted trim angles which were displayed in Fig. 16. The obtained spray apex height is illustrated in Fig. 21. It is shown that for the planing hull of β=10, the spray apex height has a range of $4.03<Z'<4.25$, while maximum spray apex height occurs at C_V=5. Before this speed coefficient, spray apex height increases, but after this speed coefficient, it decreases. Furthermore, for the planing hull of β=15, the range of the parameter (*Z'*) is between 2.70 m to 2.90 m. Finally, for the deadrise angle β=15, maximum spray height is generated at C_V =5.5.

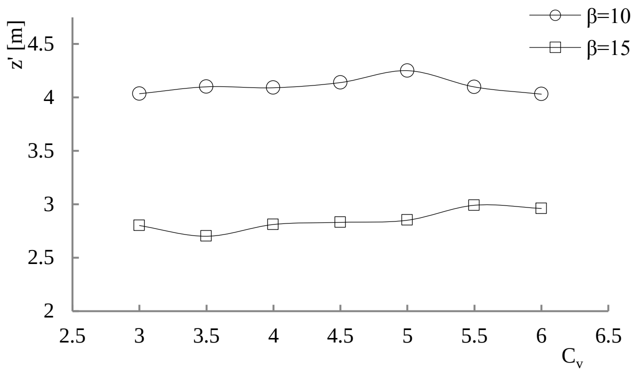

Fig. 21: Spray apex height (Z') for two planing hulls at various speed coefficients

Variation of longitudinal position of spray apex is illustrated in Fig. 22 for both vessels with different deadrise angles. For the planing hull with smaller deadrise i.e. β=10, this parameter (*X'*) has larger values and varies from 14.29 m to 33.25 m. For the hull with β=15, *X'* varies from 12.87 m to 29.2 m.

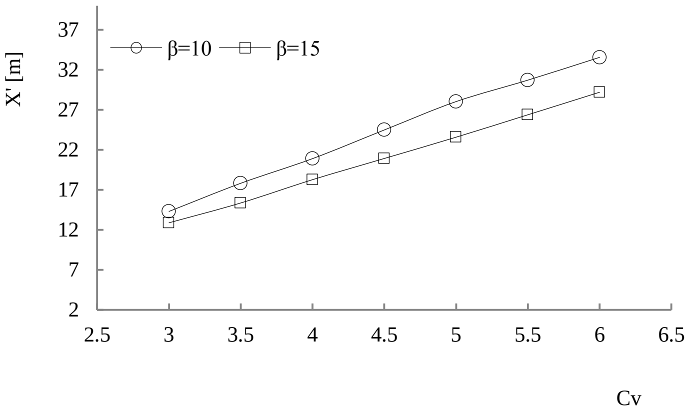

Fig. 22: Longitudinal position of spray apex (X') for two planing hulls as a function of speed coefficient

Finally, the lateral position of the spray apex (*Y'*) is studied and the resulting plot for two hulls of β=10 and β=15 are displayed in Fig. 23.

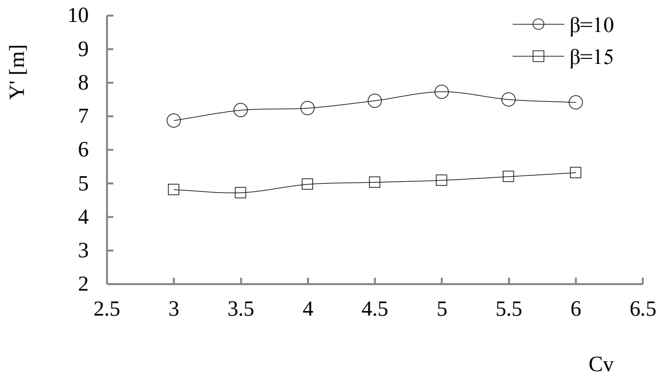

Fig. 23: Lateral position of the spray apex (Y') for two planing hulls as a function of speed coefficient

For the planing hull of β=10, the lateral position of the spray apex decreases after reaching a maximum value which occurs at C_V=5. Values of the lateral position of the spray apex change from 6.68 m to 7.73 m for β=10. Finally, the lateral position of the spray apex of the planing boat of β=15 is studied. In terms of variation of *Y'* for this hull (β=15), there is an initial decreasing trend at C_V=3 and an increasing at C_V =3.5 and values of *Y*' is between 4.72 m to 5.32 m.

8.3 Resistance and effective power

The Resulting resistance of the proposed mathematical model for the case where not all the forces pass through the center of gravity has been determined for a planing hull whose parameters are presented in Table 3 and the calculated resistance is illustrated in Fig. 24.

Table 3: Parameters of the modeled planing hull.

Parameter	Value
Mass	*27000 Kg*
LCG	*8.81m*
VCG	*0.61 m*
L/B	*5*
Deadrise (β)	*15°*
F	*0.1524 m*
ϵ	*4°*
C_V	*3-6*

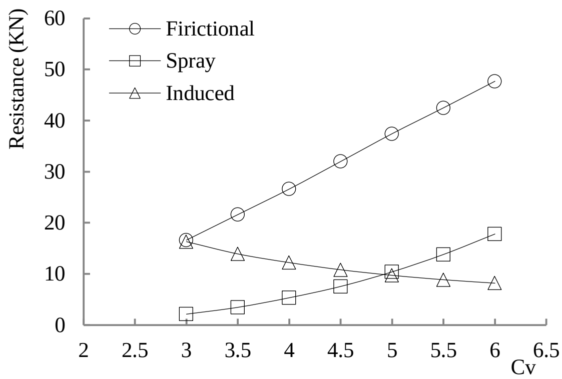

Fig. 24: Resistance of the planing hull of β=15, M=27000 Kg, LCG=8.81, B=4.27 m, VCG=0.61, F=0.154 m, ϵ=4°.

It is shown that frictional resistance increases as the speed coefficient increases and it has the most contribution to the resistance. In terms of contribution of the spray resistance, it can be concluded that at C_V=3, the spray resistance has the lowest contribution in total resistance, but this resistance increases with an increase in the speed coefficient while the induced resistances decrease. Finally, at C_V=5.5, the value of spray resistance exceeds the value of induced resistance.

Total resistance and effective power of the planing hulls whose parameters are presented in Tables 1 and 3 have also been compared in Figs. 25 and 26, respectively.

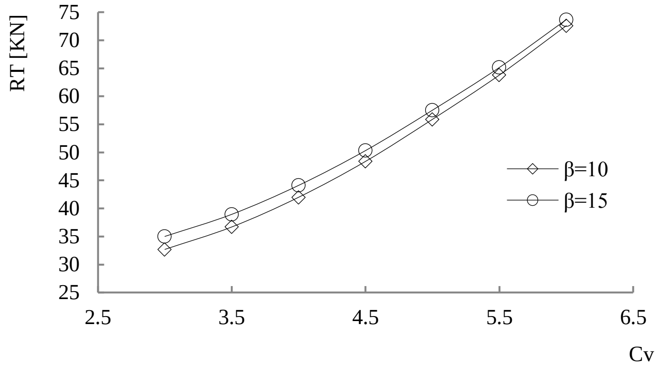

Fig. 25: Comparison of the total resistance for two planing hulls with different deadrise angles and same conditions (Tables 1 and 3)

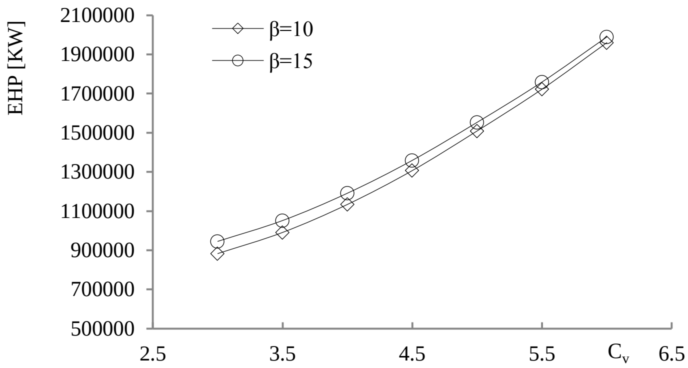

Fig. 26: Comparison of the effective power for two planing hulls with different deadrise angles and same conditions (Tables 1 and 3)

Total resistance increases for both cases. The hull with smaller deadrise angle has less resistance at all speed coefficients compared against the hull with larger deadries angle. Effective power of both crafts behaves similar to the total resistance, as speed increases.

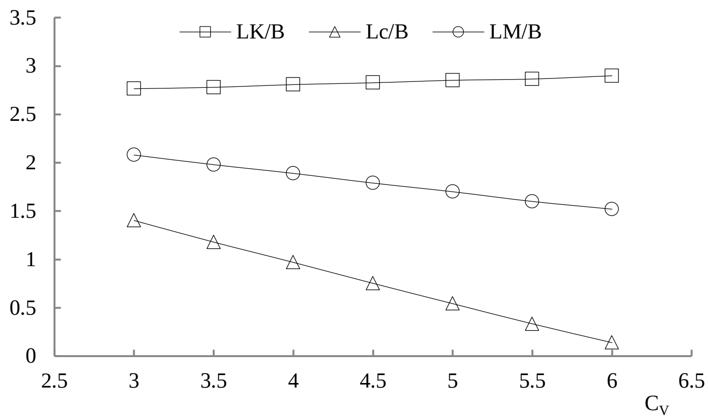

Fig. 27: Comparison of the effective power for two planing hulls with different deadrise angles and same conditions (Tables 1 and 3)

8.4 Wetted length and stagnation line angle

Different wetted lengths of the planing boat in Table 3 have been computed. These lengths are non-dimensional mean wetted length (L_M/B or λ), keel wetted length (L_K/B), and chine wetted length (L_C/B), and their values are displayed in Fig. 27. This figure indicates that mean wetted length and chine wetted length decrease as the speed increases. However, keel wetted length increases by an increase in speed coefficient. As for the slope of the resultant plots, rate of increase in keel wetted length is small while the slope of the curve of mean wetted length as a function of C_V, decreases at a faster rate. Meanwhile, it is shown that the rate of decrease in the resultant plot for the chine wetted length is much greater than the rate of decrease in the resultant plot of the mean wetted length.

Angles between the stagnation line and the center line are computed and plotted in Fig. 28.

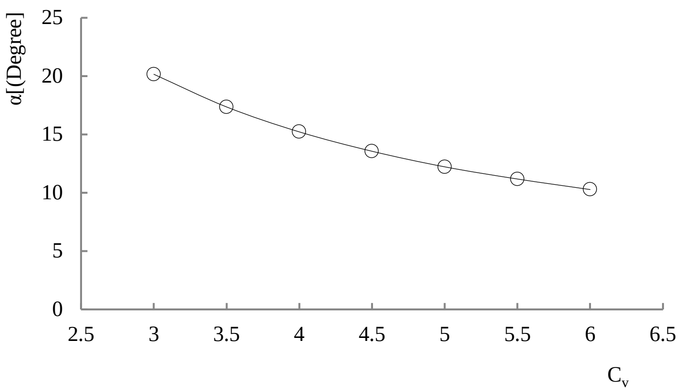

Fig. 28: Angle between the stagnation line and the center line for the planing vessel in Table 3

These angles range from 20.14 degrees to 10.27 degrees and they decrease by an increase in C_V. The slope of the resultant plot of α as a function of the speed coefficient decreases by an increase in the speed coefficient.

8.5 Overall assessment of the proposed method

The proposed mathematical model has some specific characteristics which are explained in this section. Accordingly, some of these points are summarized in Table 4. As shown in this table, the proposed mathematical model is easily adjustable for modeling the planing hulls with variable deadrise and beam from the stern to the bow of the craft. For predicting the trim angle of these crafts, the beam and deadrise of each section are required. Once these values are assigned, it becomes easily possible to use Equations (1) through (9) to compute pressure. However, these factors were not intended as targets in the current study.

Spray resistance and spray apex are two important outputs in the study of a planing hull that are added to the proposed model. Furthermore, two of the useful features observed in the proposed method are the dynamic and hydrostatic pressures over the bottom of the planing hulls which are computed in the outlined procedure. By using these values, forces acting on the body can be determined and utilized for the structural design of these types of vessels. Comparison of the results of the suggested model and Savitsky's method for a prismatic planing boat indicates that the model does not possess the ability to predict the trim angle in good agreement with Savitsky's method at C_V<3.

The proposed model and Savitsky's computational procedure both need the same inputs for predicting the running trim angles. Both models also use the same computational procedure for evaluating this angle and the mean wetted length. However, the manner in which lift forces are calculated is found to be different. The proposed method utilizes empirical equations for the pressure, while Savitsky's method uses empirical equations for the lift.In terms of some limits of the mean wetted length, the proposed method can be used for planing hulls without any limit for λ. However, Savitsky's computational method (1964) cannot be used for λ>4.

Table 4: Comparison of the proposed model and Savitsky's method (1964)

	Present Model	Savitsky's method (1964)
Inputs	Δ, LCG, B, VCG, Propeller Location, V	Δ, LCG, B, VCG, Propeller Location, V
Variable Geometry (β, B)?	Can be adjusted	Can't handle
Limit for speed angles?	$C_V<3$	No limit
Computation of Spray apex?	Yes	Can be found by using the related equations
Computation of Spray apex?	Yes	Can be found by using the related equations
3D Pressure Distribution	Yes	No
Mathematical Formulation for computing the lift force	Empirical equations for pressure distribution	Empirical equations for lift
Method for predicting the running trim angle	Computational procedure	Computational procedure
Can be used for cases in which all forces pass through CG?	Yes	Yes
Limit for mean wetted length	No limit	$\lambda>4$
Can be used for cases in which no all forces pass through CG?	Yes	Yes

Finally, when all is said and done, the new proposed method may be claimed as an improvement of Savitsky's method in order to facilitate the determination of total pressure distribution over the bottom of a planing hull which leads to prediction of its performance, spray apex, wetted surface area, and effective power among other hydrodynamic characteristics.

9. Conclusion

A mathematical model is proposed which uses the total pressure distribution for determining the hydrodynamic characteristics of a planing hull. A computer program is developed and accuracy of the proposed method is investigated by comparing the computed results against those predicted by Savitsky's method. Favorable accuracy is displayed for the calculated lift coefficient. There is also good agreement between the predicted trim angles after C_V=3 and those reported by Savitky's method. The obtained resistance through the developed computational procedure is compared against the Savitsky's method and good accuracy is observed at C_V>3. To complete the validation, the results of the proposed method is compared against available experimental data and favorable accuracy of the method is observed at C_V>3 for performance prediction. The proposed mathematical model is utilized in four major parts and some important results are concluded which are as follows:

- Planing hulls with larger deadrise angle have larger trim angles. It is shown that values of the pressure before and after the maximum pressure area are the main reason for this event. It is also showed that hydrodynamic load contribution in generation of the lift decreases as LCG moves to the bow.
- A study is performed for two boats with different deadrise angles and variation of height of the spray apex is studied. Resultant plots of the spray apex height shows the complex nature of the spray apex height. For the first craft with β=10, the height of this point increases initially and decreases after reaching C_V=5. However, for the planing hull in which deadrise angle is equal to 15, there are two declinations at C_V=3.5 and C_V=5.5. Longitudinal position of the spray apex is also investigated for these hulls. The longitudinal position of the spray apex increases by an increase in speed coefficient for both hulls. Finally, lateral position of the spray apex is examined. Variations of the lateral position for the spray apex are the same as the variation of the height of spray apex for the hull with β=10. However, for the hull with β=15, it is not the same as spray apex height. For this vessel, there is a decreasing trend before C_V=3.5 and an increasing trend after this speed coefficient.

- The spray resistance is added to the model and total resistance is determined. It is shown that the spray resistance has its lowest value at C_V=3 while frictional resistance is the greatest. At C_V=5.5, the resistance of the whisker spray reaches larger values than the induced resistance it produces. It is shown that planing hull with smaller deadrise angle has smaller resistance at the same condition.
- Non-dimensional wetted lengths including L_M/B, L_K/B and L_C/B are studied and it is concluded that mean wetted length and chine wetted length decreases as C_V increases. The keel wetted length has a direct relation with the speed coefficient. Changes in the angle between the stagnation line and the keel (α) are further investigated and it is concluded that angle α has an inverse relation with C_V.

Finally, it should be emphasized that more broad validation is needed with a certain variety of planing hulls before this method can be confidently applied for practical design of planing boats.

Mathematical modeling of the stepped planing hulls, effects of geometry of the step and number of the steps on the performance of planing hulls are the most important studies which can be conducted later. Performance prediction of planing hulls with variable deadrise angle and beam may also be considered as future studies, while the value of bottom pressure found in this study can be used for mathematical modeling of the planing hull motion in the head sea.

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