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ESTIMATION OF INVENTORY FOR DETERIORATING ITEMS WITH NON-LINEAR DEMAND CONSIDERING DISCOUNT AND WITHOUT DISCOUNT POLICY

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Abstract:

This paper develops an inventory model for deteriorating items consisting of the ordering cost, unit cost, opportunity cost, deterioration cost and shortage cost. In this inventory model instead of linear demand function nonlinear exponential function of time for deteriorating items with deterioration rate has been considered. The effects of inflation and cash flow are also taken into account under a trade-credit policy of discount and without discount with time. In order to validate the model, numerical examples have been solved by bisection method deploying MATLAB. Further, in order to estimate the cash flow the sensitivity of different parameters is considered.

Keywords: Inventory, deterioration, non-linear demand, inflation, trade-credit policy

NOMENCLATURE

D(t)	:	$b\rho^t,\ (b>0,\ 0>\rho>1)$is the demand rate at any time t
T	:	Cycle length
Q_i	:	Order quantity in the $(i+1)th$ cycle
I	:	Inventory Carrying charge
C(t)	:	Unit cost of the item at any time t
A(t)	:	Ordering cost of the item at any time t
h	:	Inflation rate per unit time
r	:	Opportunity cost per unit time
θ	:	Rate of deterioration $0 \le \theta < 1$

1. Introduction

Inventory or stock refers to the goods and materials that a business holds for the ultimate purpose of resale (or repair). Inventory management is a science primarily about specifying the shape and percentage of stock goods. It is required at different locations within a facility or within many locations of a supply network to precede the regular and planned course of production and stock of materials. The scope of inventory management concerns the fine lines between replenishment lead time, carrying costs of inventory, asset management, inventory forecasting, inventory valuation, inventory visibility, future inventory price forecasting, physical inventory, available physical space for inventory, quality management, replenishment, returns and defective goods, and demand forecasting. Balancing these competing requirements leads to optimal inventory levels, which is an ongoing process as the business needs shift and react to the wider environment. Inventory management involves a retailer seeking to acquire and maintain a proper merchandise assortment while ordering, shipping, handling, and related costs are kept in check. It also involves systems and processes that identify inventory requirements, set targets, provide replenishment techniques, report actual and projected inventory status and handle all functions related to the tracking and management of material. This would include the monitoring of material moved into and out of stockroom locations and the reconciling of the inventory balances.

Supply Chain Management (SCM) is an essential element for operational efficiency. SCM can be applied to customer satisfaction and company success, as well as within societal settings, including medical missions; disaster relief operations and other kinds of emergencies; cultural evolution; and it can help improve quality of

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life. Because of the vital role SCM plays within organizations, employers seek employees with an abundance of SCM skills and knowledge.

Deterioration of physical goods is one of the important factors in any inventory and production system. The deteriorating items with shortages have received much attention of several researches in the recent year because most of the physical goods undergo decay or deterioration over time. Commodities such as fruits, vegetables and food stuffs are kept in store by direct decay trick from reduction. The demand function relates price and quantity. It tells how many units of a goods will be purchased at different prices. In general, at higher prices, less will be purchased. Thus, the graphical representation of the demand function has a negative slope. The market demand function is calculated by adding up all of the individual consumers' demand functions. Economic order quantity (EOQ) model is the method that provides the company with an order quantity. By using this model, the companies can minimize the costs associated with the ordering and inventory holding. In 1913, Ford W. Harris developed this formula whereas R. H. Wilson is given credit for the application and in-depth analysis on this model.

The model is used to calculate the optimal quantity which can be purchased or produced to minimize the cost of both the carrying inventory and the processing of purchase orders or production set-ups. Basu et al. (2007) proposed a general inventory model with due consideration to the factors of time dependent partial backlogging and time dependent deterioration. Sharma, et al. (2013) established an inventory model for deteriorating items, the rate of deterioration following the Weibull distribution with two parameters. The demand rate is time dependent. Ghare and Schrader (1963) industrialized a model for an exponentially decaying inventory. Inventory models with a time dependent rate of deterioration were considered by Covert and Philip (1973), Mishra (1975) and Deb and Chaudhuri (1986). In these models shortages are completely backlogged.

In developing mathematical inventory model, it is assumed that payments will be made to the supplier for the goods immediately after receiving the consignment. In day-to-day dealings, it is observed that the suppliers offer different trade credit policies to the buyers. One such trade credit policy is "α/T_1 net T" which means that a $\alpha\%$ discount on sale price is granted if payments are made within T_1 days and the full sale price is due within $T(> T_1)$ days from the date of invoice if the discount is not taken. Ben-Horim and Levy (1982), Chung (1989), Aggarwal and Jaggi (1994) discussed trade-credit policy of type "α/T_1 net T" in their models. Aggarwal et al. (1997) investigated an inventory model taking into consideration inflation, time- value of money and a trade credit policy "α/T_1 net T" with a constant demand rate. Lal and Staelin (1994) examined an optimal discounting pricing policy. In this paper, an attempt has been made to extend the model of Aggarwal et al. (1997) to an inventory of deteriorating items at a constant rate $\theta(0 < \theta < 1)$. Also the demand rate is taken to be non-linear instead of a constant demand rate. Sensitivity of the optimal solution is examined to see how far the output of the model is affected by changes in the values of its input parameters.

So far I know in literature inventory with non-linear demand for deteriorating items has not considered. That is why in our model we have considered deteriorating with shortage inventory as a non-linear demand function. The objectives of this study are to (i) develop an inventory model for deteriorating items with deterioration rate when the demand is considered as a nonlinear function of time and, (ii) determine an optimal solution of the inventory model with cash flow. Further, it is considered that a M_1percent discount of sale price is granted if payments are made within M_1 days and the full sale price is due within M days from the date of invoice if the discount is not taken.

1.1 Outline of the paper

In this paper we have introduced several elementary properties of the stationary inventory process and literature review in Section 1. We have also constructed the mathematical formulation and solved it in Section 2. Section 3 describes the numerical results and graphically representation of the formulated model. Finally, we have summarized the results and proposed directions for future research in Section 4.

2. Mathematical Formulation

The exponential function has the form: $D(t) = b\rho^t$,where $\rho \neq 1$ is a constant called the base of the exponential function, $b > 0$ and $t$is the independent variable. Thus exponential function has a constant base raised to a variable exponent. In economics exponential functions are important for looking at growth or decay.

Examples are the value of an investment that increases by a constant percentage each period, sales of a company that increase at a constant percentage each period, models of economic growth or models of the spread of an epidemic.

Mathematically$D'(t) = \frac{dD(t)}{dt} > 0$ and $D''(t) = \frac{d^2D(t)}{dt^2} \leq 0$

Following assumptions are made:

1. Lead time is zero.
2. Shortage is not allowed.
3. Replenishment is instantaneous.
4. The time horizon of the inventory system is infinite.

Let $A(t)$ and $C(t)$ be the ordering cost and unit cost of the item at any time t. Then $A(t) = A(0)e^{ht}$ and $C(t) = C(0)e^{ht}$, assuming continuous compounding of inflation. Let $I_i(t)$ be the instantaneous inventory level at any time t_1 in the t_{i+1}th cycle. The differential equation governing the instantaneous states of $I_i(t)$ in the interval $[iT, (i+1)T]$ is

$$\frac{dI_i(t)}{dt} + \theta I_i(t) = b\rho^t,\ iT \leq t \leq (i+1)T \tag{2.1}$$

$$\text{Where } I_i(iT) = Q_i \text{ and } I_i((i+1)T) = 0, \text{ i=0,1,2….} \tag{2.2}$$

Now from equation (2.1)

$$\Rightarrow\ I_i(t) = \frac{log\rho}{\theta + log\rho}\frac{b\rho^t}{\log\rho} + e^{-\theta t}C_i \tag{2.3}$$

$$\text{When } I_i(iT) = Q_i \Longrightarrow C_i = (Q_i - \frac{b\rho^{iT}}{\theta + log\rho})e^{i\theta T}$$

$$\text{When } I_i((i+1)T) = 0 \therefore Q_i = \frac{b}{\theta + log\rho}\left[\rho^{iT} - \rho^{(i+1)T}e^{\theta T}\right] \tag{2.4}$$

$$\therefore I_i(t) = \frac{b}{\theta + \log\rho}\left[\rho^t - \rho^{iT}e^{(iT-t)\theta}\right] + Q_i e^{(iT-t)\theta} \tag{2.5}$$

$$iT \leq t \leq (i+1)T, i = 0,1,2,3 \ldots$$

Let $t_0, t_1, t_2, \ldots\ldots$….be the replenishment points and $t_{i+1} - t_i = T$ so that $t_i = iT, i = 0,1,2,3 \ldots\ldots$….

2.1 With discount

Here purchases made at time t_i are paid after M_i days and purchase price in real terms for $Q_i$units at time t_i in the t_{i+1}th cycle is $Q_i(1-\alpha)C(t_i)e^{-hM_1}$

The present worth of cash-flows for the t_{i+1}th cycle is

$$PV_i^{(d)}(T) = [A(t_i) + Q_i C(t_i)(1-\alpha)e^{-hM_1} + IC(t_i)(1-\alpha)e^{-hM_1}\int_{iT}^{(i+1)T} I_i(t)e^{-rt}dt]e^{-rt_i}$$

$$= A_1 + A_2 + A_3, say$$

Where $A_1 = A(t_i)e^{-rt_i} = A(0)e^{-iRT}$; (assuming R=r-h and using $t_i = iT$),

$$A_2 = Q_i C(t_i)(1-\alpha)e^{-hM_1}e^{-rt_i}$$

$$\Longrightarrow A_2 = \frac{bC(0)(1-\alpha)e^{-hM_1}}{\theta + log\rho}\rho^{iT}e^{-iRT} - \frac{bC(0)(1-\alpha)e^{-hM_1}}{\theta + log\rho}\rho^{(i+1)T}e^{-iRT}e^{\theta T}$$

$$A_3 = [IC(t_i)(1-\alpha)e^{-hM_1}\int_{iT}^{(i+1)T} I_i(t)e^{-rt}dt]e^{-rt_i}$$

Now, $$\int_{iT}^{(i+1)T} I_i(t)e^{-rt}dt = \frac{b(e^{-(r+\theta)T}-1)}{(\theta+\log\rho)(r+\theta)}\rho^{iT}e^{-irt} + \frac{b(\rho^T e^{-rT}-1)}{(\theta+\log\rho)(log\rho - r)}\rho^{iT}e^{-irt}$$ (2.6)

$$-\frac{b\left(e^{-(r+\theta)T}-1\right)}{(\theta+\log\rho)(r+\theta)}\rho^{iT}e^{-irt} + \frac{b\left(e^{-rT}-e^{\theta T}\right)}{(\theta+\log\rho)(r+\theta)}\rho^{(i+1)T}e^{-irt}$$

Therefore,

$$PV_i^{(d)}(T) = A(0)e^{-iRT} + \frac{bC(0)(1-\alpha)e^{-hM_1}}{\theta + log\rho}\rho^{iT}e^{-iRT} - \frac{bC(0)(1-\alpha)e^{-hM_1}}{\theta + log\rho}\rho^{(i+1)T}e^{-iRT}e^{\theta T} + IC(0)(1-\alpha)e^{-hM_1}\left[\frac{b(\rho^T e^{-rT}-1)}{(\theta+\log\rho)(log\rho - r)}\rho^{iT} + \frac{b\left(e^{-rT}-e^{\theta T}\right)}{(\theta+\log\rho)(r+\theta)}\rho^{(i+1)T}\right]e^{-iPT} \quad (2.7)$$

taking P=2r-h

The present worth of all future cash flows is $PV_\infty^{(d)}(T) = \sum_{i=0}^{\infty} PV_i^{(d)}(T)$

$$= A(0)\frac{1}{1-e^{-RT}} + \frac{bC(0)(1-\alpha)e^{-hM_1}}{\theta + log\rho}\frac{1}{(1-\rho^T e^{-RT})} - \frac{bC(0)(1-\alpha)e^{-hM_1}}{\theta + log\rho}\frac{\rho}{(1-\rho^T e^{-RT})}e^{\theta T} + \frac{bIC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)(log\rho - r)}\frac{(\rho^T e^{-rT}-1)}{(1-\rho^T e^{-PT})} + \frac{\rho bIC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)(r+\theta)}\frac{\left(e^{-rT}-e^{\theta T}\right)}{(1-\rho^T e^{-PT})} \quad (2.8)$$

The solution of the equation $\frac{dPV_\infty^{(d)}(T)}{dT} = 0$ gives the optimum value of T provided it satisfies the condition

$$\frac{d^2 PV_\infty^{(d)}(T)}{dT^2} > 0 \quad (2.9)$$

Now $\frac{dPV_\infty^{(d)}(T)}{dT} = 0$ yields the equation

$$-A(0)\frac{Re^{-RT}}{(1-e^{-RT})^2} + \left[\frac{bC(0)(1-\alpha)(log\rho - R)e^{-hM_1}}{(\theta+log\rho)} - \frac{\rho bC(0)(1-\alpha)(log\rho - R)e^{-hM_1}e^{\theta T}}{(\theta+log\rho)}\right]\frac{\rho^T e^{-RT}}{(1-\rho^T e^{-RT})^2} - \frac{\rho\theta bC(0)(1-\alpha)e^{-hM_1}}{(\theta+log\rho)}\frac{e^{\theta T}}{(1-\rho^T e^{-RT})} + \frac{bIC(0)(1-\alpha)e^{-hM_1}}{(\theta+log\rho)(log\rho - r)}$$

$$\frac{(1-\rho^T e^{-PT})(log\rho - r)\rho^T e^{-rT} + (\rho^T e^{-rT}-1)(log\rho - P)\rho^T e^{-PT}}{(1-\rho^T e^{-PT})^2} - \frac{b\rho IC(0)(1-\alpha)e^{-hM_1}}{(\theta+log\rho)(r+\theta)}$$

$$\frac{(1-\rho^T e^{-PT})\left(re^{-rT}+\theta e^{\theta T}\right) + \left(e^{-rT}-e^{\theta T}\right)(P-log\rho)\rho^T e^{-PT}}{(1-\rho^T e^{-PT})^2} = 0 \qquad (2.10)$$

This equation being highly nonlinear cannot be solved analytically. It can be solved numerically for given parameter values. Its solution gives the optimum Tquantities $Q_i(i = 0,1,2 \ldots \ldots.)$from and $PV_\infty^{(d)}(T)^*$, the optimum present value of all future cash flows from (2.8).

2.2 Without discount

Here purchases made at time $t_i$in the t_{i+1}th cycle is $Q_i C(t_i)e^{-hM}$

The present worth of cash flows for the t_{i+1}th cycle is

$$PV_i^{(wd)}(T) = \left[A(0) + \frac{bC(0)e^{-hM}}{(\theta+log\rho)}\rho^{iT}e^{-iRT} - \frac{bC(0)e^{-hM}}{(\theta+log\rho)}e^{\theta T}\rho^{(i+1)T}e^{-iRT} + \{\frac{bIC(0)e^{-hM}}{(\theta+log\rho)(log\rho-r)}(\rho^T e^{-rT} - 1)\}\right]\rho^{iT}e^{-iPT} + \left[\frac{bIC(0)e^{-hM}}{(\theta+log\rho)(r+\theta)}\left(e^{-rT}-e^{\theta T}\right)\right]\rho^{(i+1)T}e^{-iPT}, \qquad (2.11)$$

(taking $P = 2r - h$) i=0,1,2,…….

The present worth of all future cash flows is

$$PV_\infty^{(wd)}(T) = \sum_{i=0}^{\infty} PV_i^{(wd)}(T) = A(0)\frac{1}{1-e^{-RT}} + \frac{bC(0)(1-\alpha)e^{-hM}}{\theta+log\rho}\frac{1}{(1-\rho^T e^{-RT})} - \frac{bC(0)e^{-hM}}{\theta+log\rho}\frac{\rho}{(1-\rho^T e^{-RT})}e^{\theta T} + \frac{bIC(0)e^{-hM}}{(\theta+\log\rho)(log\rho - r)}\frac{(\rho^T e^{-rT}-1)}{(1-\rho^T e^{-PT})} + \frac{\rho bIC(0)e^{-hM}}{(\theta+\log\rho)(r+\theta)}\frac{(e^{-rT}-e^{\theta T})}{(1-\rho^T e^{-PT})} \qquad (2.12)$$

The solution of the equation $\frac{dPV_\infty^{(wd)}(T)}{dT} = 0$ gives the optimum value of T provided it satisfies the condition

$$\frac{d^2 PV_\infty^{(wd)}(T)}{dT^2} > 0 \qquad (2.13)$$

Now $\frac{dPV_\infty^{(wd)}(T)}{dT} = 0$ yields the equation

$$-A(0)\frac{Re^{-RT}}{(1-e^{-RT})^2}+\left[\frac{bC(0)(log\rho-R)e^{-hM}}{(\theta+log\rho)}-\frac{\rho bC(0)(log\rho-R)e^{-hM}e^{\theta T}}{(\theta+log\rho)}\right]\frac{\rho^T e^{-RT}}{(1-\rho^T e^{-RT})^2}-\frac{\rho\theta bC(0)e^{-hM}}{(\theta+log\rho)}\frac{e^{\theta T}}{(1-\rho^T e^{-RT})}+\frac{bIC(0)e^{-hM}}{(\theta+log\rho)(log\rho-r)}$$

$$\frac{(1-\rho^T e^{-PT})(log\rho-r)\rho^T e^{-rT}+(\rho^T e^{-rT}-1)(log\rho-P)\rho^T e^{-PT}}{(1-\rho^T e^{-PT})^2}-\frac{b\rho IC(0)e^{-hM}}{(\theta+log\rho)(r+\theta)}$$

$$\frac{(1-\rho^T e^{-PT})\left(re^{-rT}+\theta e^{\theta T}\right)+\left(e^{-rT}-e^{\theta T}\right)(P-log\rho)\rho^T e^{-PT}}{(1-\rho^T e^{-PT})^2}=0 \qquad (2.14)$$

This equation also needs to be solved numerically.

3. Result Discussion and Computation Analysis

In this section, a numerical example is given to illustrate this maintenance model. We consider parameter numerical example to valid our model for discount.

Let $b = 5, C(0) = 10, r = 0.04, I = 0.02, A(0) = 2000, M = 30, h = 0.02, \alpha = 0.1, \theta = 0.01, \rho = 0.5$ in appropriate units. Solving the highly non-linear equation (2.10) for with discount by Bisection method, we get the optimum value of T, $T = 170.978$ Substituting T in equation (2.8), we get the optimum value of $PV_\infty^{(d)}(T)$ as $PV_\infty^{(d)}(T)^* = 2170.407$.

We consider parameter numerical example to valid our model for without discount. Let $b = 5, C(0) = 10, r = 0.04, I = 0.02, A(0) = 2000, M = 35, h = 0.02, \theta = 0.01, \rho = 0.5$ appropriate units. Solving the equation (2.13) for without discount by the same method as in with discount case, we get the optimum value of T, $T = 170.807$. Substituting T in equation (2.8), we get the optimum value of $PV_\infty^{(wd)}(T)$ as $PV_\infty^{(wd)}(T)^* = 2170.959$. Based on the numerical examples considered above, a sensitivity analysis of T, $PV_\infty^{(d)}(T)^*$, $PV_\infty^{(wd)}(T)^*$is performed by changing (increasing or decreasing) the parameters by 10% and 50% and taking one parameter at a time, keeping the remaining parameters at their original values.

The percentage error is calculated by the formula: (measured value- actual value) *100/actual value.

4. Sensitivity Analysis

Sensitivity analysis of the model is also performed with respect to unit price of purchase, replenishment cost, inventory carrying cost, deteriorating cost and inflation rate. The model also judges the sensitive analysis of the parameters on ordering quantity and total system cost. Those results provided further insight in the nature of the problem studied and summarizes in Table 2.1 and 2.2.

Table 2.1: Sensitivity of different parameters with discount

Changing Parameters	(%) Change	T^*	$PV_{\infty}^{(d)}(T)^*$	(%) change in T^*	(%) change in $PV_{\infty}^{(d)}(T)^*$
A(0)	-50	149.108	1128.639	-12.791	-47.999
	-40	154.808	1338.683	-9.457	-38.321
	-30	159.659	1547.681	-6.620	-28.691
	-20	163.882	1755.867	-4.150	-19.099
	-10	167.622	1963.404	-1.963	-9.538
	+10	174.023	2376.962	+1.781	+9.516
	+20	176.809	2583.135	+3.411	+19.016
	+30	179.378	2788.977	+4.913	+28.500
	+40	181.761	2994.528	+6.307	+37.971
	+50	183.983	3199.824	+7.606	+47.429
b	-50	193.281	2111.643	+13.044	-2.707
	-40	187.381	2124.891	9.594	-2.097
	-30	182.411	2137.246	6.686	-1.527
	-20	178.119	2148.875	4.177	-0.992
	-10	174.345	2159.899	1.969	-0.484
	+10	167.941	2180.467	-1.776	0.463
	+20	165.176	2190.134	-3.393	0.909
	+30	162.639	2199.450	-4.876	1.338
	+40	160.296	2208.453	-6.247	1.752
	+50	158.121	2217.173	-7.519	+2.155
C(0)	-50	193.281	2111.643	+13.044	-2.707
	-40	187.381	2124.891	9.594	-2.097
	-30	182.411	2137.246	6.686	-1.527
	-20	178.119	2148.875	4.177	-0.992
	-10	174.345	2159.899	1.969	-0.484
	+10	167.941	2180.467	-1.776	0.463
	+20	165.176	2190.134	-3.393	0.909
	+30	162.639	2199.450	-4.876	1.338
	+40	160.296	2208.453	-6.247	1.752
	+50	158.121	2217.173	-7.519	+2.155
r	-50	291.113	3397.790	70.263	56.550
	-40	242.767	2610.291	41.987	20.267
	-30	211.944	2355.637	23.960	8.534
	-20	189.029	2236.697	10.557	3.054
	-10	156.294	2129.201	-8.587	-1.898
	+10	144.082	2101.584	-15.730	-3.170
	+20	133.749	2082.018	-21.773	-4.072
	+30	124.884	2067.554	-26.958	-4.738
	+40	117.189	2056.495	-31.459	-5.248
I	-50	175.909	2150.433	2.884	-0.920
	-40	174.858	2154.509	2.269	-0.732
	-30	173.842	2158.544	1.675	-0.546
	-20	172.857	2162.537	1.099	-0.362
	-10	171.903	2166.491	0.541	-0.180
	+10	170.079	2174.286	-0.525	0.178
	+20	169.205	2178.130	-1.036	0.355
	+30	168.355	2181.940	-1.533	0.531
	+40	167.528	2185.718	-2.017	0.705
	+50	166.723	2189.464	-2.489	0.878

Changing Parameters	(%) Change	T^*	$PV_{\infty}^{(d)}(T)^*$	(%) change in T^*	(%) change in $PV_{\infty}^{(d)}(T)^*$
M	-50	180.588	2203.977	-5.573	1.547
	-40	178.660	2196.805	-4.462	1.216
	-30	176.735	2189.869	-3.350	0.896
	-20	174.813	2183.162	-2.235	0.587
	-10	172.894	2176.677	-1.118	0.288
	+10	169.065	2164.346	1.120	-0.279
	+20	167.156	2158.488	2.243	-0.549
	+30	165.250	2152.827	3.367	-0.809
	+40	163.347	2147.357	4.492	-1.061
	+50	161.449	2142.073	5.620	-1.305
h	-50	130.186	2116.476	-23.858	-2.485
	-40	136.895	2123.462	-19.933	-2.162
	-30	144.226	2131.847	-15.646	-1.776
	-20	152.270	2142.042	-10.941	-1.306
	-10	161.142	2154.619	-5.752	-0.727
	+10	181.950	2190.633	6.417	0.931
	+20	194.277	2217.195	13.626	2.155
	+30	208.238	2253.154	21.792	3.812
	+40	224.212	2303.732	31.135	6.142
	+50	242.734	2378.502	41.968	9.588
α	-50	169.254	2176.048	-1.008	0.259
	-40	169.591	2174.929	-0.810	0.208
	-30	169.932	2173.807	-0.611	0.156
	-20	170.277	2172.679	-0.409	0.104
	-10	170.625	2171.545	-0.206	0.052
	+10	171.334	2169.262	0.208	-0.052
	+20	171.695	2168.112	0.419	-0.105
	+30	172.060	2166.957	0.633	-0.158
	+40	172.429	2165.795	0.848	-0.212
	+50	172.803	2164.629	1.068	-0.266
θ	-50	230.092	2065.298	34.574	-4.842
	-40	214.712	2084.253	25.578	-3.969
	-30	201.544	2104.347	17.877	-3.043
	-20	190.080	2125.471	11.172	-2.070
	-10	179.974	2147.524	5.262	-1.054
	+10	162.904	2194.032	-4.721	1.088
	+20	155.611	2218.319	-8.987	2.207
	+30	148.985	2243.194	-12.862	3.353
	+40	142.936	2268.592	-16.401	4.523
	+50	137.388	2294.453	-19.646	5.715
ρ	-50	216.109	2063.221	26.396	-4.939
	-40	205.507	2080.166	20.195	-4.157
	-30	195.993	2098.918	14.630	-3.293
	-20	187.219	2119.852	9.499	-2.329
	-10	178.942	2143.461	4.658	-1.241
	+10	163.171	2201.606	-4.565	3.131
	+20	155.374	2238.375	-9.125	1.437
	+30	147.433	2282.688	-13.770	5.173
	+40	139.160	2337.666	-18.609	7.706
	+50	130.301	2408.604	-23.791	10.975

Table 2.2: Sensitivity of different parameters without discount

Changing Parameters	(%) Change	T^*	$PV_{\infty}^{(wd)}(T)^*$	(%) change in T^*	(%) change in $PV_{\infty}^{(wd)}(T)^*$
A(0)	-50	148.941	1129.043	-12.802	-47.993
	-40	154.640	1339.123	-9.465	-38.316
	-30	159.489	1548.151	-6.625	-28.688
	-20	163.711	1756.367	-4.153	-19.097
	-10	167.451	1963.930	-1.964	-9.536
	+10	173.851	2377.536	1.782	9.515
	+20	176.637	2583.731	3.413	19.013
	+30	179.206	2789.593	4.917	28.495
	+40	181.588	2995.165	6.312	37.965
	+50	183.809	3200.479	7.612	47.422
b	-50	193.107	2112.013	13.056	-2.715
	-40	187.208	2125.303	9.602	-2.103
	-30	182.238	2137.696	6.692	-1.532
	-20	177.946	2149.361	4.180	-0.994
	-10	174.173	2160.418	1.970	-0.485
	+10	167.770	2181.049	-1.777	0.464
	+20	165.006	2190.744	-3.396	0.911
	+30	162.469	2200.089	-4.881	1.341
	+40	160.127	2209.119	-6.252	1.757
	+50	157.952	2217.864	-7.526	2.161
C(0)	-50	193.107	2112.013	13.056	-2.715
	-40	187.208	2125.303	9.602	-2.103
	-30	182.238	2137.696	6.692	-1.532
	-20	177.946	2149.361	4.180	-0.994
	-10	174.173	2160.418	1.970	-0.485
	+10	167.770	2181.049	-1.777	0.464
	+20	165.006	2190.744	-3.396	0.911
	+30	162.469	2200.089	-4.881	1.341
	+40	160.127	2209.119	-6.252	1.757
	+50	157.951	2217.864	-7.526	2.161
r	-50	290.809	3400.421	70.256	56.632
	-40	242.507	2611.769	41.977	20.304
	-30	211.721	2356.630	23.953	8.552
	-20	188.835	2237.420	10.554	3.061
	-10	156.142	2129.636	-8.585	-3.179
	+10	143.944	2101.936	-15.726	-1.903
	+20	133.624	2082.308	-21.768	-4.083
	+30	124.769	2067.795	-26.952	-4.751
	+40	117.084	2056.698	-31.453	-5.263
I	-50	175.737	2150.912	2.886	-0.923
	-40	174.686	2155.004	2.271	-0.734
	-30	173.670	2159.053	1.676	-0.548
	-20	172.686	2163.060	1.100	-0.363
	-10	171.732	2167.028	0.541	-0.181
	+10	169.908	2174.851	-0.526	0.179
	+20	169.034	2178.709	-1.037	0.357
	+30	168.185	2182.534	-1.534	0.533
	+40	167.358	2186.325	-2.019	0.707
	+50	166.553	2190.085	-2.491	0.881

Changing Parameters	(%) Change	T^*	$PV_\infty^{(wd)}(T)^*$	(%) change in T^*	(%) change in $PV_\infty^{(wd)}(T)^*$
M	-50	159.700	2210.808	-6.502	1.836
	-40	161.911	2202.205	-5.207	1.439
	-30	164.128	2193.929	-3.910	1.058
	-20	166.349	2185.970	-2.609	0.691
	-10	168.576	2178.316	-1.305	0.338
	+10	173.042	2163.886	1.308	-0.325
	+20	175.281	2157.089	2.619	-0.638
	+30	177.525	2150.558	3.933	-0.939
	+40	179.772	2144.284	5.248	-1.228
	+50	182.024	2138.258	6.567	-1.506
h	-50	128.841	2120.718	-24.568	-2.314
	-40	135.738	2127.104	-20.531	-2.020
	-30	143.276	2134.837	-16.118	-1.663
	-20	151.551	2144.313	-11.273	-1.227
	-10	160.680	2156.087	-5.928	-0.684
	+10	182.108	2190.114	6.616	0.882
	+20	194.809	2215.392	14.052	2.046
	+30	209.201	2249.761	22.478	3.629
	+40	225.672	2298.300	32.121	5.865
	+50	244.771	2370.336	43.302	9.183
θ	-50	229.881	2065.539	34.585	-4.856
	-40	214.510	2084.557	25.586	-3.979
	-30	201.351	2104.713	17.882	-3.051
	-20	189.895	2125.900	11.175	-2.075
	-10	179.796	2148.014	5.263	-1.056
	+10	162.740	2194.644	-4.722	1.091
	+20	155.453	2218.991	-8.988	2.212
	+30	148.833	2243.925	-12.864	3.361
	+40	142.788	2269.380	-16.403	4.533
	+50	137.245	2295.299	-19.648	5.727
ρ	-50	215.934	2063.416	26.420	-4.954
	-40	205.332	2080.417	20.213	-4.170
	-30	195.819	2099.232	14.643	-3.303
	-20	187.046	2120.236	9.507	-2.336
	-10	178.770	2143.923	4.662	-1.2453
	+10	163.001	2202.261	-4.569	1.441
	+20	155.206	2239.152	-9.133	3.141
	+30	147.266	2283.612	-13.781	5.189
	+40	138.995	2338.771	-18.624	7.729
	+50	130.139	2409.942	-23.809	11.008

Figures 2.1 (a), 2.2 (a), 2.3(a), 2.4(a), 2.5(a) and 2.6(a) show the estimated significant values of the parameters ordering cost, unit cost, opportunity cost, inventory carrying charge, inflation rate and deterioration rate with discount. On the other hand, Figures 2.1 (b), 2.2 (b), 2.3(b), 2.4(b), 2.5(b), and 2.6(b) show the estimated significant values of the parameters ordering cost, unit cost, opportunity cost, inventory carrying charge inflation rate and deterioration rate without discount.

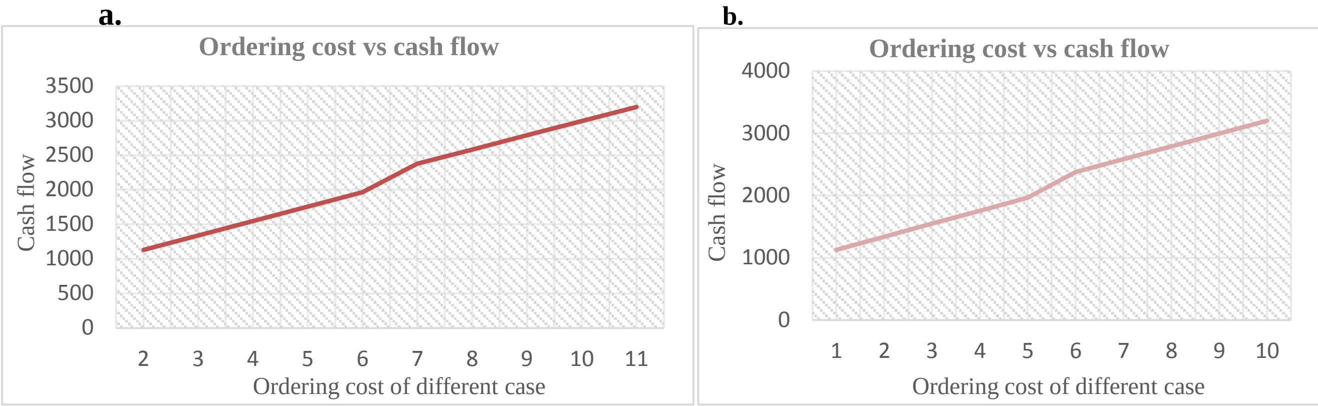

Figure 2.1: Effect of ordering cost on cash flow; (a) Discount Case, (b) Without discount Case

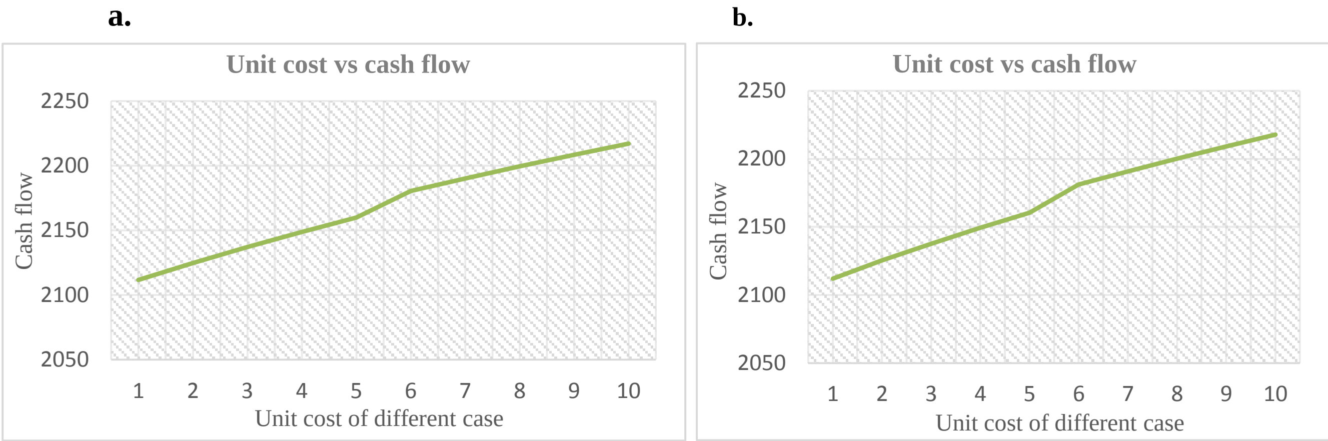

Figure 2.2: Effect of unit cost on cash flow: (a) Discount Case; (b) Without discount case

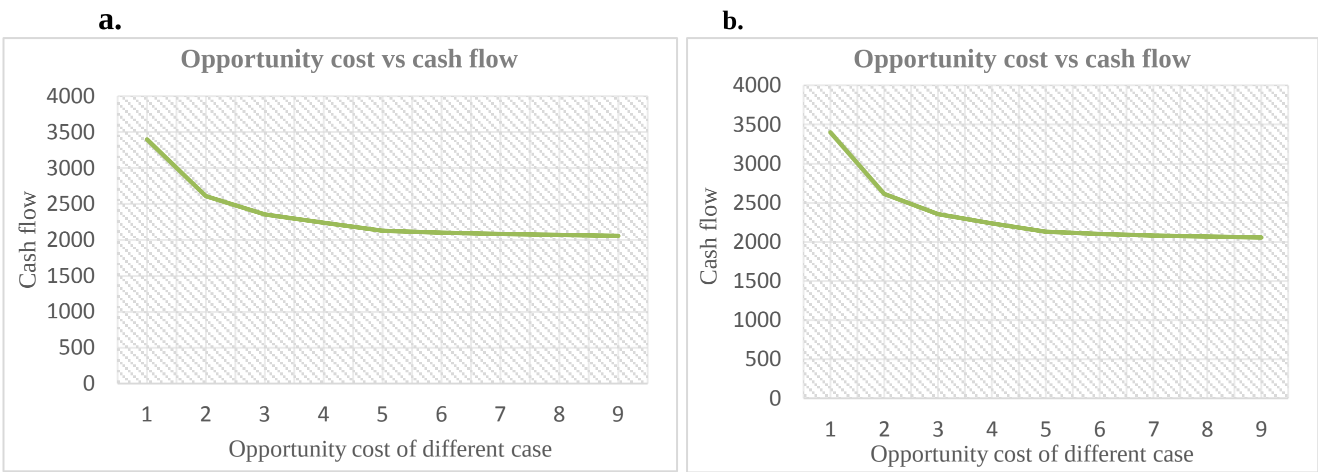

Figure 2.3: Effect of Opportunity cost on cash flow: (a) Discount Case; (b) Without discount Case

a. **b.**

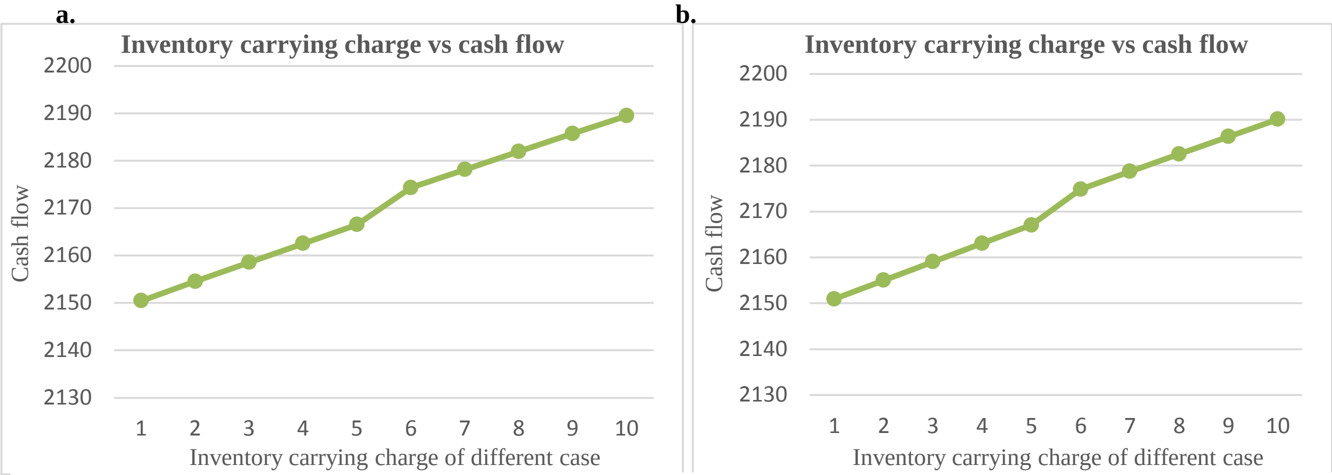

Figure2.4: Effect of Inventory carrying charge on cash flow: (a) Discount Case; (b) Without discount case

a. **b.**

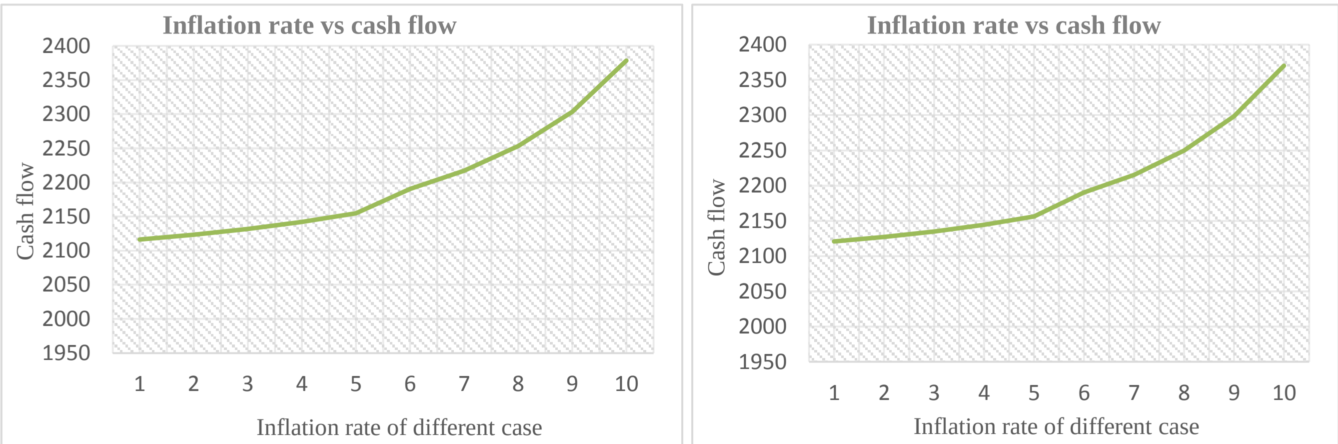

Figure2.5: Effect of Inflation rate on cash flow: (a) Discount case; (b) Without discount case

a. **b.**

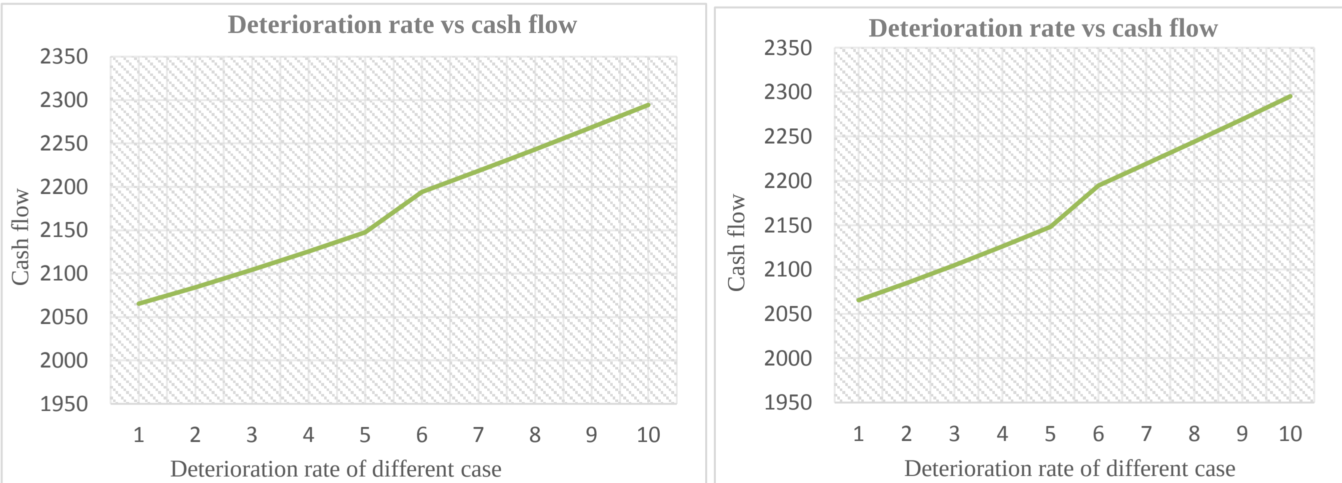

Figure 2.6: Effect of deterioration rate on cash flow: (a) Discount Case; (b) Without discount case

It is observed that if the value of parameter ordering cost, unit cost, inflation rate, inventory carrying charge and deterioration rate increase then the cash flows of ordering cost, unit cost, inflation rate, inventory carrying charge and deterioration rate increase in both case discount and without discount (see Figures 2.1, 2.2, 2.3, 2.5 and 2.6).

On the other hand, the value of opportunity cost increase then the cash flows of opportunity cost decrease in both case discount and without discount (see Figure 2.3).

Further, discount and without discount case economic order quantity and incurred cost decrease as ordering cost increases (see Tables 2.1 and 2.2), but this increment is nonlinear.

5. Conclusion

The model has developed here deals with the optimum replenishment policy of a deteriorating item in the presence of inflation and a trade credit policy. It is observed that there is no significant change in the optimum value of the present worth of all future cash flows in the discount case and without discount case. It is observed that if the ordering cost and unit cost decrease, then the cash flow also decrease both cases. On the other hand, in both cases cash flow increases if opportunity cost decreases. Hence the discount case is considered to be better economically.

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Appendix A

Assuming $r > h$, we have

$$\sum_{i=0}^{\infty} e^{-iRT} = \frac{1}{1-e^{-RT}}, \sum_{i=0}^{\infty} e^{-iPT} = \frac{1}{1-e^{-PT}},$$

$$\sum_{i=0}^{\infty} \rho^{iT} e^{-iRT} = \frac{1}{1-\rho^{T} e^{-RT}}, \sum_{i=0}^{\infty} \rho^{iT} e^{-iPT} = \frac{1}{1-\rho^{T} e^{-PT}},$$

$$\sum_{i=0}^{\infty} \rho^{(i+1)T} e^{-iRT} = \frac{\rho}{1-\rho^{T} e^{-RT}}, \sum_{i=0}^{\infty} \rho^{(i+1)T} e^{-iPT} = \frac{\rho}{1-\rho^{T} e^{-PT}},$$