Journal of Naval Architecture and Marine Engineering
June, 2016
http://dx.doi.org/10.3329/jname.v13i1.24387 http://www.banglajol.info

APPROXIMATE ANALYTICAL SOLUTIONS OF MHD VISCOUS FLOW DUE TO A SHRINKING SHEET

Vishwanath B. Awati

Department of Mathematics, Rani Channamma University. Belagavi-591 156, India, E-mail: await_vb@yahoo.com

Abstract:

The paper presents the semi-numerical solution for the magnetohydrodynamic (MHD) viscous flow due to a shrinking sheet caused by boundary layer of an incompressible viscous flow. The governing three partial differential equations of momentum equations are reduced into ordinary differential equation (ODE) by using a classical similarity transformation along with appropriate boundary conditions. Both nonlinearity and infinite interval demand novel mathematical tools for their analysis. We use fast converging Dirichlet series and Method of stretching of variables for the solution of these nonlinear differential equations. These methods have the advantages over pure numerical methods for obtaining the derived quantities accurately for various values of the parameters involved at a stretch and also they are valid in much larger parameter domain as compared with HAM, HPM, ADM and the classical numerical schemes.

Keywords: Magneto-hydrodynamics (MHD), boundary layer flow, shrinking sheet, Dirichlet series, Powell's method, method of stretching variables.

NOMENCLATURE

a	shrinking constant
f	similarity function
B_0	strength of the magnetic field [wm^{-2}]
M	Hartmann number
m	shrinking parameter
u	velocity component along the x-axis [$m\ s^{-1}$]
v	velocity component along the y-axis [$m\ s^{-1}$]
w	velocity component along the z-axis [$m\ s^{-1}$]
x	coordinate along the sheet [m]
y	coordinate across the sheet [m]
z	co-ordinate normal to the sheet [m]
f_w	suction parameter
W	suction velocity
Greek symbols	
α	amplification factor
υ	kinematic viscosity [$m^2 s^{-1}$]
σ	electrical conductivity [$mho\ \mathrm{m}^{-1}$]
ρ	density [$kg\ m^{-3}$]
μ	dynamic viscosity
η	similarity variable

1. Introduction

The boundary layer of viscous flow induced by a moving boundary has important applications in many engineering fields (Crane, 1970, Fisher, 1976, Altan et al., 1979). Such flows occur in the extrusion of a polymer sheet from a die or in the drawing of plastic films. In the manufacture of these sheets, the melt issues from a slit and is subsequently stretched to achieve the desired thickness. The mechanical properties of the final product strictly depend on the stretching and cooling rates in the process. The pioneering works of Sakiadis (1961a and 1961b) give various aspects of boundary layer flow on a continuously stretching surface with constant speed and later other aspects have been investigated by several authors in the field. Specifically Crane's problem (1970) for flow of an incompressible viscous fluid past a stretching sheet has become classic in the literature. It admits an exact analytical solution. The Uniqueness of the exact analytical solution is discussed by Mcleod and Rajagopal (1987). Gupta and Gupta (1977) examined the stretching flow subject to suction or injection. The flow inside a stretching channel or tube has been analyzed by Brady and Acrivos (1987) and the flow outside the stretching tube by Wang (1988). In another paper, Wang (1984) extended the flow analysis to the three dimensional axi-symmetric stretching surface. The unsteady flows induced by stretching film have been also discussed by Wang (1990) and Usha and Sridharan (1995).

1813-8235 (Print), 2070-8998 (Online) Received on: July 2015

All the above mentioned investigations deal with the stretching flow problems. The phenomena of velocities on the boundary towards a fixed point are known as shrinking phenomena, which often occur in the situation such as rising shrinking balloon. These studies are initiated by Wang (1990).Miklavcic and Wang(2006) proved the existence and uniqueness for steady viscous hydrodynamic flow due to a shrinking sheet for a specific value of the suction parameter. From the continuity of Cranes stretching sheet solution would induce far field suction towards the sheet, while the shrinking sheet would cause velocity away from the sheet. On the physical ground vorticity of the shrinking sheet is not confined within the boundary layer and the flow is unlikely to exist unless adequate suction on the boundary is imposed. The purpose of the present article is to study the properties of the flow due to a shrinking sheet with suction.

Recently, Nadeem and Hussain (2009) presented the MHD flow of viscous fluid on a nonlinear porous shrinking sheet with homotopy analysis method. A closed form exact solutions of MHD viscous flow over a shrinking sheet is given by Fang and Zhang (2009). Sajid and Hayat (2009), investigated, MHD viscous flow due to a shrinking sheet. The cases of two dimensional and axi-symmetric shrinking sheet have been discussed using homotopy analysis method (HAM). Ali et al. (2010) discussed boundary layer flow and heat transfer due to permeable shrinking sheet with prescribed surface heat flux using Keller-box method. Noor et al. (2010) examined the simple non-perturbative solution for MHD viscous flow due to a shrinking sheet by series solution using Adomain decomposition method (ADM). Raftari and Yildirim (2011) examined the MHD viscous flow due to a shrinking sheet by employing homotopy perturbation method (HPM) and Pade' approximants. Bhattacharyya (2011) analyzed the effects of heat source/sink on the steady two dimensional MHD boundary layer flow and heat transfer over a shrinking sheet with wall mass suction using finite difference method. Swati and Gorla (2012), examined the unsteady MHD boundary layer flow of an upper convicted Maxwell fluid past a stretching sheet with first order constructive / destructive chemical reaction. Ali et al.(2013), have discussed the dual solutions for MHD flow on a non-linear porous shrinking sheet in a viscous fluid. Aman et al. (2013), have analysed the steady two-dimensional stagnation point flow over a linear stretching /shrinking sheet in a viscous and incompressible fluid in the presence of magnetic field. Kumar (2013), have discussed MHD boundary layer flow on heat and mass transfer over a stretching sheet with slip effect. Vajravelu and Prasad (2014) have analysed various aspects of MHD stretching/ shrinking sheet problems involving boundary layer theory. Ahmad et al. (2015), considered the numerical solution of MHD flow and heat transfer through a porous medium over a stretching/ shrinking surface with suction. Sandeep and Sulochana (2015) have discussed the dual solutions for unsteady mixed convection flow of MHD micropolar fluid over a stretching/shrinking sheet with non-uniform heat source /sink. Vishwanath et al. (2015) have discussed semi-numerical solutions of MHD flow over a linearly stretching sheet.

The present investigation is to analyze the magneto-hydrodynamic (MHD) viscous flow caused by a shrinking sheet. The solution of the resulting third order nonlinear boundary value problem with an infinite interval is obtained using Dirichlet series method and method of stretching of variables. We seek solution of the general equation of the type

$$f''' + A\, ff'' + B\, f'^2 + C\, f' = 0 \tag{1}$$

with the relevant boundary conditions

$$f(0) = \alpha_1 = f_w, \qquad f'(0) = \beta_1, \qquad f'(\infty) = 0 \tag{2}$$

where *A*, *B* and *C* are constants and prime denotes derivative with respect to the independent variable η.

This equation admits a Dirichlet series solution; necessary conditions for the existence and uniqueness of these solutions may also be found in [(1965, 1972)]. For a specific type of boundary conditions i.e. $f'(\infty) = 0$, the Dirichlet series solution is particularly useful for obtaining solution and the derived quantities exactly. A general discussion of the convergence of the Dirichlet series may also be found in Riesz (1957). The accuracy as well as uniqueness of the solution can be confirmed using other powerful semi-numerical schemes. Sachdev et al. (2005) have analyzed various problems from fluid dynamics of stretching sheet using this approach and found more accurate solution compared with earlier numerical findings. Recently, Vishwanath et al. (2011, 2011) and Ramesh et al. (2011) have analysed the problems from MHD boundary layer flow with nonlinear stretching sheet using these methods and found more accurate results compared with the classical numerical methods. In this article, we present Dirichlet series solution and an approximate analytical method-method of stretching of variables. This method is quite easy to use especially for nonlinear ordinary differential equations and requires less computer time compared with pure numerical methods and easy to solve compared with other approximate methods(for example, Homotopy perturbation method (HPM) Pade' technique, Adomain decomposition methods (ADM).

The present paper is structured as follows. In section 2 the mathematical formulation of the proposed problem with relevant boundary conditions is given. Section 3 is devoted to the solution of the problem using Dirichlet series. Section 4 gives the solution by means of method of stretching of variables. In Section 5, detailed results obtained are compared with the corresponding numerical schemes and Section 6 is about the conclusion.

2. Mathematical Formulation of the problem

The continuity and momentum equations of viscous incompressible fluid in the presence of body forces are

$$\frac{\partial \rho}{\partial t}+\nabla.\left(\rho \bar{V}\right)=0 \tag{3}$$

$$\rho\left(\frac{\partial \bar{V}}{\partial t}+\left(\bar{V}.\nabla\right)\ \bar{V}.\right)=f-\nabla p+\mu\nabla^{2}\bar{V} \tag{4}$$

where p is the pressure, ρ is the density of the fluid and $\bar{V}=\left(u(x,y,z),v(x,y,z),w(x,y,z)\right)$ is the velocity field of the fluid.

The governing equations for steady laminar three-dimensional MHD viscous flow due to shrinking sheet are derived from three-dimensional momentum equations (4) which consists of a continuity equation (3) which reflects the viscous incompressible flow. The fluid is electrically conduction in the presence of magnetic field of strength B_0 is applied in the z-direction and induced magnetic field is neglected.The electromagnetic body force is given as $f=-\sigma B_0^2\left(u,v,0\right)$. Under the above assumptions, Equations (3) and (4) become the resulting three-dimensional boundary layer equations (Sajid & Hayat (2009))

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{5}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho}u \tag{6}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)-\frac{\sigma B_0^2}{\rho}v \tag{7}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\upsilon\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \tag{8}$$

where $\upsilon=\mu/\rho$ is the kinematic viscosity,(i.e. μ is the dynamic viscosity) and σ is the electrical conductivity of the fluid. The relevant boundary conditions for the present flow are

$$u=-ax,\qquad \mathrm{v}=-a\left(m-1\right)y,\qquad w=-W,\ \text{at}\ y=0$$
$$u\to 0\quad \text{as}\ y\to\infty \tag{9}$$

in which $a>0$ is the shrinking constant, W is the suction velocity. For m=1 the sheet shrinks in x-direction only for m=2 it shrinks axi-symmetrically. Eq. (5)-(8) along with the boundary conditions Eq. (9) admit similarity solution. We use following similarity variables

$$u=xaf'\left(\eta\right),\quad v=ya\left(m-1\right)f'\left(\eta\right),\quad w=-m\sqrt{av}\,f\left(\eta\right),\quad \eta=\sqrt{\frac{a}{v}}z. \tag{10}$$

The continuity Eq. (5) is identically satisfied and Eq. (8) can be integrated to get

$$\frac{p}{\rho} = \nu \frac{\partial w}{\partial z} - \frac{w^2}{2} + \text{ constant} \tag{11}$$

The Eq. (5)-(7) and (9) using Eq. (10) are reduce to the following nonlinear ordinary differential equation (2009)

$$f''' + m\, ff'' - f'^2 - M^2 f' = 0,\ ' = \frac{d}{d\eta}, \tag{12}$$

and the boundary conditions are

$$\begin{aligned} &f = f_w, \quad f' = -1, \quad \text{at } \eta = 0 \\ &f' \to 0 \text{ as } \eta \to \infty \end{aligned} \tag{13}$$

where $f_w = \dfrac{W}{m\sqrt{a\nu}}$ and $\mathrm{M}^2 = \dfrac{\sigma B_0^2}{\rho a}$.

3. Dirichlet Series Solution

We use Dirichlet series which is an elegant semi-numerical scheme, to solve the problem exactly. We seek Dirichlet series solution of Eq. (1) satisfying last boundary condition $f'(\infty) = 0$ automatically in the form (Kravchenko&Yablonskii [(1965, 1972)])

$$f = \gamma_1 + \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i e^{-i\gamma\eta} \tag{14}$$

where γ and a are parameters which are to be determined. Substituting Eq. (14) into Eq. (1), we get

$$\sum_{i=1}^{\infty}\left\{-\gamma^2 i^3 + A\gamma\gamma_1 i^2 - Ci\right\} b_i a^i e^{-i\gamma\eta} + \frac{6\gamma^2}{A}\sum_{i=2}^{\infty}\sum_{k=1}^{i-1}\left\{Ak^2 + Bk(i-k)\right\} b_k b_{i-k} a^i e^{-i\gamma\eta} = 0 \tag{15}$$

For $i=1$, we have $\gamma_1 = \dfrac{\gamma^2 + C}{A}$. (16)

Substituting Eq. (16) into Eq. (15) the recurrence relation for obtaining coefficients is given by

$$b_i = \frac{6\gamma^2}{Ai(i-1)\left\{\gamma^2 i - C\right\}}\sum_{k=1}^{i-1}\left\{Ak^2 + Bk(i-k)\right\} b_k b_{i-k} \tag{17}$$

For $i=2, 3, 4,.....$. If the Eq. (14) converges absolutely when $\gamma > 0$ for some η_0, this series converges absolutely and uniformly in the half plane $\mathrm{Re}\,\eta \geq \mathrm{Re}\,\eta_0$ and represents an analytic $\dfrac{2\pi}{\gamma}$ periodic function $f = f(\eta_0)$ such that $f'(\infty) = 0$ (Kravchenko&Yablonskii (1965)).

The Eq. (14) contains two free parameters namely a and γ . These unknown parameters are determined from the remaining boundary conditions of Eq. (2) at $\eta = 0$

$$f(0) = \frac{\gamma^2 + C}{A\gamma} + \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i = \alpha_1 \tag{18}$$

and

$$f'(0) = \frac{6\gamma^2}{A}\sum_{i=1}^{\infty}(-i) b_i a^i = \beta_1 \tag{19}$$

The solution of the above transcendental Eq. (18) and Eq. (19) yield constants a and γ . The solution of the above transcendental equations is equivalent to the unconstrained minimization of the functional

$$\left[\frac{\gamma^2 + C}{A\gamma} + \frac{6\gamma}{A}\sum_{i=1}^{\infty} b_i a^i - \alpha_1\right]^2 + \left[\frac{6\gamma^2}{A}\sum_{i=1}^{\infty}(-i) b_i a^i - \beta_1\right]^2 \tag{20}$$

We use Powell's method of conjugate directions (Press et al. (1987)) which is one of the most efficient techniques for solving unconstrained optimization problems. This helps in finding the unknown parameters a and γ uniquely for different values of the parameters A, B, C, α_1 and β_1. Alternatively, Newton's method is also used to determine the unknown parameters a and γ accurately.

The shear stress at the surface of the problem is given by

$$f''(0) = \frac{6\gamma}{A} \sum_{i=1}^{\infty} b_i a^i (i\gamma)^2 \tag{21}$$

The velocity profiles of the problem is given by

$$f'(\eta) = \frac{6\gamma^2}{A} \sum_{i=1}^{\infty} (-i) b_i a^i e^{-i\gamma\eta} \tag{22}$$

4. Method of Stretching of Variables

Many nonlinear ODE arising in MHD problems are not amenable for obtaining analytical solutions. In such situations, attempts have been made to develop approximate methods for the solution of these problems. The numerical approach is always based on the idea of stretching of variables of the flow problems. Method of stretching of variables is used here for the solution of such problems. In this method, we have to choose suitable derivative function H' such that the derivative boundary conditions are satisfied automatically and integration of H' will satisfy the remaining boundary condition. Substitution of this resulting function into the given equation gives the residual of the form $R(\xi,\alpha)$ which is called defect function. Using Least squares method, the residual of the defect function can be minimized (for details see Ariel, (1994)).Using the transformation $f = f_w + F$ into Eq. (1), we get

$$F''' + A\left(f_w + F\right)F'' + BF'^2 + CF' = 0, \quad ' = \frac{d}{d\eta} \tag{23}$$

and the boundary conditions (2) become

$$F(0) = 0, \quad F'(0) = -1, \quad F'(\infty) = 0 \tag{24}$$

We introduce two variables ξ and G in the form

$$G(\xi) = \alpha F(\eta) \text{ and } \xi = \alpha\eta \tag{25}$$

where α >0, is an amplification factor. In view of Eq. (25), the system (23)-(24) are transformed to the form

$$\alpha^2 G''' + A\left(f_w \alpha + G\right)G'' + BG'^2 + CG' = 0, \qquad ' = \frac{d}{d\xi} \tag{26}$$

and the boundary conditions in Eq. (24) become

$$G(0) = 0, \quad G'(0) = -1, \quad G'(\infty) = 0 \tag{27}$$

We choose a trail velocity profile

$$G' = -\exp(-\xi) \tag{28}$$

which satisfies the derivative conditions in Eq. (27). Integrating Eq. (28) with respect to ξ from 0 to ξ using conditions (27), we get

$$G = \exp(-\xi) - 1. \tag{29}$$

Substituting Eq. (29) into Eq. (26), we get the residual of defect function

$$R(\xi,\alpha) = \left(-\alpha^2 + Af_w\alpha - A - C\right)\exp(-\xi) + \left(A + B\right)\exp(-2\xi). \tag{30}$$

Using the least squares method as discussed in Ariel (1994), the Eq. (30) can be minimized for which

$$\frac{\partial}{\partial\alpha} \int_0^{\infty} R^2(\xi,\alpha)\, d\xi = 0. \tag{31}$$

Substituting Eq. (30) into Eq. (31) and solving cubic equation in α for a positive root, we get

$$\alpha = \frac{1}{6}\left(3A\, f_w \pm \sqrt{3}\sqrt{-4A+8B-12C+3A^2 f_w^2}\right). \tag{32}$$

Once the amplification factor is calculated, then using Eq. (23), original function *f* can be written as

$$f = f_w + \frac{1}{\alpha}\left(\exp(-\alpha\eta)-1\right). \tag{33}$$

with α defined in Eq. (32). Thus Eq. (33) gives the solution of Eq. (1) for all A, B and C.

5. Results and Discussion

The third order nonlinear boundary value problems with infinite domain are solved semi-numerically using powerful techniques which are Dirichlet series method and an approximate analytical method by method of stretching of variables. In this method it is important that the edge boundary layer $\eta \to \infty$ automatically satisfied. Numerical computations are performed for various values of the physical parameters involved in the equation viz. the Hartmann number M, the mass suction parameter $s = f_w$. The present solution is also validated by comparing it with the previously published work of Sajid and Hayat (2009), Ali et al. (2010) and Raftari & Yildirim (2011) as shown in Tables 1 and 2.

Table 1: Comparison of the values of $f''(0)$ obtained by the Dirichlet series method, Method of stretching of variables and other applied methods when $C = -M^2$ and s =1.

$A = m$	**Dirichlet Series Method**			Sajid& Hayat (2009)	Raftari&Yildirim (2011)	Method of stretching of variable
	a	γ	$f''(0)$			
1	0.03143	2.30278	2.30277	2.30277	2.30277	2.30278
2	0.04197	2.74674	2.89228	2.89160	2.89161	2.91485

Table 2: Comparison of the values of $f''(0)$ obtained by the Dirichlet series method, Method of stretching of variables and Keller-box method for various values of s when *A = m = 1, 2* and C= $-M^2$ = 0.25 and 5.0.

A=m	$C = -M^2$	$s = \alpha_1$	**Dirichlet Series Method**			Ali et al. (2010)	Method of stretching of variable
			a	γ	$f''(0)$		
1	-0.25	1.75	0.16667	1.00000	1.00000	1.00000	1.00000
		1.80	0.127138	1.14495	1.14495	1.1449	1.14495
		1.85	0.10667	1.25000	1.25000	1.25000	1.25000
		1.90	0.09275	1.34051	1.34051	1.3405	1.34051
						Bhattacharyya (2011)	
	-2	2.0	0.02859	2.41421	2.41421	2.41421	2.41421
		3.0	0.01528	3.30278	3.30278	3.30278	3.30278
		4.0	0.00929	4.23607	4.23608	4.23607	4.23607
						Ali et al (2010)	
2	-5	2.0	0.01510	4.64964	4.74662	4.7461	4.76887
		2.3	0.01220	5.18217	5.27083	5.2705	5.29277
		3.0	0.00788	6.46572	6.53881	6.5387	6.55903
		3.5	0.00602	7.40624	7.47093	7.4708	7.489957

The graphs for the function $f'(\eta)$ i.e. velocity profiles which corresponds to velocity component *u* and *v* are drawn against η for different values of the parameters $s = f_w$ and *M*. In all cases Figure (a) corresponds to the two-dimensional shrinking sheet and (b) corresponds to the axi-symmetric shrinking. Figs. 1and 2 presents velocity profiles which match very well with that of earlier findings depicted in their figures.

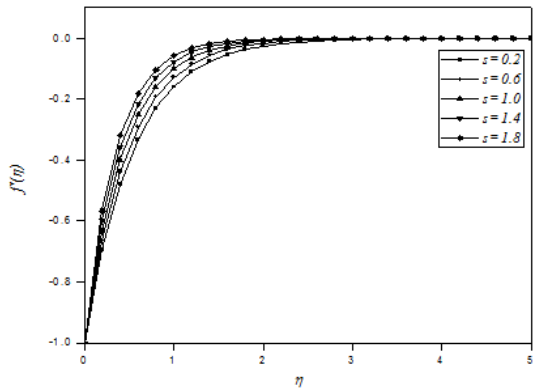

Fig. 1a: The velocity profiles for *M=2* and several values of s: *A=m=1*

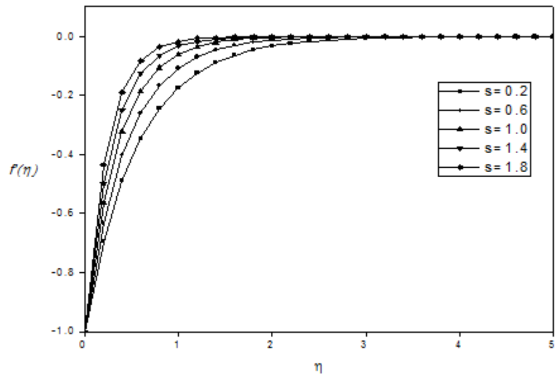

Fig. 1b: The velocity profiles for *M=2* and several values of s: *A=m=2*

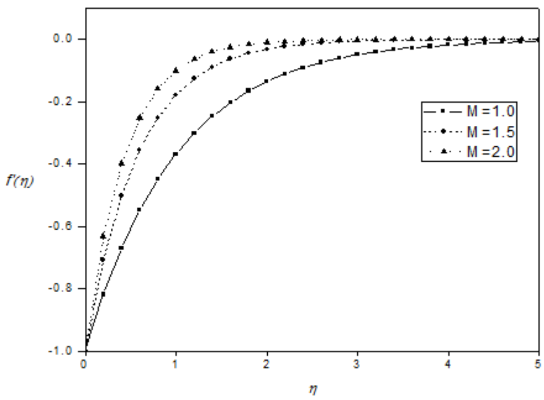

Fig. 2a: The velocity profiles for *s=2* and several values of M: *A=m=1*

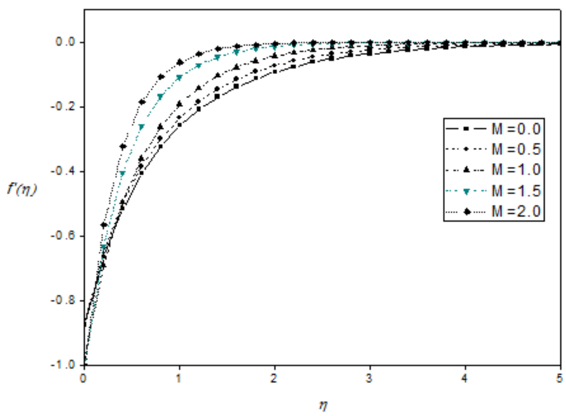

Fig. 2b: The velocity profiles for *s=1* and several values of M: *A=m=2*

In Figs. 1 and 2, which demonstrate the effects of the suction parameter $s = f_w$ and Hartmann number M on the velocity profiles for *A* = *m* =*1* and *A* = *m* =*2*. The effect of the suction parameter $s = f_w$ and Hartmann number M is to increase the velocity and decrease the boundary layer thickness by increasing the suction parameter $s = f_w$ for both two dimensional and axi-symmetric shrinking. The above computations work very well using Dirichlet series and method of stretching of variables. It is also susceptible to the computer's memory limitations and it takes very less computer memory. In this work we use Mathematica and FORTRAN compiler running on a personal computer with Pentium processor.

6. Conclusions

In this article, we describe the analysis of boundary value problem for third order nonlinear ordinary differential equation over an infinite interval arising in MHD boundary layer. The semi-numerical schemes described here offer advantages over solutions obtained by HAM, HPM and numerical methods etc. The convergence of the Dirichlet series method is given. The results are presented in Tables and graphically, the effects of the emerging parameters are discussed.

References:

Ahmad, F., Hussain, S.,Alharbi R. S. (2015): MHD flow and heat transfer through a porous medium over a stretching/ shrinking surface with suction, Science International, Vol.27, No.(2) , pp. 931-93.

Ali, F.,Nazar, R.,Arifin, N. (2010): MHD viscous flow and heat transfer induced by permeable shrinking sheet with prescribed surface heat flux, WSAS Transactions Mathematics, Vol. 5, No.9, pp. 365-375.

Ali, F. M.,Nazar, R.,Arifin, N. M.,Pop, I. (2013): Dual solutions in MHD flow on a nonlinear porous shrinking sheet in a viscous fluid, Boundary value problem, Vol.32, pp. 1-7. http://dx.doi.org/10.1186/1687-2770-2013-32

Altan, T., Oh, S.,Gergel, H. (1979): Metal forming fundamentals and applications, Metals Park, OH:American Society of Metals.

Aman, F.,Ishak, A., Pop I. (2013): MHD stagnation point flow towards a stretching / shrinking sheet with slip effects, International Communication of Heat and Mass Transfer, Vol. 47, pp. 68-72. http://dx.doi.org/10.1016/j.icheatmasstransfer.2013.06.005

Ariel, P.D.(1994): Stagnation point flow with suction ; an approximate solution, Journal of Applied Mechanics, Vol.61, No.4, pp. 976-978. http://dx.doi.org/10.1115/1.2901589

Awati, V.B.,Bujurke, N.M.,Kudenatti, R.,B. (2011): An exponential series method for the solution of the free convection boundary layer flow in a saturated porous medium, American Journal of ComputationalMathematics, Vol.1, pp.104-110.http://dx.doi.org/10.4236/ajcm.2011.12010

Awati, V.B.,Bujurke, N.M.,Kudenatti, R.B.(2011): Dirichlet series method for the solution of MHD flow over a nonlinear stretching sheet, International Journal of Applied Mathematics and Engineering Sciences, Vol. 5, No. 1, pp.07-12.

Bhattacharyya, K. (2011): Effects of heat source/ sink on MHD flow and heat transfer over a Shrinking sheet with mass suction, Chemical Engineering Research Bulletin, Vol.15, pp.12-17. http://dx.doi.org/10.3329/CERB.V151.6524

Brady, J. F.,Acrivos, A. (1981): Steady flow in a channel or tube with accelerating surface velocity. An exact analytical solution to theNavier-Stokes equations with reverse flow, Journal of Fluid Mechanics, Vol. 112, pp. 127-150.

Crane, L. J. (1970): Flow past a stretching plate, Journal of Applied Mathematics and Mechanics, Vol.21,pp. 645-647.

Fang, T., Zhang, J. (2009): Closed-form exact solutions of MHD viscous flow over a shrinking sheet, Communication in Nonlinear Science and Numerical Simulation, Vol.14, pp.2853-2857. http://dx.doi.org/10.1016/j.cnsns.2008.10.005

Fisher, E. G.(1976): Extrusion of plastics. New York ; Wiley.

Gupta, P.S., Gupta, A.S.(1977): Heat and mass transfer on a stretching sheet with suction or injection and blowing, Canadian Journal of Chemical Engineering, Vol. 55, pp.744-746. http://dx.doi.org/10.1002/cjce.5450550619

Kravchenko, T. K., Yablonskii, A. I. (1965): Solution of an infinite boundary value problem for third order equation, Differential'nyeUraneniya, Vol. 327, pp.1.

Kravchenko, T.K., Yablonskii, A.,I. (1972): A boundary value problem on a semi-infinite interval, Differential'nyeUraneniya, Vol.8, No.12, pp.2180-2186.

Kudenatti, R.B.,Awati, V.B.,Bujurke, N.M.(2011): Exact analytical solutions of class of boundary layer equations for a stretching surface, Applied Mathematics and Computation, Vol.218, pp.2952-2959. http://dx.doi.org/10.1016/j.amc.2011.08.049

McLeod, J. B.,Rajagopal, K.R. (1987): On the uniqueness of flow of a Navier-Stokes fluid due to a stretching boundary, Archive for *Rational* Mechanics and *Analysis*, Vol. 98, pp.385-393. http://dx.doi.org/10.1007/BF00276915

Miklacic, M., Wang C.Y.(2006): Viscous flow due to a shrinking sheet, Quarterly of *Applied Mathematics*, Vol. 64, pp.283-290.

Nadeem S.,Hussain, A. (2009): MHD flow of a viscous fluid on a nonlinear porous shrinkingsheet with HAM, *Applied Mathematics* and Mechanics / Engineering, Vol. 30, No. 12, pp. 1569-1578. http://dx.doi.org/10.1007/s10483-009-1208-6

Noor, N.F.M.,Kechil, S.A.,Hashim, I. (2010): Simple non-perturbative solution for MHD viscous flow due to a shrinking sheet, Communication in Nonlinear Science and Numerical Simulation, Vol. 15, pp.144-148.

Press, W.H.H., Flannery, B.P.,Teulosky, S. A.,Vetterling, W.T. (1987): Numerical Recipes in C, Cambridge. University Press, UK.

Raftari, B.,Yildirim A. (2011): A series solution of nonlinear ODE arising in MHD by HPM-Pade' technique, Computers &Mathematics with Applications, Vol.61, pp.1676-1681.
http://dx.doi.org /10.1016/ j.camwa. 2011.01.037
Riesz, S. (1957): Introduction to Dirichler series, Cambridge University Press, UK.
Rushi Kumar B. (2013): MHD boundary layer flow on heat and mass transfer over a stretching sheet with slip effect, Journal of Naval Architecture and Marine Engineering, Vol. 10, Iss. (2), pp 16-26.
http://dx.doi.org/10.3329/jname.v10i2.16400
Sachdev, P.L.,Bujurke N.M.,Awati, V.B. (2005): Boundary value problems for third order nonlinear ordinary differential equations, Studies in Applied Mathematics, Vol.115, pp. 303-318.
http://dx.doi.org/10.1111/j.1467-9590.2005.00310
Sajid, M., Hayat, T. (2009): The application of homotopy analysis method for MHD viscous flow due to shrinking sheet, Chaos Solitons and Fractals, Vol.39, pp. 1317-1323.
http://dx.doi.org/10.1016/j.chaos.2007.06.019
Sakiadis, B.C.(1961a): Boundary layer behaviour on continuous solid surfaces, American Institute of Chemical Engineers Journal, Vol. 7, pp. 26-28. http://dx.doi.org/10.1002/aic.690070108
Sakiadis, B.C. (1961b): Boundary layer behaviour on continuous solid surfaces : II, the boundary layer on a continuous flat surface, American Institute of Chemical Engineers Journal, Vol. 17, pp. 221-225.
http://dx.doi.org/10.1002/aic.690070211
Sandeep, N., Sulochana, C. (2015): Dual solutions for unsteady mixed convection flow of MHD micropolar fluid over a stretching/shrinking sheet with non-uniform heat source / sink, Engineering Science and Technology, an International Journal , Vol.18, pp.738-745. http://dx.doi.org/10.1016/j.jestch.2015.05.006
Swati, M.,Gorla, R.S.R. (2012): Unsteady MHD boundary layer flow of an upper convicted Maxwell fluid past a stretching sheet with first order constructive / destructive chemical reaction, Journal of Naval Architecture and Marine Engineering, Vol. 9(2), 123-133.http://dx.doi.org/10.3329/jname.v9i2.12541
Tadmor, Z., Klein, I. (1970): Engineering principles of plasticating extrusion polymer science and engineering series, New York, Van NostrandRaihold.
Usha, R.,Sridharan, R. (1995): The axisymmetrical motion of a liquid film on an unsteady stretching surface. Journal of Fluids Engineering, Vol. 117, pp. 81-85.http://dx.doi.org/10.1115/1.2816830
Vajravelu, K., Prasad, K. V. (2014): Keller-box method and its application, De Gruyter .
Awati, V.B., Mahesh Kumar, N.,Chavaraddi, K.B. (2015): Dirichlet series and approximate analytical solutions of MHD flow over a linearly stretching sheet, International Journal of Industrial Mathematics,Vol. 7, No. 4, pp.343-350.
Wang, C.Y. (1988): Fluid flow due to a stretching cylinder. Physics of Fluids, Vol.31, pp. 466-468.
http://dx.doi.org/org/10.1063/1.866827
Wang, C.Y. (1984): The three dimensional flow due to a stretching flat surface, Physics of Fluids, Vol.27, pp. 1915-1917.http://dx.doi.org/10.1063/1.864868
Wang, C.Y. (1990): Liquid film on an unsteady stretching sheet, Quarterly Applied Mathematics, Vol. 48, pp. 601-610.