



DUAL SOLUTIONS FOR UNSTEADY FLOW OF POWELL-EYRING FLUID PAST AN INCLINED STRETCHING SHEET

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Abstract:

This paper deals with the heat and mass transfer in unsteady flow of Powell-Eyring fluid past an inclined stretching sheet in the presence of radiation, non-uniform heat source/sink and chemical reaction with suction/injection effects. The governing equations are reduced into system of ordinary differential equations using similarity transformation and solved numerically using Runge-Kutta based shooting technique. Results display the influence of governing parameters on the flow, heat and mass transfer, friction factor, local Nusselt and Sherwood numbers. Comparisons are made with the existed studies. Present results have an excellent agreement with the existed studies. Results indicate that an increase in the chemical reaction parameter depreciates the friction factor, heat transfer rate and enhances the mass transfer rate. Dual solutions exist only for certain range of suction/injection parameters.

Keywords: Powell-Eyring fluid, non-uniform heat source/sink, radiation, chemical reaction, suction/injection.

Introduction

The flow of non-Newtonian fluids has attained a greatest importance and increasing interest in the theory of fluid mechanics. Fluids which do not obey Newton's law of motion are called Non-Newtonian fluids. Few examples of non-Newtonian fluids are tooth paste, food products, flow of blood etc. Many investigators studied electrically conducting non-Newtonian fluids of the two-dimensional magneto hydrodynamic boundary layer flows. It is assumed that non-Newtonian behaviour described by power-law model. The Powell-Eyring model is mathematically more complex, and it has advantages of power-law model. Firstly, in the case of power-law model, it is deduced from Kinematic theory of liquid rather than the empirical relation. Secondly it correctly reduces to high shear stress and Newtonian behaviour for law.

Crane (1970) investigated an exact analytical solution for the steady two dimensional flows due to a stretching surface in a quiescent fluid. The effect of variable viscosity and viscous dissipation on the flow of a third grade fluid in a uniform pipe has been studied by Massoudi and Christie (1995). Nadeem and Ali (2009) and Nadeem et al. (2010) investigated the pipe flow of non-Newtonian fluid with variable viscosity by considering no slip and partial slip conditions and found that heat transfer performance of the flow is significant in the presence of slip effect. Javed et al. (2013) studied the boundary layer flow over a stretching sheet for non-Newtonian fluid, namely the Eyring-Powell model. A theory based model for evaluating the temperature and volume fraction effects on nano fluids was presented by Hosseini and Ghader (2010). Ibrahim et al. (2008) studied the unsteady MHD free convection flow past a semi-infinite vertical plate in the presence of chemical reaction and radiation absorption. Muthucumaraswamy and Ganesan (2001) investigated the effect of heat transfer on the unsteady flow past an impulsively started vertical plate with first order chemical reaction. Eldabe et al. (2003) and Zueco and Beng (2009) discussed the effects of couple stress between two parallel plates using Eyring-Powell model. Prasad et al. (2013) presented the non-Newtonian Powell-Eyring fluid over a non-isothermal stretching sheet. Patel and Timol (2009) presented MHD Powell-Eyring fluid flow using the method of asymptotic boundary conditions. The effect of radiation on the flow of a micropolar fluid over a nonlinearly stretching sheet was discussed by Babu et al. (2015).

Anderson et al. (1992) investigated the non-Newtonian flow of a power-law fluid past a stretching surface in steady two dimensional flows. Mohankrishna et al. (2015) presented the radiation and chemical reaction effects on MHD Convective flow over a permeable stretching surface with suction and heat generation by using the shooting technique. Sandeep et al. (2016) analyzed the effect of heat and mass transfer in thermophoretic radiative hydro magnetic nano fluid flow over an exponentially stretching porous sheet. Makinde and Aziz

(2011) discussed the effect of boundary layer flow of a nano fluid past a stretching sheet. Sandeep et al. (2014) analyzed aligned magnetic field effect on unsteady convection flow over a moving vertical plate in porous medium in the presence of radiation. The unsteady two-dimensional flow of a non-Newtonian fluid over a stretching surface having a prescribed surface temperature is investigated by Mukhopadhyay et al. (2013).

Nadeem et al. (2013) investigated MHD Casson fluid flow in two lateral directions past a porous linear stretching sheet. Raju et al. (2015) investigated the effects of cross-diffusion and radiation in steady two-dimensional flow over a vertical stretching surface in the presence of aligned magnetic field. Layek et al. (2007) analysed the effect of heat and mass transfer analysis for boundary layer stagnation-point flow of stretching sheet. Hossain and Takhar (1996) worked the effect radiation on mixed convection flow with uniform surface temperature. The influence of non uniform heat source on unsteady flow of a stretching sheet was studied by Tsai et al. (2008). Peristaltic flow of non-Newtonian fluid in a symmetric channel was investigated by Hayat et al. (2014).

To the authors knowledge no studies have been reported yet on the heat and mass transfer in unsteady flow of Powell-Eyring fluid past an inclined stretching sheet in the presence of radiation, non-uniform heat source/sink and chemical reaction with suction/injection effects. In this study, we analyzed the heat and mass transfer in unsteady flow of Powell-Eyring fluid past an inclined stretching sheet in the presence of radiation, non-uniform heat source/sink and chemical reaction with suction/injection effects numerically.

2. Mathematical Formulation

Consider an incompressible, two-dimensional unsteady flow of Power-Eyring fluid past an inclined stretching sheet. The sheet makes an angle α with the vertical direction as shown in Fig. 1. The x-axis is taken along the sheet and y axis is normal to it. In addition, we considered the effects of thermal radiation, chemical reaction and non-uniform heat source/sink.

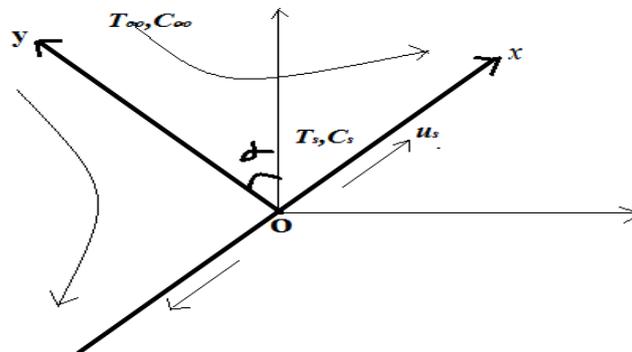


Fig. 1: Physical Model and Coordinate system

The Cauchy stress tensor in Power-Eyring fluid is given by

$$\tau_{ij} = \mu \frac{\partial u_i}{\partial x_j} + \frac{1}{\beta} \sinh^{-1} \left(\frac{1}{\gamma} \frac{\partial u_i}{\partial x_j} \right),$$

where μ is the viscosity coefficient, β and γ are the material fluid parameters. The boundary layer equations comprising the balance laws of mass, linear momentum and energy can be written as Hayat al. (2015)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \left(\nu + \frac{1}{\rho \tilde{\beta} \gamma} \right) \frac{\partial^2 u}{\partial y^2} - \frac{1}{2 \rho \tilde{\beta} \gamma^3} \left(\frac{\partial u}{\partial y} \right)^2 \frac{\partial^2 u}{\partial y^2} + g_0 [\beta_r (T - T_{\infty}) + \beta_c (C - C_{\infty})] \cos \alpha, \tag{2}$$

$$\rho c_p \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} + q''', \tag{3}$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} - k_l (C - C_\infty), \tag{4}$$

where t is the time, $\nu = (\mu/\rho)$ is the kinematic viscosity, k is the thermal conductivity of the fluid, ρ is the fluid density, T is the fluid temperature, C is the fluid concentration, C_p is the specific heat, g_o is the acceleration due to gravity, β_T and β_C is the volumetric coefficient of thermal and mass exponential, D_m is the molecular diffusivity of the species concentration, k_l is the chemical reaction parameter,

$$q_r = \frac{16\sigma^* T_\infty^3}{3k^*} \frac{\partial T}{\partial y}$$

is the linearized radiative heat flux, α is the inclined angle, k^* is the mean absorption

coefficient, σ^* is the Stefan-Boltzmann constant, q''' is the non-uniform heat source ($q''' > 0$) or sink ($q''' < 0$) per unit volume. The non-uniform heat source/sink, q''' is modeled by the following expression

$$q''' = \frac{ku_s(x,t)}{x\nu} [A^*(T_s - T_\infty)f' + B^*(T - T_\infty)], \tag{5}$$

in which A^* and B^* are the coefficients of space and temperature dependent heat source/sink, respectively. Here two cases arise. For internal heat generation $A^* > 0$ and $B^* > 0$ and for internal heat absorption, we have $A^* < 0$ and $B^* < 0$.

The surface velocity is denoted by $u_s(x,t) = \frac{bx}{(1-at)}$, whereas the surface temperature

$$T_s(x,t) = T_\infty + T_{ref} \frac{bx^2}{2\nu} (1-at)^{-3/2}$$

and surface concentration $C_s(x,t) = C_\infty + C_{ref} \frac{bx^2}{2\nu} (1-at)^{-3/2}$. Here

b (stretching rate) and a are positive constants having dimension time. Also T_{ref} , C_{ref} are constant reference temperature and concentration respectively.

The boundary conditions are taken as follows:

$$u = u_s(x,t), v = v_s, T = T_s(x,t), C = C_s(x,t) \text{ at } y = 0, \tag{6}$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty,$$

By introducing the similarity transformations

$$u = \frac{bx}{(1-at)} f'(\eta), v = -\sqrt{\frac{\nu b}{(1-at)}} f(\eta), \theta = \frac{T - T_\infty}{T_s - T_\infty}, \tag{7}$$

$$\eta = \sqrt{\frac{b}{\nu(1-at)}} y, \phi = \frac{C - C_\infty}{C_s - C_\infty}, k_l = \frac{k_0}{(1-at)},$$

Equation (1) is identically satisfied and equations (2)-(6) become

$$(1 + \Gamma) f''' - (f')^2 + ff'' - \Gamma \beta f''^2 - f''' - \varepsilon \left(f' + \frac{1}{2} \eta f'' \right) + Gr\theta \cos \alpha + Gc\phi \cos \alpha = 0, \tag{8}$$

$$\left(1 + \frac{4}{3} R \right) \theta'' + Pr \left(f\theta' - 2f'\theta - \frac{1}{2} \varepsilon (3\theta + \eta\theta') \right) + A^* f' + B^* \theta = 0, \tag{9}$$

$$\frac{1}{Sc} \phi'' - \frac{1}{2} \varepsilon (3\phi + \eta\phi') - 2\phi f' + f\phi' - Kr\phi = 0, \tag{10}$$

Boundary conditions are

$$f = S, f' = 1, \theta = 1, \phi = 1 \text{ at } \eta = 0, \tag{11}$$

$$f' \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } \eta \rightarrow \infty,$$

where prime denotes differentiation with respect to η , f is the dimensionless stream function, θ is the dimensionless temperature, ϕ is the dimensionless concentration and the dimensionless numbers are

$$\Gamma = \frac{1}{\mu\tilde{\beta}\gamma}, R = \frac{4\sigma^* T_\infty^3}{kk^*}, \beta = \frac{\rho u_s^3}{\mu x \gamma^2}, Gr = \frac{g_0 \beta_T (T_s - T_\infty) x^3 / \nu^2}{u_s^2 x^2 / \nu^2} = \frac{Gr_x}{Re_x^2}, S = -\frac{v_0}{\sqrt{\nu b}},$$

$$Gc = \frac{g_0 \beta_C (C_s - C_\infty) x^3 / \nu^2}{u_s^2 x^2 / \nu^2} = \frac{Gc_x}{Re_x^2}, \varepsilon = \frac{a}{b}, Pr = \frac{\mu c_p}{k}, Sc = \frac{\nu}{D_m}, Kr = \frac{k_0}{b},$$

where Γ and β are the dimensionless material fluid parameters, R is the radiation parameter, Gr is the thermal Grashof number, Gc is the mass Grashof number, ε is the unsteadiness parameter, Pr is the Prandtl number and Sc is the Schmidt number, Kr is the chemical reaction parameter and S is the suction/injection parameter, where $S > 0$ for suction and $S < 0$ for injection.

For engineering interest the coefficient of skin friction, local Nusselt and Sherwood numbers are defined as

$$C_f Re_x^{1/2} = f''(0), \tag{13}$$

$$Nu_x Re_x^{-1/2} = -\left(1 + \frac{4}{3}R\right)\theta'(0), \tag{14}$$

$$Sh_x Re_x^{-1/2} = -\phi'(0), \tag{15}$$

where $Re_x = \frac{u_s x}{\nu}$ is the local Reynolds number.

3. Results and Discussion

Eqs. (8) - (10) with the boundary conditions (11) have been solved numerically using Runge-Kutta based shooting technique. The results obtained shows the influence of the non-dimensional governing parameters, namely radiation parameter R , Material fluid parameter Γ , unsteadiness parameter ε , Inclined angle α , Chemical reaction parameter Kr , Schmidt number Sc , Mass Grashof number Gc and non-uniform heat source/sink parameters A^* and B^* on velocity, temperature, concentration, friction factor, Nusselt and Sherwood numbers. For numerical results we considered $\Gamma = \beta = Kr = 0.2, \varepsilon = Gc = R = 0.5, \alpha = \pi / 4, Pr = Gr = 1, \eta = 3, A^* = B^* = 0.1, Sc = 0.6$. These values are kept as common in entire study except the varied values as displayed in the respective figures and tables.

Figs. 2 and 3 depict the influence of radiation parameter on velocity and temperature profiles of the flow. It is evident from the figures that an increase in the radiation parameter enhances the velocity and temperature profiles of the flow. It is also observed that enhancements in the velocity and temperature fields are significantly high in injection case. Generally, an increase in the radiation releases the heat energy to the flow field. This causes to enhance the velocity and thermal boundary layers. In view of this we can conclude that influence of radiation is more significant as $R \rightarrow 0 (R \neq 0)$ and it can be neglected as $R \rightarrow \infty$. This agrees the general physical behavior of the radiation parameter.

Figs. 4 and 5 illustrate the influence of inclined angle on the velocity and temperature profiles for suction and injection cases. It is observed from the figures that raise the values of inclined angle depreciates the velocity field and enhances the temperature field. This is due to the fact that at $\alpha = 0$ the sheet is in vertical direction and maximum gravitational force acts on the flow. As $\alpha \rightarrow \pi / 2$ the sheet takes horizontal direction, the strength of buoyancy forces decreases and hence reduces the velocity boundary layer and enhances the thermal boundary layer. Figs. 6 and 7 depict the effect of material fluid parameter on velocity and temperature profiles of the flow for both suction and injection cases. It is clear from the figures that the increase in the value of Γ enhances the velocity boundary layer and depreciates the thermal boundary layer thickness. It is also noticed that the enhancement in the velocity field is more on injection case compared with suction case.

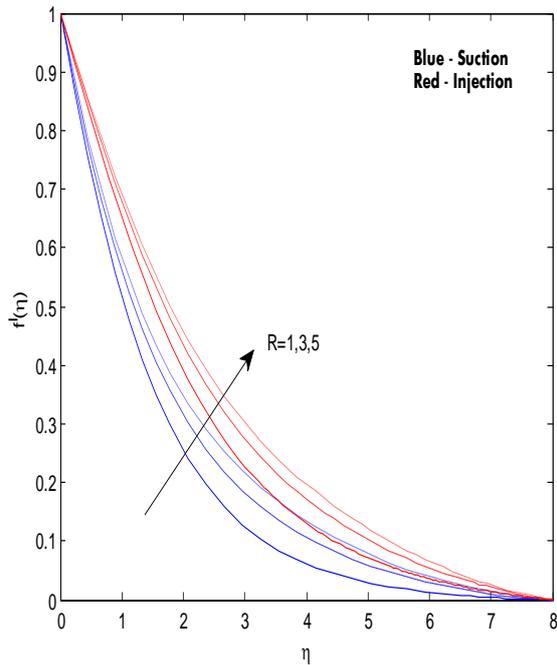


Fig. 2: Velocity field for different values of radiation parameter R

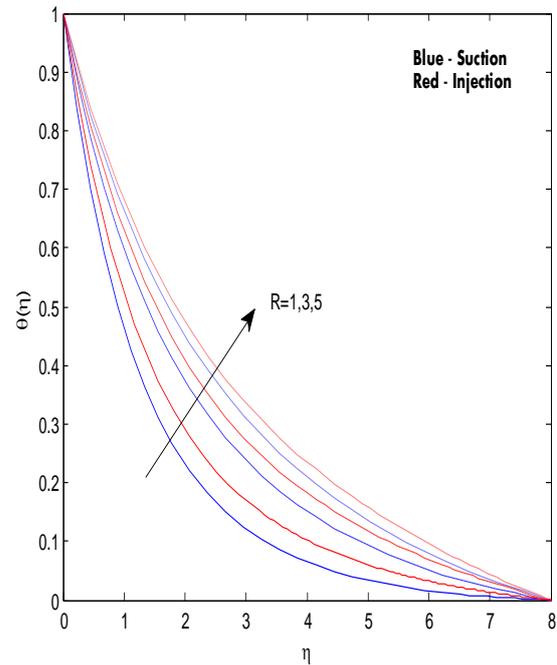


Fig. 3: Temperature field for different values of radiation parameter R

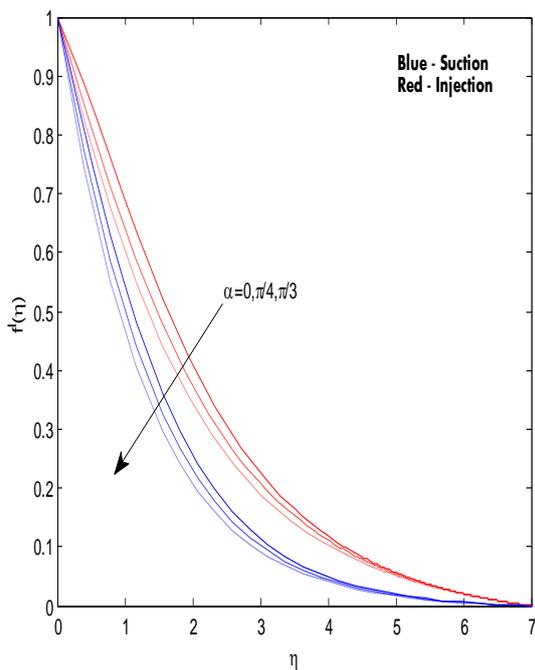


Fig. 4: Velocity field for different values of inclined angle α

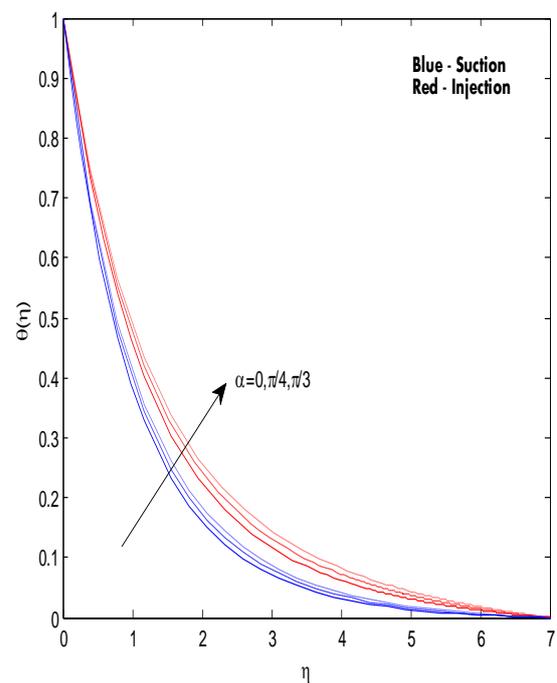


Fig. 5: Temperature field for different values of inclined angle α

Figs. 8 and 9 represent the influence of unsteadiness parameter on the velocity and temperature profiles of the flow for both suction and injection cases. We noticed an interesting result that the enhancement in the value of unsteadiness parameter depreciates the velocity profiles of the flow in injection case and increases the velocity of the flow in suction case. It is also observed the decrease in temperature profiles in both cases by increase in the value of unsteadiness parameter. This is due to the fact that increase in unsteadiness parameter reduces the

velocity boundary layer in injection case but in suction case it works in opposite manner. At the same time unsteadiness parameter causes to reduce the thermal boundary layer thickness and enhances the heat transfer rate.

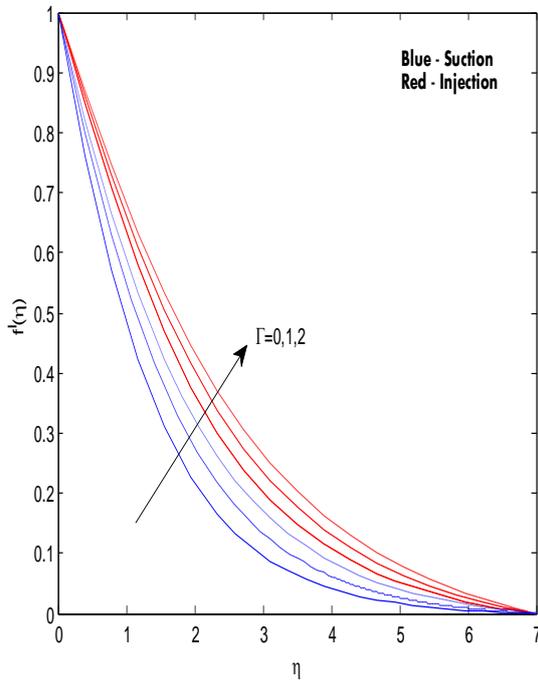


Fig. 6: Velocity field for different values of material fluid parameter Γ

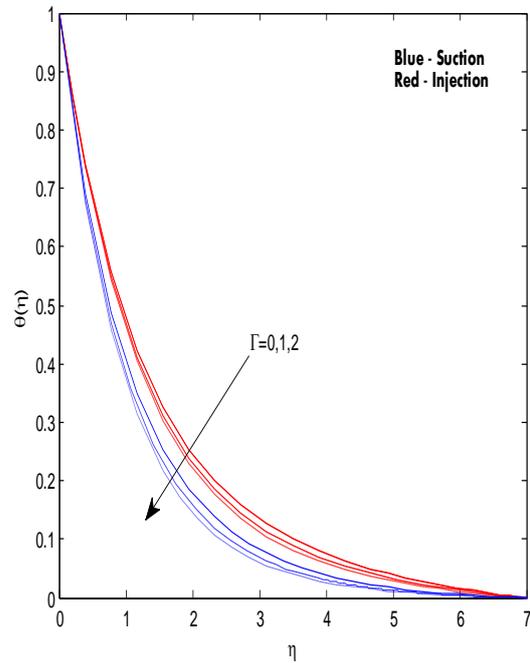


Fig. 7: Temperature field for different values of material fluid parameter Γ

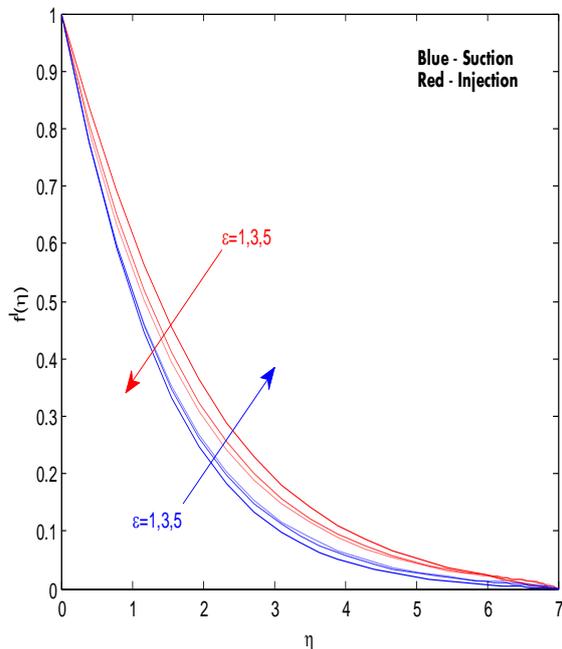


Fig. 8: Velocity field for different values of unsteadiness parameter ϵ

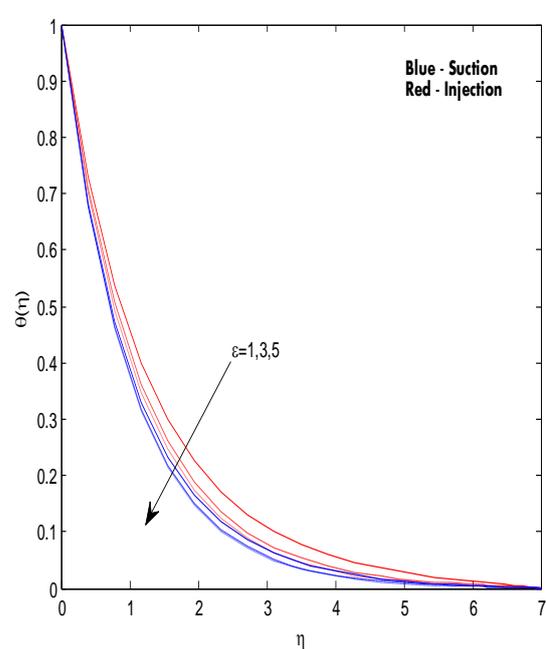


Fig. 9: Temperature field for different values of unsteadiness parameter ϵ

Figs. 10-13 illustrate the effect of non-uniform heat source/sink parameters on the velocity and temperature profiles of the flow. It is evident from the figures that an increase in the values of A^* and B^* enhances the velocity and temperature profiles of the flow. This may happen due to the fact that the positive values of A^* and B^* acts like heat generators. Generating the heat means releases the heat energy to the flow. This help

to enhance the velocity and thermal boundary layer thickness. It is also observed that the velocity and temperature profiles shown better enhancement in injection case compared with suction case.

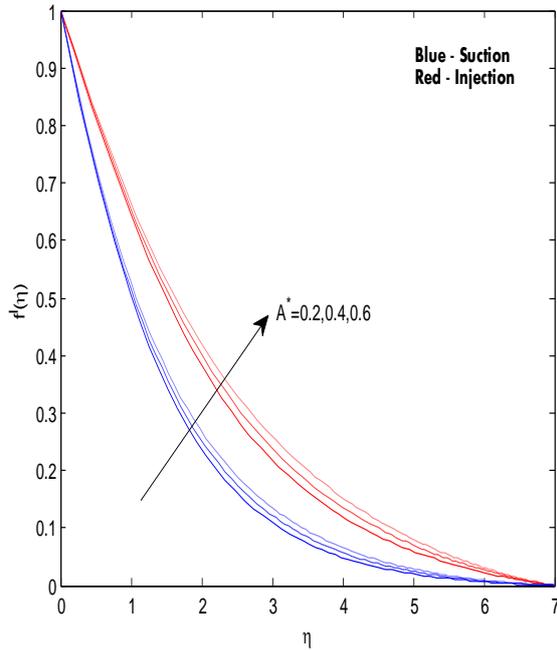


Fig. 10: Velocity field for different values of non-uniform heat source/sink parameter A^*

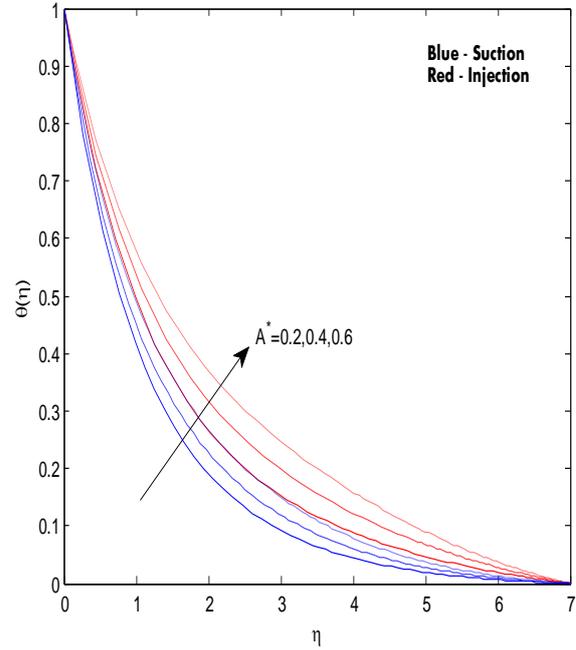


Fig. 11: Temperature field for different values of non-uniform heat source/sink parameter A^*

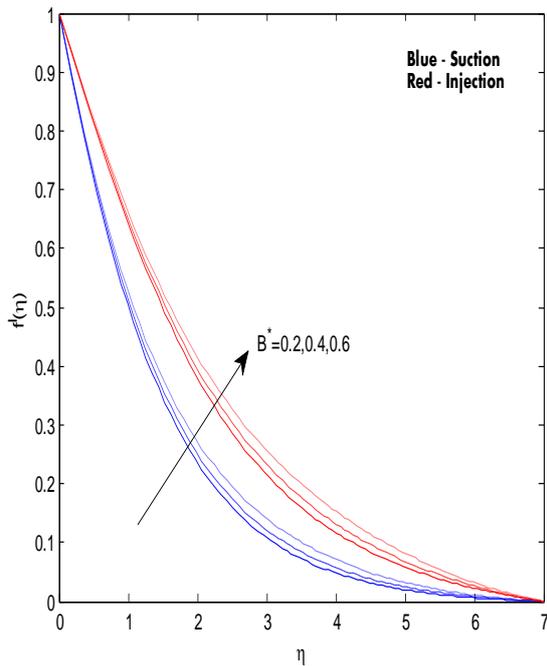


Fig. 12: Velocity field for different values of non-uniform heat source/sink parameter B^*

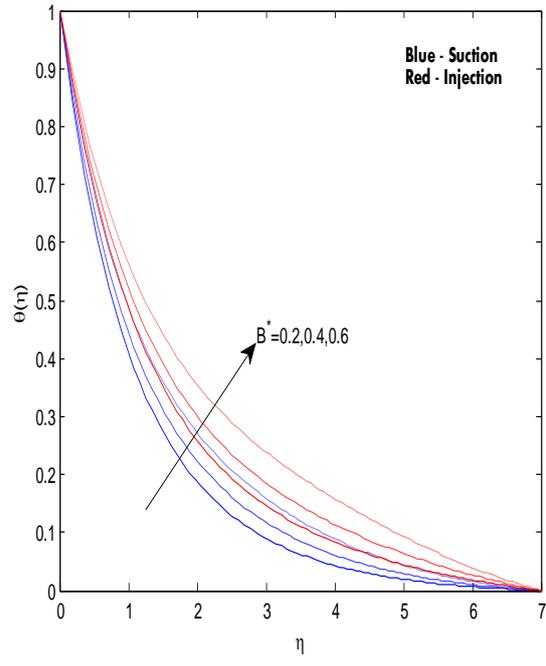


Fig. 13: Temperature field for different values of non-uniform heat source/sink parameter B^*

Figs. 14-16 display the influence of chemical reaction parameter, Schmidt number and mass Grashof number on concentration profiles of the flow. It is observed from the figures that an increase in the values of chemical reaction parameter, Schmidt number and mass Grashof number depreciates the concentration profiles of the flow and enhances the mass transfer rate. This is due to the fact that the increase in chemical reaction parameter, Schmidt number and mass Grashof number reduces the concentration boundary layer thickness.

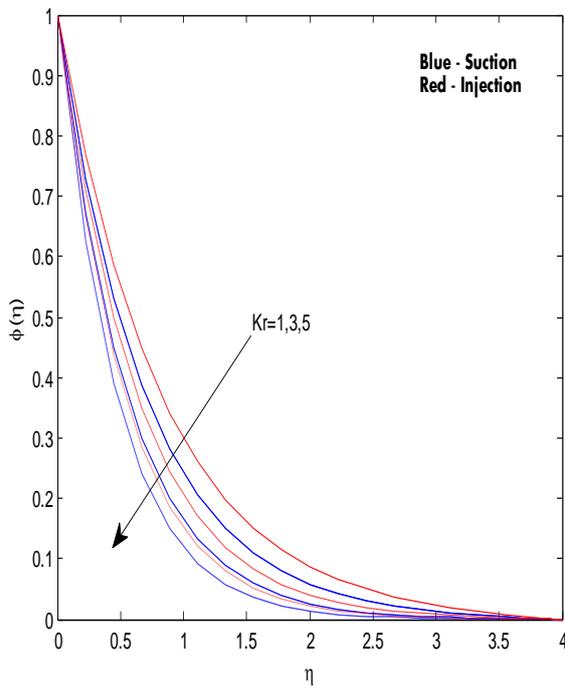


Fig. 14: Concentration field for different values of chemical reaction parameter Kr

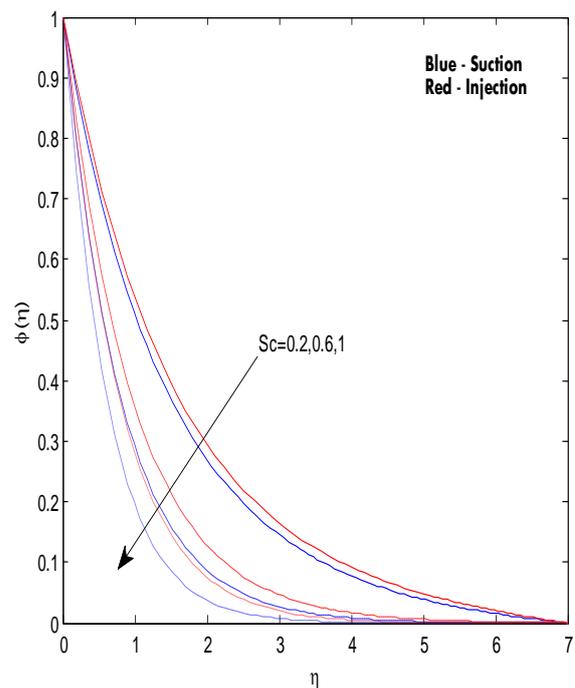


Fig. 15: Concentration field for different values of Schmidt number Sc

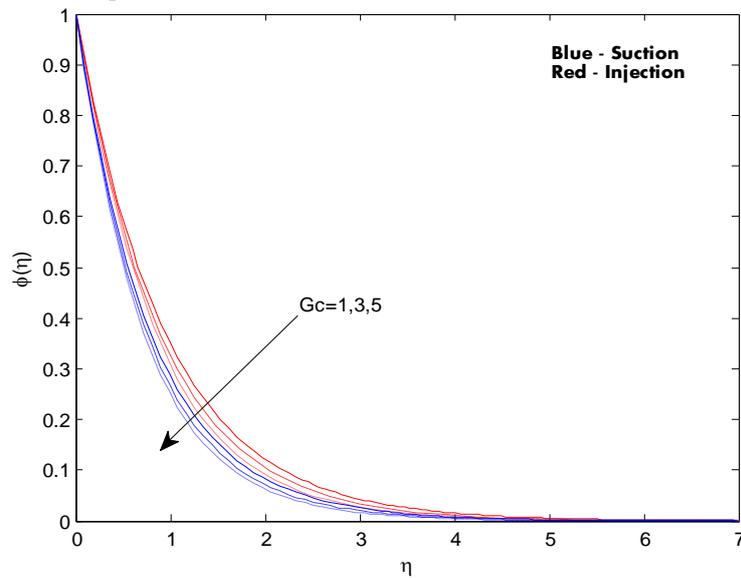


Fig. 16: Concentration field for different values of mass Grashof number Gc

Table 1 depicts the comparison of the present results with the existed results of Tsai *et al.* (2008), Hayat *et al.* (2015). Under some special conditions present results have an excellent agreement with the existed results. This shows the validity of the present results along with the accuracy of the numerical technique we used in this study. Table 2 displays the influence of non-dimensional governing parameters on friction factor, Nusselt and Sherwood numbers. It is evident from the table that an increase in the radiation parameter enhances the friction factor, mass transfer rate and reduces the heat transfer rate. A raise in the value of unsteadiness parameter showed opposite results to the radiation parameter. The enhancement in the value of material fluid parameter increases the friction factor, heat and mass transfer rate. But increase in inclined angle shows reverse results to the material parameter. An increase in chemical reaction parameter depreciates the skin friction coefficient and Nusselt number and enhances the Sherwood number. A raise in the values of non-uniform heat source/sink parameters depreciates the heat and mass transfer rate and enhances the friction factor.

Table 1: Comparison of the present with the existed studies of Tsai et.al (2008) and Hayat al. (2015) for wall temperature gradient when $\alpha = \beta = \Gamma = \varepsilon = Gr = Gc = S = R = 0$

Pr	A^*	B^*	Tsai et.al (2008)	Hayat al. (2015)	Present study
1	-1	0	-1.710937	-1.71094	-1.7109372
	-2	-1		-2.36788	-2.3678798
2	-1	0	-2.485997	-2.25987	-2.2598771
	-2	-1		-2.48600	-2.4860000

Table 2: Variation in $f''(0)$, $-\theta'(0)$ and $-\phi'(0)$ at different non-dimensional parameters

S	R	ε	Γ	α	Kr	A^* / B^*	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1	1						-0.611964	0.825574	1.254709
	3						-0.563830	0.550547	1.266303
	5						-0.540636	0.439350	1.271903
-1	1						-0.368571	0.682532	1.022146
	3						-0.346913	0.491671	1.027719
	5						-0.336007	0.404676	1.030554
1	1						-0.631188	0.999770	1.212916
	3						-0.636013	1.008084	1.132503
	5						-0.643304	1.008735	1.095664
-1	1						-0.433298	0.815952	1.010578
	3						-0.534345	0.887745	1.002026
	5						-0.574345	0.919080	1.001011
1			0				-0.665999	0.977881	1.244200
			1				-0.549080	1.006002	1.264042
			2				-0.483320	1.022728	1.276059
-1			0				-0.382825	0.776950	1.018290
			1				-0.360320	0.785661	1.024225
			2				-0.339795	0.792168	1.028788
1				0			-0.487717	1.007818	1.266435
				$\pi/4$			-0.635062	0.985020	1.249223
				$\pi/3$			-0.743444	0.966682	1.235789
-1				0			-0.248542	0.798351	1.034814
				$\pi/4$			-0.378544	0.778914	1.019606
				$\pi/3$			-0.473163	0.763538	1.007866
1					1		-0.649569	0.985466	1.429083
					3		-0.662573	0.982180	1.798841
					5		-0.670821	0.980316	2.102173
-1					1		-0.387774	0.778540	1.189780
					3		-0.398065	0.775834	1.545607
					5		-0.404897	0.774235	1.841451
1						0.2	-0.628843	0.948003	1.250751
						0.4	-0.615966	0.872625	1.253896
						0.6	-0.602492	0.795400	1.257162
-1						0.2	-0.374346	0.744978	1.020743
						0.4	-0.365781	0.676362	1.023045
						0.6	-0.357002	0.606762	1.025384

4. Conclusion

This paper presents a numerical solution for heat and mass transfer in unsteady flow of Powell-Eyring fluid past an inclined stretching sheet in presence of radiation, non-uniform heat source/sink and chemical reaction with suction/injection effects. Results display the influence of governing parameters on the flow, heat and mass transfer, friction factor, local Nusselt and Sherwood numbers. The conclusions of the present study are made as follows:

- Raise in the values of material fluid parameter enhances the heat and mass transfer rate.
- An increase in unsteadiness parameter depreciates the friction factor, Sherwood number and enhances the Nusselt number.
- Positive values of non-uniform heat source/sink parameters acts like heat generators and negative values acts like heat observers.
- Dual solutions exist only for certain range of suction/injection parameter.
- Enhancement in chemical reaction parameter depreciates the concentration profiles and increases the mass transfer rate.
- Raise in the values of non-uniform heat source/sink parameters enhances the velocity and thermal boundary layers.

References

- Anderson, H. I., Bech, K. H. and Dandapat, B. S. (1992): Magneto hydrodynamic Flow of a Power-Law Fluid over a Stretching Sheet, *International Journal of Non-Linear Mechanics*, Vol. 27, No. 6, pp. 929-936. [http://dx.doi.org/10.1016/0020-7462\(92\)90045-9](http://dx.doi.org/10.1016/0020-7462(92)90045-9).
- Babu, M. J., Gupta, R. and Sandeep, N. (2015): Effect of Radiation and Viscous Dissipation on Stagnation-Point Flow of a Micropolar Fluid over a Nonlinearly Stretching Surface with Suction/Injection, *Journal of Basic and Applied Research International*, Vol.7, No.2, pp.73-82.
- Crane, L. J. (1970): Flow Past a Stretching Plate, *Zeitschrift für angewandte Mathematik Und Physik ZAMP*, Vol. 21, No. 4, pp. 645–647. <http://dx.doi.org/10.1017/BF01587695>.
- Eldabe, N. T. M., Hassan, A. A. and Mohamed, M. A. A. (2003): Effects of Couple Stresses on the MHD of a Non-Newtonian Unsteady Flow between Two Parallel Porous Plates, *Zeitschrift für Naturforschung*, Vol. 58, No. 4, pp. 204-210. <http://dx.doi.org/10.1515/zna-2003-0405>.
- Hayat, T., Asad, S., Mustafa, M. and Alsaedi, A. (2015): Radiation Effects on the Flow of Powell- Eyring Fluid Past an Unsteady Inclined Stretching Sheet with Non-Uniform Heat Source/Sink, *PLoS ONE*, Vol. 9, No. 7, e103214. <http://dx.doi.org/10.1371/journal.pone.0103214>.
- Hayat, T., Irfan, S. S., Ahmad, B. and Mustafa, M. (2014): Effect of Slip on Peristaltic Flow of Powell-Eyring Fluid in a Symmetric Channel, *Applied Bionics and Biomechanics*, Vol. 11, No. 1-2, pp. 69-79. <http://dx.doi.org/10.3233/ABB-140087>.
- Hossain, M. A. and Takhar, H. S. (1996): Radiation Effect on Mixed Convection along a Vertical Plate with Uniform Surface Temperature, *Heat and Mass Transfer*, Vol. 31, No. 4, pp. 243-248. <http://dx.doi.org/10.1007/BF02328616>.
- Hosseini, M. and Ghader, S. (2010): A Model for Temperature and Particle Volume Fraction Effect on Nano Fluid Viscosity, *Journal of Molecular Liquids*, Vol. 153, No. 2-3, pp. 139-145. <http://dx.doi.org/10.1016/j.molliq.2010.02.003>.
- Ibrahim, F.S., Elaiw, A. M. and Bakr, A. A. (2008): Effect of the Chemical Reaction and Radiation Absorption on the Unsteady MHD Free Convection Flow Past a Semi-Infinite Vertical Permeable Moving Plate with Heat Source and Suction, *Communications in Nonlinear Science and Numerical Simulation*, Vol. 13, No. 6, pp. 1056-1066. <http://dx.doi.org/10.1016/j.cnsns.2006.09.007>.
- Javed, T., Ali, N., Abbas, Z. and Sajid, M. (2013): Flow of an Eyring-Powell Non-Newtonian Fluid over a Stretching Sheet, *Chemical Engineering Communications*, Vol. 200, No. 3, pp. 327-336. <http://dx.doi.org/10.1080/00986445.2012.703151>.
- Layek, G. C., Mukhopadhyay, S. and Samad, S. A. (2007): Heat and Mass Transfer Analysis for Boundary Layer Stagnation-Point Flow towards a Heated Porous Stretching Sheet with Heat Absorption/Generation and

- Suction/Blowing, *International Communications in Heat and Mass Transfer*, Vol. 34, No. 3, pp. 347-356. <http://dx.doi.org/10.1016/j.icheatmasstransfer.2006.11.011>.
- Makinde, O. D. and Aziz, A. (2011): Boundary Layer Flow of a Nano Fluid Past a Stretching Sheet with a Convective Boundary Condition, *International Journal of Thermal Sciences*, Vol. 50, No. 7, pp. 1326–1332. <http://dx.doi.org/10.1016/j.icheatmasstransfer.2006.11.011>.
- Massoudi, M. and Christie, I. (1995): Effect of Variable Viscosity and Viscous Dissipation on the Flow of a Third Grade Fluid in a Pipe, *International Journal of Non-Linear Mechanics*, Vol. 30, No. 5, pp. 687-699. [http://dx.doi.org/10.1016/0020-7462\(95\)00031-I](http://dx.doi.org/10.1016/0020-7462(95)00031-I).
- Mohankrishna, P., Sandeep, N. and Sugunamma, V. (2015): Effects of Radiation and Chemical Reaction on MHD Convective Flow over a Permeable Stretching Surface with Suction and Heat Generation, *Walailak Journal of Science and Technology*, Vol. 12, No. 9, pp. 831-847. <http://dx.doi.org/10.14456/WJST.2015.51>.
- Mukhopadhyay, S., De, P. R., Bhattacharyya, K. and Layek, G. C. (2013): Casson Fluid Flow over an Unsteady Stretching Surface, *Ain Shams Engineering Journal*, Vol. 4, No. 4, pp. 933-938. <http://dx.doi.org/10.1016/j.asej.2013.04.004>.
- Muthucumaraswamy, R. and Ganesan, P. (2001): First-Order Chemical Reaction on Flow Past an Impulsively Started Vertical Plate with Uniform Heat and Mass Flux, *Acta Mechanica*, Vol. 147, No. 1, pp. 45-57. <http://dx.doi.org/10.1007/BF01182351>.
- Nadeem, S. and Ali, M. (2009): Analytical Solutions for Pipe Flow of a Fourth Grade Fluid With Reynold and Vogel's Models of Viscosities, *Communications in Nonlinear Science and Simulation*, Vol. 14, No. 5, pp. 2073-2090. <http://dx.doi.org/10.1016/j.cnsns.2008.05.013>.
- Nadeem, S., Haq, R. U., Akbar, N. S. and Khan, Z. H. (2013): MHD Three-Dimensional Casson Fluid Flow Past a Porous Linearly Stretching Sheet, *Alexandria Engineering Journal*, Vol. 52, No. 4, pp. 577–582. <http://dx.doi.org/10.1016/j.aej.2013.08.005>.
- Nadeem, S., Hayat, T., Abbasbandy, S. and Ali, M. (2010): Effects of Partial Slip on a Fourth Grade Fluid with Variable Viscosity: An Analytical Solution, *Nonlinear Analysis: Real World Applications*, Vol. 11, No. 2, pp. 856–868. <http://dx.doi.org/10.1016/j.nonrwa.2009.01.030>.
- Patel, M. and Timol, M. G. (2009): Numerical Treatment of MHD Powell-Eyring Fluid Flow Using the Method of Satisfaction of Asymptotic Boundary Conditions, *Applied Numerical Mathematics*, Vol. 59, No. 10, pp. 2584- 2592.
- Prasad, K. V., Datti, P. S. and Raju, B. T. (2013): Momentum and Heat Transfer of a Non-Newtonian Eyring-Powell Fluid over a Non-Isothermal Stretching Sheet, *International Journal of Mathematical Achieve*, Vol. 4, No. 1, pp. 230-241.
- Raju, C. S. K., Sandeep, N., Sulochana, C., Sugunamma, V. and Babu, M. J. (2015): Radiation, Inclined Magnetic Field and Cross-Diffusion Effects on Flow over a Stretching Surface, *Journal of the Nigerian Mathematical Society*, Vol. 34, No. 2, pp. 169-180. <http://dx.doi.org/10.1016/j.jnnms.2015.02.003>.
- Sandeep, N. and Sulochana, C. (2016): Dual Solutions of Radiative MHD Nano Fluid over an Exponentially Stretching Sheet with Heat Generation/Absorption, *Applied Nanoscience*, Vol. 6, No. 1, pp. 131-139. <http://dx.doi.org/10.1007/s13204-015-0420-z>.
- Sandeep, N., Sugunamma, V. and Mohan Krishna, P. (2014): Aligned Magnetic Field Radiation and Rotation Effects on Unsteady Hydro Magnetic Free Convection Flow Past an Impulsively Moving Vertical Plate in a Porous Medium, *International Journal of Engineering Mathematics*, Vol. 2014, Article ID: 565162. <http://dx.doi.org/10.1155/2014/565162>.
- Tsai, R., Huang, K. H. and Huang, J. S. (2008): Flow and Heat Transfer over an Unsteady Stretching Surface with Non-Uniform Heat Source. *International Communications in Heat and Mass Transfer*, Vol. 35, No. 10, pp. 1340–1343. <http://dx.doi.org/10.1016/j.icheatmasstransfer.200807.001>.
- Zueco, J. and Beg O. A. (2009): Network Numerical Simulation Applied to Pulsatile Non-Newtonian Flow through a Channel with Couple Stress and Wall Mass Flux Effects, *International Journal of Applied Mathematics and Mechanics*, Vol. 5, No. 2, pp. 1-16.