Journal of Naval Architecture and Marine Engineering
June, 2009
DOI: 10.3329/jname.v6i1.2570 http://www.banglajol.info

EFFECT OF SUCTION AND INJECTION ON MHD THREE DIMENSIONAL COUETTE FLOW AND HEAT TRANSFER THROUGH A POROUS MEDIUM

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Abstract:

The objective of this paper is to analyze the effect of constant suction and sinusoidal injection on three dimensional couette flow of a viscous incompressible electrically conducting fluid through a porous medium between two infinite horizontal parallel porous flat plates in presence of a transverse magnetic field. The stationary plate and the plate in uniform motion are, respectively, subjected to a transverse sinusoidal injection and uniform suction of the fluid .The flow becomes three dimensional due to this type of injection velocity distribution. The governing equations of the flow field are solved by using series expansion method and the expressions for the velocity field, the temperature field, skin friction and the rate of heat transfer in terms of Nusselt number are obtained. The effects of the flow parameters on the velocity field, temperature field, skin friction and the Nusselt number have been studied and analyzed with the help of figures and tables. It is observed that a growing magnetic parameter (M) retards the main velocity (u) and accelerates the cross flow velocity (w_1) of the flow field and a growing permeability parameter (K_p) or suction / injection parameter (R_e) reverses the effect. Both Prandtl number (P_r) and the suction / injection parameter have retarding effect on the temperature field. Further, a growing suction / injection parameter diminishes both the components of skin friction at the wall while the permeability parameter enhances the x-component and reduces the z-component of the skin friction at the wall. The effect of increasing permeability parameter is to enhance the magnitude of rate of heat transfer at the wall while a growing Prandtl number (P_r) reverses the effect.

Keywords: MHD, couette flow, heat transfer, suction, sinusoidal injection, porous medium

NOMENCLATURE

B_0 uniform magnetic field
K^* permeability of the medium
K_p permeability parameter
l distance between the plates
M magnetic parameter
N_u Nusselt number
P_r Prandtl number
p^* pressure
R_e Reynolds number
T^* temperature
T dimensionless temperature
T_0 temperature at the lower plate
T_w temperature at the upper plate
U uniform velocity of the upper plate
u, v, w dimensionless velocity components
u^*, v^*, w^* velocity components along x^*, y^*, z^* direction respectively
V constant suction velocity
$v^*(z^*)$ sinusoidal injection velocity
x, y, z dimensionless Cartesian coordinates
x^*, y^*, z^* Cartesian coordinates

Greek Symbol

α thermal diffusivity
ε a small positive constant ($0<\varepsilon<<1$)
ρ density
ν coefficient of kinematic viscosity
σ electrical conductivity

1. Introduction

The phenomenon of magnetohydrodynamic couette flow with heat transfer has been a subject of growing interest in view of its possible applications in many branches of science and technology and also in industry.

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Channel flows through porous media have several engineering and geophysical applications, such as, in the field of chemical engineering for filtration and purification processes; in the field of agricultural engineering to study the underground water resources; in petroleum industry to study the movement of natural gas, oil and water through the oil channels and reservoirs.

In recent years flow through porous media has become a subject of general interest of many researchers. A series of investigations have been made by different scholars where the porous medium is either bounded by horizontal or vertical surfaces. In view of its varied theoretical and practical interests, Gulab and Mishra (1977) applied the equations of motion derived by Ahmadi and Manvi (1971) to study the unsteady MHD flow of a conducting fluid through a porous medium. Gersten and Gross (1974) studied the effect of periodic variation of suction velocity on flow and heat transfer along a plane wall. Raptis (1983) analyzed the unsteady free convective flow through a porous medium. Kaviany (1985) presented the laminar flow through a porous channel bounded by isothermal parallel plates. Singh and Verma (1995) investigated the three dimensional oscillatory flow through a porous medium with periodic permeability. Attia and Kotb (1996) discussed the MHD flow between two parallel plates with heat transfer. Chamkha (1996) analyzed the unsteady hydromagnetic natural convection in a fluid saturated porous channel.

Three dimensional free convective flow through a porous medium in presence of heat transfer was studied by Ahamed and Sharma (1997). Attia (1997) discussed the transient MHD flow and heat transfer between two parallel plates with temperature dependent viscosity. Krishna *et al*. (2004) presented the hydromagnetic oscillatory flow of a second order Rivlin-Ericksen fluid in a channel. Sharma and Yadav (2005) analyzed the heat transfer through three dimensional Couette flow between a stationary porous plate bounded by porous medium and a moving porous plate. Sharma *et al*. (2005) explained the steady laminar flow and heat transfer of a non-Newtonian fluid through a straight horizontal porous channel in the presence of heat source. Vershney and Singh (2005) presented the effect of periodic permeability on three dimensional free convective flow with heat and mass transfer through a porous medium. Jain *et al*. (2006) investigated the three dimensional couette flow with transpiration cooling through a porous medium in the slip flow regime. Recently, Das *et al*. (2008) analyzed the three dimensional couette flow and heat transfer in presence of a transverse magnetic field.

The study reported herein analyzes the effect of constant suction and sinusoidal injection on three dimensional couette flow of a viscous incompressible electrically conducting fluid through a porous medium between two infinite horizontal parallel porous flat plates in presence of a transverse magnetic field. The stationary plate and the plate in uniform motion are, respectively, subjected to a transverse sinusoidal injection and uniform suction of the fluid .The flow becomes three dimensional due to this type of injection velocity distribution. The governing equations of the flow field are solved by using series expansion method and the expressions for the velocity field, the temperature field, skin friction and the rate of heat transfer i.e. the heat flux in terms of Nusselt number are obtained. The effects of the flow parameters on the velocity field, temperature field, skin friction and heat flux have been studied and analyzed with the help of figures and tables.

2. Formulation of the Problem

Consider the three dimensional flow of a viscous incompressible electrically conducting fluid through a porous medium bounded between two infinite horizontal parallel porous plates in presence of a uniform transverse magnetic field B_0. A coordinate system is chosen with its origin at the lower stationary plate lying horizontally in x^*-z^* plane and the upper plane at a distance l from it is subjected to a uniform velocity U. The y^*-axis is taken normal to the planes of the plates. The lower and the upper plates are assumed to be at constant temperatures T_0 and T_w, respectively, with $T_w > T_0$. The upper plate is subjected to a constant suction velocity V whereas the lower plate to a transverse sinusoidal injection velocity of the form:

$$v^*(z^*) = V\,(1+\varepsilon\cos \pi z^* / l), \tag{1}$$

where ε (<<1) is a very small positive constant quantity, l is taken equal to the wavelength of the injection velocity. Due to this kind of injection velocity the flow remains three dimensional. All the physical quantities involved are independent of x^* for this fully developed laminar flow. Denoting the velocity components u^*, v^*, w^* in x^*, y^*, z^* directions, respectively, and the temperature by T^*, the problem is governed by the following equations:

$$\frac{\partial v^*}{\partial y^*} + \frac{\partial w^*}{\partial z^*} = 0, \tag{2}$$

$$v^* \frac{\partial u^*}{\partial y^*} + w^* \frac{\partial u^*}{\partial z^*} = \nu\left(\frac{\partial^2 u^*}{\partial y^{*2}} + \frac{\partial^2 u^*}{\partial z^{*2}}\right) - \frac{\nu}{K^*}u^* - \frac{\sigma B_0^2}{\rho}u^*, \tag{3}$$

$$v^* \frac{\partial v^*}{\partial y^*} + w^* \frac{\partial v^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \nu\left(\frac{\partial^2 v^*}{\partial y^{*2}} + \frac{\partial^2 v^*}{\partial z^{*2}}\right) - \frac{\nu}{K^*}v^*, \tag{4}$$

$$v^* \frac{\partial w^*}{\partial y^*} + w^* \frac{\partial w^*}{\partial z^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial z^*} + \nu\left(\frac{\partial^2 w^*}{\partial y^{*2}} + \frac{\partial^2 w^*}{\partial z^{*2}}\right) - \frac{\nu}{K^*}w^* - \frac{\sigma B_0^2}{\rho}w^*, \tag{5}$$

$$v^* \frac{\partial T^*}{\partial y^*} + w^* \frac{\partial T^*}{\partial z^*} = \alpha\left(\frac{\partial^2 T^*}{\partial y^{*2}} + \frac{\partial^2 T^*}{\partial z^{*2}}\right), \tag{6}$$

where ρ is the density, σ is the electrical conductivity, p^* is the pressure, K^* is the permeability of the porous medium, ν is the coefficient of kinematic viscosity and α is the thermal diffusivity.
The initial and the boundary conditions of the problem are

$$u^* = 0,\ v^* = V(1+\varepsilon\cos\pi z^* / l),\ w^* = 0,\ T^* = T_0^* \text{ at } y^* = 0,$$

$$u^* = U,\ v^* = V,\ w^* = 0,\ T^* = T_w^* \text{ at } y^* = l. \tag{7}$$

Introducing the following non-dimensional quantities

$$y = \frac{y^*}{l},\ z = \frac{z^*}{l},\ u = \frac{u^*}{U},\ v = \frac{v^*}{V},\ w = \frac{w^*}{V},\ p = \frac{p^*}{\rho V^2},\ T = \frac{T^* - T_0^*}{T_w^* - T_0^*}, \tag{8}$$

Equations (2) - (6) reduce to the following forms:

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{9}$$

$$v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{1}{R_e}\left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) - \frac{u}{R_e K_p} - \frac{M^2}{R_e}u, \tag{10}$$

$$v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{R_e}\left(\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right) - \frac{v}{R_e K_p}, \tag{11}$$

$$v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{R_e}\left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right) - \frac{w}{R_e K_p} - \frac{M^2}{R_e}w, \tag{12}$$

$$v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \frac{1}{R_e P_r}\left(\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right), \tag{13}$$

where

$R_e = \frac{Vl}{\nu}$ (Reynolds number), $M^2 = \frac{\sigma B_0^2}{\rho}\frac{l^2}{\nu}$ (Magnetic parameter), $P_r = \frac{\nu}{\alpha}$ (Prandtl number),

$$K_p = \frac{K^*}{l^2} \text{ (Permeability parameter).} \tag{14}$$

The corresponding boundary conditions now reduce to the following form:

$$u = 0,\quad v = 1+\varepsilon\cos\pi z,\quad w = 0,\quad T = 0 \text{ at } y = 0,$$

$$u = 1,\quad v = 1,\quad w = 0,\quad T = 1 \text{ at } y = l. \tag{15}$$

3. Method of Solution

In order to solve the problem, we assume the solutions of the following form because the amplitude ε ($<<$ 1) of the permeability variation is very small:

$$u(y, z) = u_0(y) + \varepsilon\, u_1(y, z) + \varepsilon^2 u_2(y, z) + \ldots\ldots \tag{16}$$

$v(y, z) = v_0(y) + \varepsilon\, v_1(y, z) + \varepsilon^2 v_2(y, z) + \ldots\ldots$ (17)
$w(y, z) = w_0(y) + \varepsilon\, w_1(y, z) + \varepsilon^2 w_2(y, z) + \ldots\ldots$ (18)
$p(y, z) = p_0(y) + \varepsilon\, p_1(y, z) + \varepsilon^2 p_2(y, z) + \ldots\ldots$ (19)
$T(y, z) = T_0(y) + \varepsilon\, T_1(y, z) + \varepsilon^2 T_2(y, z) + \ldots\ldots$ (20)

When $\varepsilon = 0$, the problem reduces to the two dimensional free convective MHD flow through a porous medium with constant permeability which is governed by the following equations:

$$\frac{dv_0}{dy} = 0, \tag{21}$$

$$\frac{d^2u_0}{dy^2} - v_0 R_e \frac{du_0}{dy} - \left(M^2 + \frac{1}{K_p}\right)u_0 = 0, \tag{22}$$

$$\frac{d^2T_0}{dy^2} - v_0 R_e P_r \frac{dT_0}{dy} = 0. \tag{23}$$

The corresponding boundary conditions become

$u_0 = 0,\quad v_0 = 1,\quad T_0 = 0$ at $y = 0$,
$u_0 = 1,\quad v_0 = 1,\quad T_0 = 1$ at $y = 1$. (24)

The solutions for $u_0(y)$ and $T_0(y)$ under boundary conditions (24) for this two dimensional problem are

$$u_0(y) = \frac{e^{\lambda_1 y} - e^{\lambda_2 y}}{e^{\lambda_1} - e^{\lambda_2}}, \tag{25}$$

$$T_0(y) = \frac{e^{R_e P_r y} - 1}{e^{R_e P_r} - 1}, \tag{26}$$

with $v_0 = 1,\ w_0 = 0,\ p_0 =$ constant, (27)

where

$$\lambda_1 = \frac{1}{2}\left[R_e + \sqrt{R_e^2 + 4\left(M^2 + \frac{1}{K_p}\right)}\right],\ \lambda_2 = \frac{1}{2}\left[R_e - \sqrt{R_e^2 + 4\left(M^2 + \frac{1}{K_p}\right)}\right].$$

When $\varepsilon \neq 0$, substituting Equations (16)-(20) into Equations (9) - (13) and comparing the coefficients of like powers of ε, neglecting those of ε^2, we get the following first order equations with the help of Equation (27):

$$\frac{dv_1}{dy} + \frac{dw_1}{dz} = 0, \tag{28}$$

$$v_1 \frac{du_0}{dy} + \frac{du_1}{dy} = \frac{1}{R_e}\left(\frac{d^2u_1}{dy^2} + \frac{d^2u_1}{dz^2}\right) - \frac{1}{R_e}\left(M^2 + \frac{1}{K_p}\right)u_1, \tag{29}$$

$$\frac{dv_1}{dy} = -\frac{dp_1}{dy} + \frac{1}{R_e}\left(\frac{d^2v_1}{dy^2} + \frac{d^2v_1}{dz^2}\right) - \frac{v_1}{R_e K_p}, \tag{30}$$

$$\frac{dw_1}{dy} = -\frac{dp_1}{dz} + \frac{1}{R_e}\left(\frac{d^2w_1}{dy^2} + \frac{d^2w_1}{dz^2}\right) - \frac{1}{R_e}\left(M^2 + \frac{1}{K_p}\right)w_1, \tag{31}$$

$$v_1 \frac{dT_0}{dy} + \frac{dT_1}{dy} = \frac{1}{R_e P_r}\left(\frac{d^2T_1}{dy^2} + \frac{d^2T_1}{dz^2}\right). \tag{32}$$

The corresponding boundary conditions are

$u_1 = 0,\quad v_1 = \cos\pi z,\quad w_1 = 0,\quad T_1 = 0$ at $y = 0$,
$u_1 = 0,\quad v_1 = 0,\quad w_1 = 0,\quad T_1 = 0$ at $y = 1$. (33)

Equations (28) - (32) are the linear partial differential equations which describe the MHD three-dimensional flow through a porous medium. For solution, we shall first consider three Equations (28), (30) and (31) being independent of the main flow component u_1 and the temperature field T_1. Following Das *et al*. (2008), we assume v_1, w_1 and p_1 of the following form:

$$v_1(y,z) = v_{11}(y)\cos \pi z\,, \tag{34}$$

$$w_1(y,z) = \frac{1}{\pi} v'_{11}(y)\sin \pi z\,, \tag{35}$$

$$p_1(y,z) = p_{11}(y)\cos \pi z\,, \tag{36}$$

where the prime in $v'_{11}(y)$ denotes the differentiation with respect to y. Expressions for $v_1(y, z)$ and $w_1(y, z)$ have been chosen so that the equation of continuity (28) is satisfied. Substituting these expressions (34) - (36) into (30) and (31) and solving under corresponding transformed boundary conditions, we get the solutions of v_1, $w_1$and p_1 as:

$$v_1(y,z) = \frac{1}{A}\left[A_1 e^{m_1 y} + A_2 e^{m_2 y} + A_3 e^{\pi y} + A_4 e^{-\pi y}\right]\cos \pi z\,, \tag{37}$$

$$w_1(y,z) = -\frac{1}{\pi A}\left[A_1 m_1 e^{m_1 y} + A_2 m_2 e^{m_2 y} + A_3 \pi e^{\pi y} - A_4 \pi e^{-\pi y}\right]\sin \pi z\,, \tag{38}$$

$$p_1(y,z) = -\frac{1}{\pi R_e A}\left[A_5 e^{\pi y} + A_6 e^{-\pi y}\right]\cos \pi z\,, \tag{39}$$

where

$$m_1 = \frac{R_e}{2} + \sqrt{\frac{R_e^2}{4} + \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \qquad m_2 = \frac{R_e}{2} - \sqrt{\frac{R_e^2}{4} + \left(\pi^2 + M^2 + \frac{1}{K_p}\right)},$$

$$A = (\pi - m_1)(\pi + m_2)\left[e^{m_2-\pi} + e^{m_1+\pi}\right] + (m_1 + \pi)(m_2 - \pi)\left[e^{\pi+m_2} + e^{m_1-\pi}\right] - 2\pi(m_2 - m_1)\left[e^{m_1+m_2} + 1\right],$$

$$A_1 = -2\pi m_2 + \pi(m_2 + \pi)e^{\pi+m_2} - \pi(\pi - m_2)e^{m_2-\pi}\,, A_2 = 2\pi m_1 - \pi(\pi + m_1)e^{m_1-\pi} - \pi(m_1 - \pi)e^{m_1+\pi}$$

,

$$A_3 = -m_1(m_2 + \pi)e^{m_2-\pi} + m_2(m_1 + \pi)e^{m_1-\pi} - \pi(m_2 - m_1)e^{m_1+m_2}\,,\; A_5 = A_3\left(\pi R_e + M^2 + \frac{1}{K_p}\right),$$

$$A_4 = m_1(m_2 - \pi)e^{m_2+\pi} - m_2(m_1 - \pi)e^{m_1+\pi} - \pi(m_2 - m_1)e^{m_1+m_2}\,,\; A_6 = A_4\left(\pi R_e - M^2 - \frac{1}{K_p}\right).$$

To solve Equations (29) and (32) for u_1 and T_1, we assume

$$u_1(y) = u_{11}(y)\cos \pi z\,, \tag{40}$$

$$T_1(y,z) = T_{11}(y)\cos \pi z\,. \tag{41}$$

Substituting the values of u_1 and T_1 from Equations (40) and (41) into Equations (29) and (32), we get

$$u''_{11} - R_e u'_{11} - \left(\pi^2 + M^2 + \frac{1}{K_p}\right)u_{11} = R_e v_{11} u_0\,, \tag{42}$$

$$T''_{11} - R_e P_r T'_{11} - \pi^2 T_{11} = R_e P_r v_{11} T_0\,, \tag{43}$$

where the primes denote the differentiation with respect to y.
The corresponding boundary conditions are
u_{11}=0, T_{11}=0 at y =0,
u_{11}=0, T_{11}=0 at y =1. 44)

Solving Equations (42) and (43) under the boundary conditions (44) and using Equations (40) and (41), we get

$$u_1 = \Big[\; B_1 e^{-m_3 y} - B_2 e^{m_4 y} + \frac{R_e}{A\left(e^{\lambda_1} - e^{\lambda_2}\right)}\Big\{\; B_3 e^{2m_1 y} + B_4 e^{(m_1+m_2)y} - B_5 e^{2m_2 y} + B_6 e^{(m_1+\pi)y}$$
$$- B_7 e^{(m_1-\pi)y} - B_8 e^{(m_2+\pi)y} + B_9 e^{(m_2-\pi)y} \;\Big\}\Big]\cos \pi z \tag{45}$$

$$T_1 = \Big[D_1 e^{m_5 y} + D_2 e^{m_6 y} + \frac{R_e^2 P_r^2}{A\left(e^{R_e P_r} - 1\right)}\Big\{D_3 e^{(m_1 + R_e P_r)y} + D_4 e^{(m_2 + R_e P_r)y}$$
$$+ D_5 e^{(m_2 + R_e P_r)y} + D_6 e^{(\pi + R_e P_r)y} \Big\}\;\Big]\cos \pi z$$

(46)

where

$$m_3 = -\frac{R_e}{2} + \sqrt{\frac{R_e^2}{4} + \left(\pi^2 + M^2 + \frac{1}{K_p}\right)}, \quad m_4 = \frac{R_e}{2} + \sqrt{\frac{R_e^2}{4} + \left(\pi^2 + M^2 + \frac{1}{K_p}\right)},$$

$$m_5 = \frac{R_e P_r}{2} + \sqrt{\frac{R_e^2 P_r^2}{4} + \pi^2}, \qquad m_6 = \frac{R_e P_r}{2} - \sqrt{\frac{R_e^2 P_r^2}{4} + \pi^2},$$

$$A_7 = \frac{R_e}{A\left(e^{\lambda_1} - e^{\lambda_2}\right)}\left\{B_3 + B_4 - B_5 + B_6 - B_7 - B_8 + B_9\right\},$$

$$A_8 = \frac{R_e}{A\left(e^{\lambda_1} - e^{\lambda_2}\right)}\left\{B_3 e^{2m_1} + B_4 e^{m_1+m_2} - B_5 e^{2m_2} + (1+\pi)B_6 e^{m_1+\pi} - 2B_7 e^{m_1-\pi}\right\},$$

$$A_9 = \frac{R_e^2 P_r^2}{A\left(e^{R_e P_r} - 1\right)}\left\{D_3 + D_4 + D_6 + D_7\right\},$$

$$A_{10} = \frac{R_e^2 P_r^2}{A\left(e^{R_e P_r} - 1\right)}\left\{D_3 e^{m_1+R_eP_r} + D_4 e^{m_2+R_eP_r} + D_6 e^{\pi+R_eP_r} + D_7 e^{-\pi+R_eP_r}\right\},$$

$$B_1 = \left(\frac{A_7 - A_8 e^{m_3}}{1 - e^{m_3+m_4}} - A_7\right), B_2 = \left(\frac{A_7 - A_8 e^{m_3}}{1 - e^{m_3+m_4}}\right), B_3 = \frac{A_1}{m_1(2m_1 + m_3)}, B_4 = \frac{(A_2 - A_1)}{m_2(m_1 + m_2 + m_3)},$$

$$B_5 = \frac{A_2}{(2m_2 + m_3)(2m_2 - m_4)}, B_6 = \frac{A_3}{\pi(m_1 + \pi + m_3)}, B_7 = \frac{A_4}{\pi(m_1 - \pi + m_3)},$$

$$B_8 = \frac{A_3}{(m_2 + \pi + m_3)(m_2 + \pi - m_4)}, B_9 = \frac{A_4}{(m_2 - \pi + m_3)(m_2 - \pi - m_4)}, D_1 = \left(-A_9 - \frac{A_9 - A_{10}e^{-m_5}}{e^{m_6-m_5} - 1}\right),$$

$$D_2 = \left(\frac{A_9 - A_{10}e^{-m_5}}{e^{m_6-m_5} - 1}\right), D_3 = \frac{A_1}{(m_1 + R_eP_r - m_5)(m_1 + R_eP_r - m_6)},$$

$$D_4 = \frac{A_2}{(m_2 + R_eP_r - m_5)(m_2 + R_eP_r - m_6)}, D_5 = \frac{A_3}{(m_2 + R_eP_r - m_5)(m_2 + R_eP_r - m_6)},$$

$$D_6 = \frac{A_3}{(\pi + R_eP_r - m_5)(\pi + R_eP_r - m_6)}, \; D_7 = \frac{A_4}{(R_eP_r - \pi - m_5)(R_eP_r - \pi - m_6)}.$$

Substituting the values of u_0, u_1, T_0 and T_1 from Equations (25), (45), (26) and (46) in Equations (16) and (20), the solutions for velocity and temperature are given by

$$u = \frac{e^{\lambda_1 y} - e^{\lambda_2 y}}{e^{\lambda_1} - e^{\lambda_2}} + \varepsilon\Big[\, B_1 e^{-m_3 y} - B_2 e^{m_4 y} + \frac{R_e}{A\left(e^{\lambda_1} - e^{\lambda_2}\right)}\Big\{\, B_3 e^{2m_1 y} + B_4 e^{(m_1+m_2)y} - B_5 e^{2m_2 y} + B_6 e^{(m_1+\pi)y}$$
$$- B_7 e^{(m_1-\pi)y} - B_8 e^{(m_2+\pi)y} + B_9 e^{(m_2-\pi)y} \,\Big\}\Big]\cos \pi z \,, \tag{47}$$

$$T = \frac{e^{R_eP_r y} - 1}{e^{R_eP_r} - 1} + \varepsilon\Big[D_1 e^{m_5 y} + D_2 e^{m_6 y} + \frac{R_e^2 P_r^2}{A\left(e^{R_eP_r} - 1\right)}\Big\{D_3 e^{(m_1+R_eP_r)y}$$
$$+ D_4 e^{(m_2+R_eP_r)y} + D_5 e^{(m_2+R_eP_r)y} + D_6 e^{(\pi+R_eP_r)y}\Big\}\Big]\cos \pi z \tag{48}$$

3.1. Skin friction

The x- and z- components of skin friction at the wall are given by

$$\tau_x = \left(\frac{du_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{du_1}{dy}\right)_{y=0} \quad \text{and} \quad \tau_z = \varepsilon\left(\frac{dw_1}{dy}\right)_{y=0} \tag{49}$$

3.2. Rate of heat transfer

The rate of heat transfer i.e. heat flux at the wall in terms of Nusselt number (N_u) is given by

$$N_u = \left(\frac{dT_0}{dy}\right)_{y=0} + \varepsilon\left(\frac{dT_1}{dy}\right)_{y=0} \tag{50}$$

4. Results and Discussion

The magnetohydrodynamic three dimensional couette flow of a viscous incompressible electrically conducting fluid through a porous medium between two infinite horizontal parallel porous flat plates with heat transfer has been analyzed. The flow becomes three dimensional due to the injection of a transverse sinusoidal velocity distribution. The governing equations of the flow field are solved by using series expansion method and the expressions for the velocity field, temperature field, skin friction and heat flux in terms of Nusselt number are obtained. The effects of the flow parameters on the velocity field, temperature field have been studied and discussed with the help of velocity profiles shown in Figs. (1)- (5) and temperature profiles shown in Figs. (6)-(7) and the effects of the pertinent parameters on the skin friction and heat flux have been discussed with the help of Tables (1)-(4).

4.1. Main velocity field (*u*)

The main velocity of the flow field suffers a change in magnitude due to the variations in the values of magnetic parameter (M), permeability parameter (K_p) and suction / injection parameter (R_e). The magnetic parameter influences the main velocity of the flow field to a greater extent than the other parameters of the flow field. The effects of these parameters appear in Figs. (1)- (3) respectively.

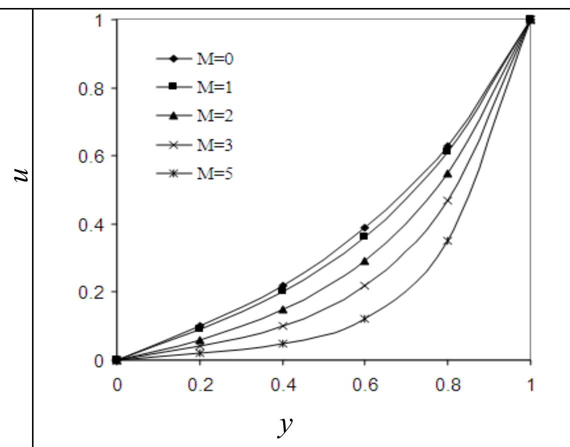

Fig. 1: Velocity profile against y for different values of M with z=0, ε=0.002, K=0.2, Re=0.2

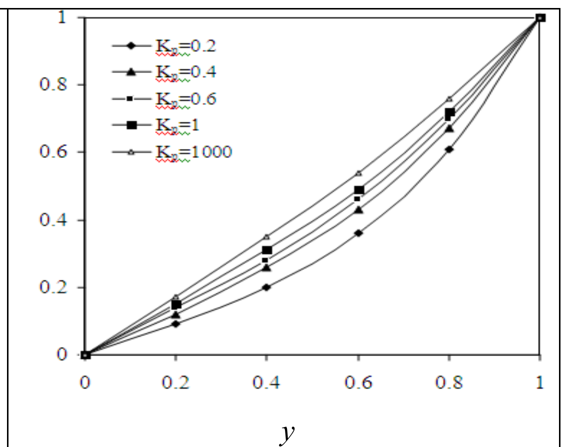

Fig. 2: Velocity profile against y for different values of Kp with z=0, ε=0.002, M=1, Re=0.2

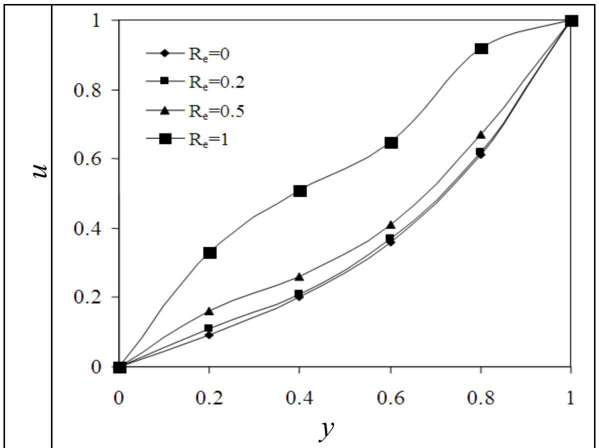

Fig. 3: Velocity profile against y for different values of Re with z=0, ε=0.002, M=1, KP=0.2

4.1.1. Effect of magnetic parameter (m)

The effect of the magnetic parameter on the main velocity of the flow field is shown in Fig. (1). Curve with *M*=0 corresponds to the case of non-MHD flow. For a given value of M, the main velocity increases slowly from zero to its maximum value as we proceed from the inlet section. But in case of non-MHD flow (*M*=0), there is a rapid increase in velocity. Comparing the curves of Fig. (1), it is observed that the effect of growing magnetic parameter is to retard the main velocity of the flow field due to the magnetic pull of the Lorentz force acting on the flow field. Higher this value, the more prominent is the reduction in velocity. Our results closely match with those obtained in case of Das *et al*. (2008).

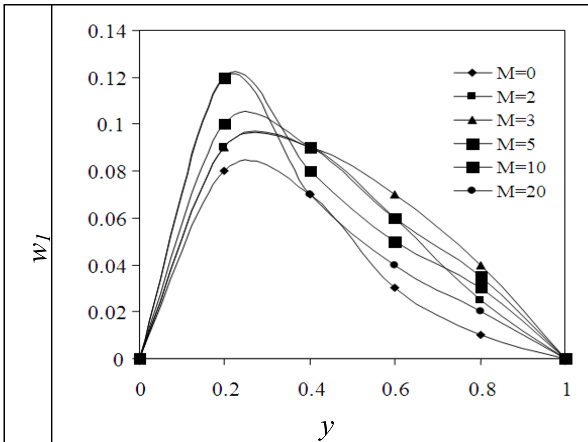

Fig. 4: Cross flow velocity profile against y for different values of M with z=0.5, ε=0.2, Kp=0.2, Re=1

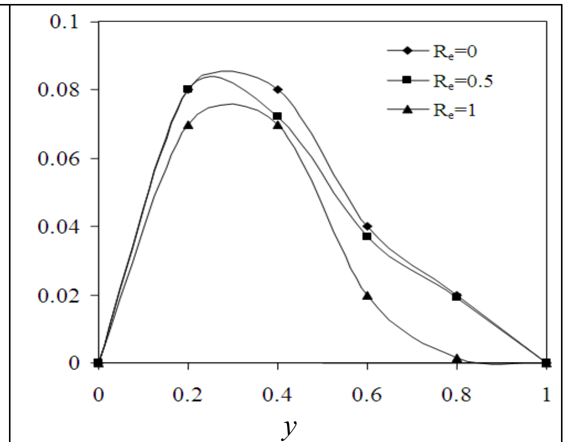

Fig. 5: Cross flow velocity profile against y for different values of Re with z=0.5, ε=0.2, M=1, Kp =0.2

4.2.2. Effect of suction / injection parameter (r_e)

The effect of suction / injection parameter (R_e) on the cross velocity of the flow field is shown in Fig. (5) for three different values of the suction / injection parameter (R_e=0, 0.5, 1).The cross flow velocity is found to decrease with the increase of suction / injection parameter.

4.3. Temperature field (*T*)

The major change in the temperature field is due to the variation of Prandtl number and the suction / injection parameter. These variations are shown in Figs. (6) and (7) respectively. Both suction / injection parameter and the Prandtl number have substantial effect on the temperature field. The temperature profiles agree with the results of Das *et al*. (2008).

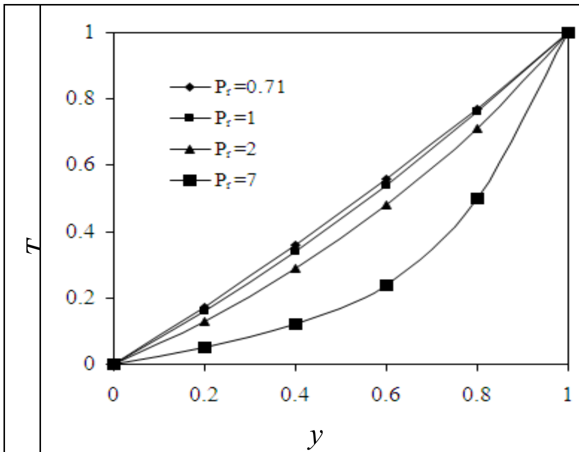

Fig. 6: Temperature profile against y for different values of Pr with z=0, ε=0.002, M=1, Kp=0.5, Re=0.2

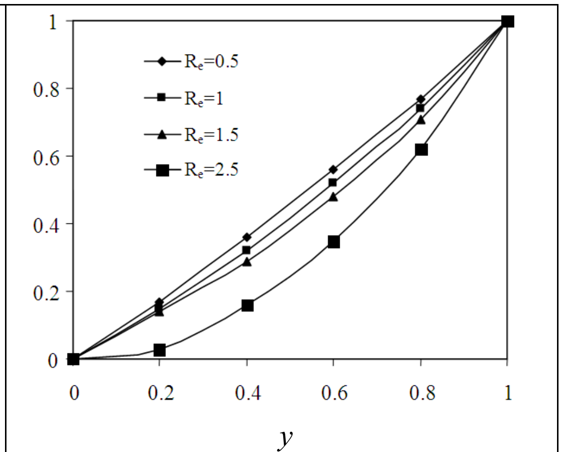

Fig. 7: Temperature profile against y for different values of Re with z=0, ε=0.002, M=1, Pr=0.71

4.3.1. Effect of Prandtl number (P_r)

In Fig. (6) we analyze the effect of Prandtl number (P_r) on the temperature of the flow field. Fig. (6) is a plot of temperature against the non-dimensional distance for different values of P_r (= 0.71, 1, 2, 7). A comparison of the curves of the said figure shows that a growing Prandtl number decreases the temperature of the flow field at all points. With the increase of Prandtl number, the molecular motion of the fluid elements is lowered and therefore, the flow field suffers a decrease in temperature at all points.

4.3.2. Effect of suction / injection parameter (r_e)

Fig. (7) depicts the effect of suction / injection parameter on the temperature of the flow field. In presence of growing suction / injection, the temperature of the flow field is found to decrease. Further, in absence of suction / injection (R_e=0) the temperature profile becomes very much linear.

4.4. Skin friction (τ)

The skin friction at the wall for different values of suction / injection parameter (R_e) and the permeability parameter (K_p) are presented in Tables (1) and (2) respectively. A growing suction / injection parameter reduces both the components of skin friction at the wall while the permeability parameter enhances the x-component and decreases the *z*-component of the skin friction at the wall.

Table 1: Values of skin friction at the wall for different values of suction / injection parameter (R_e) with M=1, K_p=0.2, z=0 & ε =0.002

R_e	τ_x	τ_z
0	0.4261	0.8589
0.01	0.4238	0.8579
0.2	0.3805	0.8376
0.5	0.3121	0.8062
1.0	0.1782	0.7540

Table 2: Values of skin friction at the wall for different values of permeability parameter (K_p) with R_e=0.2, M=1, z=0 & ε =0.002

K_p	τ_x	τ_z
0.2	0.3805	0.8376
0.4	0.5871	0.6991
0.6	0.6599	0.5897
0.8	0.7009	0.4387
1.0	0.7271	0.2063

4.5. Rate of heat transfer (N_u)

The rates of heat transfer i.e. the heat flux in terms of Nusselt number (N_u) for different values of permeability parameter (K_p) and the Prandtl number (P_r) are entered in Tables (3) and (4) respectively. The permeability parameter is found to enhance the magnitude rate of heat transfer at the wall while the Prandtl number (P_r) shows the reverse effect.

Table 3: Values of rate of heat transfer at the wall for different values of permeability parameter (K_p) with R_e=0.5, M=1, P_r=0.71, z=0 & ε =0.002

K_p	N_u
0.2	-1.5152
0.4	-1.5170
0.8	-1.5238
1.0	-1.5284

Table 4: Values of rate of heat transfer at the wall for different values of Prandtl number (P_r) with R_e= 0.5, M =1, K_p=0.2, z=0 & ε =0.002

P_r	N_u
0.71	-1.552
1	-0.7744
2	-0.0152
7	-0.0017

5. Conclusion

The present investigation brings out the following interesting features of physical interest on the main flow velocity, cross flow velocity and temperature of the flow field:

a) The magnetic parameter (M) retards the main velocity (u) at all points of the flow field due to the magnetic pull of the Lorentz force acting on the flow field and accelerates the cross velocity (w_1) of the flow field.

b) A growing permeability parameter (K_p) and the suction / injection parameter (R_e) accelerate the main velocity of the flow field while the effect reverses for cross flow velocity of the flow field.

c) The effect of increasing suction / injection parameter (R_e) and the Prandtl number (P_r) is to reduce the temperature of the flow field at all points.

d) A growing suction / injection parameter diminishes both the components of skin friction at the wall while the permeability parameter enhances the x-component and reduces the z-component of the skin friction at the wall.

e) The effect of increasing permeability parameter is to enhance the magnitude of rate of heat transfer at the wall while a growing Prandtl number (P_r) reverses the effect.

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