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COMBINED EFFECTS OF VISCOUS DISSIPATION AND TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY ON MHD FREE CONVECTION FLOW WITH CONDUCTION AND JOULE HEATING ALONG A VERTICAL FLAT PLATE

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Abstract:

Combined effects of viscous dissipation and temperature dependent thermal conductivity on MHD free convection flow with heat conduction and Joule heating along a vertical flat plate have been described in the present work. The governing boundary layer equations with associated boundary conditions for this phenomenon are converted to non-dimensional form using a suitable transformation. The resulting non-linear partial differential equations are then solved using the implicit finite difference method with Keller–box scheme. The numerical results in terms of the skin friction coefficient, the surface temperature, the velocity and the temperature profiles over the whole boundary layer are shown graphically for different values of the Prandtl number Pr, the magnetic parameter M, the thermal conductivity variation parameter γ, viscous dissipation parameter N and the Joule heating parameter J. Numerical results of the local skin friction co-efficient and the surface temperature profile for different values of N are presented in tabular form.

Keywords: Joule heating, MHD, conduction, variable thermal conductivity, viscous dissipation, natural convection.

NOMENCLATURE

b	plate thickness
C_{fx}	local skin friction coefficient
C_p	specific heat at constant pressure
f	dimensionless stream function
g	acceleration due to gravity
h	dimensionless temperature
H_0	applied magnetic field strength
l, L	length of the plate and reference length respectively
T	temperature of the interface
T_b, T_f	temperature outside the plate and of the fluid respectively
T_∞	temperature of the ambient fluid
$\bar{u}$, $\bar{v}$	velocity component along x, y direction
u , v	dimensionless velocity component along x, y direction
$\bar{x}$, $\bar{y}$	Cartesian co-ordinates
x , y	dimensionless Cartesian co-ordinates

Greek symbols

β	volumetric coefficient of thermal expansion
γ	thermal conductivity variation parameter
η	similarity variable
θ	dimensionless temperature profile
κ_s , κ_f	thermal conductivity of the solid and fluid respectively
κ_∞	thermal conductivity of the ambient fluid
μ , ν	dynamic and kinematic viscosities of the fluid respectively
ρ	density of the fluid
σ	electric conductivity of the fluid
τ_w , ψ	shearing stress and stream function respectively

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1. Introduction

In the study of the structure of stars and planets or in cooling of nuclear reactors, natural convection flow is often encountered. Joule heating in electronics and in physics refers to the increase in temperature of a conductor as a result of resistance to an electrical current flowing through it. But at an atomic level, Joule heating is the result of moving electrons colliding with atoms in a conductor, whereupon momentum is transferred to the atom, increasing its kinetic energy. When similar collisions cause a permanent structural change, rather than an elastic response, the result is known as electro migration. The increase in the kinetic energy of the ions manifests itself as heat and a rise in the temperature of the conductor. Hence energy is transferred from electrical power supply to the conductor and any materials with which it is in thermal contact.

Magnetohydrodynamic (MHD) was originally applied to astrophysical and geophysical problems, where it is still very important, but more recently to the problem of fusion power, where the application is the creation and containment of hot plasmas by electromagnetic forces, since material walls would be destroyed. Astrophysical problems include solar structure, especially in the outer layers, the solar wind bathing the earth and other planets, and interstellar magnetic fields. The primary geophysical problem is planetary magnetism, produced by currents deep in the planet, a problem that has not been solved to any degree of satisfaction. Flow of electrically conducting fluid in presence of magnetic field and the effect of temperature dependent thermal conductivity on MHD flow and heat conduction problems are important from the technical point of view and such types of problems have received much attention by many researchers.

The effect of pressure stress work and viscous dissipation in some natural convection flows has been analyzed by Joshi and Gebhart (1981). Pozzi and Lupo (1988) investigated the coupling of conduction with laminar convection along a flat plate. Hossain (1992) analyzed the viscous and Joule heating effects on MHD free convection flow with variable plate temperature. Merkin and Pop (1996) studied the conjugate free convection on a vertical surface. Elbashbeshy (2000) also discussed the effect of free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field. MHD free convection flow of visco-elastic fluid past an infinite porous plate was investigated by Chowdhury and Islam (2000). Amin (2003) analyzed combined effect of viscous dissipation and Joule heating on MHD forced convection over a non isothermal horizontal cylinder embedded in a fluid saturated porous medium.

Mamun et al. (2007) studied combined effect of conduction and viscous dissipation on MHD free convection flow along vertical flat plate. Alim et al. (2007) investigated the Joule heating effect on the coupling of conduction with MHD free convection flow from a vertical flat plate. Alam et al. (2007) studied viscous dissipation effects on MHD natural convection flow over a sphere in the presence of heat generation. Alim et al. (2008) studied the combined effect of viscous dissipation & Joule heating on the coupling of conduction & free convection along a vertical flat plate. Rahman et al. (2008) investigated the effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat conduction.

In all the aforementioned analyses the combined effects of temperature dependent thermal conductivity and viscous dissipation with Joule heating have not been studied. The present study is to incorporate the idea that the combined effects of variable thermal conductivity and Joule heating on MHD free convection boundary layer flow of electrically conducting fluid with viscous dissipation along a vertical flat plate. The governing boundary layer equations are transformed into a non-dimensional form and the resulting non-linear system of partial differential equations is reduced to local non-similarity equations, which are solved numerically by very efficient implicit finite difference method together with Keller-box scheme (1978). In the following section detailed derivations of the governing equations for the flow and heat transfer and the method of solutions along with the results and discussion are presented.

2. Formulation of the problem

At first we consider a steady, two-dimensional natural convection flow of an electrically conducting, viscous and incompressible fluid with variable thermal conductivity along a vertical flat plate of length l and thickness b (Fig. 1). It is assumed that heat is transferred from the outside surface of the plate, which is maintained at a constant temperature T_b , where $T_b > T_\infty$, the ambient temperature of the fluid. A uniform magnetic field of strength H_0 is imposed along the $\bar{y}$ -axis i.e. normal direction to the surface and $\bar{x}$ - axis is taken along the flat plate.

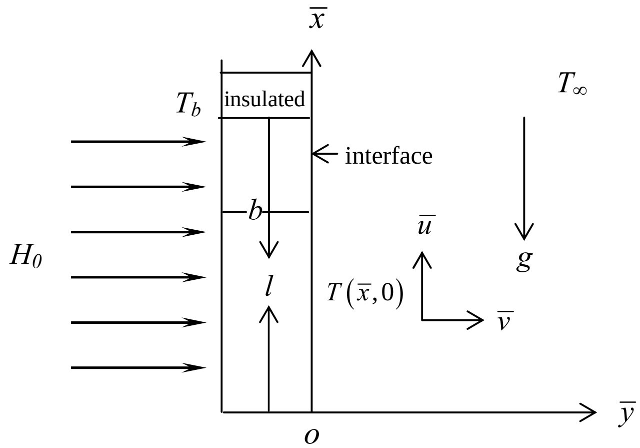

Fig. 1: Physical model and coordinate system

The governing equations of such laminar flow with Joule heating and also thermal conductivity variation along a vertical flat plate under the Boussinesq approximations for the present problem can be written as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T_f - T_\infty) - \frac{\sigma H_0^2 \bar{u}}{\rho} \tag{2}$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}} + \bar{v}\frac{\partial T_f}{\partial \bar{y}} = \frac{1}{\rho C_p}\frac{\partial}{\partial \bar{y}}\left(\kappa_f \frac{\partial T_f}{\partial \bar{y}}\right) + \frac{\sigma H_0^2}{\rho C_p}\bar{u}^2 + \frac{\nu}{C_p}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \tag{3}$$

Here β is coefficient of volume expansion. The temperature dependent thermal conductivity, which is used by Rahman (2008), as follows

$$\kappa_f = \kappa_\infty[1 + \delta(T_f - T_\infty)] \tag{4}$$

where κ_∞ is the thermal conductivity of the ambient fluid and δ is defined as $\delta = \frac{1}{\kappa_f}\left(\frac{\partial \kappa}{\partial T}\right)_f$. The appropriate boundary conditions to be satisfied by the above equations are

$$\left.\begin{array}{ll} \bar{u} = 0, & \bar{v} = 0 \\ T_f = T(\bar{x},0), & \dfrac{\partial T_f}{\partial \bar{y}} = \dfrac{\kappa_s}{b\kappa_f}(T_f - T_b) \end{array}\right\} \quad on \quad \bar{y} = 0,\ \bar{x} > 0$$
$$\bar{u} \to 0, \quad T_f \to T_\infty \quad as \quad \bar{y} \to \infty, \quad \bar{x} > 0 \tag{5}$$

It is observed that Equations (2) and (3) together with the boundary conditions (5) are non-linear partial differential equations. Then Equations (1) to (3) will be non-dimensionalized by using the following dimensionless variables

$$x = \frac{\bar{x}}{L},\ y = \frac{\bar{y}}{L}Gr^{1/4},\ u = \frac{\bar{u}L}{\nu}Gr^{-1/2},\ v = \frac{\bar{v}L}{\nu}Gr^{-1/4},\ \theta = \frac{T_f - T_\infty}{T_b - T_\infty},$$
$$Gr = \frac{g\beta L^3(T_b - T_\infty)}{\nu^2} \tag{6}$$

where $L = \dfrac{\nu^{2/3}}{g^{1/3}}$ is reference length , Gr is the Grashof number, θ is the non-dimensional temperature, $\nu = \mu / \rho$ is kinematic viscosity. Substituting the relations (6) into the Equations (1) to (3), the following non-dimensional equations are obtained

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u\,\frac{\partial u}{\partial x} + v\,\frac{\partial u}{\partial y} + M\,u = \frac{\partial^2 u}{\partial y^2} + \theta \tag{8}$$

$$u\,\frac{\partial \theta}{\partial x} + v\,\frac{\partial \theta}{\partial y} = \frac{1}{\Pr}(1+\gamma\,\theta)\frac{\partial^2 \theta}{\partial y^2} + \frac{\gamma}{\Pr}\left(\frac{\partial \theta}{\partial y}\right)^2 + J\,u^2 + N\left(\frac{\partial u}{\partial y}\right)^2 \tag{9}$$

where $\Pr = \dfrac{\mu\, C_p}{\kappa_\infty}$ is the Prandtl number, $M = \dfrac{\sigma\, H_0{}^2 L^2}{\mu\, Gr^{1/2}}$ is the dimensionless magnetic parameter, $\gamma = \delta(T_b - T_\infty)$ is the non-dimensional thermal conductivity variation parameter, $J = \dfrac{\sigma\, H_0{}^2\, \nu\, Gr^{1/2}}{\rho\, C_p (T_b - T_\infty)}$ is the dimensionless Joule heating parameter and $N = \dfrac{\nu^2\, Gr}{L^2 C_p\,(T_b - T_\infty)}$ is the non-dimensional viscous dissipation parameter. The corresponding boundary conditions (5) then take the following form

$$\begin{aligned} &u = 0,\;\; v = 0,\;\;\; \theta - 1 = (1+\gamma\,\theta)\; p\,\frac{\partial \theta}{\partial y} \quad on \quad y = 0,\;\; x > 0 \\ &u \to 0,\;\;\; \theta \to 0 \qquad as \;\; y \to \infty,\;\; x > 0 \end{aligned} \tag{10}$$

where $p = \left(\dfrac{\kappa_\infty b}{\kappa_s L}\right) Gr^{1/4}$ is the conjugate conduction parameter.

To solve Equations (8) and (9) subject to the boundary conditions (10) the following transformations are proposed by Merkin & Pop (1996)

$$\begin{aligned} \psi &= x^{4/5}\,(1+x)^{-1/20}\, f(x,\eta) \\ \eta &= y\; x^{-1/5}\,(1+x)^{-1/20} \\ \theta &= x^{1/5}\,(1+x)^{-1/5}\, h(x,\eta) \end{aligned} \tag{11}$$

here η is the similarity variable and ψ is the stream function which satisfies the continuity equation and is related to the velocity components in the usual way as $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x$. Thus the following equations are obtained

$$\begin{aligned} &f''' + \frac{16+15x}{20(1+x)}\, f\, f'' - \frac{6+5x}{10(1+x)}\, f'^2 - M\, x^{2/5}(1+x)^{1/10} f' \\ &\qquad\qquad + h = x\,(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}) \end{aligned} \tag{12}$$

$$\begin{aligned} &\frac{1}{\Pr}\, h'' + \frac{\gamma}{\Pr}\left(\frac{x}{1+x}\right)^{1/5} h\, h'' + \frac{\gamma}{\Pr}\left(\frac{x}{1+x}\right)^{1/5} h'^{\,2} + \frac{16+15x}{20(1+x)}\, f\, h' \\ &\quad + J\, x^{7/5}(1+x)^{1/10} f'^2 - \frac{1}{5(1+x)}\, f' h + N\, x\, f''^2 = x\left(f'\,\frac{\partial h}{\partial x} - h'\,\frac{\partial f}{\partial x}\right) \end{aligned} \tag{13}$$

where prime denotes partial differentiation with respect to η. The boundary conditions as mentioned in Equation (10) then take the following form

$$f(x,0) = f'(x,0) = 0$$

$$h'(x,0) = \frac{x^{1/5}(1+x)^{-1/5}h(x,0) - 1}{(1+x)^{-1/4} + \gamma x^{1/5}(1+x)^{-9/20}h(x,0)} \tag{14}$$

$$f'(x,\infty) \to 0, \quad h(x,\infty) \to 0$$

From the process of numerical computation, it is important to calculate the values of the surface shear stress in terms of the skin friction coefficient. This can be written in the following non-dimensional form

$$C_f = \frac{Gr^{-3/4}L^2}{\mu\nu}\tau_w \tag{15}$$

where $\tau_w[= \mu(\partial\bar{u}/\partial\bar{y})_{\bar{y}=0}]$ is the shearing stress. Using the new variables described in (6), the local skin friction co-efficient can be written as

$$C_{fx} = x^{2/5}(1+x)^{-3/20} f''(x,0) \tag{16}$$

The numerical values of the surface temperature profile are obtained from the following relation

$$\theta(x,0) = x^{1/5}(1+x)^{-1/5} h(x,0) \tag{17}$$

3. Method of Solution

This paper investigates the combined effect of thermal conductivity variation and viscous dissipation of electrically conducting fluid with free convection flow along a vertical flat plate in presence of Joule heating and strong magnetic field. Along with the boundary condition (14), the solution of the parabolic differential equations (12) and (13) will be found by using the implicit finite difference method together with Keller-box scheme (1978) which is well documented by Cebeci and Bradshaw (1984) and widely used by Keller and Cebeci and Hossain et al. (1999).

4. Results and Discussion

The values of Prandtl number *Pr* are considered to be 0.73, 1.73, 2.97 and 4.24. The velocity, the temperature, the local skin friction coefficient and the surface temperature profiles obtained from the solutions of Equations (12) and (13) are depicted in Figs. 2 to 11. Numerical computation are carried out for a wide range of magnetic parameter M = 0.01, 1.00, 2.40, 3.50, Joule heating parameter J = 0.01, 0.10, 0.19, 0.23, thermal conductivity variation parameter γ = 0.01, 0.20, 0.60, 0.94 and viscous dissipation parameter N = 0.01, 0.50, 1.00, 1.80.

The interaction of the magnetic field and moving electric charge carried by the flowing fluid induces a force, which tends to oppose the fluid motion. In Fig. 2(a), it is shown that the magnetic field acting along the horizontal direction retards the fluid velocity with γ = 0.02, J = 0.01, N = 0.1 and Pr = 1.73. Here position of peak velocity moves toward the interface with the increasing *M.* From Fig. 2(b), it can be observed that the temperature within the boundary layer increases for the increasing values of M from 0.01 to 3.50. The magnetic field decreases the temperature gradient at the wall and increases the temperature in the flow region.

The effect of thermal conductivity variation parameter γ on the velocity and the temperature profiles within the boundary layer with M = 0.02, J = 0.01, N = 0.01 and Pr = 4.24 are shown in Figs. 3(a) and (b), respectively. The velocity and the temperature profiles increase within the boundary layer with the increasing values of γ. It means that the velocity boundary layer and the thermal boundary layer thickness increase for large value of γ. Moreover, the maximum values of the velocity are 0.2989, 0.3084, 0.3275 and 0.3425 for γ = 0.01, 0.20, 0.60 and 0.94 respectively and each of which occurs at η = 1.0425. It is observed that the velocity increases by 12.73% when γ increases from 0.01 to 0.94. The highest values of the temperature are 0.6925, 0.7015, 0.7160 and 0.7246 for γ = 0.01, 0.20, 0.60, and 0.94 respectively and each of which occurs at the surface. It is observed that the temperature increases by 4.43% when γ increases from 0.01 to 0.94.

Figs. 4(a) and (b) illustrate the velocity and the temperature profiles for different values of Prandtl number *Pr* with M = 0.02, J = 0.01, N = 0.02 and γ = 0.1. From Fig. 4(a), it can be observed that the velocity decreases as well as its position moves toward the interface with the increasing *Pr*. It is seen that the velocity decreases by 52.45% when *Pr* increases from 0.73 to 4.24. From Fig. 4(b), it is seen that the temperature profile shifts downward with the increasing *Pr*. The temperature decreases by 16.57% for increasing values of *Pr* at the interface.

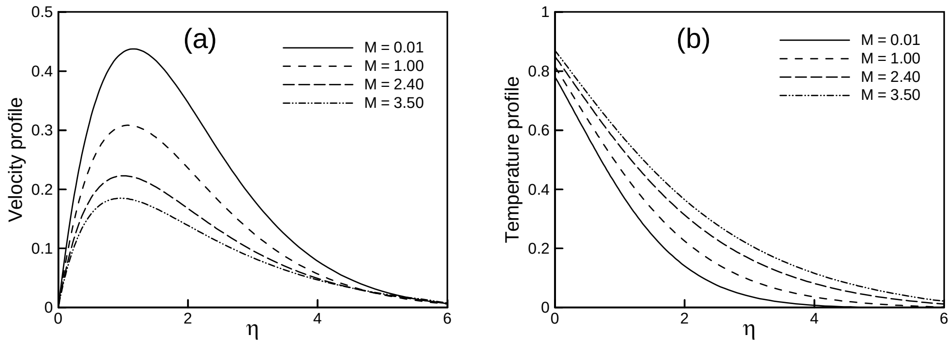

Fig. 2. Variation of (a) Velocity and (b) temperature profiles for varying of M against η with γ = 0.02, J = 0.01, N = 0.1 and Pr = 1.73.

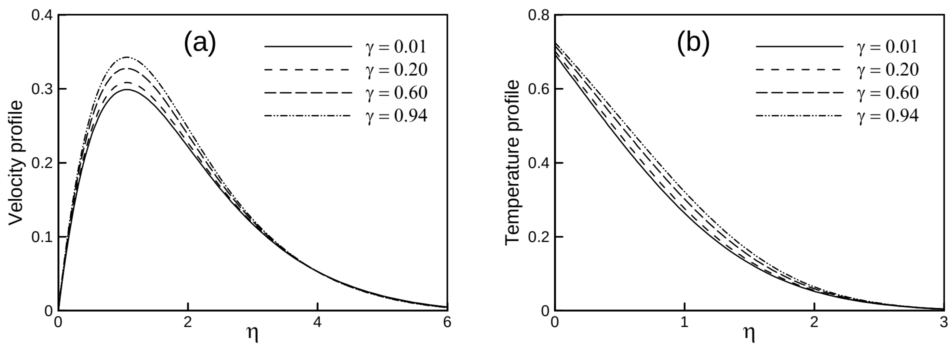

Fig. 3. Variation of (a) Velocity and (b) temperature profiles for varying of γ against η with M = 0.02, J = 0.01, N = 0.01 and Pr = 4.24.

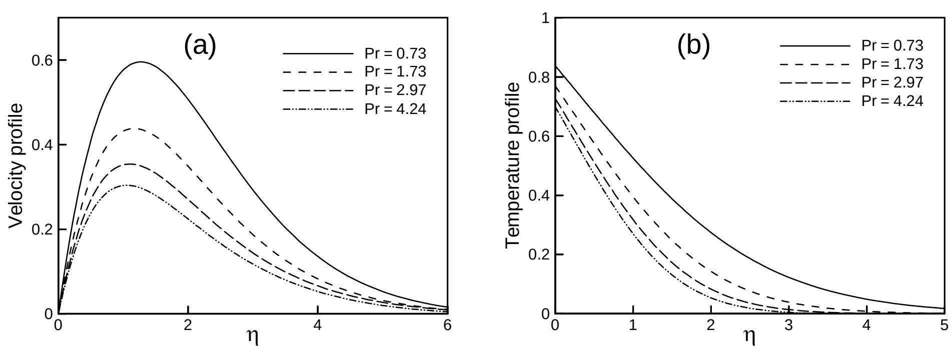

Fig. 4. Variation of (a) Velocity and (b) temperature profiles for varying of Pr against η with γ = 0.1, J = 0.01, N = 0.02 and M = 0.02.

The effect of Joule heating parameter J on the velocity and the temperature profiles within the boundary layer with M = 2.6, γ = 0.01, N = 0.01 and Pr = 1.73 are shown in Fig. 5(a) and (b) respectively. The velocity and the temperature increase within the boundary layer with the increasing values of J. Moreover, the maximum values of the velocity are 0.2128, 0.2146, 0.2165 and 0.2173 for J = 0.01, 0.10, 0.19 and 0.23 respectively and each of which occurs at η = 1.0107. It is seen that the velocity increases by 2.07% when J increases from 0.01 to 0.23. Also the largest values of the temperature are 0.8457, 0.8488, 0.8520 and 0.8534 for J = 0.01, 0.10, 0.19, and

0.23 respectively and each of which occurs at the surface. It is observed that temperature increases by 0.90% when *J* increases from 0.01 to 0.23.

Figs. 6(a) and (b) illustrate the velocity and the temperature profiles for different values of viscous dissipation parameter *N* with *M* = 2.6, *J* = 0.01, *Pr* = 0.73 and γ = 0.02. From Fig. 6(a), it can be observed that the velocity increases as well as its position moves toward the interface with the increasing *N*. It is seen that the velocity increases by 10.18% when *N* increases from 0.01 to 1.80. From Fig. 6(b), it is clear that the temperature profile shifts upward with the increasing *N*. Also the temperature increases by 10.90% for increasing values of *N* at the interface. It is also observed that the temperature at the interface varies due to the conduction within the plate.

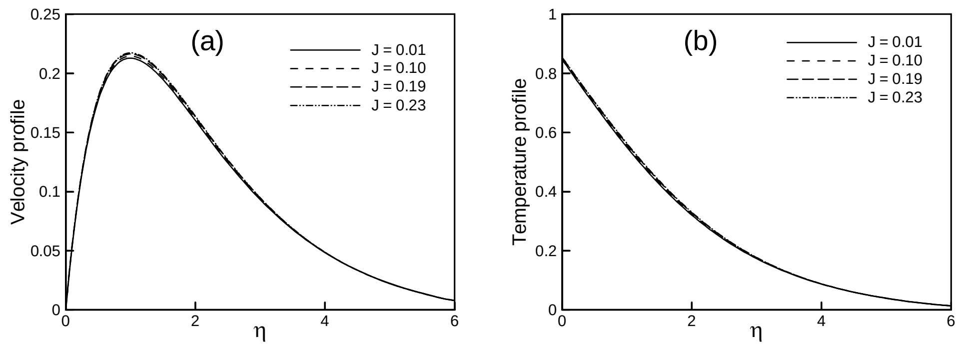

Fig. 5. Variation of (a) Velocity and (b) temperature profiles for varying of *J* against η with γ = 0.01, *N* = 0.01, *Pr* = 1.73 and *M* = 2.6.

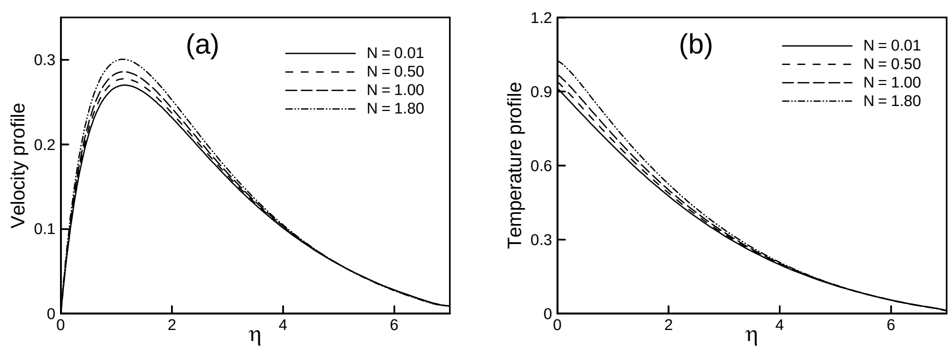

Fig. 6. Variation of (a) Velocity and (b) temperature profiles for varying of *N* against η with γ = 0.02, *Pr* = 0.73, *M* = 2.6 and *J* = 0.01.

The variation of the local skin friction coefficient C_{fx} and the surface temperature θ(x, 0) for different values of *M* with *Pr* = 1.73, *J* = 0.01, *N* = 0.1 and γ = 0.02 at different positions are illustrated in Figs. 7(a) and (b), respectively. It is observed from Fig. 7(a) that the increasing values of *M* from 0.01 to 3.50 leads to a decrease the skin friction factor of 85.87%. Again Fig. 7(b) shows that the surface temperature increases 58.62% due to such values of *M*. The magnetic field acts against the flow and reduces the skin friction coefficient and produces the temperature at the interface.

Figs. 8(a), (b) illustrate the effect of γ on the skin friction coefficient and the surface temperature profile against x with *M* = 0.02, *J* = 0.01, *N* = 0.01 and *Pr* = 4.24. It is seen that the skin friction coefficient increases monotonically along the upward direction of the plate for a particular value of γ. It is also observed that the skin friction coefficient and surface temperature profile increase by 1.07% and 7.92% respectively for the increasing values of γ from 0.01 to 0.94.

The effect of *Pr* on the skin friction coefficient C_{fx} and the surface temperature θ(x, 0) against *x* with *M* = 0.02, *J* = 0.01, *N* = 0.02 and γ = 0.1 are shown in Figs. 9(a) and (b) respectively. It can be shown from Fig. 9(a) that the skin friction coefficient increases monotonically for each value of *Pr*. Fig. 9(b) states that the surface temperature decreases owing to the increasing of *Pr*. It can also be noted that the skin friction and the surface temperature profile decrease by 25.91% and 4.51% for the increasing values of *Pr* from 0.73 to 4.24 respectively.

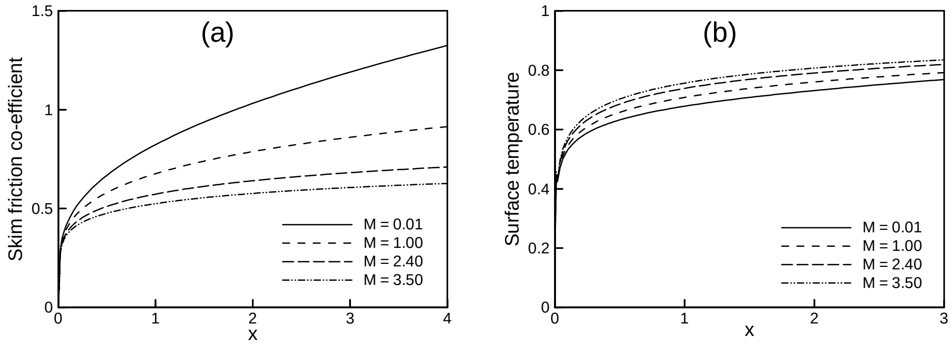

Fig. 7. Variation of (a) Skin friction and (b) surface temperature profile for varying of M against x with γ = 0.02, Pr = 1.73, N = 0.1 and J = 0.01.

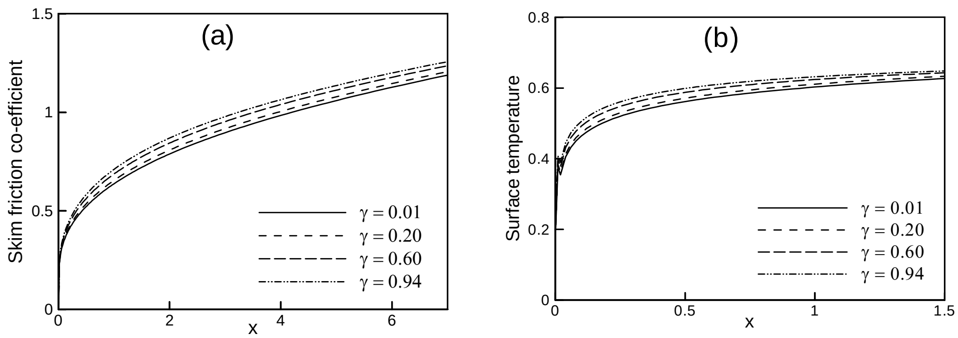

Fig. 8. Variation of (a) Skin friction and (b) surface temperature profile for varying of γ against x with M = 0.02, Pr = 4.24, N = 0.01 and J = 0.01.

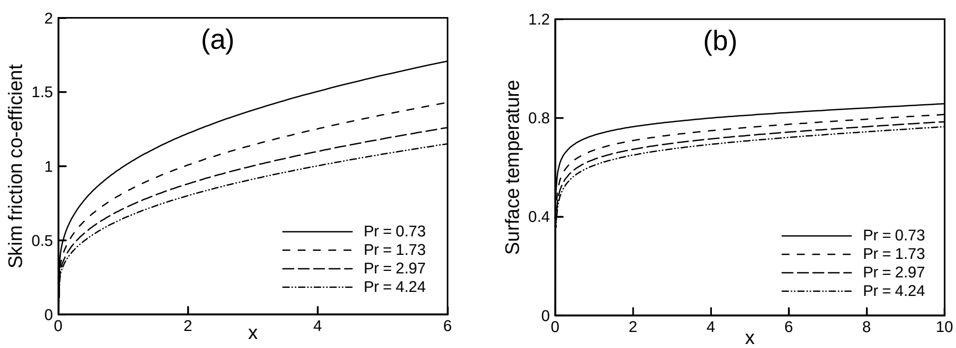

Fig. 9. Variation of (a) Skin friction and (b) surface temperature profile for varying of Pr against x with M = 0.02, γ = 0.1, N = 0.02 and J = 0.01.

Figs. 10(a) and (b) deal with the effect of J on the skin friction coefficient C_{fx} and the surface temperature θ(x, 0) against x with M = 2.6, Pr = 1.73, N = 0.01 and γ = 0.01. It can be noted that the skin friction coefficient and the surface temperature profile increase by 53.11% and 42.18% for the increasing values of J from 0.01 to 0.23 respectively.

Figs. 11(a) and (b) deal with the effect of N on the skin friction coefficient and the surface temperature against x with M = 2.6, Pr = 0.73, J = 0.01 and γ = 0.02. It can be observed from Fig. 11(a) that the local skin friction coefficient increases for the increasing N. It can also be noted that the skin friction coefficient and the surface temperature profile increase by 88.23% and 86.41% for the increasing values of N from 0.01 to 1.80

respectively. This is because the higher value of N accelerates the fluid flow and increases the temperature as mentioned in Figs. 6(a) and (b) respectively.

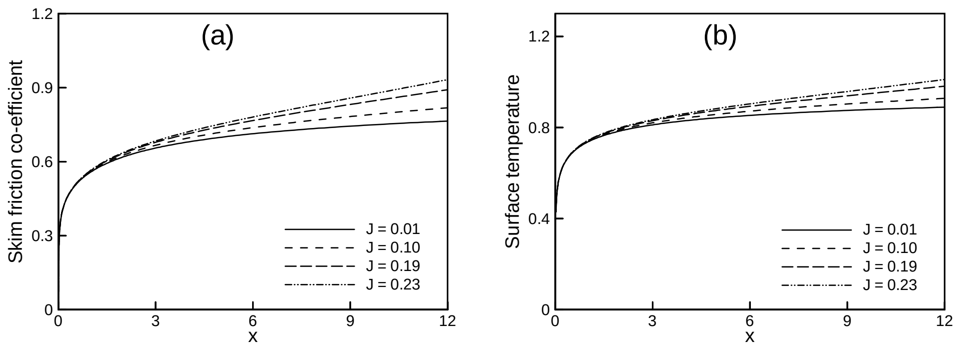

Fig. 10. Variation of (a) Skin friction and (b) surface temperature profile for varying of J against x with M = 2.6, Pr = 1.73, N = 0.01 and γ = 0.01.

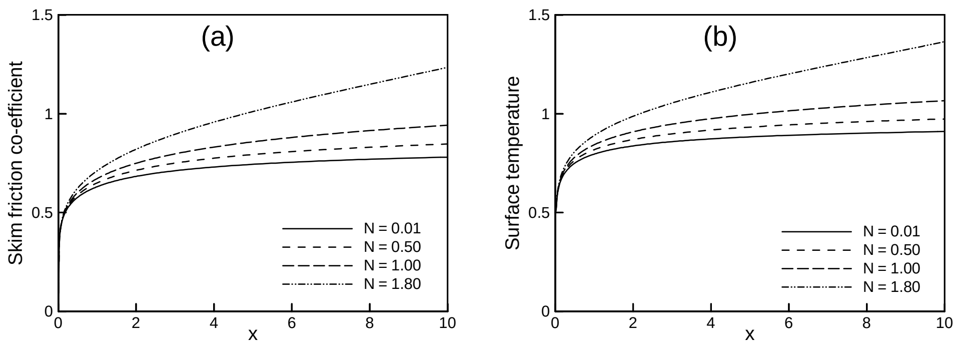

Fig. 11. Variation of (a) Skin friction and (b) surface temperature profile for varying of N against x with M = 2.6, Pr = 0.73, J = 0.01 and γ = 0.02.

Table 1 depicts the comparison of present numerical results of the surface temperature θ (x, 0) with those obtained by Pozzi and Lupo (1988) and Merkin and Pop (1996). Here, the parameters M, γ, N and J are ignored and Pr = 0.733 with $x^{1/5} = \xi$ is chosen. It is clearly shown that there is an excellent agreement among the present results with the solutions of Pozzi and Lupo in 1988 and Merkin and Pop in 1996.

Table 1: Comparison of present numerical results of the surface temperature profile with Pr = 0.733, M = 0.0, J = 0.0, γ = 0.0 and N = 0.0

$x^{1/5} = \xi$	Surface Temperature $\theta(x, 0)$		
	Pozzi and Lupo (1988)	Merkin and Pop (1996)	Present result
0.7	0.651	0.651	0.625
0.8	0.684	0.686	0.664
0.9	0.708	0.715	0.695
1.0	0.717	0.741	0.721
1.1	0.699	0.762	0.744
1.2	0.640	0.781	0.763

In Table 2 the numerical values of the skin friction co-efficient C_{fx} and the surface temperature profile θ (x, 0) against x for different values of N while M = 2.6, Pr = 0.73, J = 0.01 and γ = 0.02. It is observed from the table that the values of the skin friction co-efficient increases at different position of x for N = 0.01, 0.50, 1.00, 1.80. Near the axial position x = 5.6929, the rate of increase of the local skin friction co-efficient is 28.06% as N changes from 0.01 to 1.80. Furthermore, it is noted that the numerical values of the surface temperature profile increase for increasing values of N. It is also seen that the same axial position of x, the rate of increase of the surface temperature profile is 25.20% as N changes from 0.01 to 1.80. The rate of increase in the values of the skin friction coefficient and the surface temperature profile become higher than that of the upstream values.

Table 2: Skin friction coefficient and surface temperature profile against x for varying of N with other controlling parameters M = 2.6, γ = 0.02, J = 0.01 and Pr = 0.73

x	N = 0.01		N = 0.50		N = 1.00		N = 1.80	
	C_{fx}	θ	C_{fx}	θ	C_{fx}	θ	C_{fx}	θ
0.3678	0.5548	0.7288	0.5640	0.7406	0.5739	0.7532	0.5906	0.7750
1.1752	0.6435	0.8060	0.6654	0.8304	0.6900	0.8584	0.7354	0.9112
2.3756	0.6958	0.8469	0.7295	0.8821	0.7698	0.9251	0.8515	1.0146
3.4792	0.7219	0.8664	0.7633	0.9086	0.8149	0.9619	0.9272	1.0818
4.5494	0.7388	0.8790	0.7861	0.9263	0.8469	0.9880	0.9879	1.1362
5.6929	0.7519	0.8888	0.8045	0.9405	0.8738	1.0100	1.0452	1.1882
7.7112	0.7681	0.9008	0.8280	0.9588	0.9103	1.0400	1.1366	1.2723
8.8791	0.7751	0.9060	0.8385	0.9669	0.9274	1.0541	1.1868	1.3193
10.0179	0.7807	0.9103	0.8471	0.9737	0.9421	1.0662	1.2354	1.3652
12.0026	0.7887	0.9163	0.8597	0.9836	0.9642	1.0846	1.3213	1.4478
15.2684	0.7983	0.9236	0.8756	0.9962	0.9941	1.1098	1.4764	1.6009

5. Conclusion

The combined effects of variable thermal conductivity and viscous dissipation on MHD free convection flow with heat conduction and Joule heating along a vertical flat plate have been studied numerically. The coupled effects of natural convection and conduction required that the temperature and the heat flux be continuous at the interface. From the present investigation the following conclusion may be drawn

i) The velocity profile within the boundary layer increases for decreasing values of M, Pr and increasing values of γ, N and J.

ii) The temperature profile within the boundary layer increases for the increasing values of M, γ, J, N and decreasing values of Pr.

iii) The local skin friction coefficient decreases for the increasing values of M, Pr and decreasing values of γ, N and J.

iv) An increase in the values of γ, J, N and M leads to an increase in the surface temperature. Moreover, this profile decreases for increasing values of Pr.

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