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MHD NON-DARCIAN FLOW DUE TO HORIZONTAL STRETCHING SHEET EMBEDDED IN A POROUS MEDIUM WITH THERMAL STRATIFICATION EFFECTS

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Abstract:

The aim of present study is to analyze the non- Darcian effects on unsteady non- linear MHD flow of an incompressible, electrically conducting and viscous fluid over a horizontal stretching sheet embedded in a porous medium with heat source, viscous dissipation and thermal stratification. The dimensionless governing equations have been solved numerically by using 4th order Runge - Kutta method with shooting technique. The effects of pertinent parameters on velocity and temperature are depicted graphically and discussed in details.

***Keywords*:** Viscous dissipation, thermal stratification, heat source, Runge-Kutta method, non-Darcy flow.

NOMENCLATURE

u, v	velocity components
f, f'	dimensionless velocity components
B_0	Magnetic field intensity
k	Permeability of medium
c_f	Form or drag coefficient
Q'	Rate of heat generation/absorption
C_p	Specific heat
a, n	Stretching rate and constant
Re_x	Reynolds number
Pr	Prandtl number
M	Magnetic parameter
q_w	Heat transfer from sheet
Ec	Eckert number
F_s	Local inertia coefficient
T	temperature
x, y	directional coordinate along and normal the stretching sheet
C_f	Skin friction coefficient
Nu	Nusselt number
k_1	Porous parameter
U	Dimension less free stream velocity

Greek symbols

υ	kinematic viscosity
σ	Electric conductivity
ρ	Density
η	Similarity variable
μ	Dynamic viscosity
α	Thermal conductivity
θ	Dimensionless temperature
ψ	Stream function

Subscript

w	Wall
∞	Free stream

1. Introduction

During the last decade fluid flow in porous media has an important bearing in many areas of reservoir engineering, such as petroleum, environmental and groundwater hydrology. Darcy' law also describes the phenomena of fluid flow in porous medium which is valid in a limited range of low velocities but at the high flow rate, inertia effect and turbulence become important and cause non Darcian flow. This type of flow in pours medium has many practical applications such as filtration, transpiration cooling, geothermal and biomechanical process. Many attempts have been made to study the non Darcian flow. Singh et al. (2011) studied the non-Darcian effects on natural convection flow in a vertical channel partially filled with porous

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medium. Taklifi et al. (2012) investigated non-Darcian flow through a non-isothermal vertical surface embedded in a porous medium. A study of Variable Fluid Viscosity of non-Darcian flow over a moving vertical plate in a porous medium with suction and viscous dissipation is carried out by Animasaun et al. (2014) and an analysis by Pal and Mondal (2010, 2011) made to study the effects of variable viscosity, Soret-Dufour, chemical reaction and thermal radiation on MHD non-Darcy unsteady mixed convective heat and mass transfer over a stretching sheet. Singh et al. (2011) dealt with Darcy–Brinkman–Forchheimer extended model with heat generation/absorption. Finite element analysis of hydromagnetic flow and heat transfer of a heat generation fluid over a surface embedded in a non-Darcian porous medium in the presence of chemical reaction was studied by Mohamed et al. (2009). Santhosh et al. (2009) analyzed non-Darcy models for mixed convection in a porous cavity using a multigrid approach and similar model was used by Hooman et al. (2008) to discuss about the effects of temperature-dependent viscosity on Be´nard convection in a porous medium.

Stratified fluids are found everywhere in nature. There are many examples of stratified fluids including thermal stratification of reservoirs and oceans, salinity stratification rivers, groundwater reservoirs, and oceans, heterogeneous mixtures in industrial, food, and manufacturing processing, density stratification of the atmosphere.

Basically the term stratification is related with the variation in the density field and density differences causes due to temperature and pressure differences. Thermal stratification effects on nonlinear hydromagnetic flow over a vertical stretching surface with a power-law velocity was studied by Kandasamy et al. (2007) and effects of thermal stratification on flow and heat transfer past a porous vertical stretching surface presented by Mukhopadhyay et al. (2012). Although the effect of stratification of the medium in a porous medium is important, very little work has been reported in the literature. Saha et al. (2004) considered the thermally stratified media for natural convection flow with combined buoyancy effects. Madhu et al. (2015) discussed the effects of viscous dissipation and thermal stratification on chemical reacting fluid flow over a vertical stretching surface with heat source.

The present study is an analysis on MHD flow using Darcy Forchheimer model with thermal stratification over a stretching sheet. Some similarity transformations are used to solve the governing equations by 4th order Runge – Kutta integration method with shooting technique and the effects of various parameters such as Eckert number, magnetic field parameter, Grashof number, thermal stratification parameter, porous parameter and inertia parameter are shown in figures and analyzed in detail.

2. Mathematical Formulation

Considered a two dimensional incompressible fluid flow over a stretching sheet embedded in porous medium where the x axis taken in the direction of flow along stretching sheet and y axis is normal to it. A magnetic field of strength B_0 is applied transversely to the x axis and the induced magnetic field due to motion of electrically conducting fluid is negligible. It is assumed that sheet is stretched with velocity $U(x) = ax$. Moreover, the sheet is maintained at the temperature T_w and T_∞ is the temperature of ambient fluid. Under the above stated physical situation the governing equations of continuity and energy under the Darcy-Forchheimer model are:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\upsilon}{k}u - \frac{c_f}{\sqrt{k}}u^2 \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 u}{\partial y^2} + \frac{Q^{'}}{\rho C_p}(T - T_\infty) + \frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

where u and v are velocity component in x and y directions respectively, υ is the kinematic viscosity, σ is

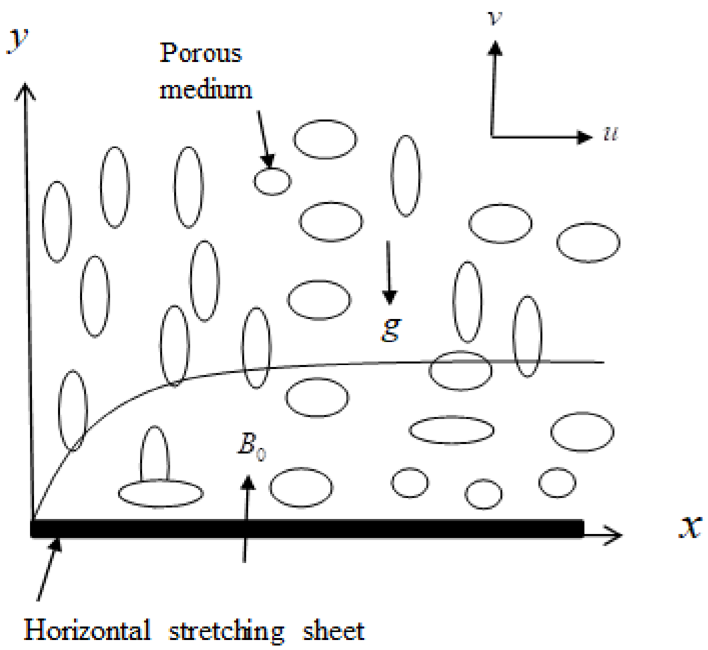

Fig. 1: Physical model for mathematical formulation

the electric conductivity, ρ is the density of the fluid, k is the permeability of the medium, c_f is the form of drag coefficient, which is independent of viscosity and other physical properties of the fluid , α is the thermal conductivity of porous medium, Q' is the rate of heat generation/absorption, μ is the dynamic viscosity and C_p is the specific heat at constant pressure. The flow is subjected to the following boundary conditions

$$u = U(x) = ax, \quad v = 0, \quad T = T_w(x) \qquad \text{at} \quad y = 0$$

$$u = 0, \quad T = T_\infty(x) = (1-n)T_0 + nT_w(x) \qquad \text{at} \quad y \to \infty \tag{4}$$

where a is constant called stretching rate and n is a constant such that $0 \le n \le 1$. The n defined as thermal stratification parameter is equal to $\frac{m_1}{m_1+1}$ (Nakayama et al.,1989), where m_1 is a constant.

To solve the governing equation (see Acharya et al., 1999) similarity transformations are used,

$$\psi = \left(\upsilon x U(x)\right)^{1/2} f(\eta)$$

$$\eta = \left(\frac{U(x)}{\upsilon x}\right)^{1/2} y \tag{5}$$

The velocity components are given by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

The dimensionless parameters are defined as follows:

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{7}$$

$$\mathrm{Re}_x = \frac{U\,x}{\upsilon} \quad \text{Reynolds number} \tag{8}$$

$$\mathrm{Pr} = \frac{\mu C_p}{k} = \frac{\upsilon}{\alpha} \quad \text{Prandtl number} \tag{9}$$

$$M = \frac{\sigma B_0^2}{\rho a} \quad \text{Magnetic parameter} \tag{10}$$

$$Q = \frac{Q'}{\rho C_p a} \quad \text{Heat generation/absorption parameter} \tag{11}$$

$$Ec = \frac{U^2}{C_p (T_w - T_\infty)} \quad \text{Eckert number} \tag{12}$$

$$k_1 = \frac{\upsilon}{k\,a} \quad \text{Porous parameter} \tag{13}$$

$$Fs = \frac{c_f}{\sqrt{k}} x \quad \text{Local inertia parameter} \tag{14}$$

The wall shear stress τ_w, may be expressed in terms of the local skin friction coefficient, C_f as

$$C_f (\mathrm{Re}_x)^{1/2} = f''(0) \tag{15}$$

The local Nusselt number which are defined as

$$Nu = \frac{x q_w}{k(T_w - T_\infty)} \tag{16}$$

where q_w is the heat transfer from the sheet is given by

$$q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{17}$$

Using the non-dimensional variables, we get from Equations (16) and (17) as

$$Nu / \mathrm{Re}_x^{1/2} = -\theta'(0) \tag{18}$$

Also temperature variation of the surface is considered $T_w - T_\infty = Nx^n$ and Equations (2)-(3) are transformed to

$$f''' + ff'' - (f')^2 - (M + k_1) f' - Fs(f')^2 = 0 \tag{19}$$

$$\theta'' - \mathrm{Pr}\, nf' \left[\theta + \frac{n}{1-n}\right] + f\,\mathrm{Pr}\,\theta' + Q\,\mathrm{Pr}\,\theta + \mathrm{Pr}\,Ec(f'')^2 = 0 \tag{20}$$

The boundary conditions (4) become

$$\begin{aligned} &f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1, \\ &f'(\infty) = 0\ \ \theta(\infty) = 0 \end{aligned} \tag{21}$$

3. Results and Discussions

The transformed governing equations (19)-(20) with boundary conditions (21) are integrated by using the 4th order Runge - Kutta method with shooting technique on MATLAB R2007b (7.5.0.342)(32 bit). In a shooting method the boundary value problem first converted into initial value problem, the missing terms in initial condition, $f''(0)$ and $\theta'(0)$ are assumed and taking initial guesses for $f''(0)$ and $\theta'(0)$ until the boundary conditions at infinite are satisfied. We select the appropriate finite value $\eta_\infty = 8$ which depending upon the physical parameters.

The influence of the various physical parameters are shown graphically in Fig. 2 – Fig. 13 by considering $Ec = 0.01,\ M = 0.5,\ \Pr = 0.7,\ Fs = 0.1,\ n = 0.1,\ k_1 = 0.5,$ and $Q = -0.5.$

Figs. 2 - 3 illustrate the effects of magnetic parameter M on velocity and temperature profiles. It is clear in Fig. 2 that if magnetic field parameter increases, the fluid velocity $f'(\eta)$ decreases and also clear in Fig. 3 that the temperature $\theta(\eta)$ increases with an increasing of magnetic field parameter. As M increases, the retarding force, which resists the flow, also increases and resultant there is a deceleration in velocity and increment in temperature.

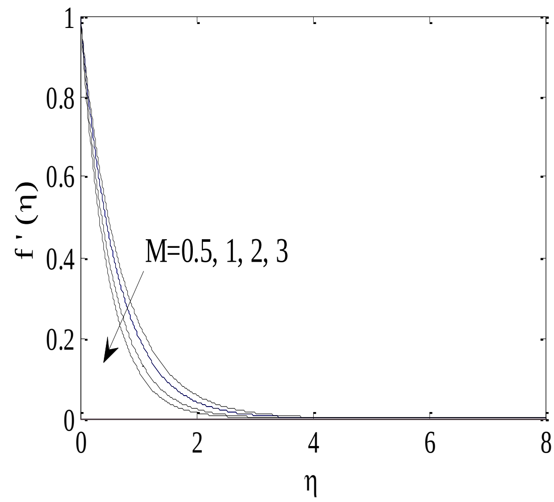

Fig. 2: Velocity profile for different values of M

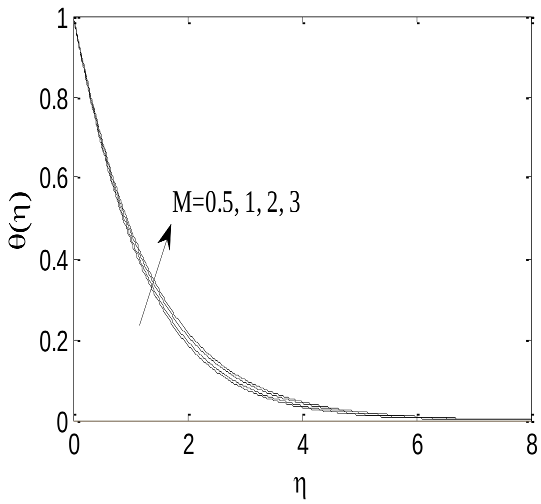

Fig. 3: Temperature profile for different values of M

The influence of inertia parameter Fs on the velocity and temperature are plotted in Figs. 4- 6. It is obvious that fluid normal $f(\eta)$ and axial velocity $f'(\eta)$ decrease with an increasing of inertia parameter but temperature $\theta(\eta)$ is increasing and the increment is very small. As inertia parameter increases the resistance of the flow increases, causing the fluid flow in porous medium slow down and the temperature increases.

Figs. 7-9 show the effect of the porous parameter k_1 on fluid normal velocity $f(\eta)$, velocity $f'(\eta)$ and temperature $\theta(\eta)$. It is clear from Figs. 7 and 8 that the fluid velocity $f(\eta)$ and $f'(\eta)$ decrease with an increase in porous parameter also it appears in Fig.9 that the temperature increases with an increasing of porous parameter. The increase in porous parameter of the fluid is due to enhancement of viscosity of the fluid or decrease in permeability of porous medium. This will result a reduction in velocity of fluid and enhancement in temperature.

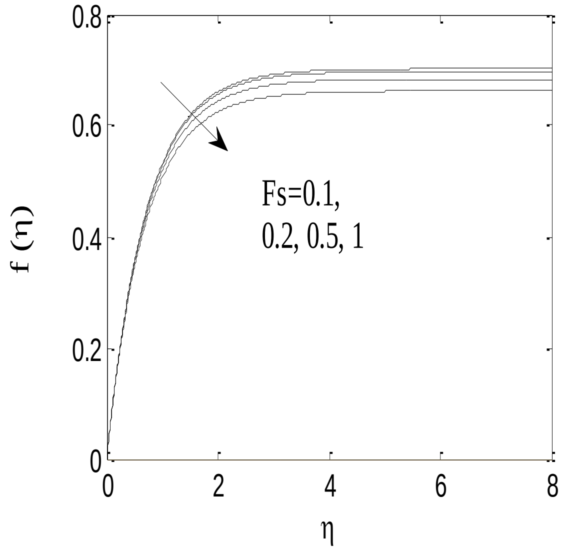

Fig. 4: Normal velocity $f(\eta)$ profile for different values of Fs

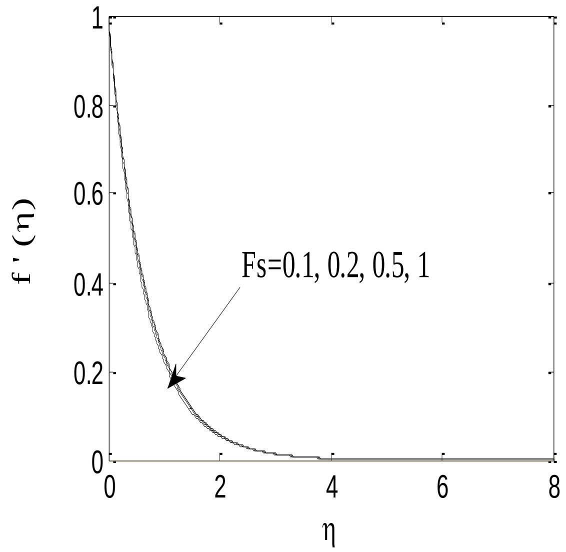

Fig. 5: Velocity profile $f^{'}(\eta)$ for different values of Fs

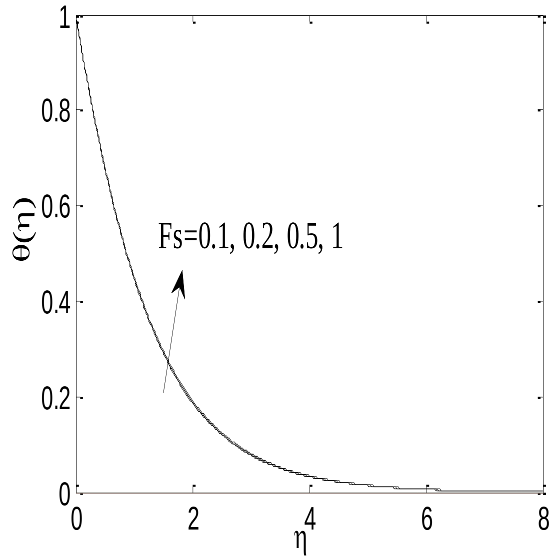

Fig. 6: Temperature profile for different values of Fs

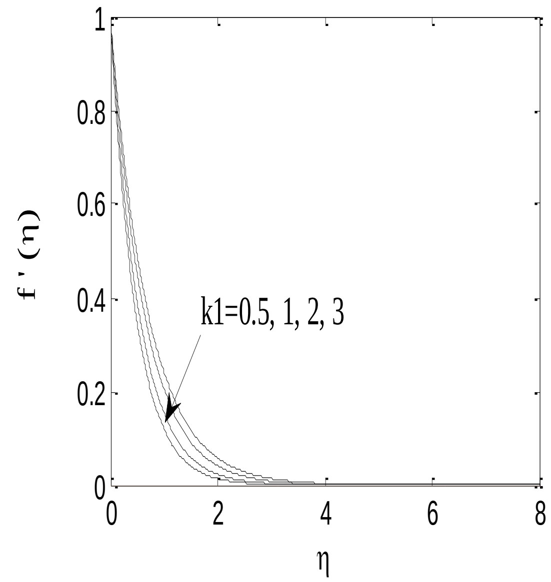

Fig.7: Velocity profile $f^{'}(\eta)$ for different values of k_1

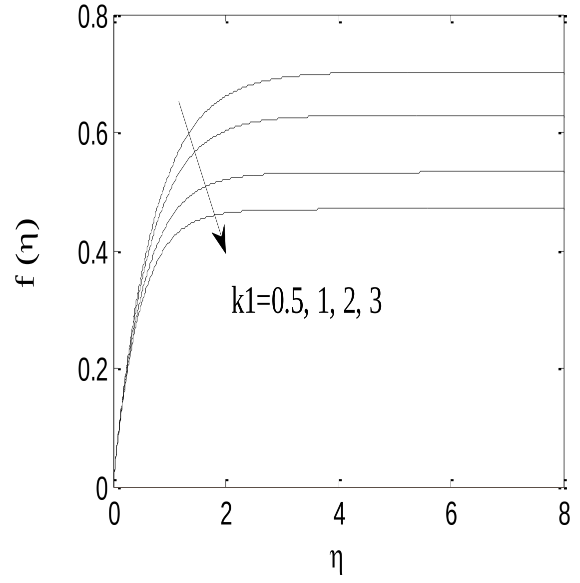

Fig.8: Velocity profile $f(\eta)$ for different values of k_1

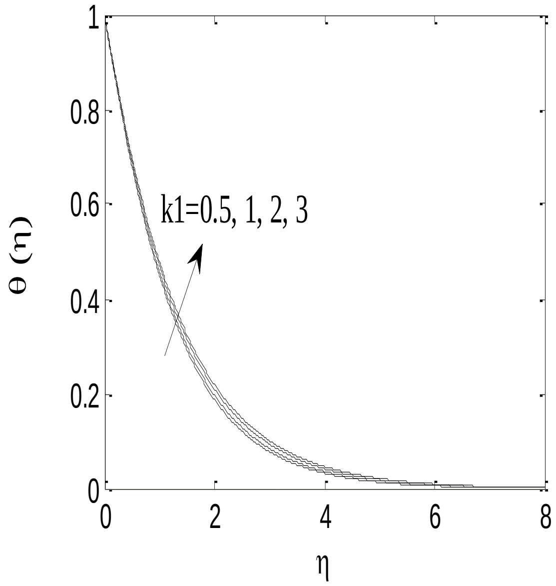

Fig. 9: Temperature profile for different values of k_1

Fig. 10 shows the effect of thermal stratification parameter n on temperature profile $\theta(\eta)$. It is observed that the temperature decreases with an increase in the thermal stratification parameter. This is due to the fact that increase in thermal stratification parameter means increase in free stream temperature or decrease in surface temperature which results decrease in the thermal boundary layer thickness.

Fig. 11 reveals the effect of Prandtl number on temperature profile $\theta(\eta)$ respectively. It is shown that the temperature decreases with an increase in Prandtl number. As expected, viscous force increases and thermal diffusivity reduces with increase of Prandtl number. The influence of heat absorption/generation parameter on the temperature is plotted in Fig. 12. It is seen that temperature increased as the heat absorption/generation parameter increases.

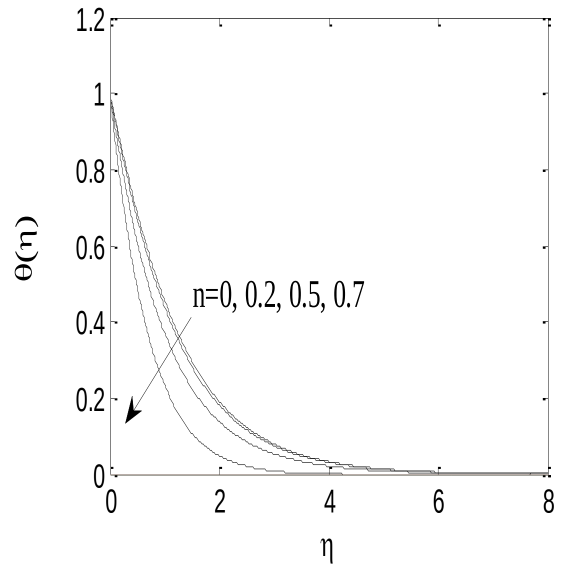

Fig.10 Temperature profile for different values of n

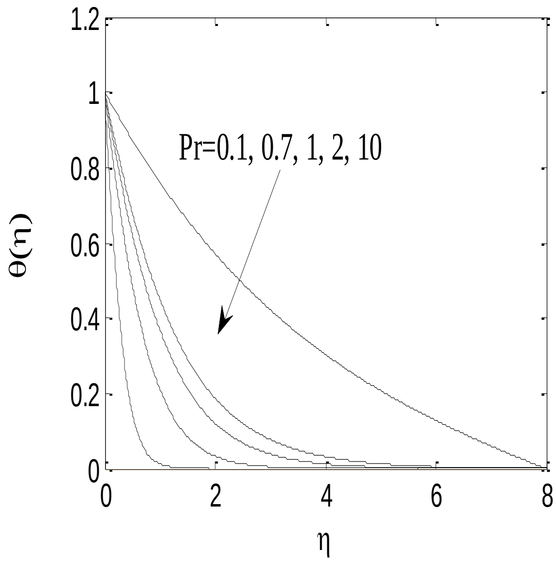

Fig.11 Temperature profile for different values of Pr

Fig. 13 depicts the temperature profile $\theta(\eta)$ for different values of Eckert number. It is noticed that an increase in the Eckert number results an increase in the temperature profile.

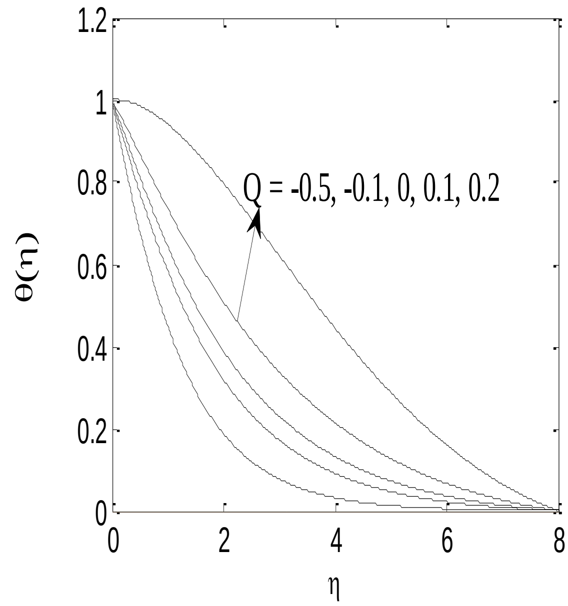

Fig. 12: Temperature profile for different values of Q

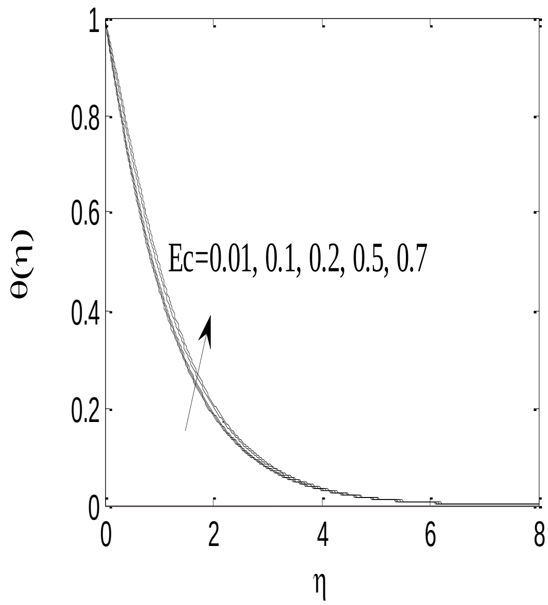

Fig. 13: Temperature profile for different values of Ec

4. Conclusion

In this paper, an analysis for MHD non-Darcian flow due to stretching sheet embedded thermally stratified medium with viscous dissipation, heat source and thermal stratification is presented. The main finding of the present study is summarized as follows:

- As increasing the inertia, porous and magnetic parameters, reduced the velocity of fluid but enhanced the temperature.
- The higher values of the thermal stratification parameter result in lower temperature profile.
- The temperature of the fluid increases with increase in the values of Eckert number and heat generation/absorption parameter but decrease with an increase in Prandtl number.

The current study of thermal stratification effects on MHD non-Darcy flow over a horizontal stretching sheet embedded in a porous medium has wide industrial applications as well as in many branches of engineering.

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