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# NATURAL CONVECTION FLOW IN A SQUARE CAVITY WITH INTERNAL HEAT GENERATION AND A FLUSH MOUNTED HEATER ON A SIDE WALL

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## Abstract:

*In this study natural convection flow in a square cavity with heat generating fluid and a finite size heater on the vertical wall have been investigated numerically. To change the heat transfer in the cavity, a heater is placed at different locations on the right vertical wall of the cavity, while the left wall is considered to be cold. In addition, the top and bottom horizontal walls are considered to be adiabatic and the cavity is assumed to be filled with a Bousinessq fluid having a Prandtl number of 0.72. The governing mass, momentum and energy equations along with boundary conditions are expressed in a normalized primitive variables formulation. Finite Element Method is used in solution of the normalized governing equations. The parameters leading the problem are the Rayleigh number, location of the heater, length of the heater and heat generation. To observe the effects of the mentioned parameters on natural convection in the cavity, we considered various values of heater locations, heater length and heat generation parameter for different values of Ra varying in the range $10^2$ to $10^5$. Results are presented in terms of streamlines, isotherms, average Nusselt number at the hot wall and average fluid temperature in the cavity for the mentioned parameters. The results showed that the flow and thermal fields through streamlines and isotherms as well as the rate of heat transfer from the heated wall in terms of Nusselt number are strongly dependent on the length and locations of the heater as well as heat generating parameter.*

***Keywords:*** Finite Element Method, location of heater, length of heater, natural convection and square cavity.

## NOMENCLATURE

| | |
|---|---|
| $C_p$ | Specific heat of fluid at constant pressure |
| $g$ | Gravitational acceleration ($ms^{-2}$) |
| $Gr$ | Grashof number |
| $h$ | Convective heat transfer coefficient ($Wm^{-2}K^{-1}$) |
| $H$ | Height of the cavity (m) |
| $\kappa$ | Thermal conductivity of fluid ($Wm^{-1}K^{-1}$) |
| $l$ | Dimensional length of the heater (m) |
| $L$ | Dimensionless length of the heater |
| $N$ | The non-dimensional distances either along $X$ or $Y$ direction acting normal to the surface |
| $Nu$ | Nusselt number |
| $p$ | Dimensional pressure ($Nm^{-2}$) |
| $P$ | Dimensionless pressure |
| $Pr$ | Prandtl number |
| $q'''$ | Volumetric rate of heat generation ($W/m^3$) |
| $Q_0$ | Heat generation constant |
| $Ra$ | Rayleigh number |
| $T$ | Dimensional temperature (K) |
| $\Delta T$ | Dimensional temperature difference (K) |
| $u$, $v$ | Dimensional velocity components ($ms^{-1}$) |
| $U$, $V$ | Dimensionless velocity components |
| $\bar{V}$ | Cavity volume ($m^3$) |
| $x$, $y$ | Cartesian co-ordinates (m) |
| $X$, $Y$ | Dimensionless Cartesian coordinates |
| *Greek symbols* | |
| $\alpha$ | Thermal diffusivity ($m^2s^{-1}$) |
| $\beta$ | Thermal expansion coefficient ($K^{-1}$) |
| $\nu$ | Kinematic viscosity ($m^2s^{-1}$) |
| $\theta$ | Non dimensional temperature |

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$\rho$ Density of the fluid (kgm$^{-3}$)

$\mu$ Dynamic viscosity of the fluid (m$^2$s$^{-1}$)

$\lambda$ The internal heat generation parameter

$\psi$ Stream function

*Subscripts*

av average

$h$ heated wall

$c$ cold

*Abbreviation*

Pt Top position

Pm Middle position

Pb Bottom position

## 1. Introduction

Natural convection in both externally and internally heated enclosures has been conducted to investigate the flow and thermal fields as well as heat transfer characteristics in closed cavities in the past several decades. The study of such phenomena is important due to its relevance to a wide variety of application area in engineering and science. Some of these include in nuclear reactor cores, fire and combustion modelling, electronic chips and semiconductor wafers etc. A number of studies have been conducted to investigate the flow and heat transfer characteristics in closed cavities in the past. Acharya (1985) studied natural convection in an inclined enclosure containing internal energy sources and cooled from below. Chadwick *et al*. (1991) presented natural convection in a discretely heated enclosure for single and multiple heater configurations experimentally and analytically. Fusegi *et al.* (1992) performed a numerical study on natural convection in a square cavity by using a high-resolution finite difference method. The authors considered differentially heated vertical walls and uniform internal heat generation in the cavity. Natural convection heat transfer in rectangular cavities heated from the bottom had been investigated by Hasnaoui *et al*. (1995). Ganzarolli and Milanez (1995) investigated the natural convection in rectangular enclosures heated from below and cooled from the sides. Turkoglu and Yucel (1995) made a numerical study using control volume approach for the effect of heater and cooler locations on natural convection on cavities. The authors indicated that for a given cooler position, average Nusselt number increases as the heater is moved closer to the bottom horizontal wall. Aydin and Yang (2000) numerically investigated natural convection of air in a two-dimensional rectangular enclosure with localized heating from below and symmetrical cooling from the side walls. Deng *et al*. (2002) studied the interaction between discrete heat sources in horizontal natural convection enclosures. Rahman and Sharif (2000) studied the effects of aspect ratio in detail for an inclined rectangular enclosure. Hossain and Rees (2005) considered unsteady laminar natural convection flow of water subject to density inversion in a rectangular cavity formed by isothermal vertical walls with internal heat generation. Saeid and Yaacob (2006) studied natural convection in a square cavity with special sidewall temperature variation. Dixit and Babu (2006) investigated natural convection of air in a square cavity by using lattice Boltzmann method. Varol *et al* (2006) studied natural convection in a triangular enclosure with flush mounted heater on the wall. Vajravelu and Hadjinicolaou (1993) studied heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation.

On the basis of the literature review, it appears that no work was reported on the natural convection flow in a closed cavity with internal heat generation and a flush mounted heater on a sidewall. Therefore, due to its practical interest in the engineering fields, the topic needs to be further explored. Hence, the purpose of the present study is to numerically examine the effects of the heater positions, length and heat generation parameter on the flow and thermal fields as well as heat transfer characteristics in the cavity. The obtained numerical results are presented graphically in terms of streamlines, isotherms, average Nusselt number at the heated wall and average fluid temperature in the cavity. In addition, the average Nusselt number at the heated wall and average fluid temperature in the cavity are also presented in tabular form to illustrate the interesting features of the solution.

## 2. Formulation of the Problem

Consider a steady-state two-dimensional square cavity of height $H$ filled with viscous incompressible fluid as shown in Fig. 1. It is assumed that the top and bottom walls of the cavity are adiabatic. The left wall of the cavity is maintained at constant cold temperature $T_c$, while a flush heater is mounted on the vertical right side wall, which has a constant hot temperature $T_h$. The other parts of the vertical right side wall are considered adiabatic. The effect of temperature dependent heat generation in the flow region has also been taken into account Vajravelu and Hadjinicolaou (1993). The volumetric rate of heat generation q‴ [W/m$^3$], is

$$q''' = \begin{cases} Q_0\left(T - T_c\right), & for\, T \geq T_c \\ 0, & for\, T < T_c \end{cases} \tag{1}$$

Where $Q_0$ is a heat generation constant which may be either positive or negative. This source term represents the heat generation when $Q_0 > 0$ and the heat absorption when $Q_0 < 0$.

Steady two-dimensional laminar free convective flow of viscous incompressible fluid having constant properties is assumed where buoyancy effect is included through Boussinesq approximation. The gravitational acceleration acts in the negative y-direction. All solid boundaries are assumed to be rigid no-slip walls. Under these assumptions the equations of continuity, momentum and energy are:

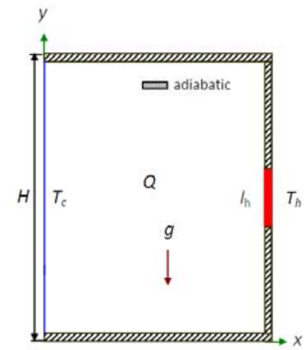


**Fig. 1:** Physical model and Co-ordinate system

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{3}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho g\beta\left(T-T_c\right) \tag{4}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{Q_0}{\rho C_p}\left(T-T_c\right) \tag{5}$$

where $x$ and $y$ are the distances measured along the horizontal and vertical directions respectively; $u$ and $v$ are the velocity components in the $x$ and $y$ directions respectively; $T$ denotes the fluid temperature and $T_c$ denotes the reference temperature for which buoyant force vanishes, $p$ is the pressure and $\rho$ is the fluid density, $g$ is the gravitational acceleration $\beta$ is the volumetric coefficient of thermal expansion, $C_p$ is the specific heat at constant pressure.
To make the above equations dimensionless, we introduce the following non-dimensional variables

$$X=\frac{x}{H},\ Y=\frac{y}{H},\ L=\frac{l_h}{H},U=\frac{uH}{\upsilon},\ V=\frac{vH}{\upsilon},\ P=\frac{pH^2}{\rho\upsilon^2},\ \theta=\frac{T-T_c}{T_h-T_c} \tag{6}$$

where $\nu\left(=\mu/\rho\right)$ is the reference kinematic viscosity and $\theta$ is the non-dimensional temperature. After substitution of dimensionless variables (6) into equations (2)-(5) leads to:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{7}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{8}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{Ra}{Pr}\theta \tag{9}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)+\lambda\theta \tag{10}$$

In the above equations $Ra$ is the Rayleigh number, $Pr$ is the Prandtl number and $\lambda$ is the heat generation parameter defined respectively by the following relations

$$Ra=\frac{\beta\left(T_h-T_c\right)H^3}{\alpha\nu},\ Pr=\frac{\nu}{\alpha}\ \text{ and }\ \lambda=\frac{Q_0H^2}{\mu c_p} \tag{11}$$

The dimensionless form of the boundary conditions can be written as:

At the top and bottom walls: $U=0, V=0, \frac{\partial\theta}{\partial N}=0$, At the left vertical wall: $U=0, V=0,\ \theta=0$

At the right vertical wall: $U=0,\ V=0, \theta=1$ (on the heater), $U=0,\ V=0,\ \frac{\partial\theta}{\partial N}=0$ (on the unheated part)

where $N$ is the non-dimensional distances either along $X$ or $Y$ direction acting normal to the surface.

The average Nusselt number at the heated wall of the cavity based on the dimensionless quantities may be expressed by $Nu = -\frac{1}{L}\int_0^L \frac{\partial\theta}{\partial X}\,dY$ (12)

and the average temperature of the fluid in the cavity is defined by $\theta_{av} = \int \theta \, d\bar{V} / \bar{V}$ (13)

where $\bar{V}$ is the cavity volume.

The non-dimensional stream function is defined as

$$U = \frac{\partial\psi}{\partial Y},\ V = -\frac{\partial\psi}{\partial X} \quad (14)$$

## 3. Numerical Technique

The governing equations along with boundary conditions are solved through the Galerkin finite element formulation while the continuum domain is divided into a set of non-overlapping regions called elements. Six node triangular elements with quadratic interpolation functions for velocity as well as temperature and linear interpolation functions for pressure are utilized to discretize the physical domain. Moreover, interpolation functions in terms of local normalized element coordinates are employed to approximate the dependent variables within each element. Substitution of the obtained approximations into the system of the governing equations and boundary conditions yield a residual for each of the conservation equations. These residuals are reduced to zero in a weighted sense over each element volume using the Galerkin method. Details of the method are available in Taylor and Hood (1973) and Dechaumphai (1999).

### *3.1 Grid refinement check*

To allow grid independent examination, the numerical procedure has been conducted for different grid resolutions. Table 1 demonstrates the influence of number of grid points for a test case of fluid confined within the present configuration. In order to determine independence of each solution on the size of the grid, solutions for various mesh sizes have been conducted and values of average Nusselt number as well as average bulk temperature have been calculated. The results show that the grid system of 42894 nodes and 4834 elements is fine enough to obtain accurate results.

**Table 1:** Results of grid independence examination

| Nodes (Elements) | 26540 (2910) | 31810 (3530) | 41126 (4626) | 42894 (4834) | 56579 (6444) |
|---|---|---|---|---|---|
| $Nu$ | 5.698792 | 5.706400 | 5.807881 | 5.829540 | 5.837540 |
| $\theta_{av}$ | 0.310757 | 0.310723 | 0.310696 | 0.310646 | 0.310544 |

### *3.2 Code validation*

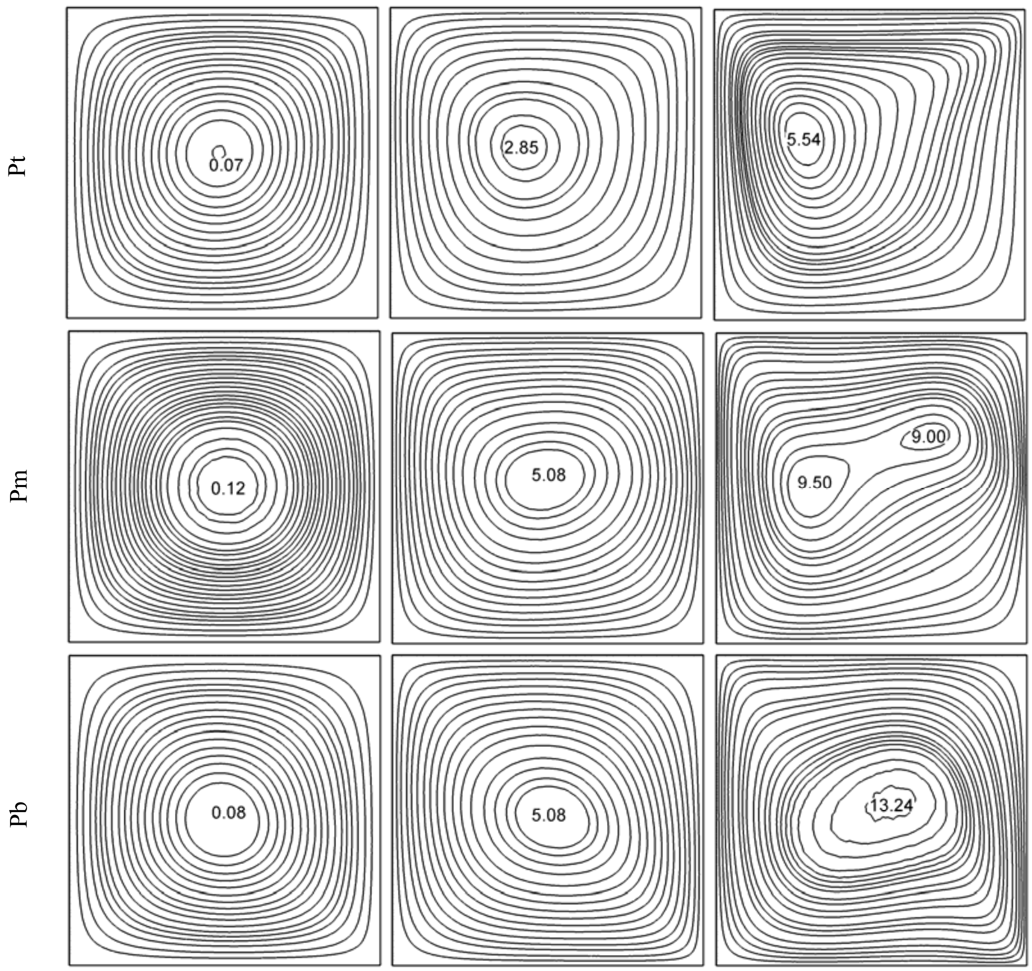
The validity of the computer code developed has been already verified for the problem of laminar natural convection in a square cavity having inner obstacle and differentially heated vertical walls. For detail see Rahman *et al* (2009)

## 4. Results and Discussion

Laminar natural convection heat transfer and fluid flow are studied in a square enclosure for different parameters, including heater length and position, Rayleigh number and heat generation parameter. Position of heater was tested for three cases: top position (Pt), middle position (Pm) and bottom position (Pb) of the right vertical wall as shown in Fig. 1. The other mentioned parameters having the following ranges: length of the heater from 0.1 to 0.4, Rayleigh number from $10^2$ to $10^5$ and heat generation parameter from 0.0 to 20.0. Air is chosen as a fluid with Prandtl number 0.71. The streamlines, isotherms, average Nusselt number at the hot wall and average fluid temperature in the cavity are plotted for the cavity with partially heated vertical wall for different aforementioned parameters. Moreover, the variation of the average Nusselt number at the heated surface and average fluid temperature in the cavity are also highlighted in tabular form.

### *4.1 Effect of heater locations*

The location of heater is a significant parameter on natural convection as indicated by Varol et al. [13]. Consequently, in this study, the location of the heater was considered as a parameter and results are exposed in Figs. 2-4. Fig. 2 shows the streamlines for different locations of the heater and Rayleigh numbers, while $L = 0.1$ and $\lambda = 0.0$. When the heater is located on the bottom of the right vertical wall (position Pb), a circle shaped circulation cell is found in the cavity at low values of *Ra* (= $10^2$), due to domination of conduction effects. It can also be seen from the Fig. 2 that for the smaller Rayleigh number (= $10^2$) the pattern of the circulation cell are almost the same for different locations of the heater as stated earlier. In addition, at lower values of *Ra* the value of stream function is maximum, when the heater is considered at the middle of the wall. However, with the increasing of Rayleigh number, *Ra* to $10^4$ natural convection becomes dominant; as a result flow velocity increases and the circle shaped circulation cell becomes an ellipse shaped for the different location of the heater considered. Finally, for higher values of *Ra* (= $10^5$), there is a strong effect of buoyancy force in the flow field as shown in the right column of the Fig. 2.




$Ra = 10^2$ $Ra = 10^4$ $Ra = 10^5$

**Fig. 2:** Streamlines for the various location of the heater and Rayleigh numbers while $L = 0.1$ and $\lambda = 0.0$.

Fig. 3 shows the isotherms for different locations of the heater and Rayleigh numbers, while $L = 0.1$ and $\lambda = 0.0$. It can be seen from the figure that the isotherms appear parallel to the vertical walls for the lowest values of *Ra* (= $10^2$) and the aforesaid locations of the heater. However, with the increasing of Rayleigh number to $10^4$, the isotherms show that the vertical stratification of isotherms breaks down and becomes nonlinear the aforementioned locations of the heater. Further as *Ra* increases to $10^5$, the nonlinearity in the isotherms becomes higher and plume-like temperature distribution is observed.

In addition, a thermal boundary layer starts to develop near the cold wall. This is because, at the highest Rayleigh number, natural convection is more effective than that of conduction. On the other hand, at the lower values of Ra (= $10^2$, $10^4$) the isotherms near the left wall are almost similar but dissimilar near the right wall for different locations of the heater. In addition, at the higher value of Ra (= $10^5$) the isotherms near the both vertical wall are unlike for different locations of the heater.

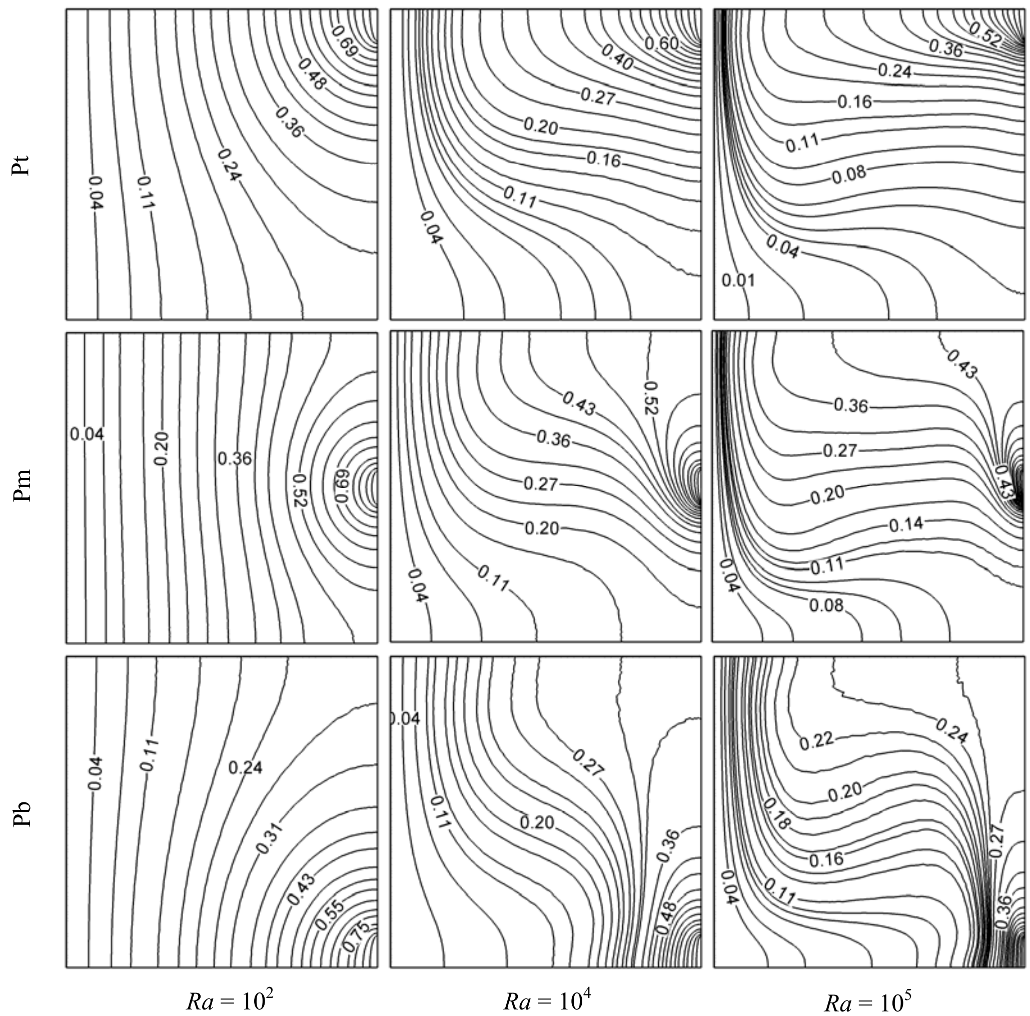



**Fig. 3:** Isotherms for the various locations of heater and Rayleigh numbers, while $L = 0.1$ and $\lambda = 0.0$.

The variation of the average Nusselt number *Nu* at the heated surface and average temperature $\theta_{av}$ of the fluid in the cavity against Rayleigh number *Ra* for different locations of the heater is depicted in Fig. 4. From this figure it is observed that with increasing *Ra* the values of *Nu* increase for the mentioned positions of the heater, this is due to the increase of the buoyancy force.

On the other hand, for a fixed value of *Ra*, the value *Nu* is maximum when heater is at the middle position of the wall. This is supported from the obtained actual numerical values of *Nu* as shown in Table 2. Moreover, the values of average fluid temperature $\theta_{av}$ decreases slowly with increasing *Ra* for the mentioned positions of the heater and for a fixed value of *Ra*, the value of $\theta_{av}$ is lower for the case, when heater is at the top position of the wall. This is also supported from the obtained numerical values of $\theta_{av}$ as shown in Table 3.

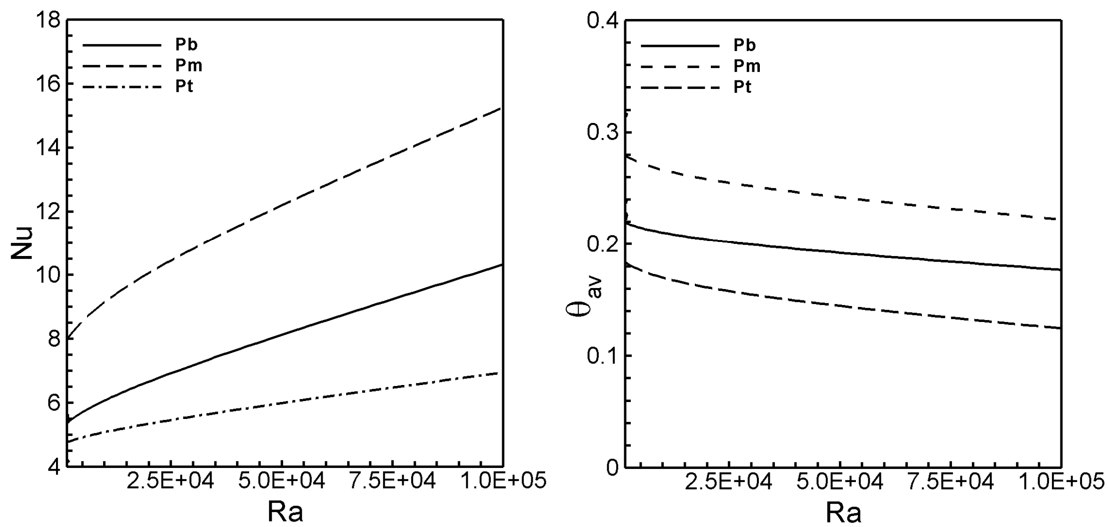



**Fig. 4:** Effect of heater locations on (i) average Nusselt number and (ii) average fluid temperature in the cavity, while $L = 0.1$ and $\lambda = 0.0$.

**Table 2:** Average Nusselt number for different values of *Ra* and locations of heater

| *Ra* | *Nu* | | |
|---|---|---|---|
| | Pb | Pm | Pt |
| $10^2$ | 4.2053650 | 5.493231 | 4.202375 |
| $10^3$ | 4.2989110 | 5.807881 | 4.258308 |
| $10^4$ | 6.0686650 | 9.144790 | 5.080145 |
| $10^5$ | 10.319102 | 15.247889 | 6.946737 |

**Table 3:** Average fluid temperature for different values of *Ra* and locations of heater

| *Ra* | $\theta_{av}$ | | |
|---|---|---|---|
| | Pb | Pm | Pt |
| $10^2$ | 0.231033 | 0.315346 | 0.228985 |
| $10^3$ | 0.238299 | 0.310646 | 0.218354 |
| $10^4$ | 0.210192 | 0.266373 | 0.169851 |
| $10^5$ | 0.176795 | 0.222114 | 0.124435 |

## *4.2 Effect of heater length*

Fig. 5 shows the streamlines for different values of heater length and Rayleigh numbers. In this case, heater is located at Pm and $\lambda = 0.0$. It can be seen from the figure that the streamlines for $Ra = 10^2$ and $L = 0.1$ has a closed circular path, which indicates conduction is the mode of heat transfer.

When the streamlines for $Ra = 10^2$ and higher values of heater length $L$ (= 0.2, 0.3 and 0.4) are compared with that for $Ra = 10^2$ and $L = 0.1$, it is observed that the patterns of the streamlines are almost similar but the values of the stream functions at the core of the circular cell increases with increasing the length of the heater. Further, for $Ra = 10^4$ and the aforesaid values of the heater length the patterns of the streamlines are same as those for $Ra = 10^2$. Meanwhile, it is seen that the values of the stream functions at the core of the circular cell increase with increasing the Rayleigh number at the mentioned values of $L$. This is because the effect of convection on heat transfer increases with increasing the Rayleigh number. Next, when $Ra = 10^5$, the flow changes its pattern from a uni-cellular vortex to a bi-cellular vortex for the different values of $L$.

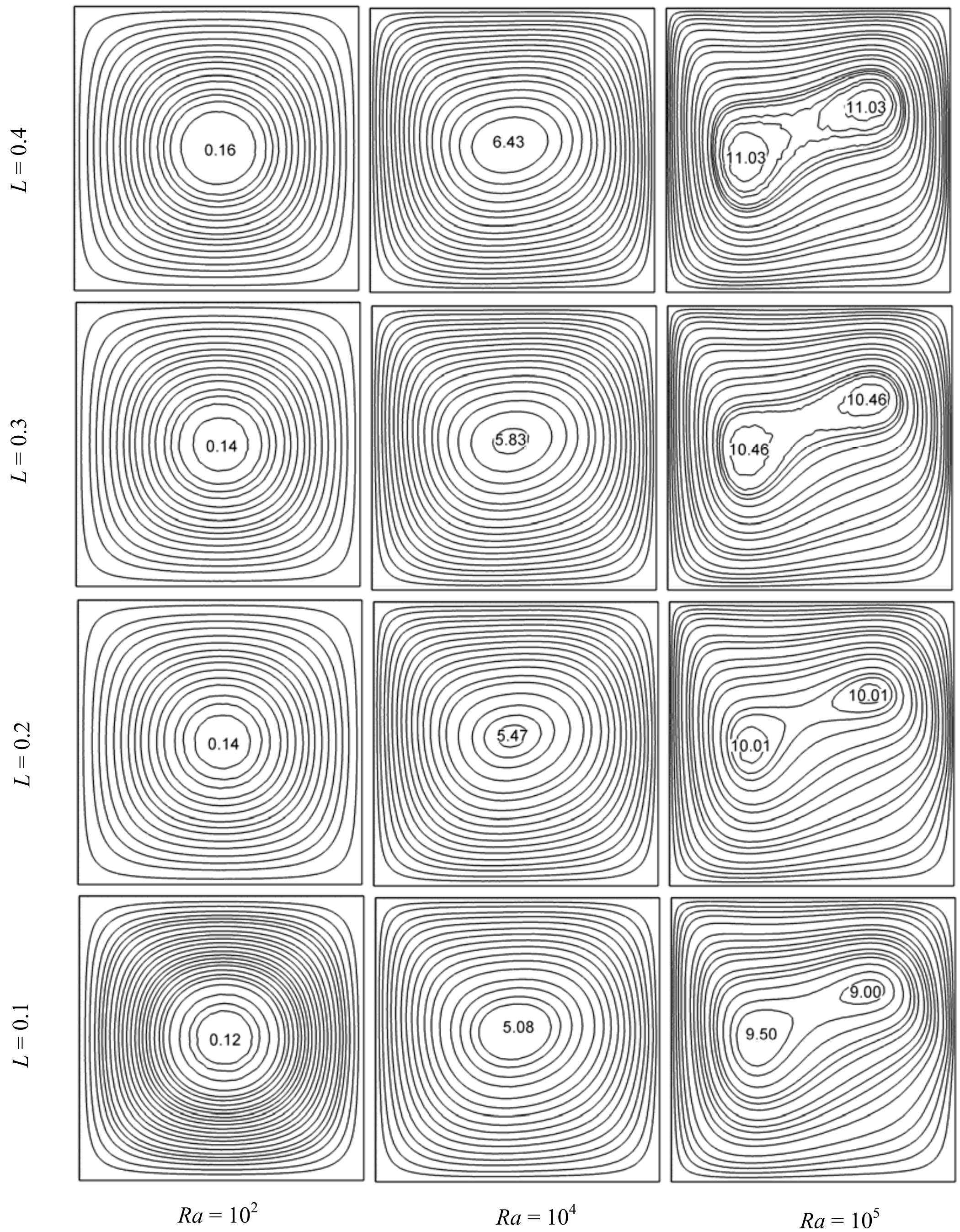



**Fig. 5:** Streamlines for different values of heater length and Rayleigh numbers, while $\lambda = 0.0$ and heater is at Pm.

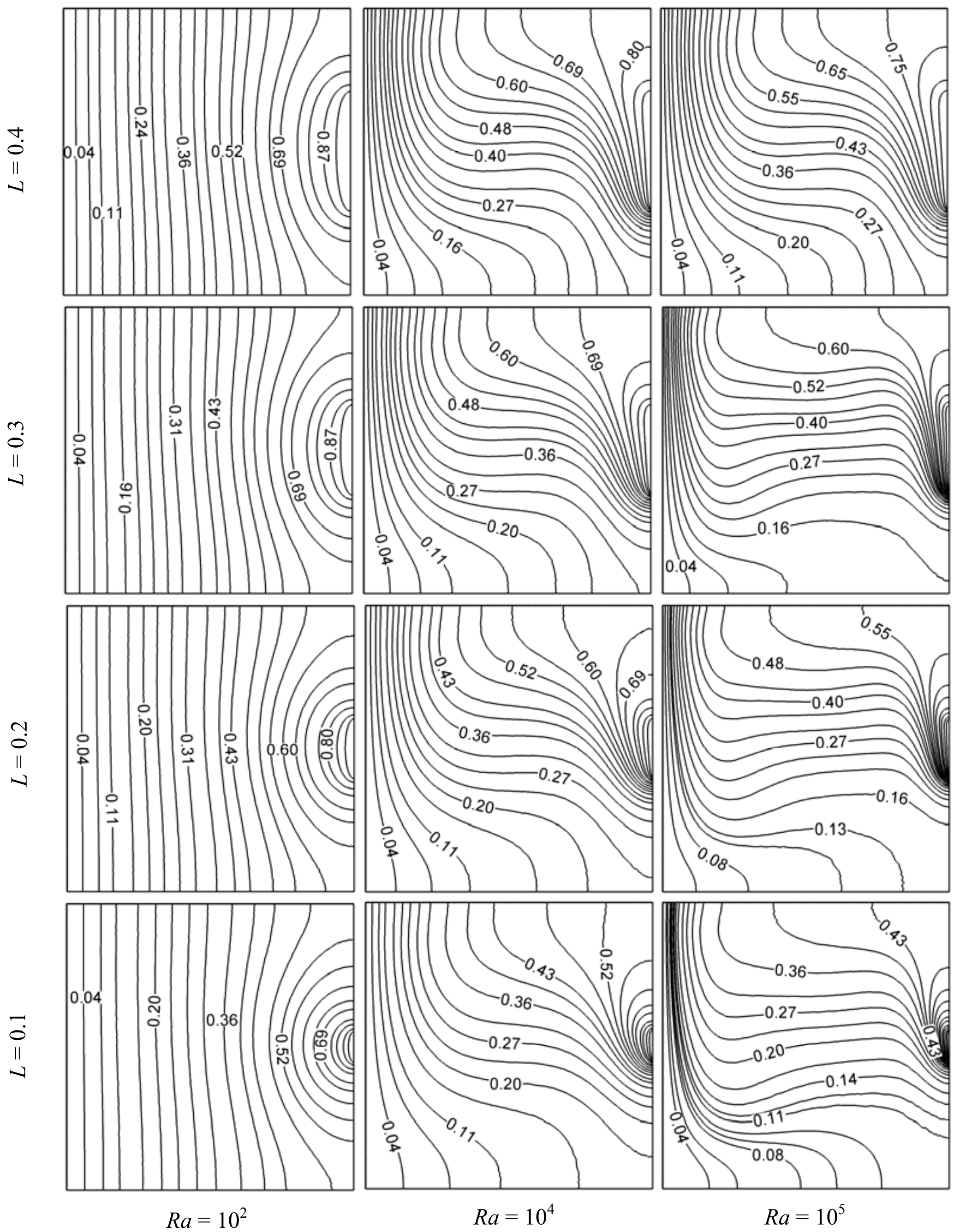



**Fig. 6:** Isotherms for different values of heater length and Rayleigh numbers, while $\lambda = 0.0$ and heater is at Pm

The effects of heater length on average Nusselt number $Nu$ at the hot wall and average temperature $\theta_{av}$ of the fluid in the cavity is shown in Fig. 7, with heater location is at the position of Pm and $\lambda$ = 0.0. From these figures, it is observed that the average Nusselt number $Nu$ at the hot wall goes up very rapidly and average temperature $\theta_{av}$ of the fluid in the cavity goes down slowly with increasing $Ra$ due to the increasing effect of convection for all values of $L$. Moreover, the value of $Nu$ is found maximum and the value of $\theta_{av}$ is found minimum for the lowest value of $L$ that is well documented in Tables 4-5.

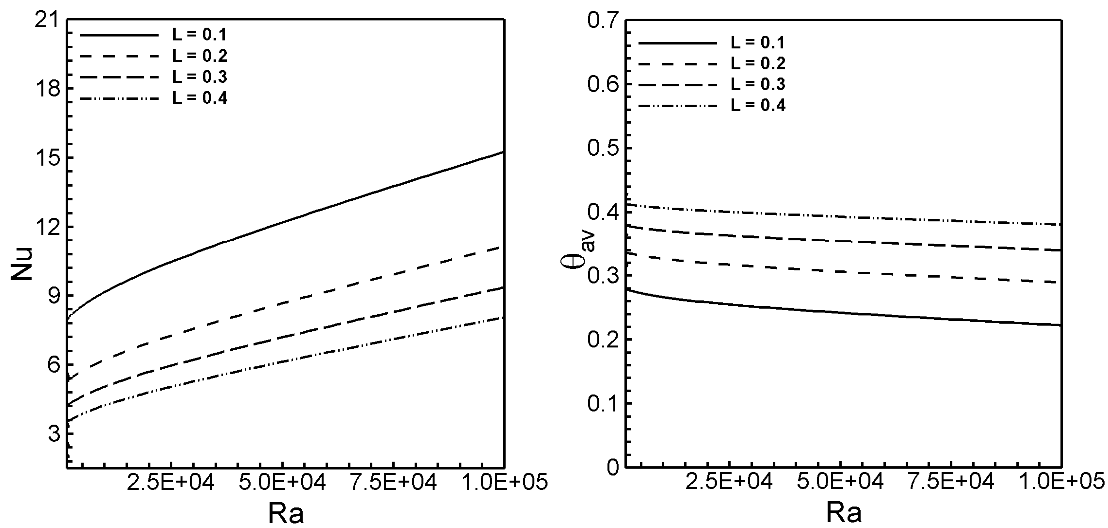



**Fig. 7:** Effect of heater length on (i) average Nusselt number and (ii) average fluid temperature in the cavity, while $\lambda$ = 0.0 and heater is at Pm.

**Table 4:** Average Nusselt number for different values of *Ra* and *L*

| *Ra* | *Nu* | | | |
|---|---|---|---|---|
| | *L* = 0.1 | *L* = 0.2 | *L* = 0.3 | *L* = 0.4 |
| $10^2$ | 5.493231 | 3.317270 | 2.498127 | 2.005268 |
| $10^3$ | 5.807881 | 3.574388 | 2.735491 | 2.214463 |
| $10^4$ | 9.144790 | 6.205870 | 5.035486 | 4.224746 |
| $10^5$ | 15.247889 | 11.137792 | 9.352302 | 8.043779 |

**Table 5:** Average fluid temperature for different values of *Ra* and *L*

| *Ra* | $\theta_{av}$ | | | |
|---|---|---|---|---|
| | *L* = 0.1 | *L* = 0.2 | *L* = 0.3 | *L* = 0.4 |
| $10^2$ | 0.315346 | 0.364608 | 0.400021 | 0.428026 |
| $10^3$ | 0.310646 | 0.360643 | 0.396946 | 0.425749 |
| $10^4$ | 0.266373 | 0.326295 | 0.370955 | 0.406439 |
| $10^5$ | 0.222114 | 0.289232 | 0.339548 | 0.380338 |

## *4.3 Effect of heat generating parameter*

Now we discuss the effect of the internal heat generation parameter ($\lambda$) on the flow and thermal fields in the cavity on taking $L$ = 0.1 and heater location is of Pm. Fig. 8 depicts the streamlines for the values of heat generation parameter $\lambda$ = 0.0, 10, 15 and 20 at the three different Rayleigh numbers. From this figure it is seen that at the lower values of *Ra* (= $10^2$) and $\lambda$ (= 0.0), a circle shaped circulation cell is found in the cavity, due to domination of conduction effects.

From this figure, it is also seen that increasing values of heat generation leads increasing value in the centre of the circulation cell at the lower values of *Ra* (= $10^2$). Moreover, the flow strength in the circulation cell also increases when the internal heat generation increases in magnitude at the higher values of *Ra* (= $10^4$, $10^5$).

The effect of different values of heat generation parameter on thermal field is depicted in Fig. 9 for considered Rayleigh number *Ra*, while flush heater placed at middle of the right vertical wall. The isotherms near the left vertical wall are almost parallel for all values of heat generating parameters $\lambda$, but isotherms near the vicinity of the right vertical wall more concentrated with the increasing values of $\lambda$ at lower value of $Ra$. The isotherm pattern of $Ra = 10^4$ for all values of $\lambda$ becomes more non linear while it compares with the lower values of $Ra$, which is expected. Besides, at higher $Ra$ ($=10^5$), a thermal spot is developed near the left vertical wall and becomes stronger but the concentrated isotherms near the heat source at the right vertical wall disappear with the increasing values of $\lambda$.

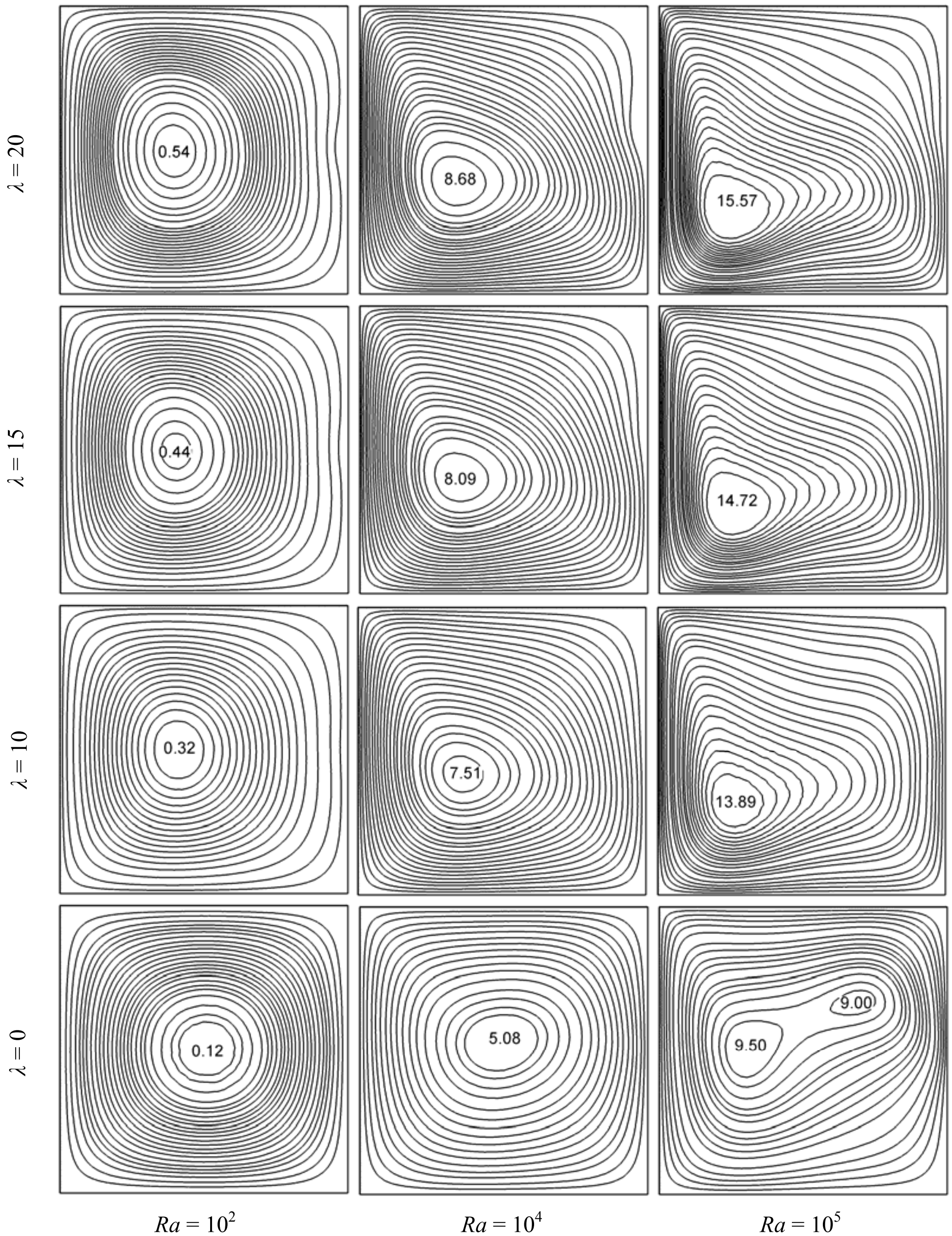



**Fig. 8:** Streamlines for different values of heat generating parameter $\lambda$ and Rayleigh numbers, while $L$ = 0.1 and heater is at Pm.

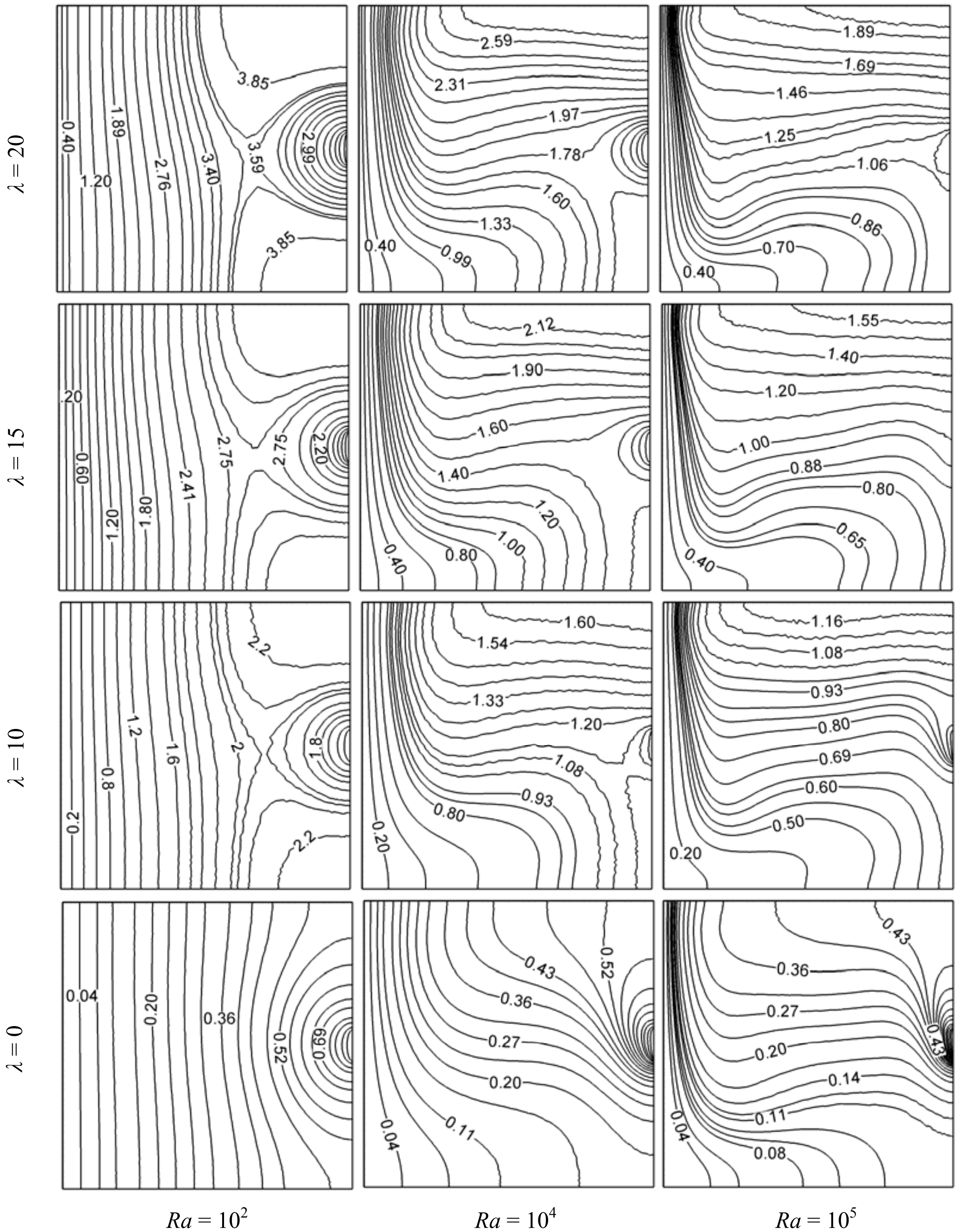



**Fig. 9:** Isotherms for different values of heat generating parameter λ and Rayleigh numbers, while $L$ = 0.1 and heater is at Pm.

The effects of heat generation parameter on the heat transfer rate along the hot wall is plotted as a function of Rayleigh number $Ra$ in Fig. 10 for the four different values of heat generating parameters λ. It is observed that average Nusselt number increases very slowly with the increase of $Ra$ for the different values of heat generating parameters λ. It is also notes that $Nu$ is always higher at λ = 0.0, as expected. This is due to the fact that, the heat generation mechanism creates a layer of hot fluid near the hot surface as a result the increasing rate of internal heat generation negates the heat transfer from the hot surface. Fig. 10 also explains average temperature of the fluid $\theta_{av}$ in the cavity as a function of Rayleigh number $Ra$ for different values of λ.

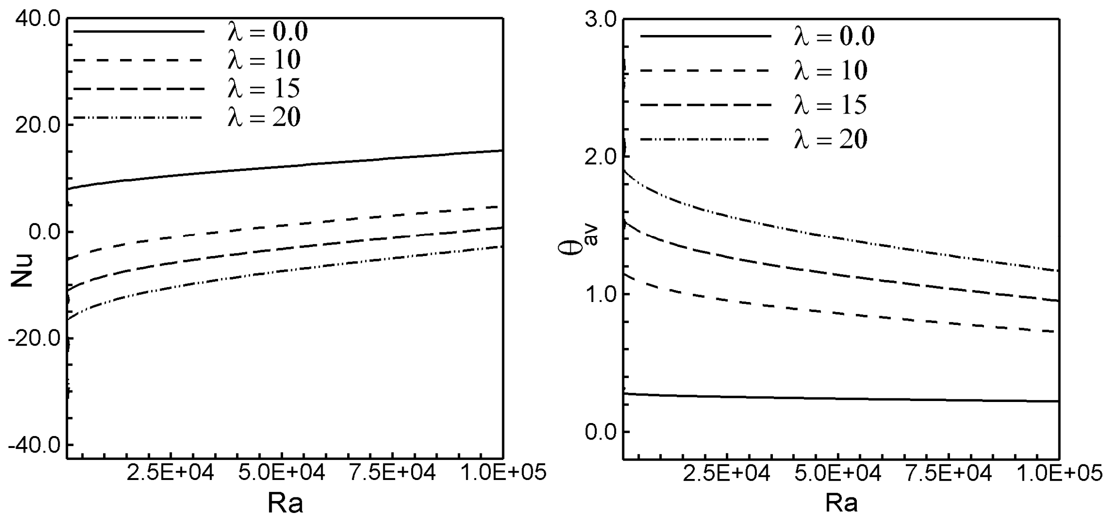



**Fig. 10:** Effect of heat generating parameter $\lambda$ on (i) average Nusselt number and (ii) average fluid temperature in the cavity, while $L$ = 0.1 and heater is at Pm.

The average temperature of the fluid in the cavity decreases with $Ra$ for the higher values of $\lambda$ (= 10, 15 and 20), and become independent of $Ra$ for the lower values of $\lambda$ (= 0.0), which is logical. Moreover, the value of $\theta_{av}$ is the highest for the highest value of $\lambda$. Finally, the numerical values of the average Nusselt number $Nu$ and average temperature fluid $\theta_{av}$ are listed in Tables 6-7.

**Table 6:** Average Nusselt number for different values of $Ra$ and $\lambda$

| $Ra$ | $Nu$ | | | |
|---|---|---|---|---|
| | $\lambda$ = 0.0 | $\lambda$ = 10.0 | $\lambda$ = 15.0 | $\lambda$ = 20.0 |
| $10^2$ | 5.493231 | -14.200905 | -23.987151 | -33.71470 |
| $10^3$ | 5.807881 | -11.255042 | -18.854166 | -26.07655 |
| $10^4$ | 9.144790 | -3.1781830 | -8.4143650 | -13.393163 |
| $10^5$ | 15.247889 | 4.766690 | 0.7610270 | -2.844285 |

**Table 7:** Average fluid temperature for different values of $Ra$ and $\lambda$

| $Ra$ | $\theta_{av}$ | | | |
|---|---|---|---|---|
| | $\lambda$ = 0.0 | $\lambda$ = 10.0 | $\lambda$ = 15.0 | $\lambda$ = 20.0 |
| $10^2$ | 0.315346 | 1.576822 | 2.204442 | 2.828985 |
| $10^3$ | 0.310646 | 1.438047 | 1.946862 | 2.431886 |
| $10^4$ | 0.266373 | 1.043528 | 1.391162 | 1.723099 |
| $10^5$ | 0.222114 | 0.723484 | 0.950063 | 1.165700 |

## 5. Conclusion

In the present study, a problem on natural convection laminar flow in a partially side heated square cavity with internal heat generation has been investigated numerically by employing Finite Element Method together with Newton's iterative technique. The results have been presented for the chosen fluid of Prandtl number $Pr$ = 0.71, location of heater (Pt, Pm and Pb), length of the heater $L$ (= 0.1, 0.2, 0.3 and 0.4), Rayleigh number $Ra$ (= $10^2$, $10^4$ $10^5$) and heat generation parameter $\lambda$ (= 0.0, 10, 15 and 20.0). From the present investigation the following conclusion may be drawn:

- Flow and temperature field are affected by the location of heater and Rayleigh numbers. When heater is located at the middle of the wall the average Nusselt number at the hot wall and average fluid temperature in the cavity become higher for the Rayleigh numbers considered.
- Fluids flow and heat transfer characteristics inside the cavity strongly depend upon the length of the heater. The average Nusselt number at the hot wall becomes higher and average fluid temperature in the cavity become lower for the lower values of the heater length at the considered Rayleigh numbers.
- The heat generation parameter has significant effect on streamlines and isotherms at the higher values of Rayleigh number. The temperature of the fluid in the cavity also increases due to the increase of the internal heat generation consequently, the rate of heat transfer from the hot wall decreases.

## References

Acharya, S. (1985): Natural convection in an inclined enclosure containing internal energy sources and cooled from below, Int. J. Heat & Fluid Flow, vol. 6, No. 2, pp. 113-121. doi:10.1016/0142-727X(85)90045-1

Chadwick, M. L., Webb, B. W., Heaton, H. S. (1991): Natural convection from two-dimensional discrete heat sources in a rectangular enclosure, Int. J. of Heat and Mass Transfer, vol. 34, No. 7, pp. 1679-1693. doi:10.1016/0017-9310(91)90145-5

Fusegi, T., Hyun, J. M., Kuwahara, K. (1992): Natural convection in a differentially heated square cavity with internal heat generation, Num. Heat Transfer Part A, vol. 21, pp. 215-229. doi:10.1080/10407789108944873

Hasnaoui, M., Bilgen, E., Vasseure, P. (1995): Natural convection heat transfer in rectangular cavities heated from below, J. Thermophysics Heat Transfer, vol. 6, pp. 225-264.

Ganzarolli, M., M., Milanez, L., F. (1995): Natural convection in rectangular enclosures heated from below and symmetrical cooled from the sides, Int. J. of Heat and Mass Transfer, vol. 38, pp. 1063-1073. doi:10.1016/0017-9310(94)00217-J

Turkoglu, H., Yucel, N. (1995): Effect of heater and cooler locations on natural convection in square cavities, Num. Heat Transfer Part A, vol. 27, pp. 351-358. doi:10.1080/10407789508913705

Aydin, O., Yang, W. J. (2000): Natural convection in enclosures with localized heating from below and symmetrical cooling from sides, Int. J. of Num. Methods Heat Fluid Flow, vol. 10, pp. 518-529. doi:10.1108/09615530010338196

Deng, Q. H., Tang, G. F., Li, Y., Ha, M. Y. (2002): Interaction between discrete heat sources in horizontal natural convection enclosures, Int. J. of Heat and Mass Transfer, vol. 45, pp. 5117-5132. doi:10.1016/S0017-9310(02)00221-1

Rahman, M., Sharif, M. A. R. (2000): Numerical study of laminar natural convection in inclined rectangular enclosures of various aspect ratios, Num. Heat Transfer Part A, vol. 37, pp. 695-710.

Hossain, M. A., Rees, D. A. S.: Natural convection flow of water near its density maximum in a rectangular enclosure having isothermal walls with heat generation, Heat and Mass Transfer, vol. 41, (2005), pp. 367-374. doi:10.1007/s00231-004-0551-3

Saeid, N. H., Yaacob, Y. (2006): Natural convection in a square cavity with special side wall temperature variation, Num. Heat Transfer Part A, vol. 49, pp. 683-697. doi:10.1080/10407780500359943

Dixit, H., N., Babu, V.: Simulation of high Rayleigh number natural convection in a square cavity using the lattice Boltzmann method, Int. J. of Heat and Mass Transfer, vol. 49, (2006), pp. 727-749. doi:10.1016/j.ijheatmasstransfer.2005.07.046

Varol, Y., Koca, A., Oztop, H., F. (2006): Natural convection in a triangular enclosure with flush mounted heater on the wall, Int. Comm. in Heat and Mass Transfer, vol. 33, pp. 951-958. doi:10.1016/j.icheatmasstransfer.2006.05.003

Vajravelu, K., Hadjinicolaou, A. (1993): Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation, Int. Comm. in Heat and Mass Transfer, vol. 20, (1993), pp. 417-430. doi:10.1016/0735-1933(93)90026-R

Taylor, C., Hood, P. (1973): A numerical solution of the Navier-Stokes equations using finite element technique, Computer and Fluids 1, vol. 1, pp. 73-89.

Dechaumphai, P. (1999): Finite Element Method in Engineering, second ed., Chulalongkorn University Press, Bangkok

Rahman, M. M., Alim, M. A. Mamun, M. A. H. (2009): Finite element analysis of mixed convection in a rectangular cavity with a heat-conducting horizontal circular cylinder, Nonlinear Analysis: Modeling and Control, vol.14, No.2, pp. 217-247.