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AN ANALYTICAL STUDY OF LINEAR STABILITY ANALYSIS ON SORET DRIVEN FERROTHERMOHALINE CONVECTION IN A DARCY POROUS MEDIUM WITH MFD VISCOSITY AND CORIOLIS FORCE

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Abstract:
The effect of magnetic field dependent (MFD) viscosity on the onset of soret driven convection in a ferromagnetic fluid layer heated from below and salted from above saturating rotating porous medium in the presence of vertical magnetic field is investigated theoretically by using Darcy model. The thermal perturbation method is employed for analytical solution. A theory of linear stability analysis and normal mode technique have been carried out to analyze the onset of convection for a fluid layer contained between two impermeable boundaries for which an exact solution is obtained.

Keywords: Coriolis force, Darcy Model, ferromagnetic fluid, MFD viscosity, perturbation technique, Soret Effect.

1. Introduction

In thermal instability problems, the instability is driven by a density difference which is caused by a temperature difference between the upper and lower planes bounding the fluid. If the fluid layer additionally has salt dissolved in it, then there are potentially two destabilizing sources for the density difference, that is the temperature field when the simultaneous presence of two or more components with different diffusivities is considered, the phenomenon of convection which arises is called thermosolutal or double diffusive convection. The magnetization of ferrofluids depends on the magnetic field, temperature, and density. Hence, any variations of these quantities induce change of body force distribution in the fluid and eventually give rise to convection in ferrofluids in the presence of a gradient of magnetic field. There have been numerous studies on thermal convection in a ferrofluid layer called ferroconvection analogous to Rayleigh-Benard convection in ordinary viscous fluids.

Sharma (1977) investigated the thermal instability of compressible fluids in the presence of rotation and magnetic field. Knobloch and Moore (1988) studied the linear stability of experimental Soret convection. Abdullah and Lindsay (1991) examined the Benard convection in a nonlinear magnetic fluid under the influence of a non-vertical magnetic field. Abdullah (1992) derived thermal instability of a non-linear magnetic fluid under the influence of both non-vertical magnetic field and Coriolis force. Thermal convection in a rotating layer of a magnetic fluid has been studied by Auernhammer and Brand (2000). Bennacer *et al.* (2003) carried out the Soret effect on convection in a horizontal porous domain under cross temperature and concentration gradients. Alam *et al.* (2006) carried out Dufour and Soret effects on mixed convection flow past a vertical porous flat plate with variable suction. Lakshmi Narayana *et al.* (2008) discussed the Soret-driven thermosolutal convection induced by inclined thermal and solutal gradients in a shallow horizontal layer of a porous medium. Sekar *et al.* (2008, 2009) derived the effect of presence of dust particles on Soret-driven ferrothermohaline convection with and without porous medium. Shivakumara *et al.* (2011) investigated the ferromagnetic convection in a rotating ferrofluid saturated porous layer.

A study of the effect of chemical reaction and radiation absorption on MHD convective heat and mass transfer flow past a semi-infinite vertical moving plate with time dependent suction derived by Singh *et al* (2011). Rana *et al.* (2011, 2012) discussed the effect of rotation on the onset of convection in Walters' (Model B') heated from

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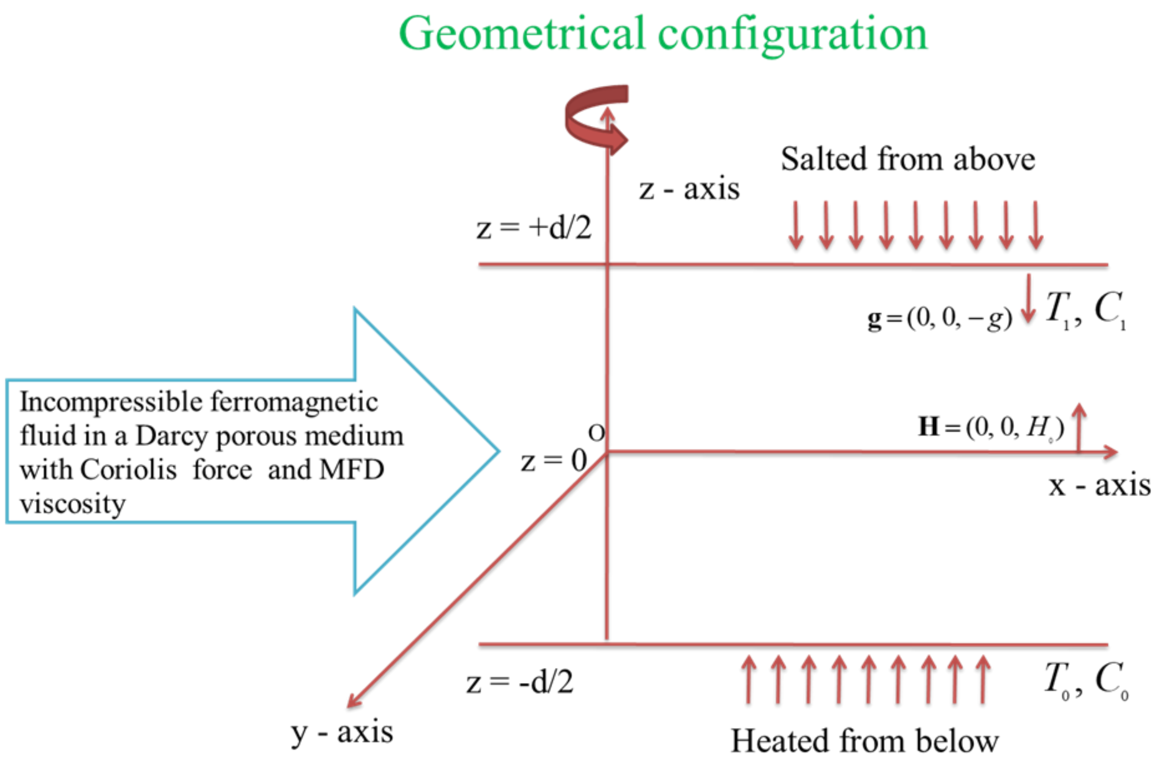
below in a Brinkman porous medium with and without dust particles. Chand (2012) analyzed the effect of rotation on triple-diffusive convection in a magnetized ferrofluid with internal angular momentum saturating a porous medium. Alloui *et al.* (2012) discussed the Double-diffusive and Soret-induced convection in a micro polar fluid layer. Malashetty *et al*. (2012) investigated the linear and non-linear double-diffusive convection in a fluid saturated porous layer with cross-diffusion effect. Vasanthakumari *et al.* (2013) investigated the effect of rotation and magnetic field on thermal instability of Compressible Walters' B' and incompressible non-newtonian viscoelastic fluid. Ram *et al.* (2014) discussed the Swirling flow of field dependent viscous ferrofluid over a porous rotating disk with heat transfer. Jana *et al.* (2014) carried out the oscillatory mixed convection in a porous medium. Singh *et al.* (2014) examined unsteady MHD free convection past an impulsively started isothermal vertical plate with radiation and viscous dissipation. Ramesh Chand and Rana (2015) investigated the magneto convection in a layer of nanofluid in porous medium. Ram *et al.* (2017) derived free convective boundary layer flow of radiating and reacting MHD fluid past a cosinusoidally fluctuating heated plate. Raju (2018) examined the effect of temperature dependent viscosity on ferrothermohaline convection saturating an anisotropic porous medium with Soret effect using the Galerkin technique. Mahajan analyzed *et al.* (2018) Penetrative Internally Heated Convection in Magnetic Fluids. Sekar *et al.* (2018) carried out the linear stability effect of densely distributed porous medium and coriolis force on soret driven ferrothermohaline convection. Prakash *et al.* (2020) found that the effect of magnetic field dependent viscosity on ferromagnetic convection in a rotating sparsely distributed porous medium – revisited. Pulkit Kumar Nadian *et al.* (2020) derived thermal instability of couple stress ferromagnetic fluid in the presence of variable gravity field, Rotation and Magnetic Field. Murugan *et al.* (2021) studied the onset of Soret driven ferrothermoconvective instability in the presence of Darcy Porous medium with Anisotropy effect and MFD viscosity. Murugan *et al.* (2022) investigated a numerical technique and effect of magnetic field dependent (MFD) viscosity on thermal instability in a ferrofluid with Coriolis force for Darcy model.

2. Mathematical formulation



An infinitely spread layer of Boussinesq ferromagnetic fluid of thickness '*d*' rotating with uniform constant angular velocity $\Omega = (0,\ 0,\ \Omega)$ along the vertical direction, is taken as *z*-axis. The entire system is heated from below and salted from above.The temperature and salinity at the bottom and top surfaces $z = \pm d/2$ are $T_0 \pm \Delta T/2$ and $S_0 \pm \Delta S/2$, respectively. Both the boundaries are taken to be impermeable and perfect conductors of heat and solute. The fluid is assumed to be incompressible fluid having a variable viscosity, given by $\mu = \mu_1(1+\boldsymbol{\delta}.\mathbf{B})$. where μ_1 is taken as viscosity of the fluids when the applied magnetic field is absent. The variation in the coefficient of the magnetic field dependent viscosity δ has been taken to be isotropic, that is, $\delta = \delta_1 = \delta_2 = \delta_3$.

The basic governing equations for the above model are

The continuity equation is

$$\nabla.\, \mathbf{q} = 0 \tag{1}$$

The modified Navier-Stokes equation is

$$\rho_o \frac{D\mathbf{q}}{Dt} = -\nabla p + \rho \mathbf{g} + \nabla.(\mathbf{HB}) + 2\rho_o(\mathbf{q}\times\boldsymbol{\Omega}) + \frac{\rho_o}{2}\nabla\left(|\boldsymbol{\Omega}\times\mathbf{r}|^2\right) - \frac{\mu_1(1+\boldsymbol{\delta}.\mathbf{B})}{k}\mathbf{q} \tag{2}$$

The modified thermal diffusivity equation is

$$\left[\rho_o C_{V,H} - \mu_o \mathbf{H}.\left(\frac{\partial \mathbf{M}}{\partial T}\right)_{V,H}\right]\frac{dT}{dt} + \mu_o T\left(\frac{\partial \mathbf{M}}{\partial T}\right)_{V,H}.\frac{d\mathbf{H}}{dt} = K_1\nabla^2 T + \phi \tag{3}$$

The Fick's diffusion equation is

$$\frac{DS}{Dt} = K_s \nabla^2 S + S_T \nabla^2 T \tag{4}$$

Maxwell's equations are

$$\nabla.\mathbf{B} = 0,\ \nabla\times\mathbf{H} = 0 \tag{5a,b}$$

Further **B**, **M** and **H** are related by

$$\mathbf{B} = \mu_0(\mathbf{M}+\mathbf{H}) \tag{6}$$

Combining Equations (5a) and (6), we get

$$\nabla.(\mathbf{M}+\mathbf{H}) = 0 \tag{7}$$

Since, the magnetization is aligned with the magnetic field and depends on the magnitude of the magnetic field, temperature and salinity, so that

$$\mathbf{M} = \frac{\mathbf{H}}{H} M(H,T,S) \tag{8}$$

The magnetic equation of state is

$$M = M_0 + \chi(H - H_0) - K(T - T_0) + K_2(S - S_0) \tag{9}$$

where $\chi = (\partial M/\partial H)_{H_0,T_0}$, $K = -(\partial M/\partial T)_{H_0,T_0}$ and $K_2 = (\partial M/\partial S)_{H_0,S_0}$.

The density equation of state is

$$\rho = \rho_0\left[1 - \alpha_t(T - T_0) + \alpha_s(S - S_0)\right] \tag{10}$$

where $\alpha_t = -(1/\rho)(\partial\rho/\partial T)$ and $\alpha_s = (1/\rho)(\partial\rho/\partial S)$.

The basic state is assumed to be the quiescent state and the basic state quantities obtained are:

$$\mathbf{q} = \mathbf{q}_b = 0,\quad \mathrm{p} = \mathrm{p}_b(\mathrm{z}),\quad \frac{\partial T}{\partial z} = -\beta_t \Rightarrow T_b = T_0 - \beta_t z,\quad \frac{\partial S}{\partial z} = \beta_S \Rightarrow S_b = S_0 + \beta_s z,$$
$$\mathbf{H}_b(Z) = \left[H_0 + \frac{\mathrm{K}(T_b - T_0)}{1+\chi} - \frac{K_2(S_b - S_0)}{1+\chi}\right]\hat{\mathbf{k}},\quad \mathbf{M}_b(Z) = \left[M_0 - \frac{\mathrm{K}(T_b - T_0)}{1+\chi} + \frac{K_2(S_b - S_0)}{1+\chi}\right]\hat{\mathbf{k}}. \tag{11}$$

3. Linear Stability Theory

The basic state is disturbed by a small thermal perturbation, consider a perturbed state such that $\mathbf{q}=\mathbf{q}'$, $\mathrm{p}=\mathrm{p}_b(\mathrm{z})+\mathrm{p}'$, $\mu=\mu_b(\mathrm{z})+\mu'$, $T=T_b(\mathrm{z})+T'$, $\mathbf{H}=\mathbf{H}_b(\mathrm{z})+\mathbf{H}'$, $\mathbf{M}=\mathbf{M}_b(\mathrm{z})+\mathbf{M}'$. where $\mathbf{q}'$, p', μ', T', $\mathbf{H}'$, and $\mathbf{M}'$ are perturbed variables and are assumed to small.

$$H_i' + M_i' = \left(1+\frac{M_0}{H_0}\right)H_i' \qquad (i=1,2) \tag{12}$$

$$H_3' + M_3' = (1+\chi)H_3' - KT' + K_2 S' + S_T K T' \tag{13}$$

Let (B_1, B_2, B_3) denote the components of $\mathbf{B}$, using Eq. (6), one gets the result $B_i = \mu_0\left(M_i' + H_i'\right)$ and Eqs. (12) and (13) become

$$B_i = \mu_0\left(1+\frac{M_0}{H_0}\right)H_i' \qquad (i=1,2) \tag{14}$$

$$B_3 = \mu_0\left[(1+\chi)H_3' - KT' + K_2 S' + S_T K T' + M_0 + H_0\right] \tag{15}$$

When of Eq. (6) is used in Eq. (2) and resulting equation is linearized with B_i (i=1, 2, 3) given by Eqs. (14) and (15), we obtain the following components

$$\rho_0\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu_0\left(M_0+H_0\right)\frac{\partial H_1'}{\partial z} + 2\rho_0 v\Omega - \frac{\mu}{k}u \tag{16}$$

$$\rho_0\frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu_0\left(M_0+H_0\right)\frac{\partial H_2'}{\partial z} - 2\rho_0 u\Omega - \frac{\mu}{k}v \tag{17}$$

$$\begin{aligned}\rho_0\frac{\partial w}{\partial t} = &-\frac{\partial p}{\partial z} + \mu_0\left(M_0+H_0\right)\frac{\partial H_3'}{\partial z} - \mu_0 H_3' K\beta_t + \frac{\mu_0 K^2\beta_t T'}{1+\chi}\left(1-S_{\mathrm{T}}\right) + \mu_0 H_3' K_2\beta_s - \frac{\mu_0 K K_2\beta_s T'}{1+\chi}\left(1-S_{\mathrm{T}}\right) \\ &-\frac{\mu_0 K K_2\beta_t S'}{1+\chi} + \frac{\mu_0 K_2^2\beta_s S'}{1+\chi} + \rho_0 g\alpha_t T' - \rho_0 g\alpha_s S' - \frac{\mu}{k}w - \frac{\mu}{k}\delta\mu_0\left(M_0+H_0\right)w\end{aligned} \tag{18}$$

Differentiating Eqs. (16)-(18) with respect to x, y and z respectively and adding, the following equation is obtained upon using Eq. (2)

$$\begin{aligned}\nabla^2 p = &\mu_0\left(M_0+H_0\right)\frac{\partial}{\partial z}\left(\nabla.\mathbf{H}'\right) + \mu_0 K_2\beta_s\frac{\partial H_3'}{\partial z} + 2\rho_0\Omega\xi + \frac{\mu_0 K^2\beta_t}{1+\chi}\left(1-S_T\right)\frac{\partial T'}{\partial z} - \frac{\mu_0 K K_2\beta_s}{1+\chi}\left(1-S_T\right)\frac{\partial T'}{\partial z} \\ &+\frac{\mu_0 K_2^2\beta_s}{1+\chi}\frac{\partial S'}{\partial z} - \frac{\mu_0 K K_2\beta_t}{1+\chi}\frac{\partial S'}{\partial z} - \mu_0 K\beta_t\frac{\partial H_3'}{\partial z} + \rho_0 g\alpha_t\frac{\partial T'}{\partial z} - \rho_0 g\alpha_s\frac{\partial S'}{\partial z} - \frac{\mu}{k}\delta\mu_0\left(M_0+H_0\right)\frac{\partial w}{\partial z}\end{aligned} \tag{19}$$

where $\mathbf{H}'$ has the components $\left(H_1', H_2', H_3'\right)$.

From Eq. (6), $\mathbf{H}' = \nabla\phi$ where ϕ is a scalar potential. Elimination of p from Eq. (16) - (18) and using Eq. (19), We get,

$$\begin{aligned}\rho_0\frac{\partial}{\partial t}\left(\nabla^2 w\right) = &\mu_0 K_2\beta_s\nabla_1^2 H_3' - 2\rho_0\Omega\frac{\partial\xi}{\partial z} - \mu_0 K\beta_t\nabla_1^2 H_3' + \rho_0 g\alpha_t\nabla_1^2 T' - \rho_0 g\alpha_s\nabla_1^2 S' + \frac{\mu_0 K^2\beta_t}{1+\chi}\left(1-S_T\right)\nabla_1^2 T' \\ &-\frac{\mu_0 K K_2\beta_s}{1+\chi}\left(1-S_T\right)\nabla_1^2 T' + \frac{\mu_0 K_2^2\beta_s}{1+\chi}\nabla_1^2 S' - \frac{\mu_0 K K_2\beta_t}{1+\chi}\nabla_1^2 S' - \frac{\mu}{k}\nabla^2 w - \frac{\mu}{k}\delta\mu_0\left(M_0+H_0\right)\nabla_1^2 w\end{aligned} \tag{20}$$

where $\nabla_1^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}$ and $\nabla^2 = \nabla_1^2 + \dfrac{\partial^2}{\partial z^2}$.

4. Normal Mode Technique

The normal mode solution of all dynamical variables can be written as

$$f(x, y, z, t) = f(z, t)e^{i(k_x x + k_y y)},\ \phi = \phi(z, t)e^{i(k_x x + k_y y)},\ w = w(z, t)e^{i(k_x x + k_y y)},$$
$$T' = \theta(z, t)e^{i(k_x x + k_y y)},\ S' = S(z, t)e^{i(k_x x + k_y y)} \tag{21}$$

with the wave number $k_0^2 = k_x^2 + k_y^2$

Using Eq. (21) in Eq. (20), one gets the vertical component of the momentum equation can be written as

$$\left(\rho_0 \frac{\partial}{\partial t} + \frac{\mu}{k}\right)\left(\frac{\partial^2}{\partial z^2} - k_0^2\right)w = \frac{\mu}{k} k_0^2 \delta\mu_0 (M_0 + H_0) w + \frac{\mu_0 K \beta_t}{1+\chi}\left[(1+\chi)\frac{\partial \phi}{\partial z} - K\theta(1 - S_T)\right]k_0^2 - 2\rho_0 \Omega \frac{\partial \xi}{\partial z}$$
$$+ \rho_0 g \alpha_s k_0^2 S + \frac{\mu_0 K_2 \beta_s}{1+\chi}\left[(1+\chi)\frac{\partial \phi}{\partial z} + K_2 S\right]k_0^2 - \rho_0 g \alpha_t k_0^2 \theta - \frac{\mu_0 K K_2}{1+\chi}\left[\beta_s (1 - S_T)\theta - \beta_t S\right]k_0^2 \tag{22}$$

$$\left(\rho_0 \frac{\partial}{\partial t} + \frac{\mu}{k}\right)\xi = 2\rho_0 \Omega \frac{\partial w}{\partial z} \tag{23}$$

where ξ is the z – component of vorticity given by $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$

The linearized perturbed temperature equation is

$$\rho_0 C_{V,H} \frac{\partial \theta}{\partial t} - \mu_0 K T_0 \frac{\partial}{\partial t}\left(\frac{\partial \phi}{\partial z}\right) = K_1\left(\frac{\partial^2}{\partial z^2} - k_0^2\right)\theta + \left[\rho_0 C_{V,H} \beta_t - \frac{\mu_0 K^2 T_0^2 \beta_t}{1+\chi} + \frac{\mu_0 K K_2 T_0 \beta_s}{1+\chi}\right]w \tag{24}$$

where $\rho_0 C = \rho_0 C_{V,H} + \mu_0 K H_0$

The salinity equation is

$$\frac{\partial S}{\partial t} + \beta_s w = K_s\left(\frac{\partial^2}{\partial z^2} - k_0^2\right)S + S_T\left(\frac{\partial^2}{\partial z^2} - k_0^2\right)\theta \tag{25}$$

The magnetic potential equation is

$$(1+\chi)\frac{\partial^2 \phi}{\partial z^2} - \left(1 + \frac{M_0}{H_0}\right)k_0^2 \phi - K\frac{\partial \theta}{\partial z} + K_2 \frac{\partial S}{\partial z} + S_T K \frac{\partial \theta}{\partial z} = 0 \tag{26}$$

The above equations can be written in dimensionless form using

$$t* = \frac{\nu t}{d^2},\ w* = \frac{wd}{\nu},\ T* = \left(\frac{K_1 a R^{1/2}}{\rho_0 C_{V,H} \beta_t \nu d}\right)\theta,\ \phi* = \left(\frac{(1+\chi) K_1 a R^{1/2}}{K \rho_0 C_{V,H} \beta_t \nu d^2}\right)\phi,\ z* = \frac{z}{d},\ a = k_0 d,$$
$$D = \frac{\partial}{\partial z*},\ S* = \left(\frac{K_s a R_s^{1/2}}{\rho_0 C_{V,H} \beta_s \nu d}\right)S,\ \gamma = \frac{\mu}{\rho_0},\ \xi^* = \frac{\xi d^2}{\gamma},\ k* = \frac{k}{d^2},\ \delta* = \mu_0 \delta H_0 (1+\chi)$$

Following the normal mode analysis, the linearized perturbation dimensionless equations for the thermosolutal convection due to Soret effect in a ferrofluid are

$$\left.\begin{aligned}&\left(\frac{\partial}{\partial t*} + \frac{1}{k*}\right)(D^2 - a^2)w* = a R^{1/2} M_1 M_5 D\phi* + a R^{1/2}\left[M_1 D\phi* - (1 + M_1(1 - S_T))T*\right] \\ &- a R^{1/2} M_1 M_5 (1 - S_T) T* + a R_S^{1/2}\left[1 + M_4 + \frac{M_4}{M_5}\right]S* - (T_a)^{1/2} D\xi* + \frac{a^2}{k*} M_3 \delta* w*\end{aligned}\right\} \tag{27}$$

$$\left(\frac{\partial}{\partial t^*} + \frac{1}{k^*}\right)\xi^* = (T_a)^{1/2} Dw^*, \tag{28}$$

$$P_r\left[\frac{\partial T*}{\partial t*} - M_2 \frac{\partial}{\partial t*}(D\phi*)\right] = (D^2 - a^2)T* + aR^{1/2}(1 - M_2 - M_2 M_5)w* \tag{29}$$

$$P_r \frac{\partial S^*}{\partial t^*} = \tau(D^2 - a^2)S^* - aR_S^{1/2} M_6 w^* + S_T \left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2} (D^2 - a^2)T^* \tag{30}$$

$$D^2\phi^* - M_3 a^2 \phi^* - (1 - S_T) DT^* + \frac{M_5}{M_6}\left(\frac{R}{R_S}\right)^{1/2} DS^* = 0 \tag{31}$$

where the non-dimensional parameters used are

$$\left.\begin{aligned} &M_1 = \frac{\mu_0 K^2 \beta_t}{(1+\chi)\rho_0 g \alpha_t},\ M_2 = \frac{\mu_0 K^2 T_0}{(1+\chi)\rho_0 C_{v,H}},\ M_3 = \frac{1 + M_0 / H_0}{(1+\chi)}, M_4 = \frac{\mu_0 K^2 \beta_s}{(1+\chi)\rho_0 g \alpha_s}, M_5 = \frac{K_2 \beta_s}{K \beta_t}, \\ &M_6 = \frac{K_S}{K_1},\ \tau = \rho_0 C_{v,H}\left(\frac{K_S}{K_1}\right), P_r = \frac{\mu C_{v,H}}{K_1},\ R_S = \frac{\rho_0 C_{v,H} \beta_s \alpha_s g d^4}{\nu K_S},\ R = \frac{\rho_0 C_{v,H} \beta_t \alpha_t g d^4}{\nu K_1}, \end{aligned}\right\} \tag{32}$$

5. Mathematical Analysis

The free-free boundary conditions on velocity, temperature, salinityand angular momentum are

$$w^* = D^2 w^* = T^* = D\phi^* = S^* = \xi^* = D\xi^* = 0 \text{ at } z^* = \pm 1/2. \tag{33}$$

The exact solutions satisfying above equation (33) are

$$w^* = Ae^{\sigma t^*}\cos \pi z^*, T^* = Be^{\sigma t^*}\cos \pi z^*, S^* = Ce^{\sigma t^*}\cos \pi z^*, D\phi^* = Ee^{\sigma t^*}\cos \pi z^*,\ \phi^* = \frac{E}{\pi} e^{\sigma t^*}\sin \pi z^*. \tag{34}$$

In this part, all the partial derivatives and asterisks are removed with use of exact solutions to find the solution of the system of homogeneous equations in (35) to (38). Using equation (5.2) in equations (27) to (31), we get

$$\left.\begin{aligned} &\left[\left(\sigma + \frac{1}{k}\right)(\pi^2 + a^2) + \frac{T_a \pi^2}{\left(\sigma + \frac{1}{k}\right)} + \frac{1}{k} a^2 M_3 \delta\right] A - aR^{1/2}\left[1 + M_1(1 - S_T) + M_1 M_5 (1 - S_T)\right] B \\ &\qquad + aR_S^{1/2}(1 + M_4 + M_4 M_5^{-1}) C + aR^{1/2}\ M_1 (1 + M_5) E = 0, \end{aligned}\right\} \tag{35}$$

$$aR^{1/2}(1 - M_2 - M_2 M_5) A - (\pi^2 + a^2 + P_r \sigma) B + P_r \sigma M_2 E = 0 \tag{36}$$

$$aR_S^{1/2} M_6 A + S_T \left(\frac{M_5}{M_6}\right)\left(\frac{R_S}{R}\right)^{1/2} (\pi^2 + a^2) B + \left[\tau(\pi^2 + a^2) + \sigma P_r\right] C = 0 \tag{37}$$

$$-R_S^{1/2}\pi^2 (1 - S_T) B + R^{1/2}\ \pi^2 M_5 M_6^{-1} C + R_S^{1/2}(\pi^2 + a^2 M_3) E = 0 \tag{38}$$

The determinant of co-efficient of *A, B, C* and *E* are vanish for the existence of non-trivial Eigen functions. Eqs. (35) – (38) lead to

$$U\sigma^4 + V\sigma^3 + W\sigma^2 + X\sigma + Y = 0 \tag{39}$$

$$U = \left(\pi^2 + a^2\right)\left(\pi^2 + a^2 M_3\right) P_r^2$$

$$V = \left(\pi^2 + a^2 M_3\right)\left[\left(\pi^2 + a^2\right)^2 (1 + \tau) + P_r \left\{\frac{2}{k}\left(\pi^2 + a^2\right) + \frac{1}{k} a^2 M_3 \delta\right\}\right] P_r$$

$$\begin{aligned} W = &\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left[\tau\left(\pi^2 + a^2\right)^2 + \mathrm{P}_r (1 + \tau)\left\{\frac{2}{k}\left(\pi^2 + a^2\right) + \frac{1}{k} a^2 M_3 \delta\right\}\right] + a^2 R\mathrm{P}_r \left(\pi^2 + a^2 M_3\right) \\ &+ a^2 R\mathrm{P}_r M_1 (1 + M_5)\left[(1 - S_\mathrm{T})\left(\pi^2 + a^2 M_3\right) + \pi^2 (1 - S_\mathrm{T} + M_5)\right] \\ &- a^2 R_s P_r \left(\pi^2 + a^2 M_3\right)\left(1 + M_4 + M_4 M_5^{-1}\right) M_6 + \left(\pi^2 + a^2 M_3\right)\left[T_a \pi^2 + \frac{1}{k^2}\left(\pi^2 + a^2\right) + \frac{1}{k^2} a^2 M_3 \delta\right] P_r^2 \end{aligned}$$

$$X = \left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)(1+\tau) P_r \left[T_a \pi^2 + \frac{1}{k^2}\left(\pi^2 + a^2\right) + \frac{1}{k^2} a^2 M_3 \delta\right]$$

$$+\tau\left(\pi^2 + a^2\right)^2 \left(\pi^2 + a^2 M_3\right)\left[\frac{2}{k}\left(\pi^2 + a^2\right) + \frac{1}{k} a^2 M_3 \delta\right] + a^2 R\tau\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left[1 + (1 - S_T) M_1 (1 + M_5)\right]$$

$$+\frac{1}{k} a^2 R \mathrm{P}_r \left[\left(\pi^2 + a^2 M_3\right)\left\{1 + M_1 (1 + M_5)(1 - S_T)\right\} + \pi^2 M_1 (1 + M_5)\left\{(1 - S_T) + M_5\right\}\right]$$

$$+a^2 R\left(\pi^2 + a^2\right) M_1 (1 + M_5)\pi^2 \left[S_T \left(\frac{M_5}{M_6}\right)^2 + \tau(1 - S_T) + M_5\right]$$

$$-a^2 R_s \left(\pi^2 + a^2 M_3\right)\left(1 + M_4 + M_4 M_5^{-1}\right)\left[\frac{1}{k} M_6 P_r + \left(\pi^2 + a^2\right)\left\{S_T \left(\frac{M_5}{M_6}\right) + M_6\right\}\right]$$

$$Y = \tau\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)^2 \left[T_a \pi^2 + \frac{1}{k^2}\left(\pi^2 + a^2 + a^2 M_3 \delta\right)\right]$$

$$+\frac{1}{k} a^2 R\tau\left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left[1 + (1 - S_T) M_1 (1 + M_5)\right]$$

$$+\frac{1}{k} a^2 R\left(\pi^2 + a^2\right) M_1 (1 + M_5)\pi^2 \left[S_T \left(\frac{M_5}{M_6}\right)^2 + \tau(1 - S_T) + M_5\right]$$

$$-\frac{1}{k} a^2 R_s \left(\pi^2 + a^2 M_3\right)\left(\pi^2 + a^2\right)\left(1 + M_4 + M_4 M_5^{-1}\right)\left[S_T \left(\frac{M_5}{M_6}\right) + M_6\right]$$

6. Stationary convection

For the steady state (i.e., the validity of principle of exchange of stability), we have $\sigma = 0$ at the margin of stability. Then the Eq. (39) helps one to obtain Eigen value R_{SC} for which a solution exists;

$$R_{SC} = \frac{N_r}{D_r}, \tag{40}$$

where

$$N_r = \left(\pi^2 + a^2\right)\left[T_a \pi^2 k + \frac{1}{k}\left(\pi^2 + a^2\right) + \frac{1}{k} a^2 M_3 \delta\right] - a^2 R_s \tau^{-1}\left(1 + M_4 + M_4 M_5^{-1}\right)\left[S_T \left(\frac{M_5}{M_6}\right) + M_6\right]$$

and

$$D_r = a^2\left[1 + (1 - S_T) M_1 (1 + M_5)\right] - \pi^2 \left[\frac{a^2 M_1 (1 + M_5)}{\pi^2 + a^2 M_3}\right]\left[S_T \left(\frac{M_5}{M_6}\right)^2 \tau^{-1} + (1 - S_T) + M_5 \tau^{-1}\right]$$

For M_1 very large, the critical magnetic thermal Rayleigh number $N_{SC} = R_{SC} M_1$ for stationary mode could be simplified as

$$N_{SC} = M_1 R_{SC} = \frac{N_r}{D_r}, \tag{41}$$

where

$$N_r = \left(\pi^2 + a^2\right)\left[T_a \pi^2 k + \frac{1}{k}\left(\pi^2 + a^2\right) + \frac{1}{k} a^2 M_3 \delta\right] - a^2 R_s \tau^{-1}\left(1 + M_4 + M_4 M_5^{-1}\right)\left[S_T \left(\frac{M_5}{M_6}\right) + M_6\right]$$

and

$$D_r = a^2\left[(1 - S_T)(1 + \mathrm{M}_5)\right] - \pi^2 \left[\frac{a^2 (1 + \mathrm{M}_5)}{\pi^2 + a^2 \mathrm{M}_3}\right]\left[S_T \left(\frac{\mathrm{M}_5}{\mathrm{M}_6}\right)^2 \tau^{-1} + (1 - S_T) + \mathrm{M}_5 \tau^{-1}\right]$$

7. Overstability

Taking $\sigma = i\sigma$ and $\sigma^2 > 0$, in Eq. (39), one gets the real value of the Rayleigh number because the Rayleigh number is not a complex number (i.e., $Im\ R_{oc} = 0$), implies that R_{oc} is a real number. Therefore, the critical Rayleigh number for oscillatory mode has been calculated using

$$R_{OC} = \frac{C_2 A_2 + B_2 D_2}{{A_2}^2 + {B_2}^2} \tag{42}$$

where

$$A_2 = -U_1\sigma_1^2 + V_1,\ B_2 = W_1\sigma_1,\ C_2 = -U_2\sigma_1^4 + W_2\sigma_1^2 - Y_1,$$

$$D_2 = V_2\sigma_1^3 - \mathrm{X}_1\sigma_1,\ \ \sigma_1^2 = \frac{-B_1 \pm sqrt({B_1}^2 - 4A_1C_1)}{2A_1}$$

$$A_1 = U_2W_1 - U_1V_2,\ B_1 = V_1V_2 + U_1X_1 - W_1W_2,\ C_1 = W_1Y_1 - V_1X_1$$

$$U_1 = a^2P_rM_1(1+M_5)\left[(1-S_T)\left(\pi^2 + a^2M_3\right) + \pi^2(1 - S_T + M_5)\right] + a^2RP_r\left(\pi^2 + a^2M_3\right)$$

$$U_2 = \left(\pi^2 + a^2\right)\left(\pi^2 + a^2M_3\right)P_r^2$$

$$V_1 = \frac{1}{k}a^2\tau\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)\left[1 + (1-S_T)M_1(1+M_5)\right]$$

$$+\frac{1}{k}a^2\left(\pi^2 + a^2\right)M_1(1+M_5)\pi^2\left[S_T\left(\frac{M_5}{M_6}\right)^2 + \tau(1-S_T) + M_5\right]$$

$$V_2 = \left(\pi^2 + a^2M_3\right)\left[\left(\pi^2 + a^2\right)^2(1+\tau) + \left\{\frac{2}{k}\left(\pi^2 + a^2\right) + \frac{1}{k}a^2M_3\delta\right\}P_r\right]P$$

$$W_1 = a^2\tau\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)\left[1 + (1-S_T)M_1(1+M_5)\right]$$

$$+a^2\left(\pi^2 + a^2\right)M_1(1+M_5)\pi^2\left[S_T\left(\frac{M_5}{M_6}\right)^2 + \tau(1-S_T) + M_5\right]$$

$$+\frac{1}{k}a^2P_r\left[\left(\pi^2 + a^2M_3\right)\left\{1 + M_1(1+M_5)(1-S_T)\right\} + \pi^2M_1(1+M_5)\left\{(1-S_T) + M_5\right\}\right]$$

$$W_2 = \left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)\left[\tau\left(\pi^2 + a^2\right)^2 + (1+\tau)P_r\left\{\frac{2}{k}\left(\pi^2 + a^2\right) + \frac{1}{k}a^2M_3\delta\right\}\right]$$

$$-a^2R_sP_r\left(\pi^2 + a^2M_3\right)\left(1 + M_4 + \frac{M_4}{M_5}\right)M_6 + \left(\pi^2 + a^2M_3\right)\left[T_a\pi^2 + \frac{1}{k^2}\left(\pi^2 + a^2\right) + \frac{1}{k^2}a^2M_3\delta\right]{P_r}^2$$

$$\mathrm{X}_1 = P_r\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)(1+\tau)\left[T_a\pi^2 + \frac{1}{k^2}\left(\pi^2 + a^2\right) + \frac{1}{k^2}a^2M_3\delta\right]$$

$$+\tau\left(\pi^2 + a^2\right)^2\left(\pi^2 + a^2M_3\right)\left[\frac{2}{k}\left(\pi^2 + a^2\right) + \frac{1}{k}a^2M_3\delta\right]$$

$$-a^2R_s\left(\pi^2 + a^2M_3\right)\left(1 + M_4 + M_4M_5^{-1}\right)\left[\frac{1}{k}M_6P_r + \left(\pi^2 + a^2\right)\left\{S_T\left(\frac{M_5}{M_6}\right) + M_6\right\}\right]$$

$$Y_1 = \tau\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)^2\left[T_a\pi^2 + \frac{1}{k^2}\left(\pi^2 + a^2\right) + \frac{1}{k^2}a^2M_3\delta\right]$$

$$-\frac{1}{k}a^2R_s\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)\left(1 + M_4 + M_4M_5^{-1}\right)\left[S_T\left(\frac{M_5}{M_6}\right) + M_6\right]$$

8. Results and discussion

The critical thermal Rayleigh number is calculated for both stationary and oscillatory modes. When $M_1 = 1000$, the classical Rayleigh problem for buoyancy-induced convection is and obtained Chandrasekhar (1961). When all the magnetic parameters M_1 to M_6 vanish, this reduces to double diffusive convection (Baines and Gill, 1969). When the salinity Rayleigh number $R_S = 0$, the critical Rayleigh number obtained by Finlayson (1970) for single component ferrofluid. When $\nabla^2 \mathbf{q} = 0$ and Soret effect is absent, the thermal Rayleigh number is identical to Vaidyanathan et al., (2002). When $\nabla^2 \mathbf{q} = 0$, $\delta = 0$, $\varepsilon = 0, Ta = 0$ and $k \to \infty$ this tends to critical Rayleigh number obtained by Vaidyanathan et al., (2005). When $\delta = 0$, $\varepsilon = 1$ and $Ta = 0$ the thermal Rayleigh number is identical to Sekar et al., (2006). When $\nabla^2 \mathbf{q} = 0$ and Soret effect is present, the critical Rayleigh number calculated in Hemalatha (2014). When $\delta = 0$, one gets the thermal Rayleigh number is identical to Sekar et al., (2016). When $k = (k_1, k_1, k_2)$, the critical Rayleigh number derived by Sekar et al., (2019).

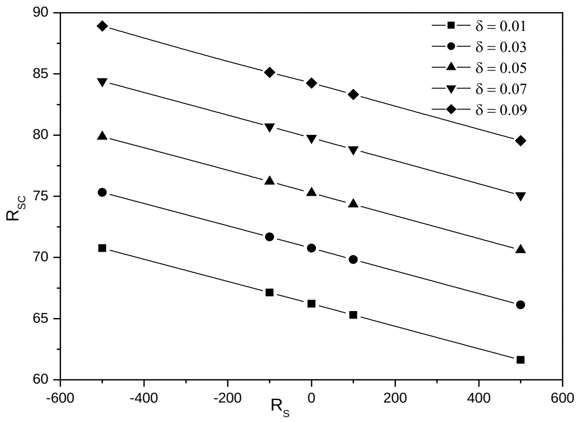


Figure 1. Variation of R_{SC} versus R_S for various δ, $\tau = 0.03$, $k = 0.001$, $S_T = -0.002$ and $M_3 = 5$.

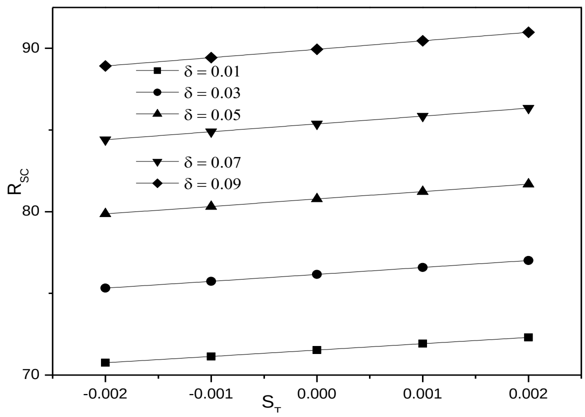


Figure 2. Variation of R_{SC} versus S_T for various δ, $\tau = 0.03$, $k = 0.001$, $R_S = -500$ and $M_3 = 5$.

Figure 1 represent the variation of R_{SC} versus R_S for different values of δ. When the salinity Rayleigh number R_S increases from -500 to 500, the critical magnetic Rayleigh number R_{SC} decreases. Therefore the system gets a destabilizing behaviour. It is observed that the MFD viscosity parameter δ is found to stabilize the system.

Figure 2 indicates the variation of the critical magnetic Rayleigh number R_{SC} with respect to the Soret parameter S_T for various δ. It is found that the increase in Soret effect stabilizes the system, thereby delaying the onset of convection. The figure exhibits a stabilizing trend. This is due to the fact that the modulation of the salinity gradient by temperature gradient promotes stabilization. Positive values of S_T stabilizethe system which is more pronounced. The stabilizing behavior of δ is seen from Figure, as would mean adding salt from the top.

Figure 3 gives the variation of the critical Rayleigh number R_{SC} versus the non-buoyancy magnetization parameter M_3 for different MFD viscosity parameter δ. It is seen from the figure that as the value of M_3 increases from 5 to 25, the value of R_{SC} decreases for small value $\delta = 0.01$, thus the convective system has a destabilizing effect for $\delta = 0.01$. whereas for higher values of δ (0.05, 0.07 and 0.09), R_{SC} gets increasing values. In this situation, the system has a stabilizing behavior which is increasing slowly.

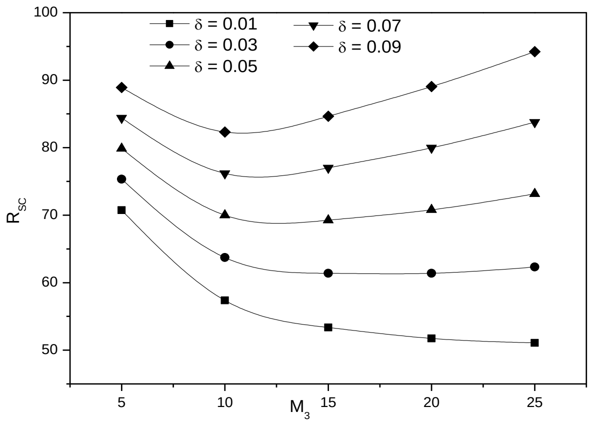


Figure 3.Variation of R_{SC} versus M_3 for various δ, $\tau = 0.03$, $k = 0.001$, $R_S = -500$, and $S_T = -0.002$.

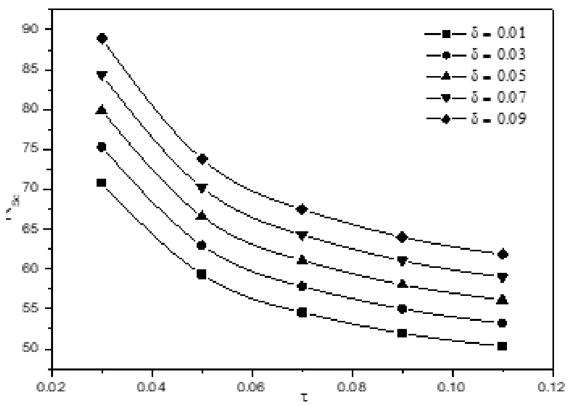


Figure 4. Variation of R_{SC} versus τ for various δ, $R_S = -500$, $k = 0.001$, $S_T = -0.002$ and $M_3 = 5$.

Figure 4 shows the variation of critical magnetic Rayleigh number R_{SC} versus the mass transport to heat transport τ for different δ. It is seen from this figure that the system destabilizes as the mass transport to heat transport τ increases. This is shown by a fall in R_{SC} values. It is observed from the figure that the magnetic field dependent viscosity δ is found to stabilize the system.

Figure 5 represents the variation of critical magnetic Rayleigh number R_{SC} versus permeability of the porous medium k for different δ. It is clear that the system destabilizes as the permeability of the porous medium k increases. This is indicated by a decrease in R_{SC} values. The reason is that as the pore size increases, it becomes easier for the flow to destabilize the system. It is observed from the figure that the magnetic field dependent viscosity δ is found to stabilize the system.

Figure 6, illustrates that as M_3 increases, the values of R_{SC} decreases for small values of δ, whereas for higher values of δ, R_{SC} decreases for lower values of M_3, and then increases for higher values of M_3. The same trend is seen from Figure 3. The destabilizing trend of R_S, k and τ is also seen from Figs. 8, 9 and 10. But stabilizing behavior of S_T is seen from Figure 7.

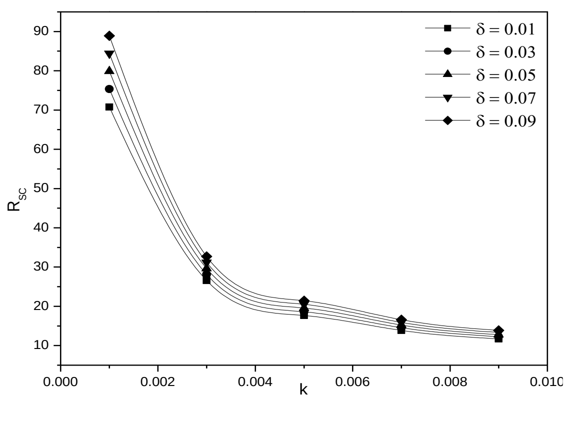


Figure 5. Variation of R_{SC} versus k for various δ, $R_S = -500$, $\tau = 0.003$, $S_T = -0.002$ and $M_3 = 5$.

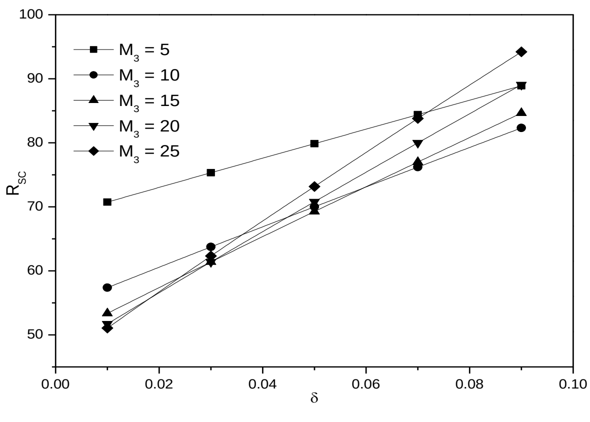


Figure 6. Variation of R_{SC} versus δ for various M_3, $R_S = -500$, $\tau = 0.003$, $S_T = -0.002$ and $k = 0.001$.

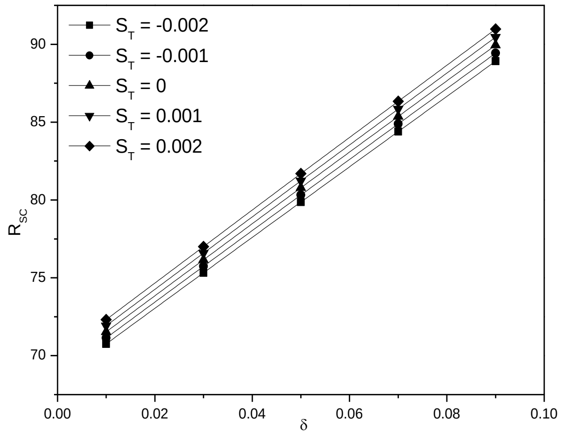


Figure 7. Variation of R_{SC} versus δ for various S_T, $R_S = -500,\ \tau = 0.003,\ M_3 = 5$ and $k = 0.001$.

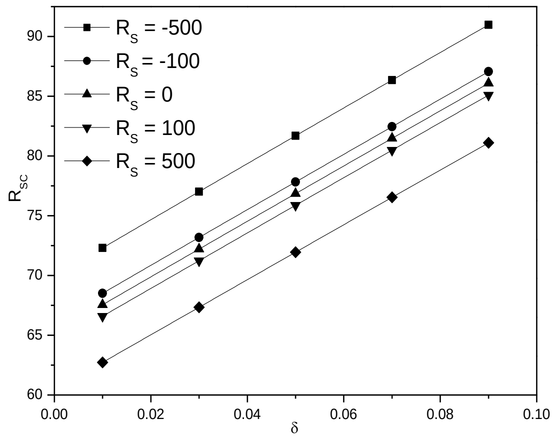


Figure 8.Variation of R_{SC} versus δ for various R_S, $S_T = -0.002,\ \tau = 0.003,\ M_3 = 5$ and $k = 0.001$.

Figures 6-10 investigate the variation of R_{SC} versus δ for different values of M_3, S_T, R_S, k and τ. From Figs. 6 - 10, one can find that as the coefficient of MFD viscosity is increased from 0.01 to 0.09, the critical magnetic Rayleigh number increases. This means that the system is stabilized through viscosity variation with respect to magnetic field. This leads to the conclusion that the MFD viscosity delays the onset of convection for ferrofluid in a densely distributed porous medium.

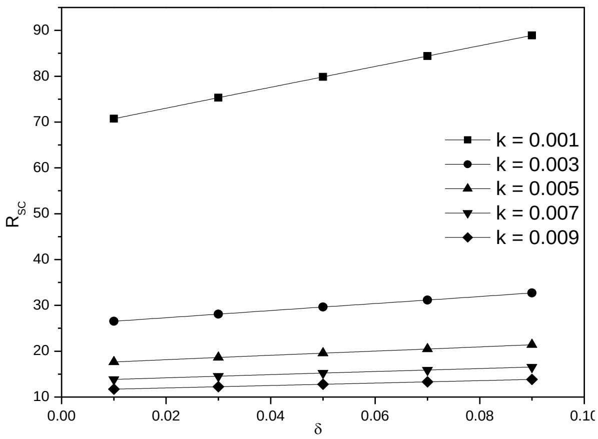


Figure 9.Variation of R_{SC} versus δ for various k, $R_S = -500,\ S_T = -0.002,\ \tau = 0.003$ and $M_3 = 5$.

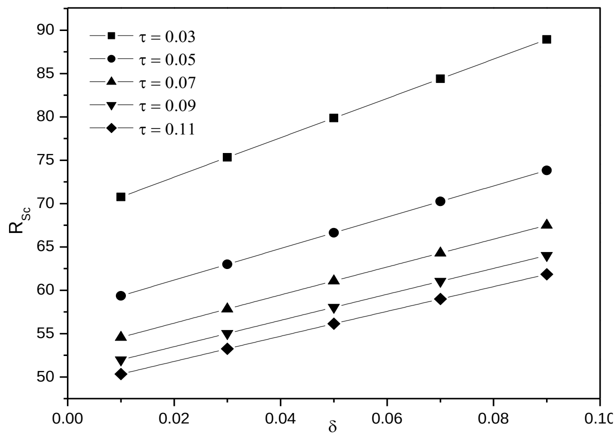


Figure 10.Variation of R_{SC} versus δ for various τ, $R_S = -500,\ S_T = -0.002,\ k = 0.001$ and $M_3 = 5$.

Figure11 indicate the variation of R_{SC} versus S_T for different values of T_a. The figure exhibits a stabilizing behavior which is not much pronounced. The stabilization is minimal when Taylor number T_a assumes values from 10^3 to 10^6, and then it increases phenomenally. This is indicated by an increase in R_{SC} values.

Figure12 is a plot of the variation of R_{SC} versus R_S for different values of T_a. This figure shows that as T_a increases, there is an increase in the values of the critical magnetic Rayleigh number R_{SC}. Therefore Taylor number leads to stability of the system.

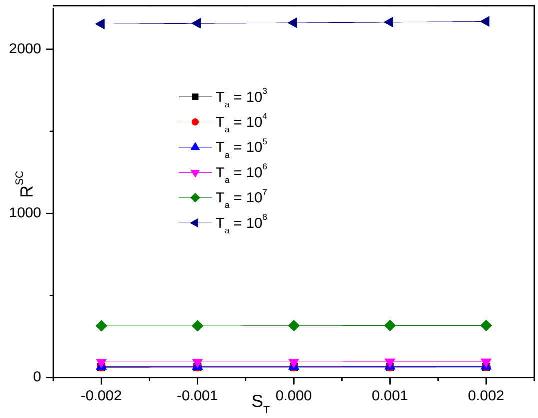


Figure 11.Variation of R_{SC} versus $S_T$for various T_a, R_S = -500, k = 0.001, $\tau = 0.03$ and M_3= 5.

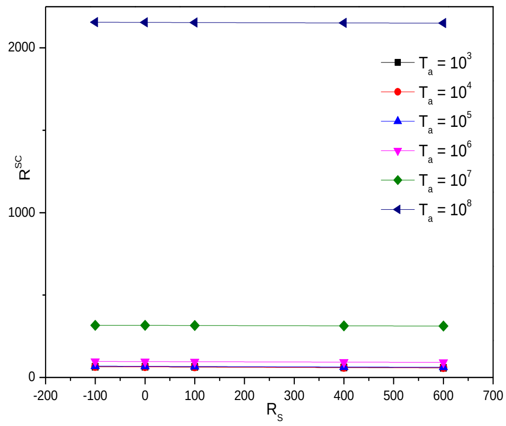


Figure 12. Variation of R_{SC} versus R_S for various T_a, R_S = -500, k = 0.001, $\tau = 0.03$ and M_3= 5.

9. Conclusions

The Soret-driven thermoconvective instability of ferromagnetic fluid layer heated from below and salted from above saturating a densely packed rotating porous medium with magnetic field dependent (MFD) viscosity has been analyzed using Darcy model. Perturbation method is applied and Normal mode analysis is adopted. In the perturbation method, due to the application of magnetic field, the system is perturbed from the basic state (quiescent state). According the governing and other equations are modified. Linear stability analysis is considered. Then Normal mode analysis is taken, Non-dimensional analysis is carried out and the exact solutions satisfying the appropriate boundary conditions are taken yielding to algebraic equations. For getting non-trivial solution for the system of linear homogeneous equations, the coefficients of the dynamic variables are equated to zero and on simplification, the expression for R_{SC} is obtained. Varying the values of the parameters in the allowable range and getting the corresponding R_{SC} values, we get the stability pattern.

Before discussing the significant results of the convective system, we turn our attention to the possible range of values of various parameters arising in the study. The Prandtl number P_r is assumed to be 0.01. The Soret parameter S_T is assumed to take values from -0.002 to 0.002, the salinity Rayleigh number R_s is varied from -500 to 500. The values of ratio of the mass transport to heat transport τ is assumed to be 0.03, 0.05, 0.07, 0.09 and 0.11. The coefficient of MFD viscosity δ is assumed from 0.01 to 0.09. The Taylor numberT_a is assumed from 10 to 10^8.The magnetization parameter M_1 is assumed to be 1000; for a very large value of M_1, the effect of magnetic mechanism is very large, when compared to buoyancy effect. For such fluids, M_2 is assumed to have negligible value and hence taken to be zero. M_3 is varied from 1 to 25 because M_3 cannot take a value less than one. $M_6$is taken to be 0.1. M_4 is the effect on magnetization due to salinity. This is allowed to vary from 0.1 to 0.5 taking values less than the magnetization parameter M_3. M_5 represents the ratio of the salinity effect on magnetic field and pyromagnetic coefficient. This is varied between 0.1 and 0.5. The permeability of porous medium k is assumed to take the values 0.001, 0.003, 0.005, 0.007, 0.009 (Darcy numbers).

A small perturbation imparted on the basic state and a linear stability is used for which normal mode technique is applied. In this investigation, it is clear that the system gets destabilized with respect to

a) variation in magnetization parameter M_3.

b) variation in salinity Rayleigh number R_s.

c) variation in the ratio of the mass transport to heat transport τ.

In order to investigate our results, we must review the results and physical explanations. It is well known that in case of Newtonian fluid the rotation introduces vorticity into the fluid. Then, the fluid moves in the horizontal planes with higher velocities. On account of this motion, the velocity of the fluid perpendicular to the planes reduces, and hence delays the onset of convection. When the fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium, free from rotation or small rate of rotation, then the permeability of porous medium has a destabilizing effect. As permeability of porous medium increases, the void space increases and as a result of this, the flow quantities perpendicular to the planes will clearly be increased. Thus, increasing Darcy's number leads to decrease in critical thermal Rayleigh number. In case of high rotation, the motion of the fluid prevails essentially in the horizontal planes. This motion is increased as permeability of porous medium increases. Thus the component of the velocity perpendicular to the horizontal planes reduces, leading to delay in the onset of convection. Hence permeability of porous medium has a stabilizing effect in the case of high rotation.

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