

# FLOW OF NON-NEWTONIAN FLUID THROUGH A PERMEABLE ARTERY HAVING NON-UNIFORM CROSS SECTION WITH MULTIPLE STENOSIS

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## Abstract:

In this paper, the effect of slip on Micropolar fluid in a circular tube of non-uniform cross-section with multiple stenosis have been studied. The coupled equations governing to the flow are calculated by using Homotopy Perturbation Method. The effects of various parameters with heights of the stenosis on the resistance to the flow and wall shear stress have been studied by deriving the expressions for the flow characteristics and their solutions have been obtained. It is found that the resistance to the flow increases with the heights of the stenosis, inclination, Thermophoresis parameter, local temperature Grashof number, local nanoparticle Grashof number, inclination and permeability constant and decreases with Brownian motion parameter. It is found that the shear stress at the wall increases with heights of the stenosis, Brownian motion parameter but decreases with local nanoparticle Grashof number, Thermophoresis parameter and permeability constant

*Keywords:* Resistance to the flow, stenosis, micropolar fluid, couple stress fluid parameters, permeability constant, wall shear stress.

NOMENCLATURE		Greek s	Greek symbols	
$L_1, L_2$	Lengths of the Stenosis	$\overline{\lambda}$	Normalized resistance to the flow	
$\delta_1, \delta_2$	Heights of the Stenosis	$ au_h$	Wall shear stress	
В	Length of the tube	$N_t$	Thermophoresis parameter	
p	Pressure	N <sub>b</sub>	Brownian motion number	
q	Flow flux	$B_r$	Local nanoparticle Grashof number	
$R_0$	Radius of the tube without stenosis	$G_r$	Local temperature Grashof number	
R	Radius of the tube with stenosis	k	Permeability constant	
w	Axial velocity	$\theta_t$	Temperature Profile	
$\Delta p$	Pressure drop	σ	Nanoparticle Phenomena	
λ	Resistance to the flow			

# 1. Introduction

Stenosis is the most common valvular heart diseases in the developed countries of the world. Vascular fluid dynamics play an important role in the development of arterial stenosis, which is one of the most extensive diseases in human being resulting to failure of the cardiovascular system. The circulation of blood gets interrupted to an extent depending upon the severity of the stenosis.

Prasad *et al.* (2010) and Prasad *et al.* (2015) have studied the peristaltic transport of nanoparticles of micropolar fluid in an inclined tube with heat and mass transport effect. Many researchers have done their work using no-slip boundary condition at the walls of the vessels. But the walls are permeable in physiological systems.

In the past, many researchers assumed the flowing blood to be Newtonian. This assumption of Newtonian behaviour of blood is acceptable for high shear rate flow. But, in some conditions, blood exhibits Non-Newtonian properties (Young, 1968, Shukla *et al.*, 1979, Padmanabhan, 1980, Hayat *et al.*, 2008, Ranadhir*et al.*, 2014 and Mandal, 2015). Most of these theoretical models studied the blood flow in a circular tube or channel having single stenosis. But in the reality, there is a possibility of forming multiple stenoses or over lapping Stenoses in the arteries. Notable researchers like Prasad *et al.* (2008), Muthu *et al.* (2008), Akbar *et al.* (2011), Awgichew *et al.* (2013), Akbar *et al.* (2013) and Raja *et al.* (2016)investigated blood flow in arteries with multiple stenosis. In all these studies they considered the wall of the tube is not flexible. He (1999) and He (2005) discussed the Homotopy perturbation technique and its applications.

The present paper considered micropolar fluid in an inclined permeable tube having non-uniform cross-section with two stenosis and investigated the effects of different parameters on pressure drop, resistance to the flow and wall shear stress.

# 2. Mathematical Formulation

Consider the steady flow of micropolar fluid through a circular tube of non-uniform cross section and two stenoses. A cylindrical polar coordinate system  $(r, \theta, z)$  is taken so that z-axis coincides with the centre line of the tube. It is assumed that the tube is inclined at an angle ' $\alpha$ ' to the horizontal axis [Fig. 1].

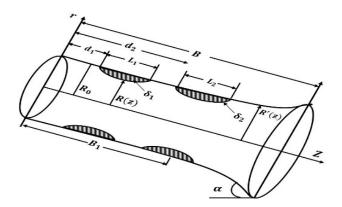


Fig.1: Geometry of an inclined tube with multiple stenoses

Assuming the stenoses are mild and develop in an axially symmetric manner. The radius of the cylindrical tube is taken as (Prasad *et al.*, (2015))

$$h = R(z) = \begin{cases} R_0 & : 0 \le z \le d_1, \\ R_0 - \frac{\delta_1}{2} \left( 1 + \cos \frac{2\pi}{L_1} \left( z - d_1 - \frac{L_1}{2} \right) \right) & : d_1 \le z \le d_1 + L_1, \\ R_0 & : d_1 + L_1 \le z \le B_1 - \frac{L_2}{2} \\ R_0 - \frac{\delta_2}{2} \left( 1 + \cos \frac{2\pi}{L_2} (z - B_1) \right) & : B_1 - \frac{L_2}{2} \le z \le B_1 \\ R^*(z) - \frac{\delta_2}{2} \left( 1 + \cos \frac{2\pi}{L_2} (z - B_1) \right) & : B_1 \le z \le B_1 + \frac{L_2}{2}, \\ R^*(z) & : B_1 + \frac{L_2}{2} \le z \le B . \end{cases}$$

The following restrictions for mild stenoses are supposed to satisfy:  $\delta_i \ll \min(R_0, R_{out}), \delta_i \ll L_i \text{ where } R_{out} = R(z) \text{ at } z = B.$ 

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Here  $L_i$  and  $\delta_i$  (i = 1,2) are the lengths and maximum heights of two stenoses (the suffixes 1 and 2 refer to the first and second stenosis respectively).

The equations for an incompressible fluid with mild stenosis are defined as  $\frac{\partial P}{\partial r} = -\frac{\cos \alpha}{r}$ 

$$\frac{N}{r}\frac{\partial}{\partial r}(rv_{\theta}) + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + (1-N)\frac{\sin\alpha}{F} + (1-N)(G_r\theta_t + B_r\sigma) = (1-N)\frac{\partial P}{\partial z}$$
(2)

$$2v_{\theta} + \frac{\partial w}{\partial r} - \frac{2-N}{m^2} \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (rv_{\theta}) \right) = 0$$
(3)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta_t}{\partial r}\right) + N_b\frac{\partial\sigma}{\partial r}\frac{\partial\theta_t}{\partial r} + N_t\left(\frac{\partial\theta_t}{\partial r}\right)^2 = 0,\tag{4}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\sigma}{\partial r}\right) + \frac{N_t}{N_b}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta_t}{\partial r}\right)\right) = 0, \qquad (5)$$

where w is the velocity in the axial direction.  $\theta_t$ ,  $\sigma$ ,  $N_b$ ,  $N_t$ ,  $G_r$  and  $B_r$  are temperature profile, nanoparticlephenomena, Brownian motion parameter, thermophoresis parameter, local temperature Grashof number and local nanoparticle Grashof number.

The non-dimensional boundary conditions are

$$\frac{\partial w}{\partial r} = 0, \frac{\partial \theta_t}{\partial r} = 0, \frac{\partial \sigma}{\partial r} = 0 \text{ at } r = 0, 
w = -k \frac{\partial w}{\partial r}, \theta_t = 0, \sigma = 0 \text{ at } r = h(z)$$
(6)

 $v_{\theta}$  is finite, w is finite at r = 0

#### 3. Solution

The solutions of the coupled Eq. (4) and (5) have been solved by using Homotopy Perturbation Method (HPM) as

$$H(q_t, \theta_t) = (1 - q_t)[L(\theta_t) - L(\theta_{10})] + q_t \left[ L(\theta_t) + N_b \frac{\partial \sigma}{\partial r} \frac{\partial \theta_t}{\partial r} + N_t \left( \frac{\partial \theta_t}{\partial r} \right)^2 \right]$$
(7)

$$H(q_t, \sigma) = (1 - q_t)[L(\sigma) - L(\sigma_{10})] + q_t \left[ L(\sigma) + \frac{N_t}{N_b} \left( \frac{1}{r} \frac{\sigma}{\sigma r} \left( r \frac{\partial \theta_t}{\partial r} \right) \right) \right], \tag{8}$$

Where  $q_t$  is the embedding parameter which has the range  $0 \le q_t \le 1$ . For our convenience,

$$L = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right)$$
 is taken as a linear operator. The initial guesses  $\theta_{10}$  and  $\sigma_{10}$  are defined as

$$\theta_{10}(r,z) = \left(\frac{r-n}{4}\right), \ \sigma_{10}(r,z) = -\left(\frac{r-n}{4}\right) \tag{9}$$

$$\theta_t(r, z) = \theta_{t_0} + q_t \theta_{t_1} + q_t^2 \theta_{t_2} + \dots \tag{10}$$

$$\sigma(r,z) = \sigma_0 + q_t \sigma_1 + q_t^2 \sigma_2 + \dots \tag{11}$$

The series (10) and (11) are convergent for most of the cases. The convergent depends on the nonlinear part of the equation. Adopting the same procedure as done by Prasad*et al.*(2015), the solution for temperature and nanoparticle phenomena can be written for  $q_t = 1$  as

$$\theta_t(r,z) = \left(\frac{r^2 - h^2}{64}\right) (N_b - N_t) \tag{12}$$

$$\sigma(r,z) = -\left(\frac{r^2 - h^2}{4}\right) \frac{N_t}{N_b}.$$
(13)

$$w(r,z) = (1-N)\left(\frac{r^2-h^2}{4} - \frac{kr}{2}\right)\left(-\frac{\sin\alpha}{F} + \frac{dP}{dz}\right) - N(r-h-k)v_{\theta} + (1-N)B_r\frac{N_t}{N_b}\left(\frac{r^4}{64} - \frac{r^2h^2}{16} + \frac{3h^4}{64} - \frac{kr^3}{16} + \frac{kr^4}{8}\right) - (1-N)G_r(N_b - N_t)\left(\frac{r^6}{2304} - \frac{r^2h^4}{256} + \frac{h^6}{288} - \frac{kr^5}{384} + \frac{krh^4}{128}\right)$$
(14)

(1)

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The dimension less flux q can be calculated as  $q = \int_0^h 2rw \, dr.$ 

By substituting the Eq. (13) in (14), the flux is given by

$$q = (1 - N)\left(\frac{h^4}{8} + \frac{kh^3}{3}\right)\left(\frac{\sin\alpha}{F} - \frac{dP}{dz}\right) + N(h^3 + kh^2)v_\theta + (1 - N)B_r\frac{N_t}{N_b}\left(h^6(0.02083) + kh^5(0.05833)\right) - (1 - N)G_r(N_b - N_t)\left(h^8(0.001627) + kh^7(0.004464)\right)$$

$$(16)$$

From the above equation,  $\frac{dP}{dz}$  can be given as  $\frac{dP}{dz} = \frac{1}{\left(\frac{h^4}{8} + \frac{kh^3}{3}\right)} \left[ -\frac{q}{1-N} + \left(\frac{h^4}{8} + \frac{kh^3}{3}\right) \frac{\sin\alpha}{F} + \frac{N}{1-N} (h^3 + kh^2) v_\theta - G_r (N_b - N_t) \left(h^8 (0.001627) + kh^7 (0.004464)\right) + \frac{N_b}{2} \left(\frac{h^4}{1-N} + \frac{kh^3}{3}\right) \frac{1}{2} \left[ -\frac{q}{1-N} + \frac{h^4}{8} + \frac{kh^3}{3} \right] \left[ -\frac{q}{1-N} + \frac{h^4}{8} + \frac{h^3}{3} \right] \left[ -\frac{q}{1-N} + \frac{h^4}{1-N} \left( -\frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right) \right] \left[ -\frac{q}{1-N} + \frac{h^4}{8} + \frac{h^3}{3} \right] \left[ -\frac{q}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right] \left[ -\frac{q}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right] \left[ -\frac{q}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right] \left[ -\frac{q}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right] \left[ -\frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right] \left[ -\frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right] \left[ -\frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right] \left[ -\frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right] \left[ -\frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right] \left[ -\frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-N} \right] \left[ -\frac{h^4}{1-N} + \frac{h^4}{1-N} + \frac{h^4}{1-$ 

The pressure drop per wave length  $\Delta p = p(0) - p(\lambda)$  is

$$\Delta p = -\int_{0}^{\infty} \frac{dz}{dz} dz$$

$$= \int_{0}^{1} \frac{1}{\left(\frac{h^{4}}{8} + \frac{kh^{3}}{3}\right)} \left[\frac{q}{1-N} - \left(\frac{h^{4}}{8} + \frac{kh^{3}}{3}\right)\frac{\sin\alpha}{F} - \frac{N}{1-N}(h^{3} + kh^{2})v_{\theta} + G_{r}(N_{b} - N_{t})\left(h^{8}(0.001627) + kh^{7}(0.004464)\right) - B_{r}\frac{N_{t}}{N_{b}}\left(h^{6}(0.02083) + kh^{5}(0.05833)\right)\right]dz$$
(18)

 $C^1 dP$ 

The resistance to the flow  $\lambda$  is defined as

$$\begin{split} \lambda &= \frac{\Delta p}{q} \\ &= \frac{1}{q} \int_0^1 \frac{1}{\left(\frac{h^4}{8} + \frac{kh^3}{3}\right)} \left[ \frac{q}{1-N} - \left(\frac{h^4}{8} + \frac{kh^3}{3}\right) \frac{\sin \alpha}{F} - \frac{N}{1-N} (h^3 + kh^2) v_\theta + G_r (N_b - N_t) \left(h^8 (0.001627) + kh^7 (0.004464)\right) - B_r \frac{N_t}{N_b} \left(h^6 (0.02083) + kh^5 (0.05833)\right) \right] dz \end{split}$$
(19)

The pressure drop in the absence of stenosis h = 1 is denoted by  $\Delta p_n$  and is obtained from Eq. (18) as

$$\Delta p_n = \int_0^1 \frac{1}{\left(\frac{1}{8} + \frac{k}{3}\right)} \left[ \frac{q}{1-N} - \left(\frac{1}{8} + \frac{k}{3}\right) \frac{\sin \alpha}{F} - \frac{N}{1-N} (h^3 + kh^2) v_\theta + G_r (N_b - N_t) ((0.001627) + k(0.004464)) - B_r \frac{N_t}{N_b} ((0.02083) + k(0.05833)) \right] dz$$

$$(20)$$

The resistance to the flow in the normal artery is denoted by  $\Delta n$ 

$$\lambda_{n} = \frac{\Delta \rho_{n}}{q} = \frac{1}{q} \int_{0}^{1} \frac{1}{\left(\frac{1}{3} + \frac{k}{3}\right)} \left[ \frac{q}{1-N} - \left(\frac{1}{8} + \frac{k}{3}\right) \frac{\sin \alpha}{F} - \frac{N}{1-N} (1+k) v_{\theta} + G_{r} (N_{b} - N_{t}) \left( (0.001627) + k(0.004464) \right) - B_{r} \frac{N_{t}}{N_{b}} \left( (0.02083) + k(0.05833) \right) \right] dz$$

$$(21)$$

The normalized resistance to the flow denoted by

$$\bar{\lambda} = \frac{\lambda}{\lambda_n} \tag{22}$$

And the wall shear stress is

$$\tau_h = -\frac{h}{2}\frac{dP}{dz}$$

2)

(15)

(17)

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$$= \frac{h}{2} \left[ \frac{1}{\left(\frac{h^4}{8} + \frac{kh^3}{3}\right)} \left[ \frac{q}{1-N} - \left(\frac{h^4}{8} + \frac{kh^3}{3}\right) \frac{\sin \alpha}{F} - \frac{N}{1-N} (h^3 + kh^2) v_\theta - G_r (N_b - N_t) (h^8 (0.001627) + kh^7 (0.004464)) + B_r \frac{N_t}{N_b} (h^6 (0.02083) + kh^5 (0.05833)) \right] \right]$$
(23)

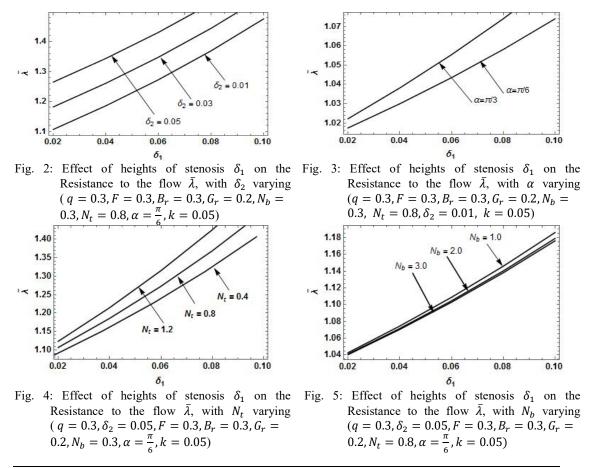
# 4. Result and Discussion

The expressions for pressure drop( $\Delta p$ ), resistance to the flow( $\bar{\lambda}$ ), and wall shear stress( $\tau_h$ ) are given by the Eq. (18), Eq. (22) and Eq. (23) respectively. Using Mathematica 9.1, the effects of various parameters on pressure drop( $\Delta p$ ), resistance to the flow( $\bar{\lambda}$ ), and wall shear stress( $\tau_h$ ) with nanoparticle phenomena have been calculated numerically, by taking

$$\frac{R^{*}(z)}{R_{0}} = \exp[\beta B^{2}(z - B_{1})^{2}],$$
 Where  $d_{1} = 0.2$ ,  $L_{1} = L_{2} = 0.2$ ,  $B_{1} = 0.7, B = 1$  and  $\beta = 0.01$ .

The effects of various parameters on the resistance to the flow  $(\bar{\lambda})$  are shown in Figures (2-8) for various values of Brownian motion number  $(N_b)$ , thermophoresis parameter  $(N_t)$ , local temperature Grashof number  $(G_r)$ , local nanoparticle Grashof number  $(B_r)$ , inclination  $(\alpha)$ . It is observed that, the resistance to the flow  $(\bar{\lambda})$ increases with the heights of the stenosis $(\delta_1 \text{ and } \delta_2)$ , local temperature Grashof number  $(G_r)$ , local nanoparticle Grashof number  $(B_r)$  and inclination  $(\alpha)$ . It is noted that, the velocity of the particles with the surrounding molecules  $(N_t)$  increases with the increases of heights of the stenosis. It is remarkable to note that, the permeability (k) of the walls of the artery increases with the increase of Resistance to the flow.

It is interesting to note that, the resistance to the flow decreases with the increase of the collision between the molecules. I.e., Brownian motion parameter  $(N_b)$ .



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The shear stress acting on the wall  $(\tau_h)$  over the height of stenosis has shown in the Figures (9-13). It is shown that, the shear stress at the wall increases with height of the stenosis. Also, it is observed that when the collision between the molecules  $(N_b)$  increases, the shear stress at the wall also increases. It is also noted that, the shear stress at the wall decreases with  $(B_r)$ , with the heat and mass transfer coefficient  $(N_t)$  and permeability constant (k).

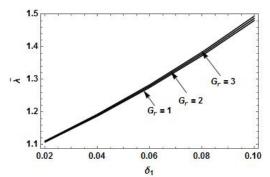


Fig. 6: Effect of heights of stenosis  $\delta_1$  on the Resistance to the flow  $\bar{\lambda}$ , with  $G_r$  varying( $q = 0.3, \delta_2 = 0.05, F = 0.3, B_r = 0.3, N_t = 0.8, N_b = 0.3, \alpha = \frac{\pi}{6}, k = 0.05$ )

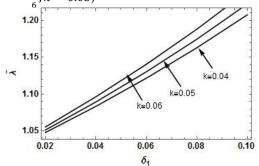
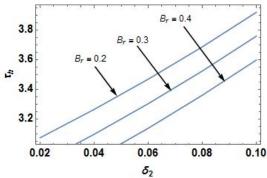


Fig. 8: Effect of heights of stenosis  $\delta_1$  on the Resistance to the flow  $\bar{\lambda}$ , with k varying( $q = 0.3, \delta_2 = 0.05, F = 0.3, G_r = 0.2, B_r = 0.3, N_t = 0.8, N_b = 0.3, \alpha = \frac{\pi}{6}$ )



1.45 1.40 1.35 1.30 Br = 0.2 1.25 Br = 0.3 1.20 Br = 0.5 1.15 1.10 0.02 0.04 0.06 0.08 0.10 δ1

Fig. 7: Effect of heights of stenosis  $\delta_1$  on the Resistance to the flow  $\bar{\lambda}$ , with  $B_r$  varying( $q = 0.3, \delta_2 = 0.05, F = 0.3, G_r = 0.2, N_t = 0.8, N_b = 0.3, \alpha = \frac{\pi}{6}, k = 0.05$ )

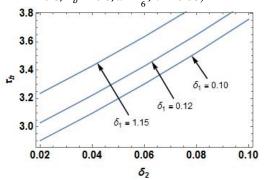
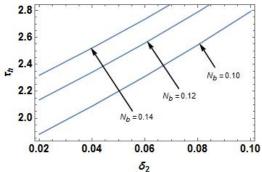


Fig.9: Effect of heights of stenosis  $\delta_2$  on wall shear stress  $\tau_h$ , with  $\delta_1$  varying( $q = 0.3, F = 0.3, G_r = 0.2, B_r = 0.3, N_t = 0.3, N_b = 0.1, \alpha = \frac{\pi}{6}, k = 0.05$ )



r Fig.11: Effect of heights of stenosis  $\delta_2$  on wall shear stress  $\tau_h$ , with  $N_b$  varying( $q = 0.3, \delta_1 = 0.1, F = 0.3, G_r = 0.2, B_r = 0.3, N_t = 0.3, \alpha = \frac{\pi}{6}, k = 0.05$ )

Fig.10: Effect of heights of stenosis  $\delta_2$  on wall shear stress  $\tau_h$ , with  $B_r$  varying  $(q = 0.3, \delta_1 = 0.1, F = 0.3, G_r = 0.2, N_t = 0.3, N_b = 0.1, F = 0.0, R_t = 0.0,$ 

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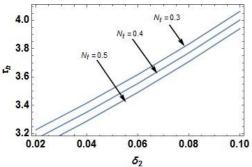


Fig.12: Effect of heights of stenosis  $\delta_2$  on wall shear stress  $\tau_h$ , with  $N_t$  varying ( $q = 0.3, \delta_1 = 0.1, F = 0.3, G_r = 0.2, B_r = 0.3, N_b = 0.1, \alpha = \frac{\pi}{6}, k = 0.05$ )

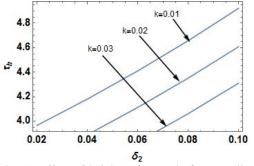


Fig.13: Effect of heights of stenosis  $\delta_2$  on wall shear stress  $\tau_h$ , with k varying  $(q = 0.3, \delta_1 = 0.1, F = 0.3, G_r = 0.2, B_r = 0.3, N_b = 0.1, \alpha = \frac{\pi}{6}, N_t = 0.3)$ 

# 5. Conclusion

A Mathematical model for steady flow of Micro-polar fluid through permeable tube of varying cross-section and is having two stenosis has been studied. The solutions for resistance to the flow, wall shear stress were obtained by using Homotopy Perturbation method.

The conclusions of this model are

- 1. The resistance to the flow increases with the heights of stenosis, inclination, Thermophoresis parameter, Local temperature Grashof number, local nanoparticle Grashof number, permeability constant, and decreases with Brownian motion parameter.
- 2. It is interesting to note that, the resistance to the flow decreases with the heights of the stenosis  $(\delta_1)$  with Brownian motion parameter. But this decrease is significant only when height of the first stenosis exceeds the value 0.04.
- 3. It is also observed that the resistant to the flow increases with heights of stenosis with local temperature Grashof number, but this increase is significant after the value of the first stenosis exceeds 0.08.
- 4. As heights of the stenosis increases, the wall shear stress increases and it is also observed that the collision between the molecules increases with the shear stress at wall.
- 5. The wall shear stress decreases with local nanoparticle Grashof number, Thermophoresis parameter and Permeability constant.

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