



GRAVITATION MODULATION IMPACT ON MHD FREE CONVECTION FLOW OF MICROPOLAR FLUID

S. Sengupta^{1*} and R. Deb²

^{1*}Department of Mathematics, Assam University, Silchar, Assam, India, E-mail: sanjib_aus2009@rediffmail.com;

²Department of Mathematics, Assam University, Silchar, Assam, India, E-mail: reshmideb27@gmail.com

Abstract

In this paper, a theoretical study is carried out on unsteady three dimensional, laminar, free convection flow of micropolar fluid with Hall effect, Joule heating and heat sink under gravitation modulation. A uniform transverse magnetic field is applied normal to the plate along the fluid region. The magnetic Reynolds number is considered to be small due to incomparability of applied and induced magnetic fields, as such the influence of induced magnetic field can be neglected. The multi parameter perturbation technique is used to solve the governed dimensionless equations. The fluid velocity, temperature and the concentration profiles are discussed with the aid of graphs and tables. The coefficient of skin friction and couple stresses is numerically computed in addition to Nusselt number and Sherwood number. The result reveals that the linear velocity increases due to escalation in gravitation modulation parameter values but for intensification in values of gravitation modulation parameter, a reverse effect is observed for rotational velocity. A comparative analysis shows that skin friction coefficient is less in micropolar fluid than the corresponding Newtonian fluid.

Keywords: Micropolar fluid, gravitation modulation, MHD, Hall current, Joule heating and heat sink.

NOMENCLATURE

\bar{C}	Species concentration	\bar{T}	Fluid temperature
\bar{C}_m	Species concentration at the plate	\bar{T}_m	Mean Temperature
C_p	Specific heat at constant pressure	\bar{T}_∞	Temperature in the free stream
\bar{C}_∞	Species concentration in the free stream	u	First component of fluid velocity (non-dimensional)
D_M	Coefficient of mass diffusion	\bar{u}	First component of fluid velocity (dimensional)
\bar{g}	Acceleration due to gravity	U_R	Real part of U
Gm	Thermal Grashof number	\bar{y}	y-Coordinate (non-dimensional)
Gr	Solutal Grashof number	y	y- Coordinate (dimensional)
K	Permeability parameter	\bar{v}	Second component of fluid velocity
k	Thermal conductivity	z	z- Coordinate (dimensional)
K_t	Chemical reaction parameter	w	Third component of fluid velocity
Nu_R	Nusselt number (real part)	\bar{v}	Second component of fluid velocity (dimensional)
Pr	Prandtl number	Greek Symbols	
Ec	Eckert number	θ	Non-dimensional temperature
Q_s	Heat sink parameter	ρ	Fluid density
s	Dimensionless material parameter	ϕ	Non-dimensional species concentration
Sc	Schmidt number	ω	Frequency of oscillation
Sh_R	Sherwood number (real part)	η	Spin gradient viscosity
Sr	Soret number	ν	Kinematic viscosity
M	Magnetic field parameter	ν_r	Rotational kinematic viscosity
m_0	Hall parameter	α	Magnetic field inclination
n	proportional parameter		
t	Time variable(non-dimensional)		
\bar{t}	Time variable (dimensional)		

parameter		θ_R	Real part of θ
ξ	Vortex viscosity	ϕ_R	Real part of ϕ
σ	Electrical conductivity		
β_T	Thermal coefficient of volumetric expansion	Subscripts	
β_c	Solutal coefficient of volumetric expansion	m	Conditions on the wall
	Gravitation modulation parameter	∞	Free stream conditions

1. Introduction

The free convective magnetohydrodynamics micropolar fluid theory is constantly attracting a significant amount of researchers due to its profound impact in science and technical arenas. The concept of micropolar fluid theory developed by Eringen (1966) was a significant generalization of classical Navier-Stokes equation. Micropolar fluids are fluids with microstructure belonging to a class of fluids non-symmetrical stress tensor. The physical instances of micro polar fluids include polymers, lubricants, fluids with additives, paints, animal blood, colloidal and suspension solutions, etc. The theory of MHD flow models under the influence of various physical aspects is developing new approaches to deal with the flow related problems. The study of magnetohydrodynamics (MHD) has important applications found in metrology, planetary magnetospheres, and solar physics and in motion of the earth's core. A comprehensive review of the MHD free convection flow problem under Hall effect in flow domain was discussed by Chamkha (1997). Takhar et al. (2001) studied the boundary layer MHD flow induced by the impulsive stretching surface. Alam et al.(2010) analyzed the problem of unsteady two-dimensional MHD convective flow of a micropolar fluid past a continuously moving porous plate under the influence of magnetic field. Gorla and Chamkha (2011) reported natural convection boundary layer flow over a non-isothermal vertical plate embedded in a porous medium. Alam and Hossain (2013) reported a new similarity approach for an unsteady two-dimensional flow of a micropolar fluid along a wedge. Rajput and Kumar (2019) analyzed the MHD flow past a vertical plate with variable wall temperature and mass diffusion under radiation and chemical reaction effects. Due to advancement of applied science, the expansion of free convective micropolar fluid flow including magnetohydrodynamical effects has great significance. A few illustrations of its various use in diverse spheres such as MHD generators, designing of cooling system of nuclear reactors, propulsion, nuclear generator, planetary magnetospheres and chemical science. Alam and Rahman (2005) analyzed MHD free convective heat and mass transfer flow with Soret and Dufor effect past a vertical porous flat plate embedded in a porous medium. Shamshuddin et al. (2018) studied MHD micropolar fluid with heat and mass transfer near to an inclined porous plate immersed in a porous medium. Sreenivasulu et al. (2018) investigated the MHD dissipative flow of a micropolar fluid under the effects of internal heat generation and viscous dissipation.

The Hall current effect plays a noteworthy role in case of strong magnetic flowing field since the electromagnetic strength creates notable effect in flowing field. Hall effect is a vital event which aids in determination of type of semiconductor. Hall effect devices procured extensive attention in flow field owing to their applications in geophysics, astrophysics and engineering section such as Hall effect sensor, turbine construction and centrifugal machines. A lots of analyst like T Srinivasacharya and Shiferaw (2009), Narayana et al. (2013), Reddy et al. (2014), Ayano and Demeke (2015), Singh (2017) etc. considered the case of Hall current effect on MHD micropolar fluid in the presence of various physical parameters and phenomena to examine the flow characteristics. In view of resolving various type of flow problems, the influence of Joule heating is relevant in designing a variety of electrical devices. Several researcher such as Eldabe and Ouaf (2006), Chamkha et al. (2011), Narayana et al. (2013) and Shamshuddin et al. (2018) analyzed the problem of Joule heating effect on MHD micropolar fluid by considering different physical factors under various sustainable conditions. The effect of heat source or heat sink in the flow process plays a vital role by designing several advanced energy convection materials. A significant use of heat source or heat sink effect was observed in welding engineering field. Several researchers like Khedr et al. (2009), Umavathi and Sultana (2012), Hayat et al. (2014), Sengupta and Sen (2013), Sengupta and Ahmed (2014,2016) and Chamkha et al. (2017) and so many .investigated heat sink effect with various flow parameters and conditions on flowing field of micropolar fluid In view of various fluid related difficulties in industrial and technological field due to rapid growth of mechanical science, the study of chemical reaction has become prominent. In many chemical science activities, chemical reactions take place between a foreign mass and the working fluid which moves due to stretching or otherwise of a surface. If the rate

of chemical reaction is directly proportional to the concentration and it occurs as a single phase volume reaction the chemical reaction is then said to be of first order and homogenous. The chemical reaction possesses its applications in chemical industry, hydrometallurgical industry, petroleum industry, cooling of nuclear reactors and so many. The hydrogenation of ethene is an example of first order chemical reaction. In this case, the reaction between ethene and hydrogen in the presence of a metal crystal like nickel produce ethane. The rate is found to be directly proportional to first power of the concentration of the reactants. Kishan and Amrutha considered the effect of chemical reaction on nonlinear flow of MHD viscous electrically conducting fluid under the influence of thermal stratification and viscous dissipation. Rout et al. (2016) analyzed the impact of chemical reaction for free convective boundary layer flow of MHD micropolar fluid motion. Pattnaik et al. (2019) considered the chemical reaction effect with a second order slip flow on MHD micropolar fluid with boundary layer flow past a stretching sheet. The impact of gravity modulation in flow field is also now receiving a substantial amount of interest in current years because of its importance in a ample branch of technical applications. The time dependent gravity modulation is of realistic impact in diverse fields such as geophysics, astrophysics, geology, chemistry and metallurgy. It has been observed that a significant amount of analyst like Sharidan et al.(2005), Pranesh et al. (2014), Rajvanshi et al. (2014) Yekasi et al. (2016), Kumar and Pranesh (2018) has worked on the effect of gravity modulation under diverse physical circumstances in micropolar fluid. The aim of the present paper is to study the effect of gravitational modulation on laminar free convection flow of incompressible micropolar fluid in the presence of uniform magnetic field. The effect of Hall effect, Joule heatings, and heat sink are also considered in this study. A multi-parameter perturbation technique is used for solving purpose of the non-dimensional governed equations. The present study finds it potential applications in the field of microscale fluid mechanics and non-Newtonian fluid mechanics. The micropolar fluid models possess huge potential applications in diverse field. For example, in fluid mechanics, the fluid models are extensively used in turbo machinery design, designing of and piping and ducting used in water etc. In medical Science, the micropolar fluid model is used in cardiovascular problems, blood pressure measurement machine design, artificial breather machine and so on.

2. Formulation of the Problem

A three dimensional unsteady flow of free convective micropolar fluid past a vertical plate in the presence of Hall current, gravitational modulo and heat sink is considered for study. To generate the mathematical modeling of the flow situation, the \bar{x} axis is taken along the length of the plate, \bar{y} -axis is considered perpendicular to the plate along the fluid region and \bar{z} - axis is taken along width of the plate. Using the fundamental principle of mass conservation, conservation of linear momentum, conservation of angular momentum, energy conservation and species continuity, a fluid model is thus developed following the fluid model of Rajvanshi et al.(2014), Yekasi et al.(2016) as:

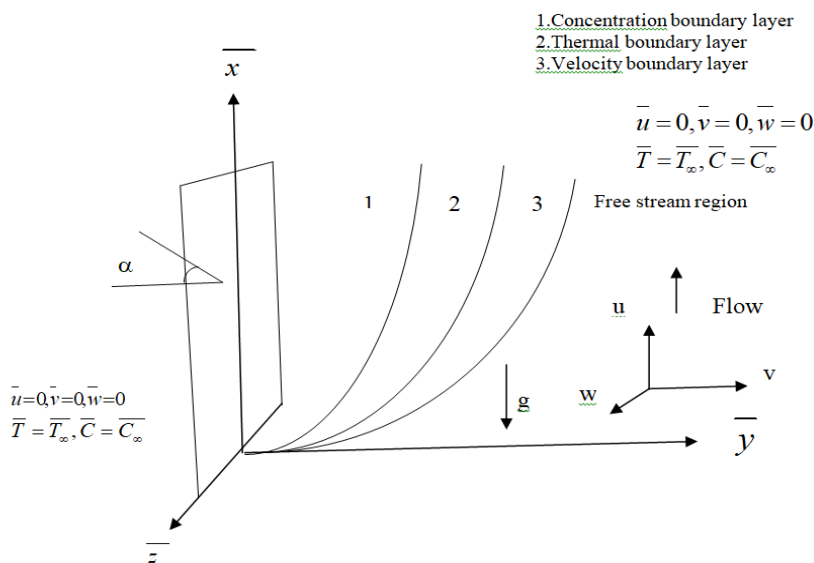


Fig. 1: Schematic representation of the flow domain and geometrical configuration

The set of dynamical equations are:

$$\frac{\partial \bar{w}}{\partial z} = 0 \tag{2.1}$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{w} \frac{\partial \bar{u}}{\partial z} = (\nu + \nu_r) \frac{\partial^2 \bar{u}}{\partial z^2} + g\beta_r (\bar{T} - \bar{T}_\infty) + g\beta_c (\bar{C} - \bar{C}_\infty) - 2\nu_r \frac{\partial \bar{N}_2}{\partial \bar{y}} + \frac{\sigma \mu_e^2 H_0^2 \sin^2 \alpha}{\rho(1+m_0^2)} (m_0 \bar{v} - \bar{u}) \tag{2.2}$$

$$\frac{\partial \bar{v}}{\partial t} + \bar{w} \frac{\partial \bar{v}}{\partial z} = (\nu + \nu_r) \frac{\partial^2 \bar{v}}{\partial z^2} - \frac{\sigma \mu_e^2 H_0^2 \sin^2 \psi}{\rho(1+m^2)} (m_0 \bar{u} + \bar{v}) + 2\nu_r \frac{\partial \bar{N}_1}{\partial z} \tag{2.3}$$

$$\rho I \left(\frac{\partial \bar{N}_i}{\partial t} + \bar{w} \frac{\partial \bar{N}_i}{\partial z} \right) = \gamma \frac{\partial^2 \bar{N}_i}{\partial z^2} \quad \forall i=1,2 \tag{2.4}$$

$$\rho C_p \left(\frac{\partial \bar{T}}{\partial t} + \bar{w} \frac{\partial \bar{T}}{\partial z} \right) = k \frac{\partial^2 \bar{T}}{\partial z^2} + \frac{\sigma \mu_e^2 H_0^2 \sin^2 \psi}{(1+m_0^2)} (\bar{u}^2 + \bar{v}^2) - Q_s (\bar{T} - \bar{T}_\infty) \tag{2.5}$$

$$\frac{\partial \bar{c}}{\partial t} + \bar{w} \frac{\partial \bar{c}}{\partial z} = D_M \frac{\partial^2 \bar{c}}{\partial z^2} + \frac{D_M K_T}{C_p C_s} \frac{\partial^2 \bar{T}}{\partial z^2} - \bar{K}_l (\bar{C} - \bar{C}_\infty) \tag{2.6}$$

The micro polar fluid theory formulation possesses an additional factor of freedom gyration to characterize the rotational motion of microstructure. As a consequence, the balance equation of angular momentum (2.4) is provided for the purpose of solving gyration. In this way, the conservation of angular momentum equation provides a means for take into consideration the effect of molecular spin. The momentum equation is based on the law of conservation of momentum. It states that the net force (F_0) acting on a fluid make a change in momentum of the flow per unit time in that direction. Mathematically, $F_0 = \frac{d}{dt}(m_0 v_0)$, where m_0 is the fluid mass, v_0 is the fluid velocity. The angular momentum equation states that the resulting torque on a rotating fluid is equal to the rate of change of momentum. Mathematically, $\tau_0 = \frac{d}{dt}(I_0 \omega_0)$, where I_0 is the rotational inertia, ω_0 is angular velocity. The angular momentum equation is thus the rotational equivalent of the angular momentum equation.

Subject to the boundary conditions are:

$$\left. \begin{aligned} \bar{u}(\bar{z}, \bar{t}) = 0 = \bar{v}(\bar{z}, \bar{t}), \bar{w}(\bar{z}, \bar{t}) = 0, \bar{N}(\bar{z}, \bar{t}) = 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty \quad \text{at } \bar{t} > 0 \\ \bar{u}(0, \bar{t}) = u_p, \bar{v}(0, \bar{t}) = 0, \bar{w}(0, \bar{t}) = -w_0(1 + \varepsilon \delta \exp(i\omega \bar{t})), \bar{N}_1(0, \bar{t}) = -n \left(\frac{\partial \bar{u}}{\partial \bar{t}} \right)_{\bar{z}=0}, \bar{T} = \bar{T}_\infty, \\ \bar{C} = \bar{C}_m + \varepsilon (\bar{C}_m - \bar{C}_\infty) e^{i\omega \bar{t}}, \bar{T}_m + \varepsilon (\bar{T}_m - \bar{T}_\infty) e^{i\omega \bar{t}}, \bar{N}_2(0, \bar{t}) = -n \left(\frac{\partial \bar{v}}{\partial \bar{t}} \right)_{\bar{z}=0} \\ , \\ \text{and } \bar{u}(\infty, \bar{t}) = 0, \bar{v}(\infty, \bar{t}) = 0, \bar{w}(\infty, \bar{t}) = 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty, \bar{N}(\infty, \bar{t}) = 0, \forall \bar{t} > 0 \end{aligned} \right\} \tag{2.7}$$

where $g = g_0 + g_1 \exp(i\omega t)$ with $g_1 = \varepsilon \delta g_0$
 g_0 : some reference acceleration due to gravity.

We introduce the following non-dimensional quantities as:

$$\begin{aligned} u = \frac{\bar{u}}{w_0}, \quad v = \frac{\bar{v}}{w_0}, \quad w = \frac{\bar{w}}{w_0}, \quad u_p = \frac{\bar{u}_p}{w_0}, \quad z = \frac{\bar{z} w_0}{\nu}, \quad t = \frac{\bar{t} w_0^2}{\nu}, \quad w = \frac{\bar{w}}{w_0^2}, \quad N_i = \frac{\bar{N}_i \nu}{w_0^2}, \quad \xi = \frac{\nu_r}{\nu}, \quad \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_m - \bar{T}_\infty}, \quad \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_m - \bar{C}_\infty}, \\ Gm = \frac{g\beta_c \nu (\bar{C}_m - \bar{C}_\infty)}{w_0^3}, \quad Gr = \frac{g\beta_r \nu (\bar{T}_m - \bar{T}_\infty)}{w_0^3}, \quad M^2 = \frac{\sigma \mu_e^2 H_0^2 \nu}{w_0^3}, \quad Ec = \frac{w_0^2}{C_p (\bar{T}_m - \bar{T}_\infty)}, \quad Q_s = \frac{\bar{Q}_s \nu}{\rho C_p w_0^2}, \quad Pr = \frac{\rho \nu C_p}{k}, \\ Sc = \frac{\nu}{D_M}, \quad Sr = \frac{D_M K_T (\bar{T}_m - \bar{T}_\infty)}{\nu C_p C_s (\bar{C}_m - \bar{C}_\infty)}, \quad K_l = \frac{\nu \bar{K}_l}{w_0^2}. \end{aligned}$$

The equations to be solved are

$$\frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} = (1 + \xi) \frac{\partial^2 u}{\partial z^2} + (Gr\theta + Gm\phi)(1 + \varepsilon\delta \exp(i\omega t)) - 2\xi \frac{\partial N_2}{\partial z} + \frac{M \sin^2 \alpha}{1 + m_0^2} (m_0 v - u) \tag{2.8}$$

$$\frac{\partial u}{\partial t} + w \frac{\partial v}{\partial z} = (1 + \xi) \frac{\partial^2 v}{\partial z^2} + 2\xi \frac{\partial N_1}{\partial z} - \frac{M \sin^2 \alpha}{1 + m_0^2} (m_0 u + v) \tag{2.9}$$

$$\frac{\partial N_i}{\partial t} + w \frac{\partial N_i}{\partial z} = \left(1 + \frac{\xi}{2}\right) \frac{\partial^2 N_i}{\partial z^2}, \quad \forall i=1,2 \tag{2.10}$$

$$\frac{\partial \theta}{\partial t} + w \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial z^2} + \frac{M^2 \sin^2 \alpha Ec (u^2 + v^2)}{1 + m_0^2} - Q_s \theta \tag{2.11}$$

$$\frac{\partial \phi}{\partial t} + w \frac{\partial \phi}{\partial z} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial z^2} + \frac{1}{Sr} \frac{\partial^2 \theta}{\partial z^2} - K_1 \phi \tag{2.12}$$

Subject to the non-dimensional boundary conditions as:

$$u=0, v=0, N_{i=0}, \theta=0, \phi=0, \forall z>0, t=0 \tag{2.13}$$

$$\text{and } u=u_p, v=0, w=-w_0(1+\varepsilon\delta \exp(i\omega t)) \tag{2.14}$$

$$\theta=1+\varepsilon \exp(i\omega t), \phi=1+\varepsilon \exp(i\omega t), N_1 = -n \left(\frac{\partial u}{\partial z}\right)_{z=0}, N_2 = -n \left(\frac{\partial v}{\partial z}\right)_{z=0} \text{ for } t>0 \text{ at } z=0 \tag{2.15}$$

$$\text{Also } u \rightarrow 0, v \rightarrow 0, w \rightarrow 0, N_i \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0, \forall t>0, z \rightarrow \infty \tag{2.16}$$

The reduced form of equations is:

$$\frac{\partial U}{\partial t} + w \frac{\partial U}{\partial z} = (1 + \xi) \frac{\partial^2 U}{\partial z^2} + (Gr\theta + Gm\phi)(1 + \varepsilon\delta \exp(i\omega t)) + 2i\xi \frac{\partial N}{\partial z} - \frac{M \sin^2 \alpha}{1 + m_0^2} U(1 + im_0) \tag{2.17}$$

$$\frac{\partial N}{\partial t} + w \frac{\partial N}{\partial z} = \left(1 + \frac{\xi}{2}\right) \frac{\partial^2 N}{\partial z^2} - 2\xi N, \tag{2.18}$$

$$\frac{\partial \theta}{\partial t} + w \frac{\partial \theta}{\partial z} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial z^2} + \frac{M^2 \sin^2 \alpha Ec}{1 + m_0^2} \bar{U} - Q_s \theta \tag{2.19}$$

$$\frac{\partial \phi}{\partial t} + w \frac{\partial \phi}{\partial z} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial z^2} + Sr \frac{\partial^2 \theta}{\partial z^2} - K_1 \phi \tag{2.20}$$

Subject to the non-dimensional boundary conditions as:

$$u(0,t)=u_p, w=-(1+\varepsilon\delta \exp(i\omega t)), N = -n \left(\frac{\partial U}{\partial z}\right)_{z=0} \tag{2.21}$$

$$\theta(0, t)=1+\varepsilon \exp(i\omega t), \phi(0,t)=1+\varepsilon \exp(i\omega t), t>0 \tag{2.22}$$

$$\text{and } U(\infty, t)=0, w(\infty, t)=0, N(\infty, t)=0, \theta(\infty, t)=0, \phi(\infty, t)=0, t>0 \tag{2.23}$$

$$\text{where } U(z, t)=u(z,t)+iv(z,t) \text{ and } N(z, t)=N_1(z,t)+iN_2(z,t)$$

3. Method of solution:

We first take trial form of solution as:

$$f(z, t) = f_0(z) + \varepsilon f_1(z) \exp(i\omega t)$$

Where, f stands for U, N, θ and ϕ respectively.

On using the above solution form in equations from (2.17)-(2.23), we obtain

$$(1 + \xi) \frac{d^2 U_0}{dz^2} + \frac{dU_0}{dz} - \frac{M^2 \sin^2 \alpha}{1 + m_0^2} (1 + im_0) U = -2\xi i \frac{dN_0}{dz} - Gr\theta_0 - Gm\phi_0 \tag{3.1}$$

$$\left(1 + \frac{\xi}{2}\right) \frac{d^2 N_0}{dz^2} + \frac{dN_0}{dz} - 2\xi N_0 = 0 \tag{3.2}$$

$$\frac{1}{Pr} \frac{d^2 \theta_0}{dz^2} + \frac{d\theta_0}{dz} - Q_s \theta_0 = \frac{M^2}{1 + m_0^2} Ec \sin^2 \alpha U_0 \bar{U}_0 \tag{3.3}$$

$$\frac{1}{Sc} \frac{d^2 \phi_0}{dz^2} + \frac{d\phi_0}{dz} - K_l \phi_0 = -Sr \frac{d^2 \theta_0}{dz^2} \tag{3.4}$$

$$(1 + \xi) \frac{d^2 U_1}{dz^2} + \delta \frac{dU_1}{dz} - \left(\frac{M^2 \sin^2 \alpha}{1 + m_0^2} (1 + im_0) + i\omega \right) U_1 = -2\xi i \frac{dN_1}{dz} - \delta (Gr\theta_0 + Gm\phi_0) - (Gr\theta_1 + Gm\phi_1) \tag{3.5}$$

$$(1 + \frac{\xi}{2}) \frac{d^2 N_1}{dz^2} + \delta \frac{dN_1}{dz} - (2\xi + i\omega) N_1 = 0 \tag{3.6}$$

$$\frac{1}{Pr} \frac{d^2 \theta_1}{dz^2} + \delta \frac{d\theta_1}{dz} - (Q_s + i\omega) \theta_1 = -\frac{M^2}{1 + m_0^2} Ec \sin^2 \alpha (U_0 \bar{U}_1 + U_1 \bar{U}_0) \tag{3.7}$$

$$\frac{1}{Sc} \frac{d^2 \phi_1}{dz^2} + \delta \frac{d\phi_1}{dz} - (K_l + i\omega) \phi_1 = -Sr \frac{d^2 \theta_1}{dz^2} \tag{3.8}$$

Subject to the set of boundary conditions as;

$$U_0 = u_p, N_0 = -n \left(\frac{\partial U_0}{\partial z} \right)_{z=0}, \theta_0 = 1, \phi_0 = 1 \text{ at } z=0 \tag{3.9}$$

$$U_0 \rightarrow 0, N_0 \rightarrow 0, \theta_0 \rightarrow 0, \phi_0 \rightarrow 0, \text{ for } z \rightarrow \infty$$

$$\text{Also, } U_1(0) = 0, N_1 = -n \left(\frac{\partial U_1}{\partial z} \right)_{z=0}, \theta_1(0) = \exp(i\omega t), \phi_1(0) = \exp(i\omega t)$$

$$\text{and } U_1(\infty) = 0, N_1(\infty) = 0, \theta_1(\infty) = 0, \phi_1(\infty) = 0 \tag{3.10}$$

We also use a second set of trial solutions as:

$$f_0(z) = f_{00}(z) + Ec f_{01}(z), f_1(z) = f_{10}(z) + Ec f_{11}(z),$$

Where, the second perturbation parameter Ec is such that $Ec \ll 1$ for an incompressible fluid.

Thus, using the second form of solutions in equations (3.1)-(3.10), we thus get the unperturbed parts as follows:

$$(1 + \xi) \frac{d^2 U_{00}}{dz^2} + \frac{dU_{00}}{dz} - \frac{M^2 \sin^2 \alpha}{1 + m_0^2} (1 + im_0) U_{00} = -2\xi i \frac{dN_{00}}{dz} - Gr\theta_{00} - Gm\phi_{00} \tag{3.11}$$

$$(1 + \frac{\xi}{2}) \frac{d^2 N_{00}}{dz^2} + \frac{dN_{00}}{dz} - 2\xi N_{00} = 0 \tag{3.12}$$

$$\frac{1}{Pr} \frac{d^2 \theta_{00}}{dz^2} + \frac{d\theta_{00}}{dz} - Q_s \theta_{00} = 0 \tag{3.13}$$

$$\frac{1}{Sc} \frac{d^2 \phi_{00}}{dz^2} + \frac{d\phi_{00}}{dz} - K_l \phi_{00} = -Sr \frac{d^2 \theta_{00}}{dz^2} \tag{3.14}$$

$$(1 + \xi) \frac{d^2 U_{01}}{dz^2} + \frac{dU_{01}}{dz} - \frac{M^2 \sin^2 \alpha}{1 + m_0^2} (1 + im_0) U_{01} = -2\xi i \frac{dN_{01}}{dz} - Gr\theta_{01} - Gm\phi_{01} \tag{3.15}$$

$$(1 + \frac{\xi}{2}) \frac{d^2 N_{01}}{dz^2} + \frac{dN_{01}}{dz} - 2\xi N_{01} = 0 \tag{3.16}$$

$$\frac{1}{Pr} \frac{d^2 \theta_{01}}{dz^2} + \frac{d\theta_{01}}{dz} - Q_s \theta_{01} = -\frac{M^2 \sin^2 \alpha}{1 + m_0^2} U_{00} \bar{U}_{00} \tag{3.17}$$

$$\frac{1}{Sc} \frac{d^2 \phi_{01}}{dz^2} + \frac{d\phi_{01}}{dz} - K_l \phi_{01} = -Sr \frac{d^2 \theta_{01}}{dz^2} \tag{3.18}$$

With the boundary conditions as:

$$U_{00} = u_p, N_{00} = -n \left(\frac{\partial U_{00}}{\partial z} \right)_{z=0}, \theta_{00} = 1, \phi_{00} = 1 \text{ at } z=0 \tag{3.19}$$

$$U_{00} \rightarrow 0, N_{00} \rightarrow 0, \theta_{00} \rightarrow 0, \phi_{00} \rightarrow 0, \text{ for } z \rightarrow \infty$$

$$U_{01} = 0, N_{01} = -n \left(\frac{\partial U_{01}}{\partial z} \right)_{z=0}, \theta_{01} = 1, \phi_{01} = 1 \text{ at } z=0 \tag{3.20}$$

$$U_{01} \rightarrow 0, N_{01} \rightarrow 0, \theta_{01} \rightarrow 0, \phi_{01} \rightarrow 0, \text{ for } z \rightarrow \infty$$

The perturbed parts are obtained as:

$$(1 + \xi) \frac{d^2 U_{10}}{dz^2} + \delta \frac{dU_{10}}{dz} - \left(\frac{M^2 \sin^2 \alpha}{1 + m_0^2} (1 + im_0) + i\omega \right) U_{10} = -2\xi i \frac{dN_{10}}{dz} - \delta (Gr\theta_{00} + Gm\phi_{00}) - (Gr\theta_{10} + Gm\phi_{10}) \tag{3.21}$$

$$(1 + \frac{\xi}{2}) \frac{d^2 N_{10}}{dz^2} + \delta \frac{dN_{10}}{dz} - (2\xi + i\omega) N_{10} = 0 \tag{3.22}$$

$$\frac{1}{Pr} \frac{d^2 \theta_{10}}{dz^2} + \delta \frac{d\theta_{10}}{dz} - (Q_s + i\omega) \theta_{10} = 0 \tag{3.23}$$

$$\frac{1}{Sc} \frac{d^2 \phi_{10}}{dz^2} + \delta \frac{d\phi_{10}}{dz} - (K_l + i\omega) \phi_{10} = -Sr \frac{d^2 \theta_{10}}{dz^2} \tag{3.24}$$

$$(1 + \xi) \frac{d^2 U_{11}}{dz^2} + \delta \frac{dU_{11}}{dz} - \left(\frac{M^2 \sin^2 \alpha}{1 + m_0^2} (1 + im_0) + i\omega \right) U_{11} = -2\xi i \frac{dN_{11}}{dz} - \delta (Gr\theta_{01} + Gm\phi_{01}) - (Gr\theta_{11} + Gm\phi_{11}) \tag{3.25}$$

$$(1 + \frac{\xi}{2}) \frac{d^2 N_{11}}{dz^2} + \delta \frac{dN_{11}}{dz} - (2\xi + i\omega) N_{11} = 0 \tag{3.26}$$

$$\frac{1}{Pr} \frac{d^2 \theta_{11}}{dz^2} + \delta \frac{d\theta_{11}}{dz} - (Q_s + i\omega) \theta_{11} = -\frac{M^2 \sin^2 \alpha}{1 + m^2} (U_{00} \bar{U}_{10} + \bar{U}_{00} U_{10}) \tag{3.27}$$

$$\frac{1}{Sc} \frac{d^2 \phi_{11}}{dz^2} + \delta \frac{d\phi_{11}}{dz} - (K_l + i\omega) \phi_{11} = -Sr \frac{d^2 \theta_{11}}{dz^2} \tag{3.28}$$

With the boundary conditions as:

$$\left. \begin{aligned} U_{10}=0, N_{10} = -n \left(\frac{\partial U_{10}}{\partial z} \right)_{z=0}, \theta_{10}=1, \phi_{10}=1 \text{ at } z=0 \\ \text{and } U_{10} \rightarrow 0, N_{10} \rightarrow 0, \theta_{10} \rightarrow 0, \phi_{10} \rightarrow 0, \text{ for } z \rightarrow \infty \\ U_{11}=0, N_{11} = -n \left(\frac{\partial U_{11}}{\partial z} \right)_{z=0}, \theta_{11}=1, \phi_{11}=1 \text{ at } z=0 \\ \text{and } U_{11} \rightarrow 0, N_{11} \rightarrow 0, \theta_{11} \rightarrow 0, \phi_{11} \rightarrow 0, \text{ for } z \rightarrow \infty \end{aligned} \right\} \tag{3.29}$$

Solving equations (1.34)–(1.51) under the boundary conditions equations (1.42)-(1.43), (1.52)–(1.53) and substituting the solutions into (1.23.1), we finally obtain,

$$U(z, t) = u(z, t) + iv(z, t)$$

Where $u(z, t) = u_a + \varepsilon(u_c \cos \omega t - u_d \sin \omega t)$, $v(z, t) = u_b + \varepsilon(u_a \cos \omega t + u_c \sin \omega t)$,

$$\begin{aligned} u_a &= \exp(-\alpha_{01} z) (c_{11} \cos \beta_{01} z + c_{12} \sin \beta_{01} z) + \exp(-m_2 z) c_3 + \exp(-m_4 z) c_5 + \exp(-m_6 z) c_7 + \exp(-m_4 z) c_9 \\ &+ Ec \left(\begin{aligned} &\exp(-\alpha_{01} z) (r_{61} \cos \beta_{01} z + r_{62} \sin \beta_{01} z) + \exp(-m_{11} z) r_1 + \exp(-\alpha_{12} z) r_3 + \exp(-\alpha_{06} z) r_5 + \exp(-\alpha_{07} z) r_7 + \exp(-\alpha_{08} z) r_9 \\ &+ \exp(-\alpha_{10} z) r_{11} + \exp(-\alpha_{02} z) (r_{13} \cos \beta_{01} z + r_{14} \sin \beta_{01} z) + \exp(-\alpha_{04} z) (r_{15} \cos \beta_{01} z + r_{16} \sin \beta_{01} z) + \exp(-\alpha_0 z) \\ &(r_{17} \cos \beta_{01} z + r_{18} \sin \beta_{01} z) + \exp(-\alpha_{02} z) (r_{19} \cos \beta_{01} z - r_{20} \sin \beta_{01} z) + \exp(-\alpha_{05} z) r_{21} + \exp(-\alpha_{04} z) (r_{23} \cos \beta_{01} z - r_{24} \sin \beta_{01} z) \\ &+ \exp(-m_{13} z) r_{29} + \exp(-\alpha_{11} z) r_{27} + \exp(-\alpha_{06} z) (r_{25} \cos \beta_{01} z - r_{26} \sin \beta_{01} z) \end{aligned} \right) \\ u_b &= \exp(-\alpha_{01} z) (c_{12} \cos \beta_{01} z - c_{11} \sin \beta_{01} z) + \exp(-m_2 z) c_4 + \exp(-m_4 z) c_6 + \exp(-m_6 z) c_8 + \exp(-m_4 z) c_{10} \\ &+ Ec \left(\begin{aligned} &\exp(-\alpha_{01} z) (r_{62} \cos \beta_{01} z - r_{61} \sin \beta_{01} z) + \exp(-m_{11} z) r_2 + \exp(-\alpha_{12} z) r_4 + \exp(-\alpha_{06} z) r_6 + \exp(-\alpha_{07} z) r_7 + \exp(-\alpha_{08} z) r_8 \\ &+ \exp(-\alpha_{10} z) r_{10} + \exp(-\alpha_{02} z) (r_{14} \cos \beta_{01} z - r_{13} \sin \beta_{01} z) + \exp(-\alpha_{04} z) (r_{16} \cos \beta_{01} z - r_{15} \sin \beta_{01} z) + \exp(-\alpha_0 z) \\ &(r_{18} \cos \beta_{01} z - r_{17} \sin \beta_{01} z) + \exp(-\alpha_{02} z) (r_{19} \cos \beta_{01} z + r_{20} \sin \beta_{01} z) + \exp(-\alpha_{05} z) r_{22} + \exp(-\alpha_{04} z) (r_{23} \cos \beta_{01} z + r_{24} \sin \beta_{01} z) \\ &+ \exp(-m_{13} z) r_{30} + \exp(-\alpha_{11} z) r_{28} + \exp(-\alpha_{06} z) (r_{25} \cos \beta_{01} z + r_{26} \sin \beta_{01} z) \end{aligned} \right) \end{aligned}$$

$$\begin{aligned}
 u_c = & \exp(-\alpha_{01}z)(d_{24} \cos \beta_{01}z + d_{25} \sin \beta_{01}z) + \exp(-\alpha_{02}z)(d_{10} \cos \beta_{02}z + d_{11} \sin \beta_{02}z) + \exp(-\alpha_{14}z)(d_{20} \cos \beta_{02}z + d_{21} \sin \beta_{02}z) \\
 & + \exp(-\alpha_{03}z)(d_{18} \cos \beta_{03}z + d_{19} \sin \beta_{03}z) + \exp(-m_4z)d_{12} + \exp(-m_6z)d_{14} + \exp(-m_4z)d_{16} \\
 & + Ec \left(\begin{aligned}
 & \exp(-\alpha_{01}z)(g_{63} \cos \beta_{01}z + g_{64} \sin \beta_{01}z) + \exp(-m_{11}z)g_1 + \exp(-\alpha_{12}z)g_3 + \exp(-\alpha_{06}z)g_5 + \exp(-\alpha_{08}z)g_7 + \exp(-\alpha_{10}z)g_9 \\
 & + \exp(-\alpha_{07}z)g_{11} + \exp(-\alpha_{02}z)(g_{15} \cos \beta_{01}z + g_{16} \sin \beta_{01}z) + \exp(-\alpha_{04}z)(g_{17} \cos \beta_{01}z + g_{18} \sin \beta_{01}z) \\
 & + \exp(-\alpha_0z)(g_{19} \cos \beta_{01}z + g_{20} \sin \beta_{01}z) + \exp(-\alpha_{02}z)(g_{21} \cos \beta_{01}z - g_{22} \sin \beta_{01}z) + \exp(-\alpha_{05}z)(g_{23}) \\
 & + \exp(-\alpha_{04}z)(g_{25} \cos \beta_{01}z - g_{26} \sin \beta_{01}z) + \exp(-\alpha_{06}z)(g_{27} \cos \beta_{01}z - g_{28} \sin \beta_{01}z) + \exp(-\alpha_{11}z)(g_{29}) + \exp(-m_{13}z)(g_{31}) \\
 & \exp(-\alpha_{03}z)(h_1 \cos \beta_{03}z + h_2 \sin \beta_{03}z) + \exp(-2\alpha_{01}z)h_3 + \exp(-\alpha_{03}z)(h_5 \cos \beta_{03}z - h_6 \sin \beta_{03}z) \\
 & + \exp(-\alpha_{04}z)(h_7 \cos \beta_{01}z - h_8 \sin \beta_{01}z) + \exp(-\alpha_0z)(h_9 \cos \beta_{03}z - h_{10} \sin \beta_{03}z) + \exp(-m_{17}z)(h_{11} \cos m_{18}z - h_{12} \sin m_{18}z) \\
 & + \exp(-\alpha_{12}z)(h_{13} \cos \beta_{02}z - h_{14} \sin \beta_{02}z) + \exp(-\alpha_{13}z)(h_{15} \cos \beta_{02}z - h_{16} \sin \beta_{02}z) + \exp(-\alpha_{15}z)(h_{17} \cos \beta_{02}z - h_{18} \sin \beta_{02}z) \\
 & + \exp(-\alpha_{04}z)(h_{19} \cos \beta_{02}z + h_{20} \sin \beta_{02}z) + \exp(-\alpha_{06}z)(h_{21}) + \exp(-\alpha_{10}z)(h_{23}) + \exp(-\alpha_{08}z)(h_{25}) + \exp(-\alpha_{07}z)(h_{29}) \\
 & + \exp(-\alpha_0z)(h_{27} \cos \beta_{02}z + h_{28} \sin \beta_{02}z) + \exp(-\alpha_{11}z)(h_{31}) + \exp(-\alpha_{16}z)(h_{33} \cos \beta_{16}z - h_{34} \sin \beta_{16}z) \\
 & + \exp(-\alpha_{17}z)(h_{35} \cos \beta_{14}z - h_{36} \sin \beta_{14}z) + \exp(-\alpha_{18}z)(h_{37} \cos \beta_{14}z - h_{38} \sin \beta_{14}z) + \exp(-\alpha_{19}z)(h_{39} \cos \beta_{14}z - h_{40} \sin \beta_{14}z) \\
 & + \exp(-\alpha_{20}z)(h_{41} \cos \beta_{20}z - h_{42} \sin \beta_{20}z) + \exp(-\alpha_{21}z)(h_{43} \cos \beta_{03}z - h_{44} \sin \beta_{03}z) + \exp(-\alpha_{23}z)(h_{45} \cos \beta_{03}z - h_{46} \sin \beta_{03}z) \\
 & \exp(-\alpha_{22}z)(h_{47} \cos \beta_{03}z - h_{48} \sin \beta_{03}z) + \exp(-m_{17}z)(h_{49} \cos m_{18}z - h_{50} \sin m_{18}z) + \exp(-\alpha_{20}z)(h_{51} \cos \beta_{20}z + h_{52} \sin \beta_{20}z) \\
 & + \exp(-\alpha_{16}z)(h_{53} \cos \beta_{16}z + h_{54} \sin \beta_{16}z) + \exp(-\alpha_{02}z)(h_{55} \cos \beta_{01}z + h_{56} \sin \beta_{01}z) + \exp(-\alpha_{12}z)(h_{57} \cos \beta_{02}z + h_{58} \sin \beta_{02}z) \\
 & + \exp(-\alpha_{21}z)(h_{59} \cos \beta_{03}z + h_{60} \sin \beta_{03}z) + \exp(-\alpha_{17}z)(h_{61} \cos \beta_{14}z + h_{62} \sin \beta_{14}z) + \exp(-\alpha_{14}z)(h_{63} \cos \beta_{01}z + h_{64} \sin \beta_{01}z) \\
 & + \exp(-\alpha_{15}z)(h_{65} \cos \beta_{02}z + h_{66} \sin \beta_{02}z) + \exp(-\alpha_{22}z)(h_{67} \cos \beta_{14}z + h_{68} \sin \beta_{14}z) + \exp(-\alpha_{19}z)(h_{69} \cos \beta_{14}z + h_{70} \sin \beta_{14}z) \\
 & + \exp(-\alpha_{13}z)(h_{71} \cos \beta_{02}z - h_{72} \sin \beta_{02}z) + \exp(-\alpha_{23}z)(h_{73} \cos \beta_{03}z + h_{74} \sin \beta_{03}z) + \exp(-\alpha_{18}z)(h_{75} \cos \beta_{14}z + h_{76} \sin \beta_{14}z)
 \end{aligned} \right)
 \end{aligned}$$

$$\begin{aligned}
 u_d = & \exp(-\alpha_{01}z)(d_{25} \cos \beta_{01}z - d_{24} \sin \beta_{01}z) + \exp(-\alpha_{02}z)(d_{11} \cos \beta_{02}z - d_{10} \sin \beta_{02}z) + \exp(-\alpha_{14}z)(d_{21} \cos \beta_{02}z - d_{20} \sin \beta_{02}z) \\
 & + \exp(-\alpha_{03}z)(d_{19} \cos \beta_{03}z - d_{20} \sin \beta_{03}z) + \exp(-m_4z)d_{13} + \exp(-m_6z)d_{15} + \exp(-m_4z)d_{17} \\
 & + Ec \left(\begin{aligned}
 & \exp(-\alpha_{01}z)(g_{64} \cos \beta_{01}z - g_{63} \sin \beta_{01}z) + \exp(-m_{11}z)g_2 + \exp(-\alpha_{12}z)g_4 + \exp(-\alpha_{06}z)g_6 + \exp(-\alpha_{08}z)g_{87} \\
 & + \exp(-\alpha_{10}z)g_{10} + \exp(-\alpha_{07}z)g_{12} + \exp(-\alpha_{02}z)(g_{16} \cos \beta_{01}z + g_{15} \sin \beta_{01}z) + \exp(-\alpha_{04}z)(g_{18} \cos \beta_{01}z - g_{17} \sin \beta_{01}z) \\
 & + \exp(-\alpha_0z)(g_{20} \cos \beta_{01}z - g_{19} \sin \beta_{01}z) + \exp(-\alpha_{02}z)(g_{22} \cos \beta_{01}z + g_{21} \sin \beta_{01}z) + \exp(-\alpha_{05}z)(g_{24}) \\
 & + \exp(-\alpha_{04}z)(g_{26} \cos \beta_{01}z + g_{25} \sin \beta_{01}z) + \exp(-\alpha_{06}z)(g_{28} \cos \beta_{01}z + g_{27} \sin \beta_{01}z) + \exp(-\alpha_{11}z)(g_{30}) \\
 & + \exp(-m_{13}z)(g_{32}) \exp(-\alpha_{03}z)(h_2 \cos \beta_{03}z - h_1 \sin \beta_{03}z) + \exp(-2\alpha_{01}z)h_4 + \exp(-\alpha_{03}z)(h_6 \cos \beta_{03}z + h_5 \sin \beta_{03}z) \\
 & + \exp(-\alpha_{04}z)(h_8 \cos \beta_{01}z + h_7 \sin \beta_{01}z) + \exp(-\alpha_0z)(h_{10} \cos \beta_{03}z + h_9 \sin \beta_{03}z) + \exp(-m_{17}z)(h_{12} \cos m_{18}z - h_{11} \sin m_{18}z) \\
 & + \exp(-\alpha_{12}z)(h_{14} \cos \beta_{02}z + h_{13} \sin \beta_{02}z) + \exp(-\alpha_{13}z)(h_{16} \cos \beta_{02}z + h_{15} \sin \beta_{02}z) + \exp(-\alpha_{15}z)(h_{18} \cos \beta_{02}z + h_{17} \sin \beta_{02}z) \\
 & + \exp(-\alpha_{04}z)(h_{20} \cos \beta_{02}z - h_{19} \sin \beta_{02}z) + \exp(-\alpha_{06}z)(h_{22}) + \exp(-\alpha_{10}z)(h_{24}) + \exp(-\alpha_{08}z)(h_{26}) + \exp(-\alpha_{07}z)(h_{30}) \\
 & + \exp(-\alpha_0z)(h_{28} \cos \beta_{02}z - h_{27} \sin \beta_{02}z) + \exp(-\alpha_{11}z)(h_{32}) + \exp(-\alpha_{16}z)(h_{34} \cos \beta_{16}z + h_{33} \sin \beta_{16}z) \\
 & + \exp(-\alpha_{17}z)(h_{36} \cos \beta_{14}z + h_{35} \sin \beta_{14}z) + \exp(-\alpha_{18}z)(h_{38} \cos \beta_{14}z + h_{37} \sin \beta_{14}z) + \exp(-\alpha_{19}z)(h_{40} \cos \beta_{14}z + h_{39} \sin \beta_{14}z) \\
 & + \exp(-\alpha_{20}z)(h_{42} \cos \beta_{20}z + h_{41} \sin \beta_{20}z) + \exp(-\alpha_{21}z)(h_{44} \cos \beta_{03}z + h_{43} \sin \beta_{03}z) + \exp(-\alpha_{23}z)(h_{46} \cos \beta_{03}z + h_{45} \sin \beta_{03}z) \\
 & + \exp(-\alpha_{22}z)(h_{48} \cos \beta_{03}z + h_{47} \sin \beta_{03}z) + \exp(-m_{17}z)(h_{50} \cos m_{18}z + h_{49} \sin m_{18}z) + \exp(-\alpha_{20}z)(h_{52} \cos \beta_{20}z - h_{51} \sin \beta_{20}z) \\
 & + \exp(-\alpha_{16}z)(h_{54} \cos \beta_{16}z - h_{53} \sin \beta_{16}z) + \exp(-\alpha_{02}z)(h_{56} \cos \beta_{01}z - h_{55} \sin \beta_{01}z) + \exp(-\alpha_{12}z)(h_{58} \cos \beta_{02}z - h_{57} \sin \beta_{02}z) \\
 & + \exp(-\alpha_{21}z)(h_{60} \cos \beta_{03}z - h_{59} \sin \beta_{03}z) + \exp(-\alpha_{17}z)(h_{62} \cos \beta_{14}z - h_{61} \sin \beta_{14}z) + \exp(-\alpha_{14}z)(h_{64} \cos \beta_{01}z - h_{63} \sin \beta_{01}z) \\
 & + \exp(-\alpha_{15}z)(h_{66} \cos \beta_{02}z - h_{65} \sin \beta_{02}z) + \exp(-\alpha_{22}z)(h_{68} \cos \beta_{14}z - h_{67} \sin \beta_{14}z) + \exp(-\alpha_{19}z)(h_{70} \cos \beta_{14}z - h_{69} \sin \beta_{14}z) \\
 & + \exp(-\alpha_{13}z)(h_{72} \cos \beta_{02}z - h_{71} \sin \beta_{02}z) + \exp(-\alpha_{23}z)(h_{74} \cos \beta_{03}z - h_{73} \sin \beta_{03}z) + \exp(-\alpha_{18}z)(h_{76} \cos \beta_{14}z - h_{75} \sin \beta_{14}z)
 \end{aligned} \right)
 \end{aligned}$$

$$\phi_R(z, t) = \phi_{0R}(z, t) + \varepsilon(\cos(\omega t)\phi_{11a} - \sin(\omega t)\phi_{11b}) = (\phi_{00}(R) + Ec\phi_{01}(R)) + \varepsilon(\cos(\omega t)\phi_{11a} - \sin(\omega t)\phi_{11b})$$

Where $\phi_{00R} = \exp(-m_6z)c_4 + c_5 \exp(-m_4z)$

$$\phi_{01R} = \left(\begin{aligned} &\exp(-m_{13}z)c_{73} + \exp(-m_{11}z)c_{51} + \exp(-\alpha_{12}z)c_{45} + \exp(-\alpha_{06}z)c_{47} + \exp(-\alpha_{08}z)c_{49} + \exp(-\alpha_{09}z)c_{51} + \exp(-\alpha_{07}z)c_{53} \\ &+ \exp(-\alpha_{10}z)c_{55} + \exp(-\alpha_{02}z)(c_{57} \cos \beta_{01}z + c_{58} \sin \beta_{01}z) + \exp(-\alpha_{04}z)(c_{59} \cos \beta_{01}z + c_{60} \sin \beta_{01}z) \\ &+ \exp(-\alpha_0z)(c_{61} \cos \beta_{01}z + c_{62} \sin \beta_{01}z) + \exp(-\alpha_{02}z)(c_{63} \cos \beta_{01}z - c_{64} \sin \beta_{01}z) + \exp(-\alpha_{05}z)c_{65} + \exp(-\alpha_{11}z)c_{71} \\ &+ \exp(-\alpha_{04}z)(c_{67} \cos \beta_{01}z - c_{68} \sin \beta_{01}z) + \exp(-\alpha_{06}z)(c_{69} \cos \beta_{01}z - c_{70} \sin \beta_{01}z) \end{aligned} \right)$$

$$\begin{aligned} \phi_{11a} &= \exp(-\alpha_{14}z)(d_{06} \cos \beta_{14}z + d_{07} \sin \beta_{14}z) + \exp(-\alpha_{03}z)(d_{08} \cos \beta_{03}z + d_{09} \sin \beta_{03}z) \\ &+ \exp(-\alpha_{14}z)(r_{77} \cos \beta_{14}z + r_{78} \sin \beta_{14}z) + \exp(-\alpha_{03}z)(r_1 \cos \beta_{03}z + r_2 \sin \beta_{03}z) + \exp(-2\alpha_{01}z)(r_3) \\ &+ \exp(-\alpha_{02}z)(r_5 \cos \beta_{01}z - r_6 \sin \beta_{01}z) + \exp(-\alpha_{04}z)(r_7 \cos \beta_{03}z - r_8 \sin \beta_{03}z) + \exp(-\alpha_0z)(r_9 \cos \beta_{01}z - r_{10} \sin \beta_{01}z) \\ &+ \exp(-m_{17}z)(r_{11} \cos m_{18}z + r_{12} \sin m_{18}z) + \exp(-\alpha_{12}z)(r_{13} \cos \beta_{02}z - r_{14} \sin \beta_{02}z) + \exp(-\alpha_{13}z)(r_{15} \cos \beta_{02}z - r_{16} \sin \beta_{02}z) \\ &+ \exp(-\alpha_{15}z)(r_{17} \cos \beta_{03}z - r_{18} \sin \beta_{03}z) + \exp(-\alpha_{04}z)(r_{19} \cos \beta_{02}z + r_{20} \sin \beta_{02}z) + \exp(-\alpha_{06}z)(r_{21}) + \exp(-\alpha_{10}z)(r_{23}) \\ &+ \exp(-\alpha_{08}z)(r_{25}) + \exp(-\alpha_0z)(r_{27} \cos \beta_{03}z + r_{28} \sin \beta_{03}z) + \exp(-\alpha_{07}z)(r_{29}) + \exp(-\alpha_{11}z)(r_{31}) + \exp(-\alpha_{16}z)(r_{33} \cos \beta_{16}z - r_{34} \sin \beta_{16}z) \\ &+ \exp(-\alpha_{03}z)(r_{35} \cos \beta_{14}z - r_{36} \sin \beta_{14}z) + \exp(-\alpha_{18}z)(r_{37} \cos \beta_{14}z - r_{38} \sin \beta_{14}z) + \exp(-\alpha_{19}z)(r_{39} \cos \beta_{14}z - r_{40} \sin \beta_{14}z) \\ &+ \exp(-\alpha_{20}z)(r_{41} \cos \beta_{03}z - r_{42} \sin \beta_{03}z) + \exp(-\alpha_{21}z)(r_{43} \cos \beta_{03}z - r_{44} \sin \beta_{03}z) + \exp(-\alpha_{23}z)(r_{45} \cos \beta_{03}z - r_{46} \sin \beta_{03}z) \\ &+ \exp(-\alpha_{22}z)(r_{47} \cos \beta_{03}z - r_{48} \sin \beta_{03}z) + \exp(-m_{17}z)(r_{49} \cos m_{18}z - r_{50} \sin m_{18}z) + \exp(-\alpha_{20}z)(r_{51} \cos \beta_{20}z + r_{52} \sin \beta_{20}z) \\ &+ \exp(-\alpha_{16}z)(r_{53} \cos \beta_{16}z + r_{54} \sin \beta_{16}z) + \exp(-\alpha_{02}z)(r_{55} \cos \beta_{01}z + r_{56} \sin \beta_{01}z) + \exp(-\alpha_{12}z)(r_{57} \cos \beta_{02}z + r_{58} \sin \beta_{02}z) \\ &+ \exp(-\alpha_{21}z)(r_{59} \cos \beta_{03}z + r_{60} \sin \beta_{03}z) + \exp(-\alpha_{17}z)(r_{61} \cos \beta_{14}z + r_{62} \sin \beta_{14}z) + \exp(-\alpha_{14}z)(r_{63} \cos \beta_{01}z + r_{64} \sin \beta_{01}z) \\ &+ \exp(-\alpha_{15}z)(r_{65} \cos \beta_{02}z + r_{66} \sin \beta_{02}z) + \exp(-\alpha_{12}z)(r_{67} \cos \beta_{03}z + r_{68} \sin \beta_{03}z) + \exp(-\alpha_{19}z)(r_{69} \cos \beta_{14}z + r_{70} \sin \beta_{14}z) \\ &+ \exp(-\alpha_{13}z)(r_{71} \cos \beta_{02}z + r_{72} \sin \beta_{02}z) + \exp(-\alpha_{23}z)(r_{73} \cos \beta_{03}z + r_{74} \sin \beta_{03}z) + \exp(-\alpha_{18}z)(r_{75} \cos \beta_{14}z + r_{76} \sin \beta_{14}z) \end{aligned}$$

$$\begin{aligned} \phi_{11b} &= \exp(-\alpha_{14}z)(d_{07} \cos \beta_{14}z - d_{06} \sin \beta_{14}z) + \exp(-\alpha_{03}z)(d_{09} \cos \beta_{03}z - d_{08} \sin \beta_{03}z) \\ &+ \exp(-\alpha_{14}z)(r_{78} \cos \beta_{14}z - r_{77} \sin \beta_{14}z) + \exp(-\alpha_{03}z)(r_2 \cos \beta_{03}z - r_1 \sin \beta_{03}z) + \exp(-2\alpha_{01}z)(r_4) \\ &+ \exp(-\alpha_{02}z)(r_6 \cos \beta_{01}z + r_5 \sin \beta_{01}z) + \exp(-\alpha_{04}z)(r_8 \cos \beta_{03}z + r_7 \sin \beta_{03}z) + \exp(-\alpha_0z)(r_{10} \cos \beta_{01}z + r_{19} \sin \beta_{01}z) \\ &+ \exp(-m_{17}z)(r_{12} \cos m_{18}z - r_{11} \sin m_{18}z) + \exp(-\alpha_{12}z)(r_{14} \cos \beta_{02}z + r_{13} \sin \beta_{02}z) + \exp(-\alpha_{13}z)(r_{16} \cos \beta_{02}z + r_{15} \sin \beta_{02}z) \\ &+ \exp(-\alpha_{15}z)(r_{18} \cos \beta_{03}z + r_{17} \sin \beta_{03}z) + \exp(-\alpha_{04}z)(r_{20} \cos \beta_{02}z - r_{19} \sin \beta_{02}z) + \exp(-\alpha_{06}z)(r_{22}) + \exp(-\alpha_{10}z)(r_{24}) \\ &+ \exp(-\alpha_{08}z)(r_{26}) + \exp(-\alpha_0z)(r_{28} \cos \beta_{03}z - r_{27} \sin \beta_{03}z) + \exp(-\alpha_{07}z)(r_{30}) + \exp(-\alpha_{11}z)(r_{32}) + \exp(-\alpha_{16}z)(r_{34} \cos \beta_{16}z + r_{33} \sin \beta_{16}z) \\ &+ \exp(-\alpha_{03}z)(r_{36} \cos \beta_{14}z - r_{35} \sin \beta_{14}z) + \exp(-\alpha_{18}z)(r_{38} \cos \beta_{14}z + r_{37} \sin \beta_{14}z) + \exp(-\alpha_{19}z)(r_{40} \cos \beta_{14}z + r_{39} \sin \beta_{14}z) \\ &+ \exp(-\alpha_{20}z)(r_{42} \cos \beta_{03}z + r_{41} \sin \beta_{03}z) + \exp(-\alpha_{21}z)(r_{44} \cos \beta_{03}z + r_{43} \sin \beta_{03}z) + \exp(-\alpha_{23}z)(r_{46} \cos \beta_{03}z + r_{45} \sin \beta_{03}z) \\ &+ \exp(-\alpha_{22}z)(r_{48} \cos \beta_{03}z + r_{47} \sin \beta_{03}z) + \exp(-m_{17}z)(r_{50} \cos m_{18}z + r_{49} \sin m_{18}z) + \exp(-\alpha_{20}z)(r_{52} \cos \beta_{20}z - r_{51} \sin \beta_{20}z) \\ &+ \exp(-\alpha_{16}z)(r_{54} \cos \beta_{16}z - r_{53} \sin \beta_{16}z) + \exp(-\alpha_{02}z)(r_{56} \cos \beta_{01}z - r_{55} \sin \beta_{01}z) + \exp(-\alpha_{12}z)(r_{58} \cos \beta_{02}z - r_{57} \sin \beta_{02}z) \\ &+ \exp(-\alpha_{21}z)(r_{60} \cos \beta_{03}z - r_{59} \sin \beta_{03}z) + \exp(-\alpha_{17}z)(r_{62} \cos \beta_{14}z - r_{61} \sin \beta_{14}z) + \exp(-\alpha_{14}z)(r_{64} \cos \beta_{01}z - r_{63} \sin \beta_{01}z) \\ &+ \exp(-\alpha_{15}z)(r_{66} \cos \beta_{02}z - r_{65} \sin \beta_{02}z) + \exp(-\alpha_{12}z)(r_{68} \cos \beta_{03}z - r_{67} \sin \beta_{03}z) + \exp(-\alpha_{19}z)(r_{70} \cos \beta_{14}z - r_{69} \sin \beta_{14}z) \\ &+ \exp(-\alpha_{13}z)(r_{72} \cos \beta_{02}z - r_{71} \sin \beta_{02}z) + \exp(-\alpha_{23}z)(r_{74} \cos \beta_{03}z - r_{73} \sin \beta_{03}z) + \exp(-\alpha_{18}z)(r_{76} \cos \beta_{14}z - r_{75} \sin \beta_{14}z) \end{aligned}$$

$$\theta_r(z, t) = \theta_{0R}(z, t) + \varepsilon(\cos(\omega t)\theta_{11a} - \sin(\omega t)\theta_{11b}) = (\theta_{00R}(z, t) + Ec\theta_{01R}(z, t)) + \varepsilon(\cos(\omega t)\theta_{11a} - \sin(\omega t)\theta_{11b})$$

Where

$$\begin{aligned} \theta_{00R} &= \exp(-m_4z) \\ \theta_{01R} &= \left(\begin{aligned} &\exp(-m_{11}z)(c_{43}) + \exp(-\alpha_{12}z)(c_{13}) + \exp(-\alpha_{06}z)(c_{15}) + \exp(-\alpha_{08}z)(c_{17}) + \exp(-\alpha_{09}z)(c_{19}) + \exp(-\alpha_{07}z)(c_{21}) + \exp(-\alpha_{10}z)(c_{23}) \\ &+ \exp(-\alpha_{02}z)(c_{25} \cos \beta_2z + c_{26} \sin \beta_2z) + \exp(-\alpha_{04}z)(c_{27} \cos \beta_2z + c_{28} \sin \beta_2z) + \exp(-\alpha_0z)(c_{30} \cos \beta_{01}z + c_{30} \sin \beta_{01}z) + \exp(-\alpha_{11}z)(c_{41}) \\ &+ \exp(-\alpha_{02}z)(c_{31} \cos \beta_2z - c_{32} \sin \beta_2z) + \exp(-\alpha_{05}z)(c_{33}) + \exp(-\alpha_{04}z)(c_{35} \cos \beta_{01}z - c_{36} \sin \beta_{01}z) + \exp(-\alpha_{06}z)(c_{37} \cos \beta_{01}z - c_{38} \sin \beta_{01}z) \end{aligned} \right) \end{aligned}$$

$$\theta_{11a} = \exp(-\alpha_{03}z) \cos(\beta_{03}z) + Ec \left(\begin{aligned} &\exp(-\alpha_{03}z)(e_{81} \cos \beta_{03}z + e_{82} \sin \beta_{03}z) + \exp(-2\alpha_{01}z)(e_1) \\ &+ \exp(-\alpha_{02}z)(e_3 \cos \beta_{01}z - e_4 \sin \beta_{01}z) + \exp(-\alpha_{04}z)(e_5 \cos \beta_{03}z - e_6 \sin \beta_{03}z) + \exp(-\alpha_0z)(e_7 \cos \beta_{01}z - e_8 \sin \beta_{01}z) \\ &+ \exp(-m_{17}z)(e_9 \cos m_{18}z + e_{10} \sin m_{18}z) + \exp(-\alpha_{12}z)(e_{11} \cos \beta_{02}z - e_{12} \sin \beta_{02}z) + \exp(-\alpha_{13}z)(e_{13} \cos \beta_{02}z - e_{14} \sin \beta_{02}z) \\ &+ \exp(-\alpha_{15}z)(e_{15} \cos \beta_{03}z - e_{16} \sin \beta_{02}z) + \exp(-\alpha_{04}z)(e_{17} \cos \beta_{02}z + e_{18} \sin \beta_{02}z) + \exp(-\alpha_{06}z)(e_{19}) + \exp(-\alpha_{10}z)(e_{21}) \\ &+ \exp(-\alpha_{08}z)(e_{23}) + \exp(-\alpha_0z)(e_{25} \cos \beta_{03}z + e_{26} \sin \beta_{03}z) + \exp(-\alpha_{07}z)(e_{27}) + \exp(-\alpha_{11}z)(e_{29}) + \exp(-\alpha_{16}z)(e_{33} \cos \beta_{16}z - e_{34} \sin \beta_{16}z) \\ &+ \exp(-\alpha_{03}z)(e_{35} \cos \beta_{14}z - e_{36} \sin \beta_{14}z) + \exp(-\alpha_{18}z)(e_{37} \cos \beta_{14}z - e_{38} \sin \beta_{14}z) + \exp(-\alpha_{19}z)(e_{39} \cos \beta_{14}z - e_{40} \sin \beta_{14}z) \\ &+ \exp(-\alpha_{20}z)(e_{41} \cos \beta_{03}z - e_{42} \sin \beta_{03}z) + \exp(-\alpha_{21}z)(e_{43} \cos \beta_{03}z - e_{44} \sin \beta_{03}z) + \exp(-\alpha_{23}z)(e_{45} \cos \beta_{03}z - e_{46} \sin \beta_{03}z) \\ &+ \exp(-\alpha_{22}z)(e_{47} \cos \beta_{03}z - e_{48} \sin \beta_{03}z) + \exp(-m_{17}z)(e_{51} \cos m_{18}z - e_{52} \sin m_{18}z) + \exp(-\alpha_{20}z)(e_{57} \cos \beta_{20}z + e_{58} \sin \beta_{20}z) \\ &+ \exp(-\alpha_{16}z)(e_{59} \cos \beta_{16}z + e_{60} \sin \beta_{16}z) + \exp(-\alpha_{02}z)(e_{61} \cos \beta_{01}z + e_{62} \sin \beta_{01}z) + \exp(-\alpha_{12}z)(e_{63} \cos \beta_{02}z + e_{64} \sin \beta_{02}z) \\ &+ \exp(-\alpha_{21}z)(e_{65} \cos \beta_{13}z + e_{66} \sin \beta_{13}z) + \exp(-\alpha_{17}z)(e_{67} \cos \beta_{14}z + e_{68} \sin \beta_{14}z) + \exp(-\alpha_{14}z)(e_{69} \cos \beta_{01}z + e_{70} \sin \beta_{01}z) \\ &+ \exp(-\alpha_{15}z)(e_{71} \cos \beta_{02}z + e_{72} \sin \beta_{02}z) + \exp(-\alpha_{22}z)(e_{77} \cos \beta_{03}z + e_{72} \sin \beta_{03}z) + \exp(-\alpha_{19}z)(e_{73} \cos \beta_{14}z + e_{74} \sin \beta_{14}z) \\ &+ \exp(-\alpha_{13}z)(e_{75} \cos \beta_{02}z + e_{76} \sin \beta_{02}z) + \exp(-\alpha_{23}z)(e_{77} \cos \beta_{03}z + e_{78} \sin \beta_{03}z) + \exp(-\alpha_{18}z)(e_{79} \cos \beta_{14}z + e_{80} \sin \beta_{14}z) \end{aligned} \right)$$

$$\theta_{11a} = \exp(-\alpha_{03}z) \sin(\beta_{03}z) + Ec \left(\begin{aligned} &\exp(-\alpha_{03}z)(e_{82} \cos \beta_{03}z - e_{81} \sin \beta_{03}z) + \exp(-2\alpha_{01}z)(e_2) \\ &+ \exp(-\alpha_{02}z)(e_4 \cos \beta_{01}z + e_3 \sin \beta_{01}z) + \exp(-\alpha_{04}z)(e_6 \cos \beta_{03}z + e_5 \sin \beta_{03}z) + \exp(-\alpha_0z)(e_8 \cos \beta_{01}z + e_7 \sin \beta_{01}z) \\ &+ \exp(-m_{17}z)(e_{10} \cos m_{18}z - e_9 \sin m_{18}z) + \exp(-\alpha_{12}z)(e_{12} \cos \beta_{02}z + e_{11} \sin \beta_{02}z) + \exp(-\alpha_{13}z)(e_{14} \cos \beta_{02}z + e_{13} \sin \beta_{02}z) \\ &+ \exp(-\alpha_{15}z)(e_{16} \cos \beta_{03}z + e_{15} \sin \beta_{02}z) + \exp(-\alpha_{04}z)(e_{18} \cos \beta_{02}z - e_{17} \sin \beta_{02}z) + \exp(-\alpha_{06}z)(e_{20}) + \exp(-\alpha_{10}z)(e_{22}) \\ &+ \exp(-\alpha_{08}z)(e_{24}) + \exp(-\alpha_0z)(e_{26} \cos \beta_{03}z - e_{25} \sin \beta_{03}z) + \exp(-\alpha_{07}z)(e_{28}) + \exp(-\alpha_{11}z)(e_{30}) + \exp(-\alpha_{16}z)(e_{34} \cos \beta_{16}z + e_{33} \sin \beta_{16}z) \\ &+ \exp(-\alpha_{03}z)(e_{36} \cos \beta_{14}z + e_{35} \sin \beta_{14}z) + \exp(-\alpha_{18}z)(e_{38} \cos \beta_{14}z + e_{37} \sin \beta_{14}z) + \exp(-\alpha_{19}z)(e_{40} \cos \beta_{14}z + e_{39} \sin \beta_{14}z) \\ &+ \exp(-\alpha_{20}z)(e_{42} \cos \beta_{03}z + e_{41} \sin \beta_{03}z) + \exp(-\alpha_{21}z)(e_{44} \cos \beta_{03}z + e_{43} \sin \beta_{03}z) + \exp(-\alpha_{23}z)(e_{46} \cos \beta_{03}z + e_{45} \sin \beta_{03}z) \\ &+ \exp(-\alpha_{22}z)(e_{48} \cos \beta_{03}z + e_{47} \sin \beta_{03}z) + \exp(-m_{17}z)(e_{52} \cos m_{18}z + e_{51} \sin m_{18}z) + \exp(-\alpha_{20}z)(e_{58} \cos \beta_{20}z - e_{57} \sin \beta_{20}z) \\ &+ \exp(-\alpha_{16}z)(e_{60} \cos \beta_{16}z - e_{59} \sin \beta_{16}z) + \exp(-\alpha_{02}z)(e_{62} \cos \beta_{01}z - e_{61} \sin \beta_{01}z) + \exp(-\alpha_{12}z)(e_{64} \cos \beta_{02}z - e_{63} \sin \beta_{02}z) \\ &+ \exp(-\alpha_{21}z)(e_{66} \cos \beta_{13}z - e_{65} \sin \beta_{13}z) + \exp(-\alpha_{17}z)(e_{68} \cos \beta_{14}z - e_{67} \sin \beta_{14}z) + \exp(-\alpha_{14}z)(e_{70} \cos \beta_{01}z - e_{69} \sin \beta_{01}z) \\ &+ \exp(-\alpha_{15}z)(e_{72} \cos \beta_{02}z - e_{71} \sin \beta_{02}z) + \exp(-\alpha_{22}z)(e_{77} \cos \beta_{03}z - e_{77} \sin \beta_{03}z) + \exp(-\alpha_{19}z)(e_{74} \cos \beta_{14}z - e_{73} \sin \beta_{14}z) \\ &+ \exp(-\alpha_{13}z)(e_{76} \cos \beta_{02}z - e_{75} \sin \beta_{02}z) + \exp(-\alpha_{23}z)(e_{78} \cos \beta_{03}z - e_{77} \sin \beta_{03}z) + \exp(-\alpha_{18}z)(e_{80} \cos \beta_{14}z - e_{79} \sin \beta_{14}z) \end{aligned} \right)$$

Some quantities of engineering interest are discussed as follows:

3.1 Nusselt number

The non-dimensional heat transfer rate at the plate $z = 0$ is given by:

$$Nu_R = -\frac{1}{Pr} \left(\frac{\partial \theta_R}{\partial z} \right)_{z=0} = -\frac{1}{Pr} (Nu_{01} + Nu_{02} + Nu_{03})$$

Where

$$Nu_{01} = -m_4 + Ec \left(\begin{aligned} &(-m_{11}c_{43} - \alpha_{12}c_{13} - \alpha_{06}c_{15} - \alpha_{08}c_{17} - \alpha_{09}c_{19} - \alpha_{07}c_{21} - \alpha_{10}c_{23} - \alpha_{02}c_{25} + c_{26}\beta - \alpha_{04}c_{27} \\ &+ \beta_2c_{28} - \alpha_0c_{29} + c_{30}\beta_{01} - \alpha_{02}c_{31} - c_{32}\beta_2 - \alpha_{05}c_{33} - \alpha_{04}c_{35} - \beta_{01}c_{36} - \alpha_{06}c_{37} - \beta_{01}c_{38} - \alpha_{11}c_{41}) \end{aligned} \right)$$

$$Nu_{02} = \varepsilon \cos \omega t \left(\begin{aligned} &(-\alpha_{03}) + Ec \left(\begin{aligned} &(-\alpha_{03}e_{81} + e_{82}\beta_{03} - 2\alpha_{01}e_1 - \alpha_{02}e_3 - \beta_{01}e_4 - \alpha_{04}e_5 - \beta_{05}e_6 - \alpha_0e_7 - \beta_{01}e_8 - m_{17}e_9 + e_{10}m_{18} - e_{11}\alpha_{12}^* - e_{12}\beta_{02}^* - e_{13}\alpha_{13} - e_{14}\beta_{02}^* - e_{15}\alpha_{15} \\ &- e_{16}\beta_{02}^* - \alpha_{04}e_{17} + e_{18}\beta_{02} - \alpha_{06}e_{19} - \alpha_{10}e_{21} - \alpha_{08}e_{23} - \alpha_{02}e_{25} + e_{26}\beta_{03} - \alpha_{07}e_{27} - \alpha_{11}e_{29} - \alpha_{16}e_{33} - \beta_{16}e_{34} - \alpha_{03}e_{35} - e_{36}\beta_{14} - \alpha_{19}e_{39} - \beta_{14}e_{40} \\ &- \alpha_{20}e_{41} - e_{42}\beta_{03} - \alpha_{21}e_{43} - e_{44}\beta_{03} - \alpha_{23}e_{45} - \beta_{05}e_{46} - \alpha_{22}e_{47} - e_{48}\beta_{03} - m_{17}e_{51} - e_{52}m_{18} - \alpha_{20}e_{57} + e_{58}\beta_{20} - \alpha_{16}e_{59} + e_{60}\beta_{16} - \alpha_{02}e_{61} + e_{62}\beta_{01} \\ &- \alpha_{12}e_{63} + e_{64}\beta_{02}^* - e_{65}\alpha_{21} + e_{66}\beta_{13} - e_{67}\alpha_{17} + e_{68}\beta_{14} - \alpha_{14}e_{69} + e_{70}\beta_{01} - \alpha_{15}e_{71} + e_{72}\beta_{02} - \alpha_{22}e_{77} + e_{72}\beta_{03} - \alpha_{19}e_{73} + e_{74}\beta_{14} - \alpha_{13}e_{75} + e_{76}\beta_{02} \\ &+ \beta_{03}e_{78} - e_{77}\alpha_{23} + \beta_{14}e_{80} - e_{79}\alpha_{18} \end{aligned} \right) \end{aligned} \right)$$

$$Nu_{02} = -\varepsilon \sin \omega t \left(\beta_{03} + Ec \begin{pmatrix} -\alpha_{03}e_{82} - e_{81}\beta_{03} - 2\alpha_{01}e_2 - \alpha_{02}e_4 + \beta_{01}e_3 - \alpha_{04}e_6 + \beta_{03}e_5 - \alpha_{08}e_7 + \beta_{01}e_7 - m_{17}e_{10} - e_{9}m_{18} - e_{12}\alpha_{12}^* + e_{11}\beta_{02}^* - e_{14}\alpha_{13} + e_{13}\beta_{02}^* - e_{16}\alpha_{15} \\ + e_{13}\beta_{02}^* - \alpha_{04}e_{18} - e_{17}\beta_{02} - \alpha_{06}e_{20} - \alpha_{10}e_{22} - \alpha_{08}e_{24} - \alpha_{06}e_{26} - e_{25}\beta_{03} - \alpha_{07}e_{28} - \alpha_{11}e_{30} - \alpha_{16}e_{34} + \beta_{16}\beta_{33} - \alpha_{03}e_{36} + e_{35}\beta_{14} - \alpha_{18}e_{38} + \beta_{14}e_{37} \\ - \alpha_{19}e_{40} + e_{39}\beta_{14} - \alpha_{20}e_{42} + e_{41}\beta_{03} - \alpha_{21}e_{44} + \beta_{03}e_{43} - \alpha_{23}e_{46} + e_{45}\beta_{03} - \alpha_{22}e_{48} - e_{52}m_{17} - \alpha_{20}e_{58} + e_{51}m_{18} - \beta_{20}e_{57} - e_{60}\alpha_{16} - \beta_{16}e_{59} - e_{62}\alpha_{02} \\ - e_{61}\beta_{01} - \alpha_{12}e_{64} - e_{63}\beta_{02} - \alpha_{21}e_{66} - e_{65}\beta_{13} - \alpha_{17}e_{68} - \beta_{14}e_{67} - \alpha_{14}e_{70} - e_{69}\beta_{01} - \alpha_{15}e_{72} - \alpha_{71}\beta_{02} - \alpha_{22}e_{072} - \alpha_{19}e_{74} - \beta_{14}e_{73} - \alpha_{15}e_{76} - e_{75}\beta_{02} \\ - \alpha_{23}e_{78} - e_{77}\beta_{03} - e_{80}\alpha_{18} - e_{79}\beta_{14} \end{pmatrix} \right)$$

3.2 Sherwood number

The non-dimensional mass transfer rate at the plate $z = 0$ is found as:

$$Sh_R = -\frac{1}{Sc} \left(\frac{\partial \phi_R}{\partial z} \right)_{z=0} = -\frac{1}{Sc} (Sh_{01} + Sh_{02} + Sh_{03})$$

Where

$$Sh_{01} = (-m_6c_4 - m_4c_3) + Ec \left(+c_{60}\beta_{01} - \alpha_0c_{61} + c_{62}\beta_{01} - \alpha_{02}c_{63} - c_{64}\beta_{01} - \alpha_{05}c_{65} - \alpha_{11}c_{71} - \alpha_{04}c_{67} - c_{68}\beta_{01} - \alpha_{06}c_{69} - c_{70}\beta_{01} \right)$$

$$Sh_{02} = \varepsilon \cos \omega t (-\alpha_{14}d_{06} + d_{07}\beta_{14} - \alpha_{05}d_{08} + d_{09}\beta_{03})$$

$$+ \varepsilon \cos \omega t \left(Ec \begin{pmatrix} -\alpha_{14}r_{77} + r_{78}\beta_{14} - \alpha_{03}r_{11} + r_2\beta_{03} - 2\alpha_{01}r_3 - \alpha_{02}r_5 - r_6\beta_{01} - \alpha_{04}r_7 - r_8\beta_{03} - \alpha_{09}r_9 - r_{10}\beta_{01} - m_{17}r_{11} + r_{12}m_{18} - \alpha_{12}r_{13} - r_{14}\beta_{02}^* - \alpha_{13}r_{15} - r_{16}\beta_{02} \\ - \alpha_{15}r_{17} - r_{18}\beta_{02} - \alpha_{04}r_{19} + r_{20}\beta_{02} - \alpha_{06}r_{21} - \alpha_{10}r_{23} - \alpha_{08}r_{25} - \alpha_{07}r_{27} + r_{28}\beta_{03} - \alpha_{07}r_{29} - \alpha_{11}r_{31} - \alpha_{16}r_{33} - r_{34}\beta_{16} - \alpha_{03}r_{35} - r_{36}\beta_{14} - \alpha_{18}r_{37} - r_{38}\beta_{14} \\ - \alpha_{19}r_{39} - r_{40}\beta_{14} - \alpha_{20}r_{41} - r_{42}\beta_{03} - \alpha_{21}r_{43} - r_{44}\beta_{03} - \alpha_{23}r_{45} - r_{46}\beta_{03} - \alpha_{22}r_{47} - r_{48}\beta_{03} - m_{17}r_{49} - r_{50}m_{18} - \alpha_{20}r_{51} + r_{52}\beta_{20} - \alpha_{16}r_{53} + r_{54}\beta_{16} - \alpha_{02}r_{55} \\ + r_{56}\beta_{01} - \alpha_{12}r_{57} + r_{58}\beta_{02}^* - \alpha_{21}r_{59} + r_{60}\beta_{03} - \alpha_{17}r_{61} + r_{62}\beta_{14} - \alpha_{14}r_{63} + r_{64}\beta_{01} - \alpha_{15}r_{65} + r_{66}\beta_{02} - \alpha_{12}r_{67} + r_{68}\beta_{03} - \alpha_{19}r_{69} + r_{70}\beta_{14} - \alpha_{13}r_{71} + r_{72}\beta_{02} \\ - \alpha_{23}r_{73} + r_{74}\beta_{03} - \alpha_{18}r_{75} + r_{76}\beta_{14} \end{pmatrix} \right)$$

$$Sh_{03} = \varepsilon \sin \omega t \left(Ec \begin{pmatrix} -\alpha_{19}r_{39} - \beta_{14}r_{40} - \alpha_{20}r_{41} - \beta_{03}r_{42} - \alpha_{21}r_{43} - \beta_{03}r_{44} - r_{45}\alpha_{23} - \beta_{03}r_{46} - r_{47}\alpha_{22} - \beta_{03}r_{48} - r_{49}m_{17} - m_{18}r_{50} - \alpha_{20}r_{51} \\ + \beta_{20}r_{52} - \alpha_{16}r_{53} + r_{54}\beta_{16} - \alpha_{02}r_{55} + \beta_{01}r_{56} - \alpha_{12}r_{57} + \beta_{02}r_{58} - \alpha_{21}r_{59} + \beta_{03}r_{60} - \alpha_{17}r_{61} + \beta_{14}r_{62} - \alpha_{14}r_{63} + \beta_{01}r_{64} \\ - \alpha_{15}r_{65} + \beta_{02}r_{66} - \alpha_{12}r_{67} + \beta_{03}r_{68} - \alpha_{19}r_{69} + \beta_{14}r_{70} - \alpha_{13}r_{71} + \beta_{02}r_{72} - \alpha_{23}r_{73} + \beta_{03}r_{74} - \alpha_{18}r_{75} + \beta_{14}r_{76} \end{pmatrix} \right)$$

3.3 Skin friction coefficient

$$Cf_R = 2(1 + \xi(1 - n)) \left(\frac{\partial U_R}{\partial z} \right)_{z=0} \\ = 2(1 + \xi(1 - n))(cf_1 + cf_2 + cf_3)$$

Where

$$cf_1 = \left(-\alpha_0c_1 + \beta_{01}c_{12} - m_2c_3 - m_4c_5 - m_6c_7 - m_4c_9 - \alpha_{01}r_{61} + \beta_{01}r_{62} - m_{11}r_{71} - \alpha_{12}r_{73} - \alpha_{06}r_{75} - \alpha_{07}r_{77} - \alpha_{09}r_{79} - \alpha_{10}r_{81} - \alpha_{02}r_{83} \right) \\ + \left(\beta_{01}r_{14} - \alpha_{04}r_{15} + \beta_{01}r_{16} - \alpha_{01}r_{17} + \beta_{01}r_{18} - \alpha_{02}r_{19} + \beta_{01}r_{20} - \alpha_{05}r_{21} - \alpha_{04}r_{23} + r_{24}\beta_{01} - \alpha_{01}r_{25} + \beta_{01}r_{26} - \alpha_{11}r_{27} - m_{13}r_{29} \right)$$

$$cf_2 = \cos \omega t \left(\begin{pmatrix} -\alpha_{01}d_{24} + \beta_{01}d_{25} - \alpha_{025}d_{10} + d_{11}\beta_{025} - m_4d_{12} - m_6d_{14} - m_4d_{16} - \alpha_{03}d_{18} + d_{19}\beta_{03} - \alpha_{14}d_{20} + d_{21}\beta_{14} \\ - \alpha_{01}g_{63} + \beta_{01}g_{64} - m_{11}g_{81} - \alpha_{12}g_{83} - \alpha_{06}g_{87} - \alpha_{10}g_{89} - \alpha_{07}g_{11} - \alpha_{02}g_{15} - \alpha_{04}g_{17} - \alpha_0g_{19} - \alpha_{02}g_{21} \\ - \beta_{01}g_{22} - \alpha_{03}g_{23} - \alpha_{04}g_{25} - \beta_{01}g_{26} - \alpha_0g_{27} - \beta_{01}g_{28} - \alpha_{11}g_{29} - g_{31}m_{13} - h_1\alpha_{03} + h_2\beta_{03} - 2\alpha_0h_3 \\ - \alpha_{02}h_5 - h_6\beta_{01} - \alpha_{04}h_7 - \beta_{01}h_8 - \alpha_0h_9 - \beta_{01}h_{10} - m_{17}h_{11} + h_{12}m_{18} - h_{13}\alpha_{12}^* - h_{14}\beta_{12}^* - \alpha_{13}h_{15} - h_{16}\beta_{02}^* \\ - \alpha_{15}h_{17} - h_{18}\beta_{02}^* - \alpha_{04}h_{19} + h_{20}\beta_{02} - \alpha_{06}h_{21} - \alpha_{10}h_{23} - \alpha_{08}h_{25} - \alpha_0h_{27} - \alpha_{07}h_{29} - \alpha_{11}h_{31} - \alpha_{16}h_{33} \\ - h_{34}\beta_{16} - \alpha_{17}h_{35} - h_{36}\beta_{14} - \alpha_{18}h_{37} - h_{38}\beta_{14} - h_{39}\alpha_{19} - h_{40}\beta_{14} - h_{41}\alpha_{20} - h_{36}\beta_{20} - \alpha_{18}h_{37} - h_{38}\beta_{14} \\ - h_{39}\alpha_{19} - h_{40}\beta_{14} - \alpha_{20}h_{41} - h_{42}\beta_{20} - \alpha_{21}h_{43} - h_{44}\beta_{03} - \alpha_{23}h_{45} - h_{46}\beta_{03} - h_{47}\alpha_{22} - h_{48}\beta_{03} - h_{49}m_{17} \\ - h_{50}m_{18} - h_{51}\alpha_{50} + h_{52}\beta_{20} - h_{53}\alpha_{16} + h_{54}\beta_{16} - h_{55}\alpha_{02} + h_{56}\beta_{01} - h_{57}\alpha_{12}^* + h_{58}\alpha_{02}^* - h_{59}\alpha_{21} + \beta_{03}h_{60} \\ - h_{61}\alpha_{17} + h_{62}\beta_{14} - h_{63}\alpha_{14} + h_{64}\beta_{01} - h_{65}\alpha_{15} + h_{66}\beta_{02} - h_{67}\alpha_{22} - h_{69}\alpha_{19} - h_{71}\alpha_{13} - h_{73}\alpha_{23} - h_{75}\alpha_{18} \\ + h_{68}\beta_{03} + h_{70}\beta_{14} + h_{72}\beta_{02}^* + h_{74}\beta_{03} + h_{76}\beta_{14} \end{pmatrix} \right)$$

$$cf_3 = -\sin \omega t \left(\begin{array}{l} -\alpha_{01}d_{25} + \beta_{01}d_{24} - \alpha_{02}^*d_{11} - \beta_{02}^*d_{10} - m_4d_{13} - m_6d_{15} - m_4d_{17} - \alpha_{03}d_{19} - d_{18}\beta_{03} - \alpha_{14}d_{21} - \beta_{14}d_{20} - \alpha_{01}g_{64} - \beta_{01}g_{63} \\ -m_{11}g_2 - \alpha_{12}g_4 - g_6\alpha_{06} - \alpha_{08}g_8 - \alpha_{10}g_{10} - \alpha_{07}g_{12} - \alpha_{02}g_{16} - \beta_{01}g_{15} - \alpha_{04}g_{16} - \beta_{01}g_{17} - \alpha_0g_{20} - \beta_{01}g_{19} - \alpha_{02}g_{22} \\ + \beta_{01}g_{21} - \alpha_{03}g_{24} - \alpha_{04}g_{26} + \beta_{01}g_{25} - g_{28}\alpha_0 + \beta_{01}g_{27} - \alpha_{11}g_{30} - m_{13}g_{32} - \alpha_{02}h_2 - \beta_{03}h_4 - 2\alpha_{01}h_4 - \alpha_{02}h_6 + \beta_{01}h_5 \\ -\alpha_{04}h_8 + \beta_{01}h_7 - \alpha_0h_{10} + \beta_{01}h_9 - h_{12}m_{17} + h_{11}m_{18} + \beta_{02}^*h_{13} - \alpha_{12}^*h_{14} - \alpha_{13}h_{16} + \beta_{02}^*h_{15} - \alpha_{15}h_{18} + \beta_{02}^*h_{17} - \alpha_{04}h_{20} \\ -\beta_{02}h_{19} - \alpha_{06}h_{22} - \alpha_{10}h_{24} - \alpha_{08}h_{26} - h_{28}\alpha_0 - \beta_{01}h_{27} - h_{30}\alpha_{07} - \alpha_{11}h_{32} + \beta_{18}h_{33} - \alpha_{10}h_{34} + \beta_{14}h_{35} - h_{36}\alpha_{17} + \beta_{14}h_{37} \\ -\alpha_{18}h_{38} + \beta_{14}h_{39} - \alpha_{19}h_{40} - \alpha_{20}h_{42} + \beta_{20}h_{41} - \alpha_{21}h_{44} + \beta_{03}h_{43} - \alpha_{23}h_{46} + \beta_{03}h_{45} - \alpha_{22}h_{48} + \beta_{03}h_{47} - m_{17}h_{50} + m_{18}h_{49} \\ -\alpha_{10}h_{52} - \beta_{20}h_{51} - \alpha_{16}h_{54} - h_{53}\beta_{16} - \alpha_{02}h_{56} - \beta_{01}h_{55} - \alpha_{12}^*h_{58} - \beta_{02}^*h_{57} - \beta_{03}h_{59} - \alpha_{21}h_{60} - \beta_{14}h_{61} - \alpha_{17}h_{62} - \beta_{01}h_{63} \\ -\alpha_{14}h_{64} - \alpha_{15}h_{66} - \beta_{03}h_{68} - \alpha_{22}h_{67} - \beta_{14}h_{69} - \alpha_{19}h_{70} - \beta_{02}^*h_{71} - \alpha_{13}h_{72} - \beta_{03}h_{74} - \alpha_{23}h_{73} - \beta_{14}h_{75} - \alpha_{18}h_{76} \end{array} \right)$$

3.4 Couple stress coefficient

$$Cm_R = \left(1 + \frac{1}{2} \xi \right) N'_R (0)$$

$$= -(c_1m_2) + \varepsilon (\cos \omega t (-\alpha_{02}d_{04} + d_{05}\beta_{02}) + \sin \omega t (\alpha_{02}d_{05} + d_{04}\beta_{02}))$$

4. Numerical Data Analysis

A numerical estimation is carried out to comprehend the substantial significance of the flow problem of the three dimensional unsteady MHD micropolar fluid with Hall effect, Joule heating and heat sink in the presence of gravitational modulation. The governing non-dimensional equations are solved with the help of perturbation technique. The results are analyzed with the help of graphs and tables. The non-dimensional velocity, temperature and concentration profiles are discussed numerically. The skin friction and couple stress coefficient in addition to the rate of heat and mass transfers are also computed numerically. In the present problem, the value of Prandtl number (*Pr*) is taken as 7.0 which relates to water (*H₂O*) at 25⁰C and Schmidt number is taken as 0.60 which relates to water vapour at 20⁰C. Prandtl number is a dimensionless physical parameter, which is the ratio between momentum diffusivity and thermal diffusivity. In mathematical strategy, Prandtl number is expressed

as, $Pr = \frac{\nu}{\alpha_0}$ where $\nu = \frac{\mu_0}{\rho}$, $\alpha_0 = \frac{k}{\rho C_p}$ and ν is the momentum diffusivity, μ_0 , α_0 , k , C_p , ρ denote the

dynamic viscosity, thermal diffusivity, thermal conductivity, specific heat and density respectively. The Prandtl number is an estimation of comparative persuasion of momentum and heat energy in a fluid flow. For $Pr \ll 1$ i.e. for small values of Prandtl number, the thermal diffusivity dominates where as for large values of Prandtl number, i.e for $Pr \gg 1$, the momentum diffusivity dominates. The value of Prandtl number for water at temperature 25⁰C, $k = 0.607 Wm^{-1}K^{-1}$, $\mu_0 = 1.002 mPas$, $C_p \square 4.182 JKg^{-1}K^{-1}$ is approximately found as 7.0. In this regard, convection is more effective in transferring energy compare to conduction i.e. the momentum diffusivity is more dominant. Since the momentum diffusivity will dominate in micropolar fluid, so in this case, the value of Prandtl number i.e $Pr \gg 1$. This is why, the value of Prandtl number (*Pr*) is assumed as 7.0 in this problem. The Schmidt number is also a dimensionless parameter, which is a measure of momentum diffusivity relative to mass

diffusivity. Mathematically, $Sc = \frac{\nu}{D_M}$ where D_M is the mass diffusivity of species and ν is the kinematic

viscosity. Schmidt number is employed to trace the fluid motion in which both momentum and mass diffusion occurs together. To justify the fluid modeling the values of other flow variables are taken arbitrarily. The effect of key flow parameters like Prandtl number (*Pr*) non-dimensional temperature profile is depicted through Fig. 2.

It is observed that the fluid temperature decreases for rise in values of Prandtl number. Due to increasing values of Prandtl number, the thermal diffusivity decreases as such the thickness of the thermal boundary layer diminishes. The temperature of fluid particles is thus seen in decreasing trend.

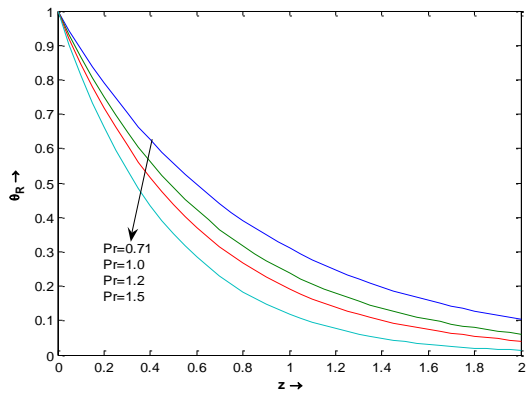


Fig. 2: Graph of temperature (θ_R) versus normal distance (z) for fixed values of $Q_s=0.5$, $K_l=0.5$, $M=0.8$, $m_0=0.2$, $n=0.5$, $\delta=0.5$, $Gr=1.5$, $Gm=0.5$, $\xi=0.5$, $\varepsilon=0.001$, $\alpha=0.7857$, $Ec=0.02$, $\omega t=1.57143$, $u_p=0.5$

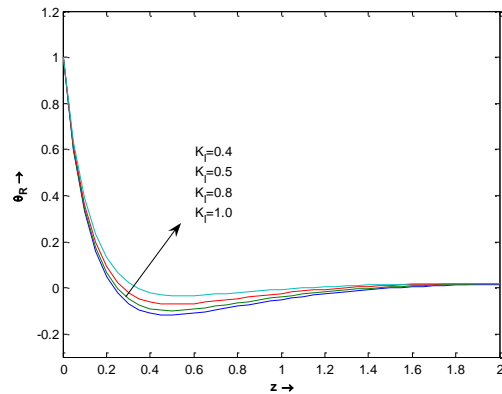


Fig. 3: Graph of temperature (θ_R) versus normal distance (z) for fixed values of $Q_s=0.5$, $M=0.6$, $m_0=0.1$, $n=0.1$, $\delta=0.2$, $Gr=3.0$, $Gm=3.0$, $\xi=0.6$, $\varepsilon=0.001$, $\alpha=0.7857$, $Ec=0.02$, $\omega t=1.57143$, $u_p=0.5$

The effects of physical parameters such as chemical reaction parameter on the temperature profile (θ_R, z) are discussed through Fig. 3. The fluid temperature is seen to increase with rise in values of chemical reaction parameter (K_l). The increase in chemical reaction parameter (K_l) causes an increase magnitude of skin friction thereby it helps in rising the amplitude of temperature wave oscillation within the thermal boundary layer. As a consequence, the temperature of fluid particles is observed in an increasing trend. It is observed from the Figs. 2-3 that fluid temperature obeys the boundary conditions as stated in (2.22-2.23).

The implication of pertinent physical parameters like Schmidt number and chemical reaction parameter on the non-dimensional concentration profile (ϕ_R, z) are demonstrated through Figs. 4 and 5 respectively.

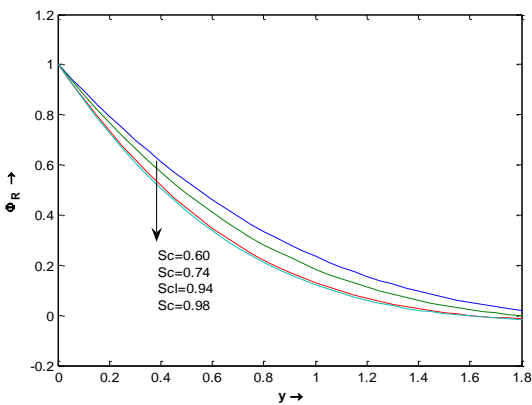


Fig. 4: Graph of concentration (Φ_R) versus normal distance (z) for fixed values of $Q_s=0.3$, $K_l=0.5$, $M=0.8$, $n=0.4$, $m_0=0.2$, $\delta=0.3$, $Gr=3.5$, $Gm=3.0$, $K=0.6$, $s=0.2$, $\varepsilon=0.001$, $\alpha=0.7857$, $\omega t=1.57143$, $Sr=0.6$, $u_p=0.5$

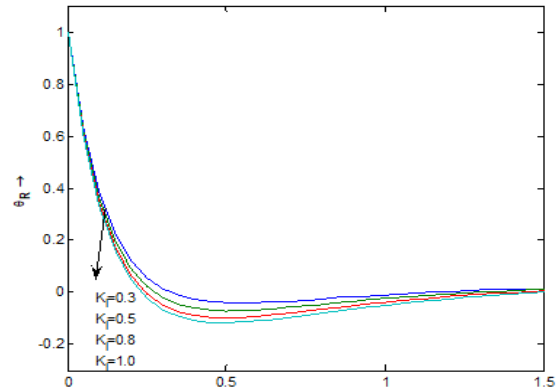


Fig. 5: Graph of concentration (Φ_R) versus normal distance (z) for fixed values of $Q_s=0.3$, $K_l=0.5$, $M=0.8$, $n=0.4$, $m_0=0.2$, $\delta=0.3$, $Gr=3.5$, $Gm=3.0$, $K=0.6$, $s=0.2$, $\varepsilon=0.001$, $\alpha=0.7857$, $\omega t=1.57143$, $Sr=0.6$, $u_p=0.5$

The concentration of fluid particles is found to decrease for increase in values of Schmidt number as well as chemical reaction parameter. In case of increasing values of Schmidt number, the fluid particles possess low diffusivity. The increasing values of Schmidt number thus helps in decreasing the Brownian mass diffusion effect. This in turn minimizes the fluid concentration within the concentration boundary layer. Due to rise in values of chemical reaction parameter, the constituents from higher concentration region i.e. plate zone diffuse to free stream region. As such, the solutal buoyancy effect reduces near plate zone. From the Figs. 4 and 5, the fluid concentration is thus followed by the boundary conditions as stated in (2.22-2.23) for the fluid concentration.

Fig. 6 demonstrates the parametric influence of Eckert number on the linear velocity profile (U_R, z). Due to upsurge in values of Eckert number, the linear velocity is seen in an increasing trend. An increase in Eckert number signifies that the kinetic energy is higher than the enthalpy energy. Thus the increasing values of Eckert number causes in increasing the thermal buoyancy force within the thermal boundary layer and thereby accelerates the linear flow rate.

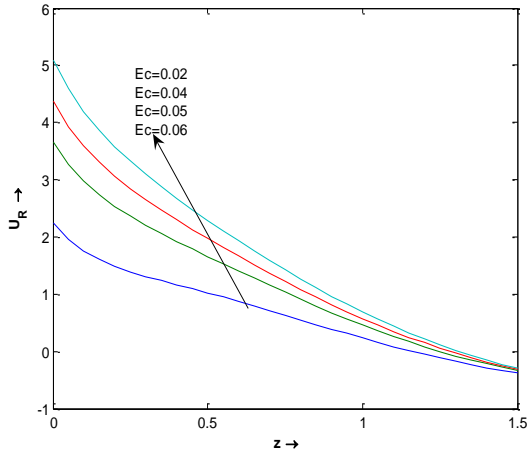


Fig. 6: Graph of linear velocity (u_R) versus normal distance (z) for fixed values of $Q_s=0.3$, $K_l=0.5$, $M=0.8$, $n=0.4$, $m_0=0.2$, $\delta=0.3$, $Gr=1.5$, $Gm=0.8$, $K=0.6$, $\xi=0.8$, $\epsilon=0.001$, $\alpha=0.7857$, $\omega=1.57143$, $Sr=0.6$, $u_p=0.5$

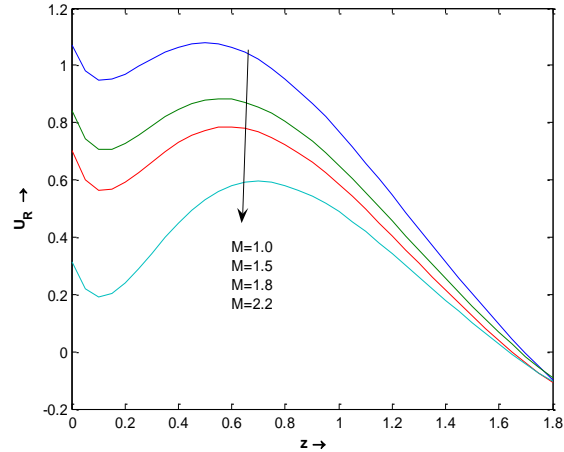


Fig. 7: Graph of linear velocity (u_R) versus normal distance (z) for fixed values of $Q_s=0.3$, $K_l=0.5$, $Sc=0.60$, $n=0.4$, $m_0=0.2$, $\delta=0.3$, $Gr=1.5$, $Gm=0.8$, $K=0.6$, $s=0.2$, $\epsilon=0.001$, $\alpha=0.7857$, $\omega=1.57143$, $Sr=0.6$, $u_p=0.5$, $Ec=0.01$

The influence of key parameter like magnetic field parameter (M) on linear velocity profile (U_R, z) and angular velocity profile (N_R, z) are demonstrated graphically by Figs. 7 and 8 respectively. It is observed that the linear velocity (U_R) decreases while the angular velocity (N_R) increases with rise in values of magnetic field parameter (M). As the Lorentz force generates within boundary layer flow due to the presence of magnetic field which works in the reverse direction of flow. This retards the linear velocity (N_R) and accelerates the rotational velocity (N_R) of fluid particles. Furthermore, increasing values of magnetic field parameter increases the resistive forces which accelerate the linear flow rate and decelerate the rotational flow rate.

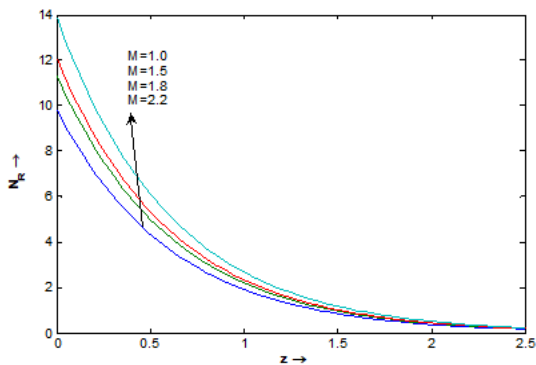


Fig. 8: Graph of angular velocity (N_R) versus normal distance (z) for fixed values of $\xi=0.5$, $m=0.3$, $\delta=0.2$, $n=0.5$, $Gr=0.5$, $Gm=1.0$, $u_p=0.5$, $\epsilon=0.001$, $\alpha=0.7857$, $\omega=1.57143$

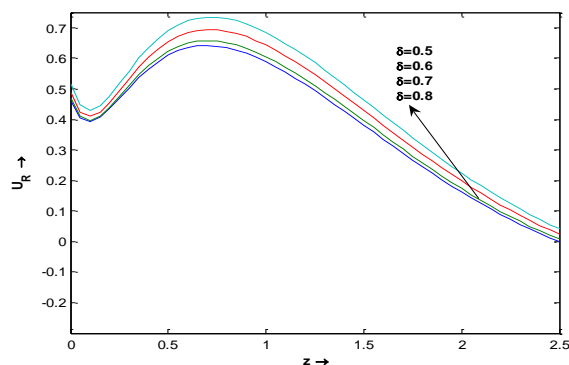


Fig. 9: Graph of linear velocity (u_R) versus normal distance (z) for fixed values of $Q_s=0.4$, $K_l=0.4$, $Sc=0.60$, $M=0.6$, $m_0=0.3$, $Gr=0.5$, $Gm=1.8$, $K=0.6$, $u_p=0.5$, $s=0.2$, $\epsilon=0.001$, $\alpha=0.7857$, $Ec=0.02$, $\omega=1.57143$, $Sr=0.5$

The effect of gravity modulation parameter (δ) on linear velocity profile (U_R, z) and the angular velocity profile (N_R, z) are expressed through Figs. 9 and 10 respectively. It is observed that the increasing values of gravity modulation parameter increase the linear velocity whereas decrease the rotational velocity. The increasing values

of gravitational modulation parameter help in accelerating the linear velocity diffusion rate and decelerating the angular velocity diffusion rate.

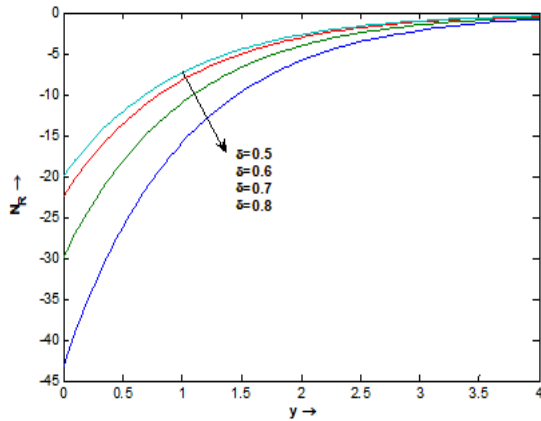


Fig. 10: Graph of angular velocity (N_R) versus normal distance (z) for fixed values of $M=0.8$, $m_0=0.2$, $n=0.3$, $Gr=0.5$, $\varepsilon=0.001$, $\alpha=0.7857$, $u_p=0.5$, $\omega t=1.57143$, $\varepsilon=0.001$

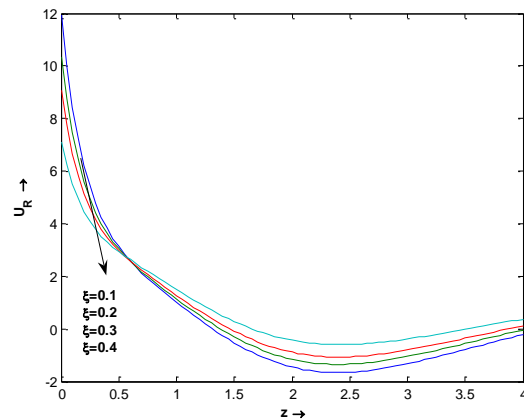


Fig. 11: Graph of linear velocity (u_R) versus normal distance (z) for fixed values of $Q_s=0.4$, $K_l=0.4$, $Sc=0.60$, $M=0.6$, $m_0=0.3$, $Gr=0.5$, $Gm=1.8$, $K=0.6$, $u_p=0.5$, $s=0.2$, $\varepsilon=0.001$, $\alpha=0.7857$, $Ec=0.02$, $\omega t=1.57143$, $Sr=0.5$

The physical impact of vortex viscosity parameter (ξ) on the linear velocity profile (U_R, z) and the angular velocity profile (N_R, z) are presented graphically through Figs. 11 and 12. Due to increase in values of vortex viscosity parameter, the angular velocity is found in decreasing trend while a fluctuating trend is viewed for the linear velocity. The increasing values of vortex viscosity parameter decrease the microrotation diffusivity. This increases the linear momentum diffusivity rate and as such the rotational flow rate decelerates and is thus decrease the rotational fluid velocity. But as the linear diffusivity effect near to the plate is less, linear momentum velocity thus takes its minimum value there in and as normal distance increases beyond 0.5, it shows surprisingly an increasing trend for increasing values of vortex viscosity parameter. It is evident from Figs. 6-12 that both the linear velocity profile (U_R, z) and the angular velocity profile (N_R, z) works in accordance with the boundary conditions as stated in (2.22-2.23).

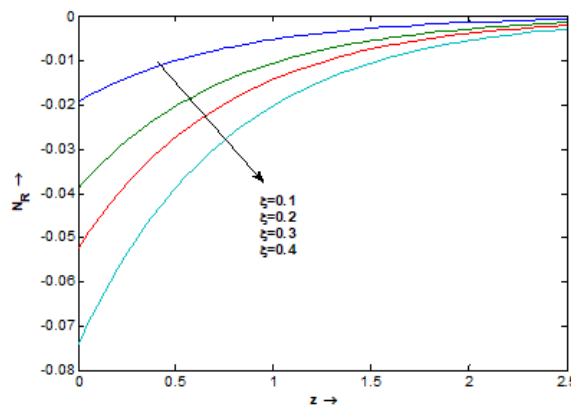


Fig. 12: Graph of angular velocity (N_R) versus normal distance (z) for fixed values of $M=0.8$, $m_0=0.02$, $\delta=0.3$, $n=0.4$, $Gr=0.5$, $Gm=0.8$, $\varepsilon=0.001$, $\alpha=0.7857$, $u_p=0.5$, $\omega t=1.57143$

In Table 1, a comparative analysis between the Newtonian fluid theory and the micropolar fluid theory is made by taking the parametric variations of heat sink, gravitational modulation parameter and chemical reaction parameter on the skin friction coefficient. It is interesting to observe that the skin friction in micropolar fluids is less than its countering Newtonian fluids. Thus, the micropolar fluid is found suitable to apply in regulating the skin friction effect.

Table 1: Comparison between micro polar fluid and Newtonian fluid theory for skin friction coefficient (Cf_R) with different parameters $t=0.2$, $\omega=7.857143$, $\varepsilon=0.01$, $n=0.5$, $K_l=0.5$, $Pr=7.0$, $Sc=0.60$, $Gr=6.0$, $Gm=4.0$, $\psi =0.5236$, $u_p=0.5$, $\xi=0.5$, $M=0.5$, $m_0=0.2$.

$Cf_R \rightarrow$		Newtonian fluid	Micropolar fluid
Q_s	0.1	-3.3949×10^2	-9.8048×10^2
	0.2	-3.0033×10^2	-9.1279×10^2
	0.3	-2.7196×10^2	-8.5805×10^2
K_l	0.3	-1.4189×10^3	-2.3055×10^3
	0.4	-3.5491×10^3	-5.2372×10^3
	0.5	-3.7512×10^3	-5.0694×10^3
m_0	0.3	-4.0252×10^2	-7.1829×10^2
	0.4	-6.3125×10^2	-7.3394×10^2
	0.6	-7.0850×10^2	-7.7608×10^2
δ	1.0	-1.4189×10^3	-2.3055×10^3
	1.5	-3.5941×10^3	-5.2372×10^3
	3.0	-3.7512×10^3	-5.0694×10^3

Table 2: Numerical values of skin friction coefficient for fixed values of $Pr=7.0$, $Sc=0.60$, $u_p=0.5$, $\omega=7.857143$, $t=0.2$, $Gr=6.0$, $Gm=4.0$, $n=0.1$, $\xi=0.5$, $Ec=0.01$

δ	Sr	Kl	α	Q_s	M	m_0	Cf_R
0.3	0.1	0.3	0.5236	0.5	0.5	0.1	-3.7772×10^2
0.3	0.1	0.5	0.5236	0.5	0.5	0.1	-2.6469×10^2
0.4	0.1	0.2	0.5236	0.5	0.5	0.1	-5.4020×10^2
0.5	0.1	0.2	0.5236	0.5	0.5	0.1	-6.3018×10^2
0.3	0.2	0.2	0.5236	0.5	0.5	0.1	-4.5432×10^2
0.3	0.5	0.2	0.5236	0.5	0.5	0.1	-4.1895×10^2
0.3	0.1	0.2	0.5236	0.5	0.5	0.1	-4.6611×10^2
0.3	0.1	0.2	0.7857	0.5	0.5	0.1	-4.2230×10^2
0.3	0.1	0.2	1.0472	0.5	0.5	0.1	-3.6827×10^2
0.3	0.1	0.2	0.5236	0.3	0.5	0.1	-4.5634×10^2
0.3	0.1	0.2	0.5236	0.4	0.5	0.1	-4.6071×10^2
0.3	0.1	0.2	0.5236	0.5	0.6	0.1	-4.7792×10^2
0.3	0.1	0.2	0.5236	0.5	0.7	0.1	-4.7524×10^2
0.3	0.1	0.2	0.5236	0.5	0.5	0.3	-4.6665×10^2
0.3	0.1	0.2	0.5236	0.5	0.5	0.4	-4.6755×10^2

The numerical data relating to the varied values of co-efficient of skin-friction (Cf_R) and couple stress co-efficient (Cm_R) for arbitrary change in values of various parameters like δ , Sr , K_l , α , Q_s , M , m (on Cf_R) and δ , ξ , N , M , α , Gr , Gm (on Cm_R) are presented through Tables 2 and 3 respectively. It is worth full in perceiving that the skin friction coefficient and couple stress coefficient are being influenced by the increasing values of varied key parameters. It is observed that, Cf_R increases only due to increase in values of δ , Q_s while Cm_R increases in presence of δ , N , α , Gr and Gm . A fluctuating effect on Cf_R and Cm_R is noted in presence of M .

Table 3: Numerical values of couple stress coefficient for fixed values of $Pr=7.0$, $Sc=0.60$, $u_p=0.5$, $\omega=7.857143$, $t=0.2$, $n=0.1$, $m_0=0.2$, $\varepsilon=0.01$

δ	ξ	N	M	α	Gr	Gm	Cm_R
0.3	0.2	0.2	0.5	0.5236	6.0	5.0	7.4592
0.3	0.3	0.2	0.5	0.5236	6.0	5.0	5.1062
0.2	0.3	0.2	0.5	0.5236	6.0	5.0	5.1060
0.4	0.3	0.2	0.5	0.5236	6.0	5.0	5.1066
0.3	0.3	0.3	0.5	0.5236	6.0	5.0	6.9266
0.3	0.3	0.4	0.5	0.5236	6.0	5.0	8.1445
0.3	0.3	0.2	0.2	0.5236	6.0	5.0	5.0953
0.3	0.3	0.2	0.4	0.5236	6.0	5.0	5.1015
0.3	0.3	0.2	0.5	0.0000	6.0	5.0	5.0932
0.3	0.3	0.2	0.5	0.5236	6.0	5.0	5.1062
0.3	0.3	0.2	0.5	0.7857	6.0	5.0	5.1193
0.3	0.3	0.2	0.5	1.0472	6.0	5.0	5.1324
0.3	0.3	0.2	0.5	0.5236	4.0	5.0	4.1802
0.3	0.3	0.2	0.5	0.5236	5.0	5.0	4.6432
0.3	0.3	0.2	0.5	0.5236	6.0	6.0	5.5693
0.3	0.3	0.2	0.5	0.5236	6.0	4.0	4.6432

Table 4: Numerical values of Nusselt number for fixed values of $Pr=7.0$, $Sc=0.60$, $u_p=0.5$, $\omega=7.857143$, $t=0.2$, $n=0.1$, $\xi=0.5$, $Ec=0.01$, $\varepsilon=0.01$.

K_l	Q_s	α	Ec	δ	M	m_0	Gr	Gm	Nu_R
0.3	0.5	0.5236	0.01	0.2	0.5	0.1	6.0	4.0	2.3496
0.5	0.5	0.5236	0.01	0.2	0.5	0.1	6.0	4.0	2.1508
0.3	0.5	0.5236	0.01	0.2	0.3	0.1	6.0	4.0	2.2676
0.3	0.5	0.5236	0.01	0.2	0.4	0.1	6.0	4.0	2.3126
0.3	0.3	0.5236	0.01	0.2	0.5	0.1	6.0	4.0	1.7267
0.3	0.4	0.5236	0.01	0.2	0.5	0.1	6.0	4.0	2.0325
0.3	0.5	0.5236	0.01	0.2	0.5	0.1	5.0	4.0	1.9215
0.3	0.5	0.5236	0.01	0.2	0.5	0.1	4.0	4.0	2.1267
0.3	0.5	0.5236	0.01	0.2	0.5	0.1	6.0	5.0	3.6701
0.3	0.5	0.5236	0.01	0.2	0.5	0.1	6.0	6.0	5.7318
0.3	0.5	0.5236	0.01	0.1	0.5	0.1	6.0	4.0	1.5029
0.3	0.5	0.5236	0.01	0.3	0.5	0.1	6.0	4.0	4.7200
0.3	0.5	0.5236	0.01	0.2	0.5	0.2	6.0	4.0	2.4200
0.3	0.5	0.5236	0.01	0.2	0.5	0.3	6.0	4.0	2.4795
0.3	0.5	0.0000	0.01	0.2	0.5	0.1	6.0	4.0	2.1761
0.3	0.5	0.7857	0.01	0.2	0.5	0.1	6.0	4.0	2.4873
0.3	0.5	1.0472	0.01	0.2	0.5	0.1	6.0	4.0	2.5827
0.3	0.5	0.5236	0.02	0.2	0.5	0.1	6.0	4.0	3.8528
0.5	0.5	0.5236	0.03	0.2	0.5	0.1	6.0	4.0	5.3560

In Tables 4 and 5, the numerical values of Nusselt number (Nu_r) as well as Sherwood number (Sh_r) are computed for various values of $\delta, \alpha, Q_s, M, m, Gr, Gm$ (on Nu_r). and $m, n, \delta, Sr, K_l, u_p, Q_s, M$ (on Sh_r). The values of Nu_r is found to increase due to positive increment in values of δ, α, Gr, Gm whereas Sh_r increases due to presence of n, δ, α, M . The computed values of Nusselt number as well as Sherwood number is thus found to be regulated by the governed flow parameters.

Table 5: Numerical values of Sherwood number for fixed values of $Pr=7.0, Sc=0.60, u_p=0.5, \omega=7.857143, t=0.2, Gr=6.0, Gm=4.0, n=0.1, \xi=0.5, Ec=0.001, \epsilon=0.001$.

m_0	n	M	K_l	Q_s	u_p	Sr	δ	α	Sh_R
0.1	0.1	0.5	0.3	1.1	0.5	0.3	0.1	0.5236	4.2154×10^4
0.1	0.1	0.5	0.3	1.1	0.5	0.3	0.3	0.5236	6.0749×10^4
0.2	0.1	0.5	0.3	1.1	0.5	0.3	0.2	0.5236	5.0631×10^4
0.3	0.1	0.5	0.3	1.1	0.5	0.3	0.2	0.5236	5.0630×10^4
0.1	0.1	0.7	0.3	1.1	0.5	0.3	0.2	0.5236	5.0633×10^4
0.1	0.1	0.9	0.3	1.1	0.5	0.3	0.2	0.5236	5.0644×10^4
0.1	0.3	0.5	0.3	1.1	0.5	0.3	0.2	0.5236	5.0665×10^4
0.1	0.4	0.5	0.3	1.1	0.5	0.3	0.2	0.5236	5.0699×10^4
0.1	0.1	0.5	0.4	1.1	0.5	0.3	0.2	0.5236	5.0602×10^4
0.1	0.1	0.5	0.6	1.1	0.5	0.3	0.2	0.5236	5.0547×10^4
0.1	0.1	0.5	0.3	0.8	0.5	0.3	0.2	0.5236	4.9813×10^4
0.1	0.1	0.5	0.3	1.3	0.5	0.3	0.2	0.5236	5.1272×10^4
0.1	0.1	0.5	0.3	1.1	0.5	0.1	0.2	0.5236	5.0632×10^4
0.1	0.1	0.5	0.3	1.1	0.5	0.4	0.2	0.5236	5.0630×10^4
0.1	0.1	0.5	0.3	1.1	0.5	0.3	0.2	0.7857	5.0634×10^4
0.1	0.1	0.5	0.3	1.1	0.5	0.3	0.2	1.0472	5.0639×10^4

In Table 6, a comparative study of the present model with the work of Deka and Soundalgekar(2006) has been made to compare the numerical values of skin friction for both the case of Prandtl number as 0.71 and 7.0. From the table, it is found that the skin friction effects in both the works are found to be decreasing due to increasing values of ωt for fixed values of certain physical parameters $Gr = 1.0, \delta = 1.0, t = 2.0, \epsilon = 0.2$. The range of values obtained are found to be close to one another in the respective papers thus justifying a good agreement of the present work to that work of Deka and Soundalgekar.

Table 6: Numerical values of skin friction for various values of physical parameters when $t=2.0, Gr = 1.0, \epsilon = 0.2, \delta = 1.0$.

ω	ωt	Pr =0.71 (Present paper)	Pr =7.0 (Present paper)	Pr =0.71 Deka and Soundalgekar (2006)paper	Pr =7.0 Deka and Soundalgekar (2006)paper
5	0	1.4755	0.1667	0.5605	0.4211
5	$\pi/6$	1.4934	0.1659	1.9936	1.8359
5	$\pi/4$	1.4885	0.1631	1.4204	1.2692
5	$\pi/3$	1.4621	0.1590	1.1337	0.9865
5	$\pi/2$	1.4621	0.1477	0.8471	0.7038
10	$\pi/4$	1.4694	0.1621	0.8539	0.7027
15	$\pi/4$	1.4585	0.1612	0.6492	0.4979

5. Conclusion

A theoretical investigation on free convection flow of unsteady MHD micropolar fluid past an inclined plate with the effects of gravitational modulation, Hall effect, Joule heating and heat sink. The investigation reveals that the fluid temperature decreases due to increase in values of Prandtl number. The concentration of fluid particles decreases with rise in values of flow parameters like Schmidt number and chemical reaction. The linear fluid velocity is found to increase due to increasing values of gravitational modulation parameter, Eckert number, but decreases for increasing values of magnetic field parameter. The rotational velocity of fluid particles is found increasing for rise in values of magnetic field parameter while it decreases for increasing values of gravitational modulation parameter. The skin friction effect is possible to regulate by using the micropolar fluid theory.

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