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EFFECTS OF VARIABLE THERMAL CONDUCTIVITY ON THE COUPLING OF CONDUCTION AND JOULE HEATING WITH MHD FREE CONVECTION FLOW ALONG A VERTICAL FLAT PLATE

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Abstract:

The effects of variable thermal conductivity on the coupling of conduction and Joule heating with MHD free convection flow along a vertical flat plate have been explained by the current work. With an aim to achieve similarity solutions of the problem posed, the developed equations are made dimensionless by using suitable transformations. The non-dimensional equations are then transformed into non-linear equations by introducing non- similarity transformations. Applying the implicit finite difference simulation along with Keller-box scheme, the resulting non-similar equations together with their corresponding boundary conditions based on conduction and convection are solved numerically. Results for the details of the velocity profile, the temperature profile, the local skin friction coefficient and the surface temperature profile are shown both on graphs and tabular form for different values of the set of parameters entering into the problem.

Keywords: Joule heating, MHD, conduction, variable thermal conductivity, natural convection.

NOMENCLATURE

b	plate thickness $[m]$
C_{fx}	local skin friction coefficient
C_p	specific heat at constant pressure $[JKg^{-1}K^{-1}]$
f	dimensionless stream function
g	acceleration due to gravity
Gr	Grashof number
h	dimensionless temperature coefficient
H_0	applied magnetic field strength
J	Joule heating parameter
L	reference length of the plate $[m]$
M	magnetic parameter
P	conjugate conduction parameter
Pr	Prandtl number
T	temperature of the interface $[K]$
T_b	temperature outside the plate $[K]$
x , y	dimensionless Cartesian co-ordinates
$\bar{x}$, $\bar{y}$	Cartesian co-ordinates $[m]$
T_f	temperature of the fluid $[K]$
T_∞	temperature of the ambient fluid $[K]$
u, v	dimensionless velocity component along x & y direction
$\bar{u}$, $\bar{v}$	velocity component along $\bar{x}$, $\bar{y}$ direction $[ms^{-1}]$

Greek symbols

β	coefficient of thermal expansion $[K^{-1}]$
γ	thermal conductivity variation parameter
η	similarity variable
θ	dimensionless temperature profile
κ_s , κ_f	thermal conductivity of the solid and fluid respectively $[Wm^{-1}K^{-1}]$
κ_∞	thermal conductivity of the ambient fluid $[Wm^{-1}K^{-1}]$
μ	dynamic viscosity of the fluid $[Kgm^{-1}s^{-1}]$
ν	kinematic viscosity $[m^2s^{-1}]$
ρ	density of the fluid
τ_w , ψ	shearing stress and stream function respectively

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1. Introduction

Joule heating in electronics and in physics refers to the increase in temperature of a conductor as a result of resistance to an electrical current flowing through it. But at an atomic level, Joule heating is the result of moving electrons colliding with atoms in a conductor, whereupon momentum is transferred to the atom, increasing its kinetic energy. When similar collisions cause a permanent structural change, rather than an elastic response, the result is known as electro migration. The increase in the kinetic energy of the ions manifests itself as heat and a rise in the temperature of the conductor. Hence energy is transferred from electrical power supply to the conductor and any materials with which it is in thermal contact. The conduction within and convection along the plate have a significant importance in many practical problems such as in ablation or perspiration cooling problems as well as in the heterogeneous chemical reaction situations. The natural convection process in the presence of heat transfer is present in various physical phenomena such as fire engineering, combustion modeling, nuclear energy, heat exchangers, petroleum reservoir etc. Flow of electrically conducting fluid in presence of magnetic field, temperature dependent thermal conductivity on MHD (magnetohydrodynamic) flow, inflow and outflow characteristics, heat transfer by conduction and convection problems are significant from the technical point of view.

Pozzi and Lupo (1988) explored the coupling of conduction with laminar convection along a flat plate. By means of two expansions, the entire thermo-fluid dynamic field was studied. The first one, describing the field in the lower part of the plate, was a regular series. The radius of convergence of which was determined by means of Pade approximant techniques. The second expansion, an asymptotic one required a different analysis because of the presence of eigensolutions. Hossain (1992) analyzed the viscous and Joule heating effects on MHD free convection flow with variable plate temperature. The author declared that temperature varied linearly with the distance from the leading edge and in the presence of uniformly transverse magnetic field. The equations governing the flow were solved and the numerical solutions were obtained for small Prandtl numbers, appropriate for coolant liquid metal, in the presence of a large magnetic field. Mamun et al. (2007) studied combined effect of conduction and viscous dissipation on MHD free convection flow along vertical flat plate. The conjugate free convection on a vertical surface was performed by Merkin and Pop (1996). Alim et al. (2007) investigated the Joule heating effect on the coupling of conduction with MHD free convection flow from a vertical flat plate.

Elbashbeshy (2000) discussed the effect of free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field. At the same year, MHD free convection flow of visco-elastic fluid past an infinite porous plate was considered by Chowdhury and Islam (2000). Alim et al. (2008) concluded the combined effect of viscous dissipation & Joule heating on the coupling of conduction & free convection along a vertical flat plate. Ahmad and Zaidi (2004) offered the magnetic effect on oberbeck convection through vertical stratum. Rahman et al. (2008) investigated the effects of temperature dependent thermal conductivity on MHD free convection flow along a vertical flat plate with heat conduction. The numerical calculation were proceed in finite-difference method and the velocity, temperature, local skin friction co-efficient and surface temperature profiles were shown by the effect of various parameters in their paper.

In all the aforesaid analyses the effects of variable thermal conductivity with Joule heating have not been considered. The present study is to integrate the idea that the effects of variable thermal conductivity and Joule heating in presence of strong magnetic field of electrically conducting fluid with free convection boundary layer flow. This problem is the extension work of Rahman and Alim (2009). Numerical results are displayed graphically over the whole boundary layer by the velocity, the temperature, the skin friction coefficient and the surface temperature profiles for a set of parameters M, γ, Pr and J. The numerical results of the local skin friction coefficient and the surface temperature profile for different values of J are also prearranged in tabular form.

2. Formulation of the Problem

At first we consider a steady, two-dimensional natural convection flow of an electrically conducting, viscous and incompressible fluid with temperature dependent thermal conductivity along a semi-infinite vertical flat plate of thickness b (Fig. 1). It is assumed that heat is transferred from the outside surface of the plate, which is maintained at a constant temperature T_b, where $T_b > T_\infty$, the temperature of the ambient fluid. A uniform magnetic field of strength H_0 is imposed along the $\overline{y}$ -axis i.e. normal direction to the surface and $\overline{x}$ - axis is chosen along the flat plate.

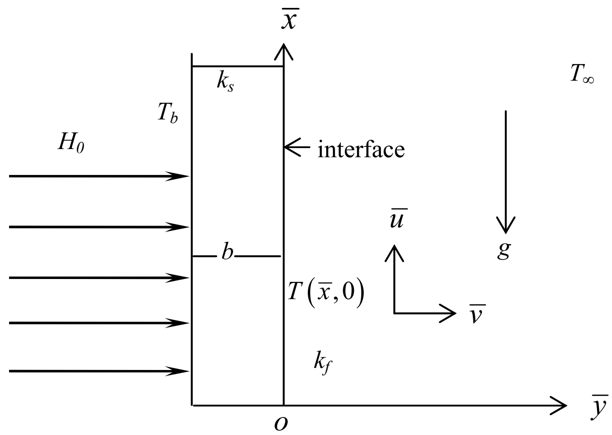

Fig. 1: Physical model and coordinate system

The governing equations (in Cartesian form) of such laminar flow with Joule heating and also thermal conductivity variation along a vertical flat plate under the Boussinesq approximations for the present problem can be written as

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{1}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \nu\frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + g\beta(T_f - T_\infty) - \frac{\sigma H_0^2\,\bar{u}}{\rho} \tag{2}$$

$$\bar{u}\frac{\partial T_f}{\partial \bar{x}} + \bar{v}\frac{\partial T_f}{\partial \bar{y}} = \frac{1}{\rho C_p}\frac{\partial}{\partial \bar{y}}(\kappa_f\frac{\partial T_f}{\partial \bar{y}}) + \frac{\sigma H_0^2}{\rho C_p}\bar{u}^2 \tag{3}$$

Here β is coefficient of volume expansion and thermal conductivity depends on temperature. It is used by Molla et al. (2005), as follows

$$\kappa_f = \kappa_\infty[1 + \delta(T_f - T_\infty)] \tag{4}$$

where κ_∞ is the thermal conductivity of the ambient fluid and δ is defined by $\delta = \frac{1}{\kappa_f}\left(\frac{\partial \kappa}{\partial T}\right)_f$.

The appropriate boundary conditions satisfying the above equations are proposed by Merkin and Pop (1996)

$$\left.\begin{aligned} &\bar{u} = 0, && \bar{v} = 0 \\ &T_f = T(\bar{x},0), && \frac{\partial T_f}{\partial \bar{y}} = \frac{\kappa_s}{b\kappa_f}(T_f - T_b) \end{aligned}\right\} \quad \text{on} \quad \bar{y} = 0,\ \bar{x} > 0$$
$$\bar{u} \to 0,\ T_f \to T_\infty \ \text{as} \quad \bar{y} \to \infty, \quad \bar{x} > 0 \tag{5}$$

It is observed that the equations (2) and (3) together with the boundary conditions (5) are non-linear partial differential equations. Then equations (1) to (3) are non-dimensionalized by using the following transformations

$$x = \frac{\bar{x}}{L},\ \ y = \frac{\bar{y}}{L}Gr^{\frac{1}{4}},\ u = \frac{\bar{u}L}{\nu}Gr^{-\frac{1}{2}},\ v = \frac{\bar{v}L}{\nu}Gr^{-\frac{1}{4}}, \theta = \frac{T_f - T_\infty}{T_b - T_\infty},$$
$$Gr = \frac{g\beta L^3(T_b - T_\infty)}{\nu^2} \tag{6}$$

where $L = \dfrac{\nu^{2/3}}{g^{1/3}}$ is reference length , Gr is the Grashof number, θ is the non dimensional temperature and $\nu = \mu / \rho$ is kinematic viscosity.

Substituting the relations (6) into the equations (1) to (3), the following non-dimensional equations are attained

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + Mu = \frac{\partial^2 u}{\partial y^2} + \theta \tag{8}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}(1+\gamma\,\theta)\frac{\partial^2 \theta}{\partial y^2} + \frac{\gamma}{Pr}\left(\frac{\partial \theta}{\partial y}\right)^2 + Ju^2 \tag{9}$$

Here $Pr = \dfrac{\mu\, C_p}{\kappa_\infty}$, $M = \dfrac{\sigma\, {H_0}^2 L^2}{\mu\, Gr^{1/2}}$, $\gamma = \delta(T_b - T_\infty)$ and $J = \dfrac{\sigma\, {H_0}^2\, \nu\, Gr^{1/2}}{\rho\, C_p (T_b - T_\infty)}$ are Prandtl number, magnetic parameter, thermal conductivity variation parameter and Joule heating parameter respectively. These parameters are all dimensionless.

The corresponding boundary conditions (5) then take the following form

$$\begin{aligned} &u = 0,\ v = 0,\ \ \theta - 1 = (1+\gamma\,\theta)\ p\frac{\partial \theta}{\partial y} \quad \text{on } \ y = 0,\ x > 0 \\ &u \to 0,\ \theta \to 0 \quad \text{as } \ y \to \infty,\ x > 0 \end{aligned} \tag{10}$$

where $p = \left(\dfrac{\kappa_\infty b}{\kappa_s L}\right) Gr^{1/4}$ is the conjugate conduction parameter.

The magnitude of p governs as magnetic parameter and thermal conductivity variation parameter in the current problem. This parameter plays an important role to determine the significance of the wall conduction resistance within the wall.

To solve the equations (8) and (9) subject to the boundary conditions (10) the following transformations are used by Alim et al. (2008)

$$\begin{aligned} \psi &= x^{4/5}\,(1+x)^{-1/20}\, f(x,\eta) \\ \eta &= y\, x^{-1/5}\,(1+x)^{-1/20} \\ \theta &= x^{1/5}\,(1+x)^{-1/5}\, h(x,\eta) \end{aligned} \tag{11}$$

here η is the similarity variable and ψ is the stream function which satisfies the continuity equation and is related to the velocity components in the usual way as $u = \partial\psi / \partial y$ and $v = -\partial\psi / \partial x$.

Thus the subsequent equations are achieved

$$\begin{aligned} f''' + \frac{16+15x}{20(1+x)} ff'' - \frac{6+5x}{10(1+x)} f'^2 - Mx^{2/5}(1+x)^{1/10} f' \\ + h = x\Big(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\Big) \end{aligned} \tag{12}$$

$$\begin{aligned} \frac{1}{\mathrm{Pr}} h'' + \frac{\gamma}{Pr}\left(\frac{x}{1+x}\right)^{1/5} h\,h'' + \frac{\gamma}{Pr}\left(\frac{x}{1+x}\right)^{1/5} h'^{\,2} + \frac{16+15x}{20(1+x)} f\,h' \\ + J\,x^{7/5}(1+x)^{1/10} f'^2 - \frac{1}{5(1+x)} f'h = x\left(f'\frac{\partial h}{\partial x} - h'\frac{\partial f}{\partial x}\right) \end{aligned} \tag{13}$$

where prime denotes partial differentiation with respect to η. The modified form represents the boundary conditions as mentioned in equation (10) given below

$$f(x,0) = f'(x,0) = 0$$
$$h'(x,0) = \frac{x^{1/5}(1+x)^{-1/5}h(x,0) - 1}{(1+x)^{-1/4} + \gamma x^{1/5}(1+x)^{-9/20}h(x,0)} \tag{14}$$
$$f'(x,\infty) \to 0,\ h(x,\infty) \to 0$$

From the development of numerical computation, it is essential to calculate the values of the surface shear stress in terms of the skin friction coefficient. It can be written in the subsequent non-dimensional form, which is used by Rahman et al. (2009)

$$C_f = \frac{Gr^{-3/4}L^2}{\mu\nu}\tau_w \tag{15}$$

where $\tau_w[=\mu(\partial\bar{u}/\partial\bar{y})_{\bar{y}=0}]$ is the shearing stress.

Using the new transformations described in (6), the local skin friction co-efficient can be specified as

$$C_{fx} = x^{2/5}(1+x)^{-3/20}f''(x,0) \tag{16}$$

The numerical value of the surface temperature profile is obtained from the following relation

$$\theta(x,0) = x^{1/5}(1+x)^{-1/5}h(x,0) \tag{17}$$

3. Method of Solution

The goal of this work is to explore the effects of thermal conductivity variation and Joule heating of electrically conducting fluid with free convection flow along a vertical flat plate in presence of strong magnetic field. The solution of the parabolic differential equations (12) and (13) along with the boundary conditions (14) are found by using the implicit finite difference simulation together with Keller- box scheme (1978). This is well predictable by Cebeci and Bradshaw (1984) and broadly used by Hossain et al. (1999).

The discretization of momentum and energy equations are carried out with respect to non-dimensional coordinates x and η to convey the equations in finite difference form by approximating the functions and their derivatives in terms of the central differences in both the coordinate directions. Then the required equations are to be linearized by using Newton's Quasi-linearization method. The linear algebraic equations can be written in a block matrix which forms a coefficient matrix. The whole procedure, namely reduction to first order followed by central difference approximations, Newton's Quasi linearization method and the block Thomas algorithm, is well known as Keller-box method.

4. Results and Discussion

The values of the Prandtl number are considered to be 0.73, 1.73, 2.97 and 4.24. In Fig. 2 to Fig. 9 the velocity, the temperature, the local skin friction coefficient and the surface temperature profiles attained from the solutions of equations (12) and (13) are depicted. Numerical computation are carried out for the ranges of magnetic parameter $0.1 \le M \le 3.7$, Joule heating parameter $0.001 \le J \le 0.480$ and thermal conductivity variation parameter $0.01 \le \gamma \le 0.51$.

The interaction of the magnetic field and moving electric charge carried by the flowing fluid induces a Lorentz force, which tends to oppose the fluid motion. In Fig. 2, it is observed that the magnetic field acting along the horizontal direction retards the fluid velocity with other fixed values γ = 0.1, J = 0.07 and Pr = 0.73. Here position of peak velocity moves toward the interface with the increase M. Consequently, the temperature within the boundary layer raises for growing values of magnetic parameter M from 0.1 to 3.7. The magnetic field decreases the temperature gradient at the wall and rises the temperature in the flow region.

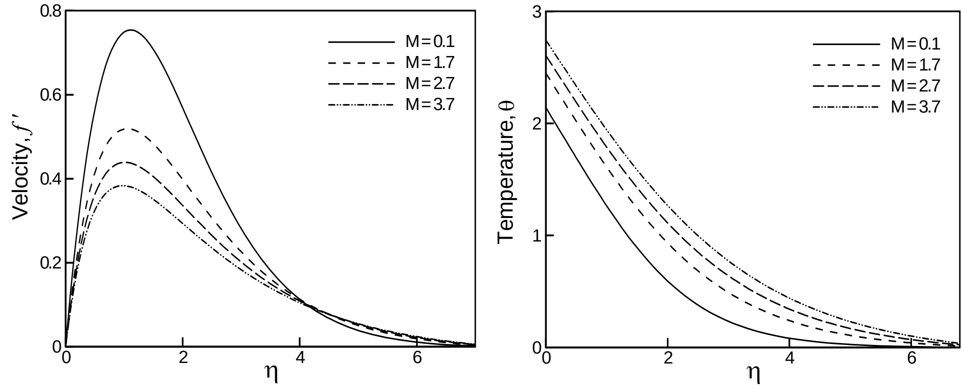

Fig. 2: Velocity and temperature profiles for different values of M with γ =0.10, J = 0.07 and Pr = 0.73

The effect of thermal conductivity variation parameter γ on the velocity and the temperature profiles within the boundary layer with M = 0.01, J = 0.01 and Pr = 0.73 is exposed in Fig. 3. The relation $\gamma = \delta(T_b - T_\infty)$ indicates that mounting values of γ increase the temperature difference between outside the plate and outside the boundary layer. Then heat is transferred rapidly from plate to fluid within the boundary layer. That's why both velocity and temperature profiles enlarge due to growing γ. It means that the velocity and the thermal boundary layer thickness raise for larger γ. Moreover, the maximum values of the velocity are 0.7371, 0.8069, 0.8396 and 0.9001 for γ = 0.01, 0.21, 0.31 and 0.51, respectively and each of which occurs at η = 1.1144. The corresponding highest values of the temperature are 2.0446, 2.1735, 2.2225 and 2.2972 respectively and they arise at the surface. The velocity and temperature profiles increase appreciably by 18.11% and 10.996% when γ changes from 0.01 to 0.51.

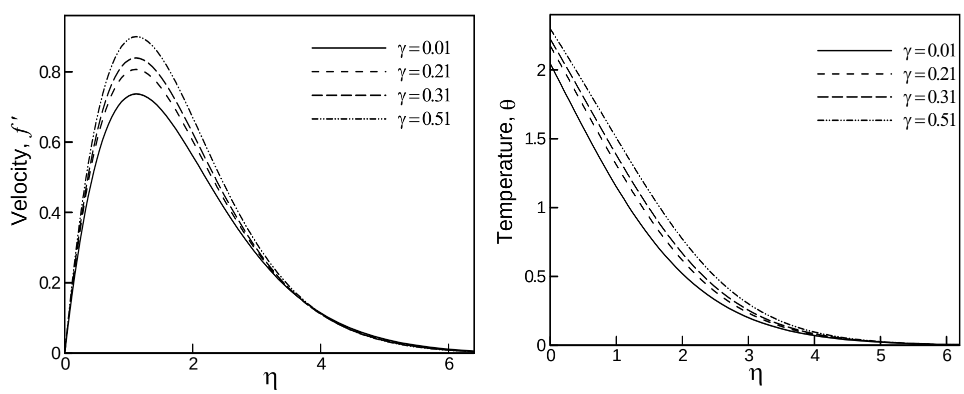

Fig. 3: Velocity and temperature profiles for different values of γ with M = 0.01, J = 0.01 and Pr = 0.73

Fig. 4 demonstrates the velocity and the temperature profiles for different values of Prandtl number Pr against η with M = 0.10, J = 0.07 and γ = 0.10. Mounting Pr increases viscosity and decrease thermal action of the fluid. If viscosity is increased, then fluid does not move freely. Because of this fact it can be noted that the velocity diminishes as well as its position moves toward the interface with increasing Pr. The velocity devalues swiftly by 52.45% when Pr rises from 0.73 to 4.24. The temperature profile shifts downward and decreases by 30.37% with growing Pr at the solid - fluid interface. Also the temperature at the interface varies due to the conduction within the plate.

The influence of Joule heating parameter J on the velocity and the temperature profiles within the boundary layer with M = 0.01, γ= 0.01 and Pr = 0.73 is depicted in Fig. 5. Due to this effect, temperature of the conductor grows and electrical energy is transferred to thermal energy. Thus the velocity and the temperature rise within the boundary layer with the escalating J. Furthermore, the greatest values of the velocity are 0.7354, 0.7476, 0.7565 and 0.7723 for J = 0.001, 0.150, 0.270 and 0.480 respectively and each of which takes place at η = 1.1144. For the analogous values of J the highest values of the temperature are 2.0424, 2.0782, 2.1077 ad 2.1608 respectively. They all arise at the surface. The corresponding velocity and temperature profiles enhance radically by 4.78% and 5.48% respectively.

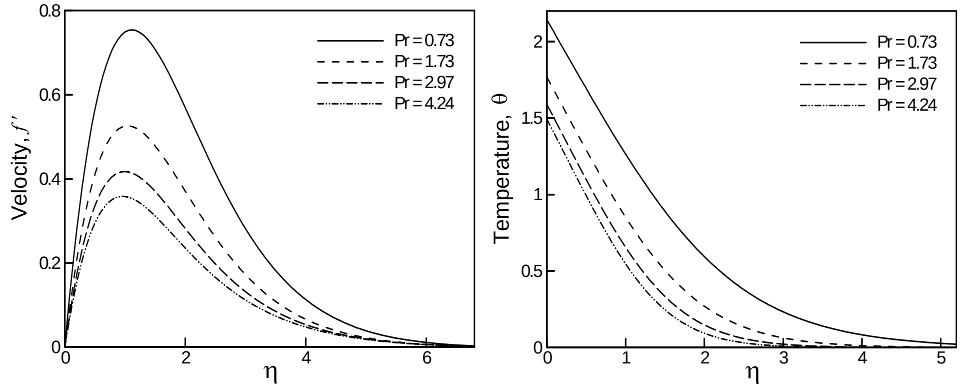

Fig. 4: Velocity and temperature profiles for different values of Pr with $\gamma = 0.10$, $J = 0.07$ and $M = 0.10$

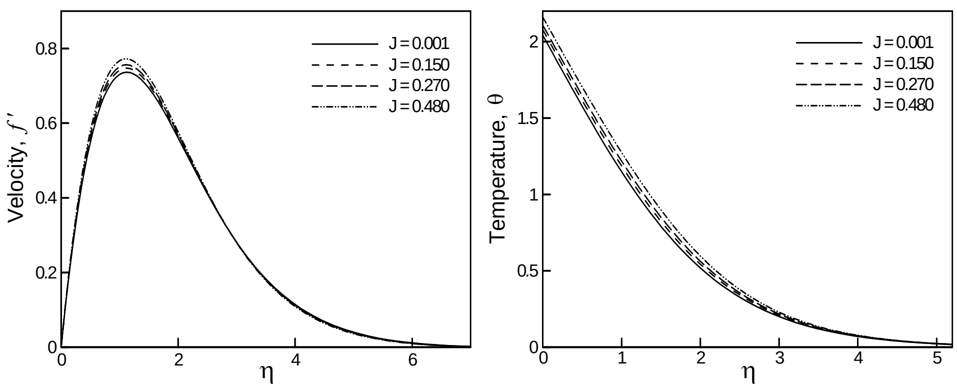

Fig. 5: Velocity and temperature profiles for different values of J with $\gamma = 0.01$, $Pr = 0.73$ and $M = 0.01$

The variation of the skin friction coefficient C_{fx} and the surface temperature $\theta\,(x,\ 0)$ for the variation of M with Pr = 0.73, J = 0.07 and γ = 0.10 at different positions is illustrated in Fig. 6. The increased of the magnetic parameter M from 0.1 to 3.7 leads to lessen the skin friction factor of 77.82% and enlarge the surface temperature by 34.08% respectively. The magnetic field acts against the flow and reduces the skin friction and produces the temperature at the interface.

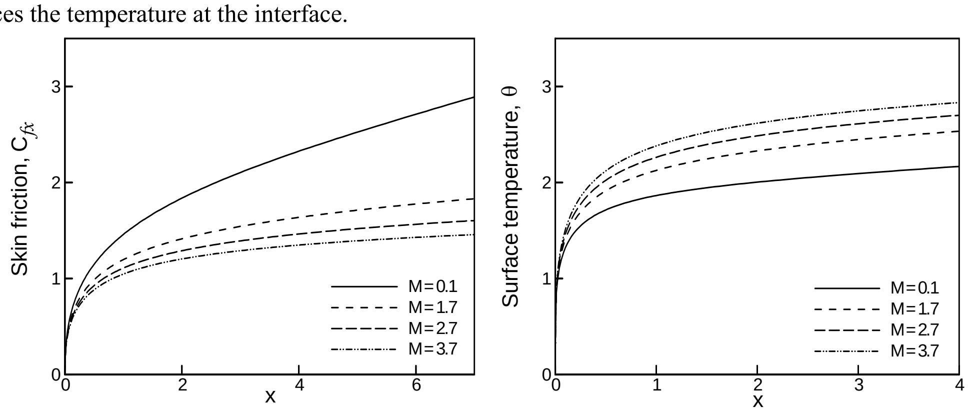

Fig. 6: Skin friction and surface temperature for different values of M with $\gamma = 0.10$, $J = 0.07$ and $Pr = 0.73$

Fig. 7 represents the effect of the thermal conductivity variation parameter γ on the local skin friction coefficient and the surface temperature profile against x with M = 0.01, J = 0.01 and Pr = 0.73. The local skin friction coefficient grows monotonically along the upward direction of the plate for a particular γ. It is also seen that C_{fx} and $\theta\,(x,\ 0)$ increase by 10.75% and 26.20% respectively due to escalating γ from 0.01 to 0.51. This is to be expected because the higher γ accelerates the fluid flow and rises the temperature as mentioned in Fig. 3.

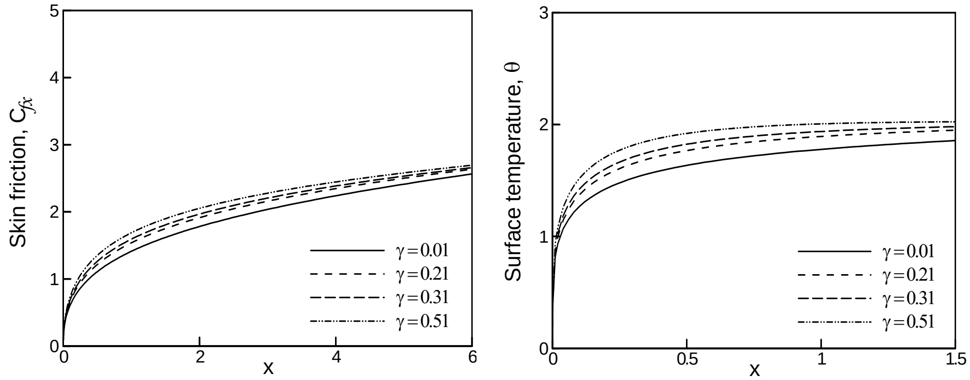

Fig. 7: Skin friction and surface temperature for different values of γ with M = 0.01, J = 0.01 and Pr = 0.73

The effect of Prandtl number Pr on the skin friction coefficient C_{fx} and the surface temperature $\theta\,(x,\ 0)$ against x with M = 0.10, J = 0.07 and γ = 0.10 is depicted in Fig. 8. It can be shown from Fig. 8 that the skin friction factor increases dramatically for each value of Pr. As well, the surface temperature reduces owing to the growing Pr. The local skin friction and the surface temperature profile diminish by 36.60% and 33.17% with mounting Pr from 0.73 to 4.24.

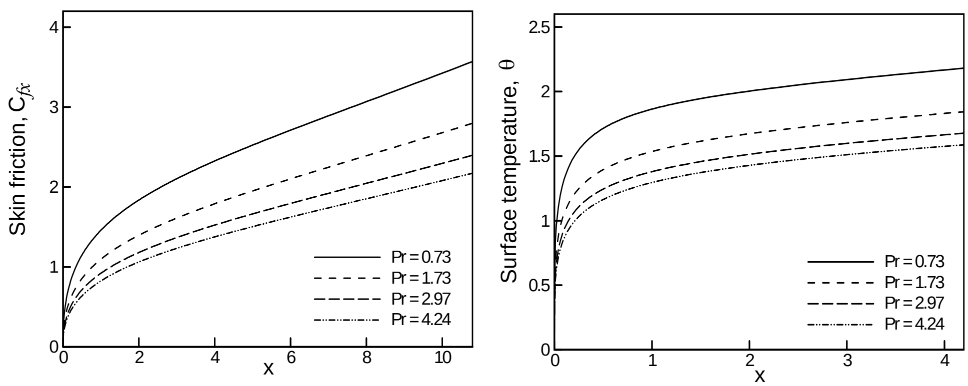

Fig. 8: Skin friction and surface temperature for different values of Pr with M = 0.10, γ = 0.10 and J = 0.07

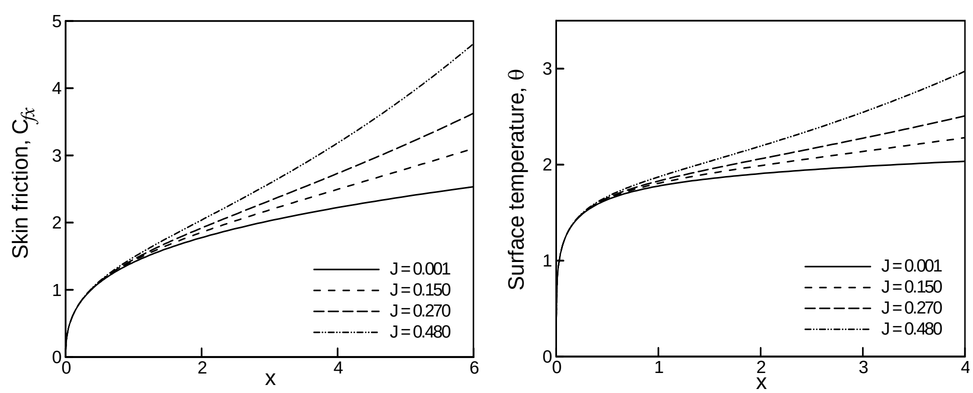

Fig. 9: Skin friction and surface temperature for different values of J with M = 0.01, γ = 0.01 and Pr = 0.73

Fig. 9 deals with the influence of Joule heating parameter J on the skin friction coefficient C_{fx} and the surface temperature $\theta\,(x,\ 0)$ with other controlling parameters M = 0.01, Pr = 0.73 and γ = 0.01. It can be noted that the local skin friction coefficient and the surface temperature profile increase radically by 93.24% and 96.94% for the rising values of J from 0.001 to 0.480 respectively.

In Table 1 the numerical values of the local skin friction co-efficient C_{fx} and the surface temperature $\theta\,(x,\,0)$ against x for different values of J while M = 0.01, Pr = 0.73 and γ = 0.01 are characterized. It is observed that the values of C_{fx} grow up rapidly at different position of x. Near the axial position x = 4.9370, the rate of increase of C_{fx} is 37.79% as J changes from 0.001 to 0.480. Consequently, the numerical values of $\theta\,(x,\,0)$ rise with mounting values of J. At the same axial position of x, the rate of increase of $\theta\,(x,\,0)$ is 40.15%. The rate of increase in the values of the skin friction coefficient and the surface temperature profiles become much higher than that of the upstream values.

Table 1: Skin friction coefficient and surface temperature profile for J with scheming parameters γ = 0.01, M = 0.01 and Pr = 0.73

	J = 0.001		J = 0.150		J = 0.270		J = 0.480	
x	C_{fx}	θ	C_{fx}	θ	C_{fx}	θ	C_{fx}	θ
0.0100	0.2447	0.5149	0.2447	0.8149	0.2447	0.8149	0.2447	0.8149
0.8881	1.3558	1.7538	1.3744	1.7779	1.3895	1.7976	1.4163	1.8329
1.3356	1.5560	1.8323	1.5943	1.8772	1.6258	1.9145	1.6820	1.9824
2.5346	1.9202	1.9504	2.0398	2.0702	2.1406	2.1746	2.3260	2.3742
3.7803	2.1846	2.0246	2.4298	2.2489	2.6427	2.4553	3.0461	2.8718
4.9370	2.3802	2.0757	2.7786	2.4209	3.1348	2.7555	3.8260	3.4635
5.5785	2.4753	2.0996	2.9741	2.5220	3.4272	2.9436	4.3167	3.8572
6.6947	2.6245	2.1361	3.3246	2.7112	3.9777	3.3143	5.2829	4.6713
7.8683	2.7641	2.1691	3.7135	2.9314	4.6226	3.7715	6.4673	5.7276
8.7021	2.8549	2.1900	4.0056	3.1030	5.1268	4.1436	7.4224	6.6187
9.2437	2.9108	2.2027	4.2033	3.2219	5.4761	4.4081	8.0959	7.2657

5. Comparison and Code Validation

Table 2 depicts the comparison of the current numerical results of the skin friction co-efficient and the surface temperature with those attained by Pozzi and Lupo (1988) and Merkin and Pop (1996). Here, the parameters M, γ and J are ignored and the Prandtl number Pr = 0.733 with $x^{1/5} = \xi$ is chosen. Excellent agreement is achieved among present results and the numerical results of Pozzi and Lupo in 1988 and Merkin and Pop in 1996 for both the profiles inside the boundary layer. These validations boost the confidence in the numerical code to carry on with the above stated objectives of the current investigation.

Table 2: Comparison of C_{fx} and $\theta\,(x,\,0)$ among the present results and that of Pozzi and Lupo (1988) and Merkin and Pop (1996)

	Skin friction co-efficient C_{fx}			Surface temperature $\theta\,(x,\,0)$		
$x^{1/5} = \xi$	Pozzi and Lupo (1988)	Merkin and Pop (1996)	Present result	Pozzi and Lupo (1988)	Merkin and Pop (1996)	Present result
0.7	0.430	0.430	0.424	0.651	0.651	0.651
0.8	0.530	0.530	0.528	0.684	0.686	0.687
0.9	0.635	0.635	0.634	0.708	0.715	0.717
1.0	0.741	0.745	0.743	0.717	0.741	0.741
1.1	0.829	0.859	0.858	0.699	0.762	0.763
1.2	0.817	0.972	0.973	0.640	0.781	0.781

6. Conclusion

The influences of thermal conductivity variation due to temperature on the coupling of conduction and Joule heating with MHD free convection flow along a vertical flat plate have been offered numerically. The coupled effects of natural convection and conduction required that the temperature and the heat flux be continuous at the interface. From the present analysis the following conclusions may be drawn:

i) The velocity profile within the boundary layer rises for devalues of magnetic parameter M, Prandtl number Pr and increasing values of thermal conductivity variation parameter γ and Joule heating parameter J.

ii) The temperature profile rises due to the escalating M, γ, J and decreasing Pr.

iii) Growing values of M, Pr and falling values of γ, J reduce the local skin friction coefficient.

iv) An increase in the values of γ, J and M leads to an increase in the surface temperature. Moreover this profile reduces with rising Pr.

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