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EFFECTS OF HEAT TRANSFER AND VISCOUS DISSIPATION ON MHD FREE CONVECTION FLOW PAST AN EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE

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Abstract:

The unsteady free convection flow of an incompressible viscous fluid past an exponentially accelerated vertical plate is analyzed by taking into an account the heat due to viscous dissipation under the influence of a uniform transverse magnetic field. The problem is governed by coupled non-linear partial differential equations. Dimensionless equations of the problem have been solved numerically by the implicit finite difference method of Crank – Nicolson's type. The effect of magnetic parameter M, Grashof number Gr, Eckert number E, time and an acceleration parameter 'a' on velocity and temperature fields are investigated through graphs. Skin friction coefficient is derived, discussed numerically and presented in tablular form.

Keywords: MHD, free convection, viscous dissipation, implicit finite difference method, exponentially accelerated plate, variable temperature.

NOMENCLATURE

A, a', a	constants
C_p	specific heat at constant pressure, $J.Kg^{-1}.K^{-1}$
Gr	thermal Grashof number
M	magnetic parameter
E	Eckert number
g	acceleration due to gravity, $m.s^{-2}$
k	thermal conductivity, $W.m^{-1}.K^{-1}$
$\Pr$	Prandtl number
T'	temperature of the fluid near the plate, K
t'	time, s
t	dimensionless time
u'	velocity of the fluid in the x' - direction $m.s^{-1}$
u_0	velocity of the plate $m.s^{-1}$
u	dimensionless velocity
y'	coordinate axis normal to the plate, m
y	dimensionless coordinate axis normal to the plate

Greek symbols

β	volumetric coefficient of thermal expansion, K^{-1}
μ	coefficient of viscosity, $Pa.S$
μ_e	magnetic permeability, Hm^{-1}
υ	kinematic viscosity, m^2s^{-1}
ρ	density of the fluid, $Kg.m-3$
σ	electrical conductivity of the fluid, VA^{-1},m^{-1}
θ	dimensionless temperature

Subscripts

w	conditions at the wall
∞	conditions in the free stream

1. Introduction

Free convection flow involving coupled heat and mass transfer occurs frequently in nature and in industrial processes. A few representative fields of interest in which combined heat and mass transfer plays an important role are designing chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, crop damage due to freezing, and environmental pollution.

Pop and Soundalgekar (1980) have investigated the free convection flow past an accelerated infinite plate. Raptis et al. (1987) have studied the unsteady free convective flow through a porous medium adjacent to a semi-infinite vertical plate using finite difference scheme. Singh and Soundalgekar (1990) have investigated the

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problem of transient free convection in cold water past an infinite vertical porous plate. Vajravelu and Sastry (1978) studied about free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. Free convection boundary layer flow of a non-Newtonian fluid along a vertical wavy wall was considered by Kumari et al. (1997). Chandran et al.(1998) have discussed the unsteady free convection flow with heat flux and accelerated boundary motion.

The most common type of body force, which acts on a fluid, is gravity so that the body force can be defined in magnitude and direction by acceleration due to gravity. Some times, electromagnetic effects are important. The electric and magnetic fields themselves obey a set of physical laws, which are expressed by Maxwell's equations. The solution of such problems requires the simultaneous solution of the equations of fluid mechanics and of electromagnetism. One special case of this type of coupling is the field known as magneto hydro dynamics (MHD).

Hydro magnetic flows and heat transfer have become more important in recent years because of its varied applications in agricultural engineering and petroleum industries. Recently, considerable attention has also been focused on new applications of magneto-hydrodynamics (MHD) and heat transfer such as metallurgical processing. Melt refining involves magnetic field applications to control excessive heat transfer rate. Other applications of MHD heat transfer include MHD generators, plasma propulsion in astronautics, nuclear reactor thermal dynamics and ionized-geothermal energy systems etc. An excellent summary of applications can be found in Huges et al. (1966). Sacheti et al. (1994) obtained an exact solution for unsteady MHD free convection flow on an impulsively started vertical plate with constant heat flux. Shankar et al. (1997) discussed the effect of mass transfer on the MHD flow past an impulsively started infinite vertical plate with variable temperature or constant heat flux. Takar (1996) analyzed the radiation effects on MHD free convection flow past a semi-infinite vertical plate using Runge-Kutta Merson quadrature. Abd-El-Naby et al. (2003) studied the radiation effects on MHD unsteady free convection flow over a vertical plate with variable surface temperature. Ramachandra Prasad et al. (2006) have studied the transient radiative hydromagnetic free convection flow past an impulsively started vertical plate uniform heat and mass flux. Samria et.al. (2004) studied about hydromantic free convection laminar flow of an elasto-viscous fluid past an infinite plate. Recently the natural convection flow of a conducting visco-elastic liquid between two heated vertical plates under the influence of transverse magnetic field has been studied by Sreehari Reddy et al. (2008).

In all these investigations, the viscous dissipation is neglected. The viscous dissipation heat in the natural convective flow is important, when the flow field is of extreme size or at low temperature or in high gravitational field. Such effects are also important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number. A number of authors have considered viscous heating effects on Newtonian flows. Mahajan et al. (1989) reported the influence of viscous heating dissipation effects in natural convective flows, showing that the heat transfer rates are reduced by an increase in the dissipation parameter. Isreal-Cookey et al. (2003) investigated the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Zueo (2007) used network simulation method (NSM) to study the effects of viscous dissipation and radiation on unsteady MHD free convection flow past a vertical porous plate. Suneetha et al. (2008) have analyzed the thermal radiation effects on hydromagnetic free convection flow past an impulsively started vertical plate with variable surface temperature and concentration is analyzed by taking into account of the heat due to viscous dissipation. Recently Suneetha et al. (2009) studied the effects of thermal radiation on the natural conductive heat and mass transfer of a viscous incompressible gray absorbing-emitting fluid flowing past an impulsively started moving vertical plate with viscous dissipation. Very recently Hiteesh (2009) studied the boundary layer steady flow and heat transfer of a viscous incompressible fluid due to a stretching plate with viscous dissipation effect in the presence of a transverse magnetic field.

The object of the present paper is to study the transient free convection flow of an incompressible dissipative viscous fluid past an exponentially accelerated vertical plate on taking into account viscous dissipative heat, under the influence of a uniform transverse magnetic field in the presence of variable surface temperature. The dimensionless governing equations are solved by using an implicit finite difference method of Crank – Nicolson's type.

2. Mathematical Analysis

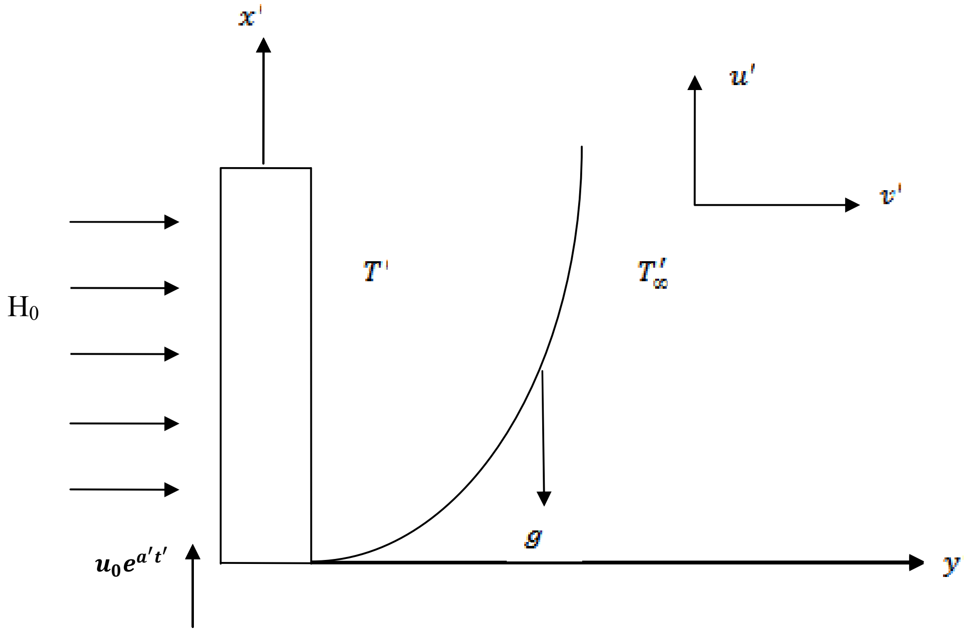

Figure (a): Flow configuration and coordonate system

Here the unsteady hydro magnetic free convective flow of viscous incompressible fluid past an exponentially accelerated infinite vertical plate with variable temperature is considered. The x'- axis is taken along the plate in the vertically upward direction and the y'– axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T'_∞. At t' > 0, the plate is exponentially accelerated with a velocity $u' = u_0 \exp(a't')$ in its own plane and the plate temperature is raised linearly with time t' under these conditions, the flow variables are functions of t' and y' alone. A uniform magnetic field of intensity H_0 is applied in the y' – direction. Therefore the velocity and the magnetic field are given by $\bar{q} = (u, 0)$ and $\bar{H} = (0, H_0)$.The fluid being slightly conducting the magnetic Reynolds number is much less than unity and hence the induced magnetic field can be neglected in comparison with the applied magnetic field in the absence of any input electric field. Then by usual Boussinesq's and boundary layer approximation the unsteady flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_\infty) + \upsilon \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma \mu_e^2 H_0^2}{\rho} u' \tag{1}$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} + \mu \left(\frac{\partial u'}{\partial y'} \right)^2 \tag{2}$$

with the following initial and boundary conditions:

$$\begin{array}{llll} u' = 0, & T' = T'_\infty; & \text{for all} & y', t' \leq 0 \\ t' > 0 : u' = u_0 \exp(a't'), & T' = T'_\infty + (T'_w - T'_\infty) A t', & \text{at} & y' = 0 \\ u' \to 0, & T' \to T'_\infty, & \text{as} & y' \to \infty \end{array}$$

Where $A = \frac{u_0^2}{\upsilon}$. (3)

On introducing the following non-dimensional quantities:

$$u=\frac{u'}{u_0},\quad t=\frac{t'u_0^2}{\upsilon},\quad y=\frac{y'u_0}{\upsilon},\quad \theta=\frac{T'-T'_\infty}{T'_w-T'_\infty},\quad M=\frac{\sigma\mu_e^2H_0^2\upsilon}{\rho u_0^2}$$

$$Gr=\frac{g\beta\upsilon\left(T'_w-T'_\infty\right)}{u_0^3},\quad \Pr=\frac{\mu C_p}{k},\quad E=\frac{u_0^2}{C_p\left(T_w-T_\infty\right)},\quad a=\frac{a'\upsilon}{u_0^2} \tag{4}$$

in Equations (1) to (3), lead to

$$\frac{\partial u}{\partial t}=Gr\theta+\frac{\partial^2 u}{\partial y^2}-Mu \tag{5}$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+E\left(\frac{\partial u}{\partial y}\right)^2 \tag{6}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{aligned} & u=0, && \theta=0, && \text{for all} && y,t\le 0 \\ t>0: \quad & u=\exp(at), && \theta=t, && \text{at} && y=0 \\ & u\to 0, && \theta\to 0 && \text{as} && y\to\infty \end{aligned} \tag{7}$$

Skin – Friction: The skin-friction is important physical parameter for this type of boundary layer flow. Knowing the velocity field, the skin-friction at the plate can be obtained, which in non-dimensional form is given by

$$\tau=-\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{8}$$

All the physical variables are defined in the nomenclature.

3. Numerical Technique

In order to solve the unsteady, non-linear coupled equations (5) & (6) under the initial and boundary conditions (7), an implicit finite difference scheme of Crank-Nicolson's type has been employed. The finite difference equation corresponding to equations (5) & (6) are as follows:

$$\frac{u_{i,j+1}-u_{i,j}}{\Delta t}=\frac{Gr}{2}\left(\theta_{i,j+1}+\theta_{i,j}\right)+\left(\frac{u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}+u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{2(\Delta y)^2}\right)$$
$$-\frac{M}{2}(u_{i,j+1}+u_{i,j}) \tag{9}$$

$$\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta t}=\frac{1}{\Pr}\left(\frac{\theta_{i-1,j+1}-2\theta_{i,j+1}+\theta_{i+1,j+1}+\theta_{i-1,j}-2\theta_{i,j}+\theta_{i+1,j}}{2(\Delta y)^2}\right)+E\left(\frac{u_{i+1,j}-u_{i,j}}{\Delta y}\right)^2 \tag{10}$$

Initial and boundary conditions take the following forms

$$\begin{aligned} & u_{i,0}=0, && \theta_{i,0}=0 && \text{for all} \quad i\ne 0 \\ & u_{0,j}=\exp(a.j.\Delta t), && \theta_{0,j}=j.\Delta t && \\ & u_{N,j}=0, && \theta_{N,j}=0, && \end{aligned} \tag{11}$$

Where N corresponds to ∞.

Here the suffix 'i' corresponds to 'y' and 'j' corresponds to 't'. Also $\Delta t=t_{j+1}-t_j$ and $\Delta y=y_{i+1}-y_i$.

Knowing the values of θ, u, at a time 't' we can calculate the values at a time $t+\Delta t$ as follows. We substitute $i=$ 1, 2, 3, ----,N-1, in equation (10) which constitute a tri-diagonal system of equations, the system can be solved by Thomas algorithm as discussed in Carnahan et al. (1969). Thus θ is known for all values of y at time $t+\Delta t$. Then knowing the values of θ and applying the same procedure with the boundary conditions, we calculate, u from equation (9). This procedure is continued to obtain the solution till desired time't'. Computations are carried out for air (Pr = 0.71) and water (Pr=7). In order to check the accuracy, we substitute

M =0 and E =0 in the equations (9) and (10), these results are coincide with exact solution by usual Laplace transform technique solved by R. Muthucumaraswamy et.al. (2008) (Vide figs. (12) and (13)).
The derivative involved in equations (8) is evaluated using five point approximation formula.

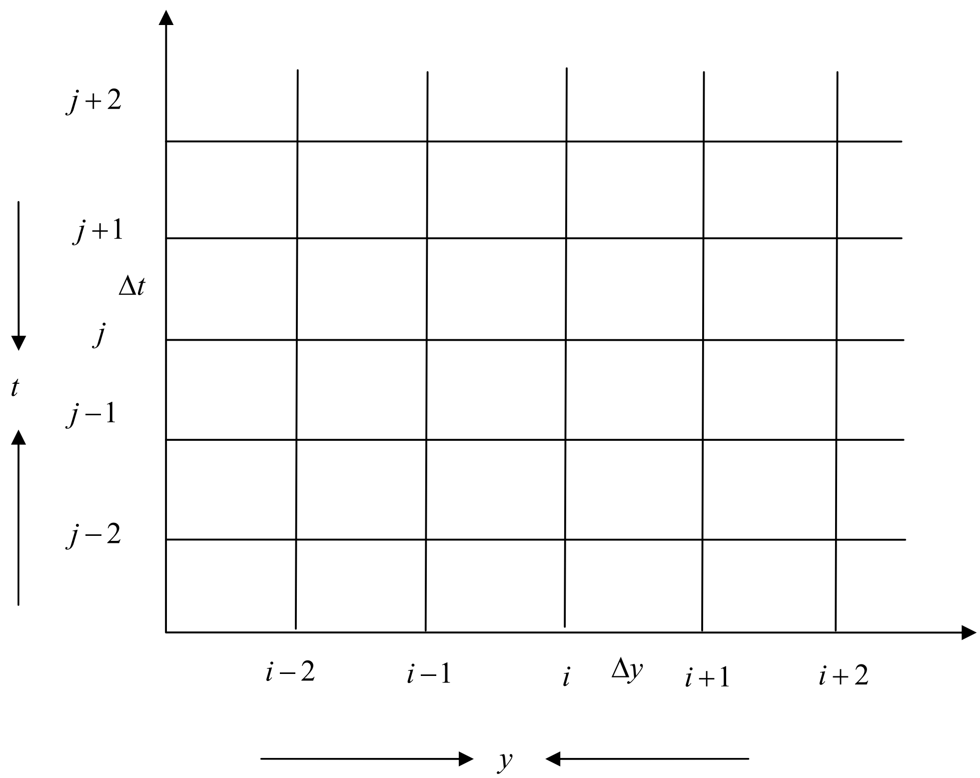

Figure (b): Finite difference grid

4. Stability Analysis

Since a finite difference procedure is being used, we wish to know the largest time – step consistent with stability. Here we are using von Neumann method as explained in Carnahan et.al. (1969) and Jain et.al. (1993) for stability. The general terms of the Fourier expansion for 'u' and 'θ' at a time arbitrarily called t = 0 are both $e^{i\beta y}$, apart from a constant. (Here, $i = \sqrt{-1}$). At a time 't' later, these terms will become:

$$u = \psi(t)e^{i\beta y}, \quad \theta = \xi(t)e^{i\beta y} \tag{12}$$

Substituting(12) in (9) and (10) and denoting values after the time step by ψ' and ξ' gives, upon simplification,

$$\psi' - \psi = (\xi' + \xi)\frac{Gr\Delta t}{2} + (\psi' + \psi)(\cos\beta\Delta y - 1)\frac{\Delta t}{(\Delta y)^2} - (\psi' + \psi)\frac{M\,\Delta t}{2} \tag{13}$$

$$\xi' - \xi = (\xi' + \xi)(\cos\beta\Delta y - 1)\frac{\Delta t}{(\Delta y)^2\,\Pr} \tag{14}$$

Now defining

$$A = (\cos\beta\Delta y - 1)\frac{\Delta t}{(\Delta y)^2} - \frac{M\,\Delta t}{2}\ ;\ B = \frac{Gr\Delta t}{2}\ ;C = (\cos\beta\Delta y - 1)\frac{\Delta t}{(\Delta y)^2\,\Pr}$$

Equations (13) and (14) can be written as:

$$(1-A)\psi' = (1+A)\psi + B(\xi' + \xi) \tag{15}$$

$$(1-C)\xi' = (1+C)\xi \tag{16}$$

After eliminating ξ' in equation (15) using equation (16), the resultant equations can be written as follows:

$$\begin{bmatrix} \psi' \\ \xi' \end{bmatrix} = \begin{bmatrix} \dfrac{1+A}{1-A} & \dfrac{2B}{(1-A)(1-C)} \\ 0 & \dfrac{1+C}{1-C} \end{bmatrix} \begin{bmatrix} \psi \\ \xi \end{bmatrix} \tag{17}$$

$$\eta' = G\eta$$

where η is the column vector whose elements are ψ and ξ. For stability, each eigen value λ_1 and λ_2 of the amplification matrix 'G' must not exceed unity in modulus. The eigen values of 'G' are $\lambda_1 = \dfrac{1+A}{1-A}$ and $\lambda_2 = \dfrac{1+C}{1-C}$. Since $A \le 0$ and $C \le 0$ we have $|1+A| \le |1-A|$ and $|1+C| \le |1-C|$. Hence $|\lambda_{1,2}| \le 1$.

Thus the implicit finite difference scheme is unconditionally stable. The local truncation error is $O(\Delta t + \Delta y^2)$ and it tends to zero when 'Δt and Δy' tend to zero. Hence the scheme is compatible. The finite difference scheme is unconditionally stable. Compatibility and stability ensures the convergence of the scheme.

5. Results and Discussion

The velocity of the flow field is found to change more or less with the variation of the flow parameters. The effect of the flow parameters on the velocity and temperature fields are analyzed with the help of graphs.

The effect of magnetic parameter 'M' on velocity 'u' when M= 0, 2, 4, Gr = 5, E = 1, Pr =0.71, a = 0.5, and t = 0.2 & t = 0.6 is illustrated in figure (1). From fig.(1) it is observed that velocity 'u' decreases as the magnetic parameter 'M' increases. It is because of that, the application of transverse magnetic field will result in a resistive type force, known as Lorentz force, which tend to resist the fluid flow and thus reduces velocity.
The effect of Grashof number (Gr) for heat transfer on the velocity of the flow field when Gr = 2,5,10, M = 2, E = 1, Pr = 0.71, a = 0.5, and t = 0.2 & 0.6 is presented in fig. (2). In the fig.(2), the velocity of the flow field is plotted against 'y' for different values of the respective Grashof numbers keeping other parameters of flow field constant. It is observed that the velocity increases with increasing values of the thermal Grashof number. This is due to enhancement in buoyancy force.

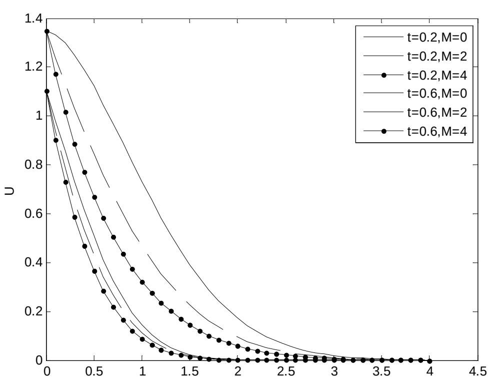

Fig. (1): Velocity profile for different values of 'M'

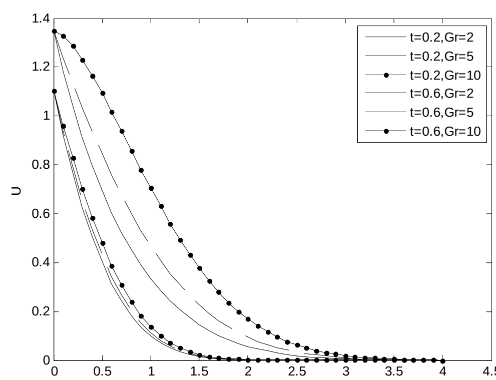

Fig. (2): Velocity profile for different values of 't'

The effect of velocity for different values of a = 0. 0.5, 0.9 and Gr = 5, M = 2, E = 1, Pr = 0.71 at t = 0.2 & t = 0.6 are studied and presented in fig.(3). It is observed that the velocity increases with increasing values of 'a'.
Fig.(4) demonstrates the velocity distribution for different values of Eckert number E when Gr = 5, M = 2, Pr = 0.71, a = 0.5, and t = 0.2 & 0.6 . It is observed that increasing values of 'E' is to increase the velocity distribution in flow region. This is due to the heat energy stored in the liquid because of the frictional heating.

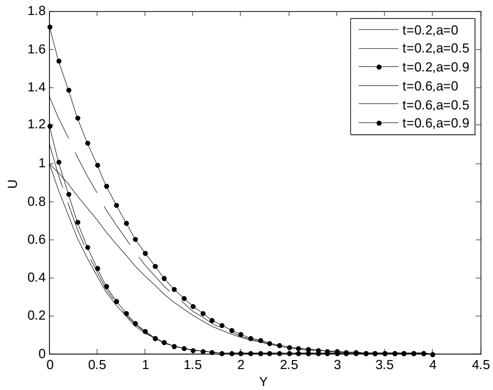

Fig. (3): Velocity profile for different values of 'a'

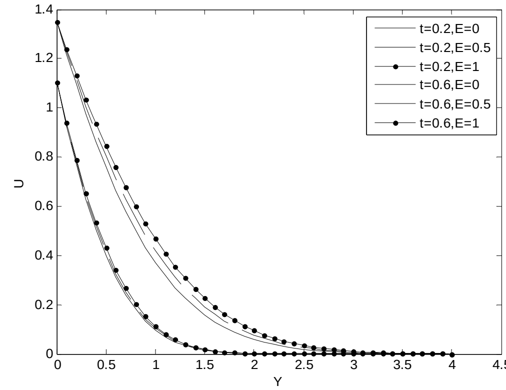

Fig. (4): Velocity profile for different values of 'E'

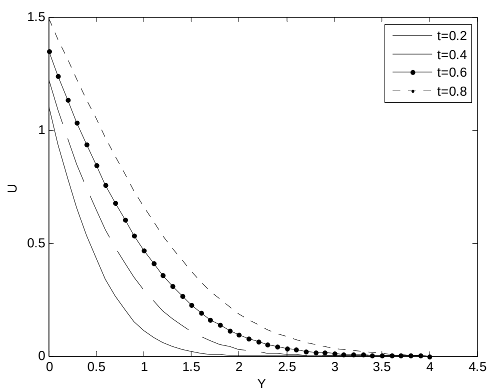

Fig. (5): Velocity profile for different values of 't'

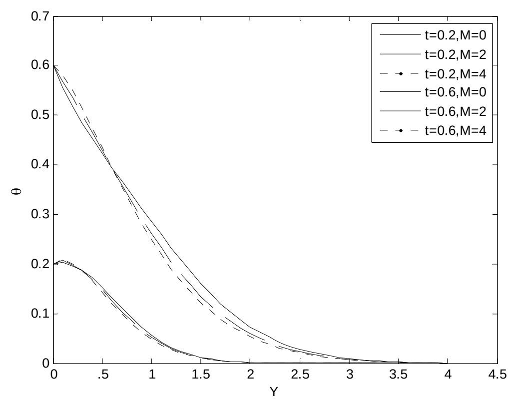

Fig. (6): Temperature profile for different values of 'M'

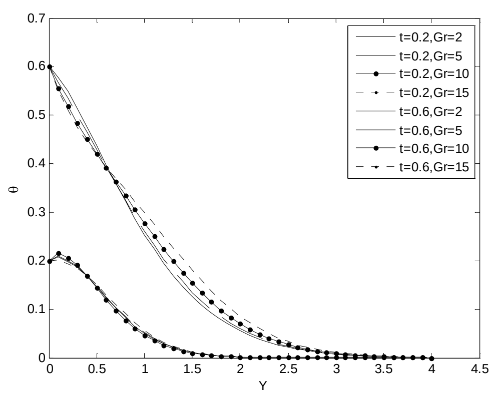

Fig. (7): Temperature profile for different values of 'Gr'

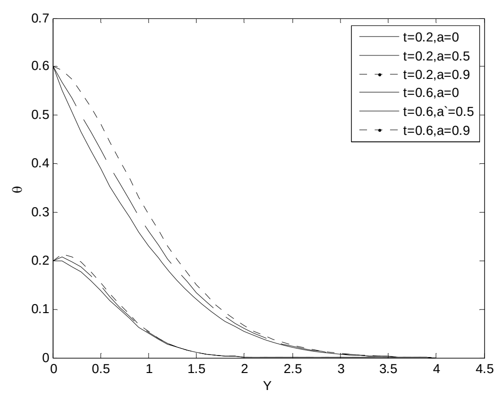

Fig. (8): Temperature profile for different values of 'a'

The velocity profiles for different values of time t = 0.2, 0.4, 0.6, 0.8, and a =0.5, Gr =5, Pr = 0.71, M =2, E = 1 are shown in fig.(5). It is observed that the velocity increases with increasing values of time.

The influence of flow parameters on the temperature are shown in the figs. (6) to (11). The effect of magnetic parameter 'M' on temperature 'θ' when M= 0, 2, 4, Gr = 5, E = 1, Pr =0.71, a = 0.5, and t = 0.2 & t = 0.6 is illustrated in fig.(6). From fig.(6) it is observed that 'θ' increases as 'M' increases in the vicinity of the plate and then decreases in the remaining flow region.

The effect of Grashof number (Gr) for heat transfer on the temperature when Gr = 2,5,10, M = 2, E = 1, Pr = 0.71, a = 0.5, and t = 0.2 & 0.6 is presented in fig (7). It is observed that the temperature decreases with increasing values of 'Gr' in the vicinity of the plate and then increases in the remaining flow region.
The effect of temperature for different values of a= 0. 0.5, 0.9 and Gr = 5, M = 2, E = 1, Pr = 0.71 at t = 0.2 & t = 0.6 are studied and presented in fig. (8). It is observed that temperature increases with increasing values of 'a'.

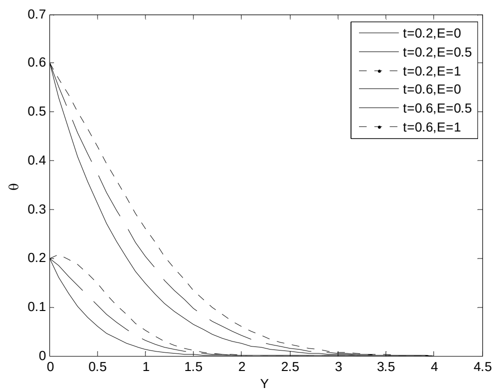

Fig. (9): Temperature profile for different values of 'E'

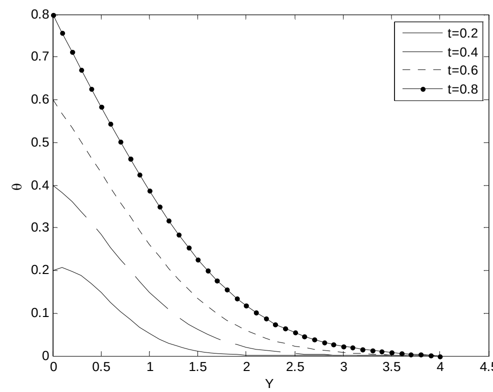

Fig. (10): Temperature profile for different values of 't'

Fig.(9) depicts the temperature distribution for different values of Eckert number 'E' when Gr = 5, M = 2, Pr = 0.71, a = 0.5, and t = 0.2 & 0.6 . It is observed that the temperature increases with increasing values of 'E.
The temperature profiles for different values of time t = 0.2, 0.4, 0.6, 0.8. Gr = 5, M = 2, E = 1, a = 0.5 are shown in fig. (10). It is observed that the temperature increases with increasing values of time.

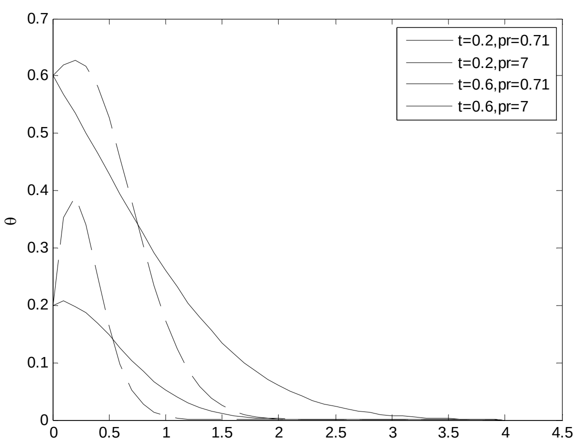

Fig. (11): Temperature profile for different values of 'Pr'

The effect of Prandtl number 'Pr' is important in temperature profiles. The temperature profiles are calculated for different values of time for water (Pr=7) and air (Pr=0.71) are shown in fig. (11). It is observed that the temperature increases with increasing time't'. Comparing both the curves of fig.(11), also observed that when Pr=7 the temperature increases in the region $0 \leq y \leq 0.7$ with maximum at y = 0.2 and then decreases in the remaining region.

In order to ascertain the accuracy of the numerical results, the present study is compared with the previous study. The velocity profiles for different values of 'a' and the temperature profiles for different values of 'Pr and time 't' are compared with the available solution of Muthucumaraswamy et.al. (2008). It is observed that the present results are in good agreement with that of Muthucumaraswamy et.al. (2008).

The numerical values of skin friction, τ is presented in Table 1.It is observed from the table that an increase in the thermal Grashof number, Eckert number, Prandtl number lead to decrease in the value of the skin-friction, but the trend is just reversed with increasing 'a, M or t'.

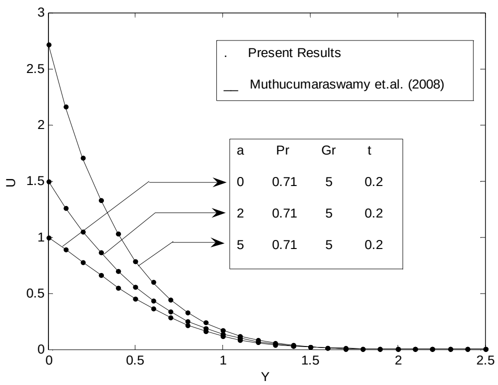

Fig. (12): Velocity profile for different values of 'a'

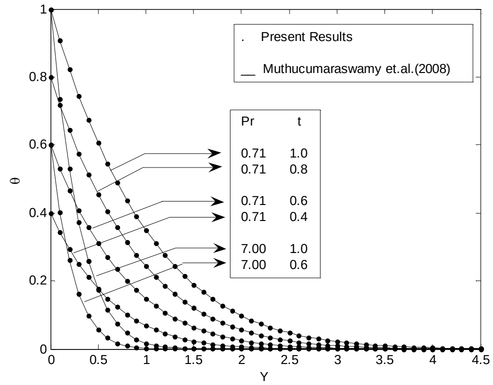

Fig. (13): Temperature profile for different values of 'Pr' and 't'

Table 1: The numerical values of skin friction, τ

Gr	a	E	M	t	Pr	τ
2	2	1	2	0.2	0.71	3.0966
5	2	1	2	0.2	0.71	2.9052
10	2	1	2	0.2	0.71	2.5906
5	0	1	2	0.2	0.71	1.4803
5	5	1	2	0.2	0.71	6.8663
5	2	0	2	0.2	0.71	3.0522
5	2	0.5	2	0.2	0.71	2.9781
5	2	1	4	0.2	0.71	3.4805
5	2	1	6	0.2	0.71	4.0005
5	2	1	2	0.4	0.71	3.6982
5	2	1	2	0.6	0.71	5.1076
5	2	1	2	0.2	7.0	2.6499

6. Conclusions

In this paper effects of heat transfer and viscous dissipation on MHD free convection flow past an exponentially accelerated vertical plate with variable surface temperature have been studied numerically. Implicit finite difference method is employed to solve the equations governing the flow. From the present numerical investigation, following conclusions have been drawn:

- Velocity increases with increase in the thermal Grashof number (Gr>0), acceleration parameter 'a', Eckert number 'E' and time 't'.
- Velocity decreases with an increase in magnetic parameter (M)
- As 'M' increases, temperature increases near the plate and decreases in the remaining flow region
- With increasing values of 'Gr', temperature decreases near the plate and increases in the remaining flow region
- Temperature increases with increasing values of 'a', Eckert number 'E' and time 't'
- Increase in the thermal Grashof number (Gr>0) , Eckert number 'E', Prandtl number 'Pr' lead to decrease in the value of the skin-friction, but the trend is just reversed with increasing 'a, M or t'.

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