Journal of Naval Architecture and Marine Engineering
Dec., 2020
http://dx.doi.org/10.3329/jname.v17i2.45674 http://www.banglajol.info

NATURAL CONVECTION OF FLUID WITH VARIABLE VISCOSITY AND VISCOUS DISSIPATION FROM A HEATED VERTICAL WAVY SURFACE IN PRESENCE OF MAGNETIC FIELD

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Abstract:

The present numerical work describes the effect of the temperature dependent variable viscosity and viscous dissipation on natural convection heat transfer boundary layer flow of a viscous incompressible electrically conducting fluid along a vertical wavy surface in presence of a transverse magnetic field. The wavy surface is maintained at uniform wall temperature that is higher than that of the ambient. A simple coordinate transformation is employed to transform the wavy surface into a flat plate. A marching finite difference scheme is used for present analysis. The numerical results, including the developments of the skin friction coefficients, the local Nusselt number, the streamlines as well as the isotherms for different values of the magnetic parameter M (= 0.0 to 2.0), viscous dissipation parameter vd (= 0.0 to 10.0), variable viscosity variation parameter ε (= 0.0 to 15.0), the amplitude-to-length ratio of the wavy surface α = 0.3 and Prandtl number Pr = 0.73 which correspond to the air at 2100^0K are presented and discussed in detail. The results of this investigation illustrated that the skin friction coefficient increase with an increase of the variable viscosity and viscous dissipation parameter, while the local Nusselt number at the heated surface decrease for increasing values of variable viscosity parameter, magnetic parameter and viscous dissipation parameter.

Keywords: Natural convection, variable viscosity, viscous dissipation, wavy surface, magnetic field, Keller-box method.

NOMENCLATURE

C_{fx}	local skin friction coefficient
C_p	specific heat at constant pressure ($Jkg^{-1}K^{-1}$)
f	dimensionless stream function
g	acceleration due to gravity (ms^{-2})
Gr	Grashof number
k	thermal conductivity of fluid ($Wm^{-1}K^{-1}$)
L	characteristic length associated with the wavy surface (m)
M	magnetic parameter
Nu_x	local Nusselt number
P	pressure of the fluid (Nm^{-2})
Pr	Prandtl number
T	temperature of the fluid in the boundary layer (K)
T_w	temperature at the surface (K)

Greek symbols

α	amplitude of the wavy surface
β	volumetric coefficient of thermal expansion(K^{-1})
β_0	applied magnetic field strength
η	dimensionless similarity variable
θ	dimensionless temperature function
ψ	stream function (m^2s^{-1})
μ	dynamic viscosity of the fluid ($Kg\ m^{-1}s^{-1}$)
ν	kinematic viscosity of the fluid (m^2s^{-1})
ρ	density of the fluid ($Kg\ m^{-3}$)
σ_0	electrical conductivity of the fluid ($\Omega^{-1}m^{-1}$)
τ_w	shearing stress
$\sigma(x)$	surface profile function defined in equation (1)

Subscripts

w	wall conditions

 Received on: Feb., 2020

T_∞	temperature of the ambient fluid (K)	∞	ambient conditions
vd	viscous dissipation parameter		
u, v	dimensionless velocity components along the (x, y) axes (ms^{-1})	***Superscripts***	
		$'$	differentiation with respect to η
x, y	axis in the direction along and normal to the tangent of the surface		

1. Introduction

The study of the flow of electrically conducting fluid in the presence of magnetic field is important from the technical point of view and such types of problems have received much attention by many researchers. Viscous dissipation is of also interest for many applications: significant temperature rises are observed in polymer processing flows such as injection molding or extrusion at high rates. The viscous dissipation effect plays an important role in natural convection in strong gravitational field processes on large scales. It is also necessary to study the heat transfer from an irregular surface because irregular surfaces are often present in many applications, such as radiator, heat exchangers and heat transfer enhancement devices. However, it is known that viscosity must be change significantly with temperature. To predict accurately the flow behavior, it is necessary to take into account of viscosity.

Charraudeau (1975) proposed the viscosity of the fluid to be proportional to a linear function of temperature. Larger scale surface non-uniformities are encountered, for example, in cavity wall insulating systems and grain storage containers. Yao (1983) studied the effects of such non-uniformities on the vertical convective boundary layer flow of a Newtonian fluid. Hossain (1992) introduced the viscous and Joule heating effects on MHD-free convection flow with variable plate temperature. The problem of free convection flow from a wavy vertical surface in presence of a transverse magnetic field was studied by Alam et al. (1997). Also the effect of free convection flow with variable viscosity and thermal diffusivity along a vertical plate in the presence of magnetic field has been discussed by Elbashbeshy (2000). Hossain et al. (2002) studied natural convection of fluid with temperature dependent viscosity from heated vertical wavy surface. Jang et al. (2003) investigated the natural convection heat and mass transfer along a vertical wavy surface. Molla and Hossain (2007) studied radiation effect on mixed convection laminar flow along a vertical wavy surface. Combined effect of viscous dissipation and Joule heating on the coupling of conduction and free convection along a vertical flat plate has been investigated by Alim et al. (2008). Jha and Ajibade (2012) investigated the effect of viscous dissipation on natural convection flow between vertical parallel plates with time-periodic boundary conditions. They solved the governing equations analytically and discovered that heat is being transferred from the fluid with viscous dissipation to the plate when the fluid has small Prandtl. Sujon and Parveen (2014) investigated natural convection flow along a vertical wavy surface with the effects of viscous dissipation and heat generation on magnetohydrodynamics. Hazarika and Konch (2016) studied the effects of variable viscosity and thermal conductivity on magnetohydrodynamic free convection dusty fluid along a vertical porous plate with heat generation. They found that viscosity and species concentration decrease and temperature increases with the increasing value of the viscosity variation parameter for both the fluid and dust particles. Quite the opposite phenomenon was seen with the thermal conductivity variation parameter. Pakdee et al. (2017) analyzed numerically the two-dimensional unsteady magnetohydrodynamic compressible flow through a porous medium. They showed that the magnetic field and variable properties considerably influences the flows that is compressible thereby affecting the heat transfer as well as the wall shear stress. Alam et al. (2018) used the finite difference method together with Newton's linearization approximation to discuss the conjugate effects of viscous dissipation and variable viscosity on free convection flow over a sphere with joule heating and heat conduction. They argued that an enhanced Joule heating causes an increase in both the velocity and temperature profiles. Recently, Rajput and Kumar (2019) analyzed the effects of radiation and chemical reaction on MHD flow past a vertical plate with variable temperature and mass diffusion. Very recently, Abiodun and Bolaji (2020) studied the approximate solution of MHD natural convection flow with variable properties induced magnetic field, viscous dissipation and Ohmic heating. They obtained that increase in the viscous dissipation of the fluid results in the increase in its velocity, temperature, current density and the heat transfer. They also found that the skin friction on an isothermal condition and a temperature equal to zero plates grows and the heat transfer increases as the magnetic parameter decreases. Hasan et al. (2020) investigated Physics of bifurcation of the natural and forced convection flow and heat transfer through a curved duct. They detected that convective heat transfers to periodic and multi-periodic flows have boosted significantly more than steady-state flows. Kumar et al. (2020) investigated heat transfer reaction on a viscous dissipative free convective radiating stream over a permeable laminate within presence of induced magnetic field. They found that velocity accelerates as heat source and Eckert number parameters increases.

To our best of knowledge, magnetic field effect with temperature dependent variable viscosity and viscous dissipation on natural convection flow along a vertical wavy surface has not been studied yet and also for considering the practical importance the present work demonstrates the issue. The developed equations are made dimensionless by using suitable transformations. The non-dimensional equations are then transformed into non-linear equations by introducing a non-similarity transformation. The resulting non-linear equations together with their corresponding boundary conditions are solved numerically by using the finite difference method along with Newton's linearization approximation. The effects of magnetic field, variable viscosity and viscous dissipation parameter on the friction factor, Nusselt number, the streamlines as well as the isotherms are studied in detail.

2. Formulation of the Problem

The boundary layer analysis outlined below allows $\bar{\sigma}(X)$ being arbitrary, but our detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$Y_w = \bar{\sigma}(X) = \alpha \sin\left(\frac{n\pi X}{L}\right) \tag{1}$$

where α is the amplitude and L is the characteristic length associated with the wavy surface.

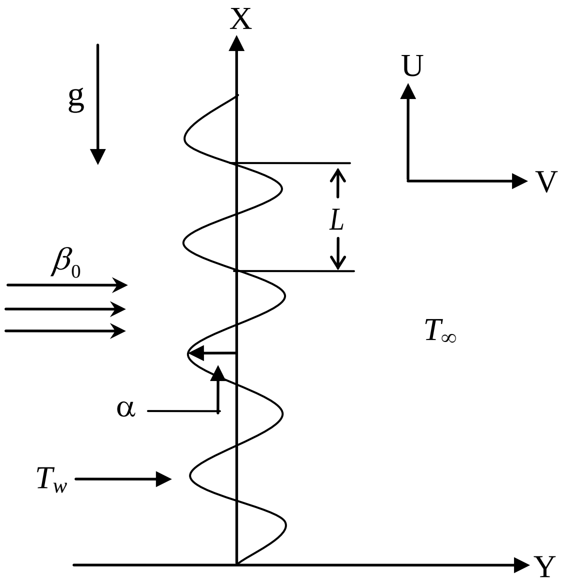

Fig. 1: The coordinate system and the physical model

The geometry of the wavy surface and the two-dimensional Cartesian coordinate system are shown in Fig. 1.

Under the usual Boussinesq approximation, the flow governed by the following boundary layer equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \frac{1}{\rho}\nabla.(\mu\nabla U) + g\beta(T - T_\infty) - \frac{\sigma_0\beta_0^2}{\rho}U \tag{3}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{1}{\rho}\frac{\partial P}{\partial Y}+\frac{1}{\rho}\nabla.(\mu\nabla V) \tag{4}$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{k}{\rho C_p}\nabla^2 T+\frac{\nu}{C_p}\left(\frac{\partial U}{\partial Y}\right)^2 \tag{5}$$

where (X, Y) are the dimensional coordinates along and normal to the tangent of the surface and (U, V) are the velocity components parallel to (X, Y), $\nabla^2 (=\partial^2/\partial x^2+\partial^2/\partial y^2)$ is the Laplacian operator, g is the acceleration due to gravity, P is the dimensional pressure of the fluid, ρ is the density, C_p is the specific heat at constant pressure and $\nu\,(=\mu/\rho)$ is the kinematic viscosity and $\mu(T)$ is the dynamic viscosity of the fluid in the boundary layer region depending on the fluid temperature, k is the thermal conductivity of the fluid, σ_0 is the electrical conductivity of the fluid, β_0 is the strength of magnetic field and β is the volumetric coefficient of thermal expansion.

The boundary conditions for the present problem are

$$\left.\begin{aligned} &U=0, V=0, T=T_w \quad at\ Y=Y_w=\bar{\sigma}(X)\\ &U=0,\quad T=T_\infty,\quad P=p_\infty \quad as \quad Y\to\infty \end{aligned}\right\} \tag{6}$$

Following Yao (1983), here introduce the following non-dimensional variables

$$x=\frac{X}{L},\ y=\frac{Y-\bar{\sigma}}{L}Gr^{\frac{1}{4}},\ p=\frac{L^2}{\rho\nu^2}Gr^{-1}P,\ u=\frac{\rho L}{\mu_\infty}Gr^{-1/2}U,\quad v=\frac{\rho L}{\mu_\infty}Gr^{-1/4}(V-\sigma_x U),$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\ \sigma_x=\frac{d\bar{\sigma}}{dX}=\frac{d\sigma}{dx},\ Gr=\frac{g\beta(T_w-T_\infty)}{\nu^2}L^3$$

Introducing the above dimensionless dependent and independent variables into Equations (2)–(5), the following dimensionless form of the governing equations are obtained:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+Gr^{1/4}\sigma_x\frac{\partial p}{\partial y}+\varepsilon(1+\sigma_x^2)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y}+(1+\sigma_x^2)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}-Mu+\theta \tag{8}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-Gr^{1/4}\frac{\partial p}{\partial y}+\varepsilon\sigma_x(1+\sigma_x^2)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y}+\sigma_x(1+\sigma_x^2)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}-\sigma_{xx}u^2 \tag{9}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}(1+\sigma_x^2)\frac{\partial^2\theta}{\partial y^2}+vd\left(\frac{\partial u}{\partial y}\right)^2 \tag{10}$$

where $\Pr=\frac{C_p\mu_\infty}{k}$ is the Prandtl number, $\varepsilon=\varepsilon^*(T_w-T_\infty)$ is the viscosity variation parameter, $vd=\frac{\nu^2 Gr}{L^2 C_p(T_w-T_\infty)}$ is the viscous dissipation parameter and $M=\frac{\sigma_0\beta_0^2 L^2}{\mu Gr^{1/2}}$ is the magnetic parameter.

There are very few forms of viscosity variation available in the literature. Among them we have considered that one which is appropriate for liquid introduced by Hossain et al.(2002) as follows:

$$\mu = \mu_\infty[1+\varepsilon^*(T-T_\infty)] \tag{11}$$

where μ_∞ is the viscosity of the ambient fluid and ε^* is a constant evaluated at the film temperature of the flow.

Eq. (9) indicates that the pressure gradient along the *y*-direction is $O(Gr^{-1/4})$, which implies that lowest order pressure gradient along *x*-direction can be determined from the inviscid flow solution. For the present problem this pressure gradient ($\partial p/\partial x = 0$) is zero. Eq. (9) further shows that $Gr^{1/4}\partial p/\partial y$ is O(1) and is determined by the left-hand side of this equation. Thus, the elimination of $\partial p/\partial y$ from Eqs. (8) and (9) leads to

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x^2}u^2+\varepsilon\left(1+\sigma_x^2\right)\frac{\partial u}{\partial y}\frac{\partial \theta}{\partial y}-\frac{M}{1+\sigma_x^2}u+\frac{1}{1+\sigma_x^2}\theta \tag{12}$$

The corresponding boundary conditions for the present problem are

$$\left.\begin{array}{lll} u=v=0, \quad \theta=1 & at & y=0 \\ u=\theta=0, \quad p=0 & as & y\to\infty \end{array}\right\} \tag{13}$$

Now introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi = x^{3/4} f(x,\eta), \quad \eta = yx^{-1/4}, \quad \theta=\theta(x,\eta) \tag{14}$$

where η is the pseudo similarity variable, θ is the dimensionless temperature and ψ is the stream function.

Introducing the transformations given in Eq. (14) into Eqs. (12) and (10) the momentum and energy equations are transformed the following forms:

$$\left(1+\sigma_x^2\right)(1+\varepsilon\theta)f'''+\frac{3}{4}ff''+\varepsilon\left(1+\sigma_x^2\right)\theta' f''+\frac{1}{1+\sigma_x^2}\theta-\frac{Mx^{1/2}}{1+\sigma_x^2}f'-\left(\frac{1}{2}+\frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right)f'^2=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \tag{15}$$

$$\frac{1}{\Pr}\left(1+\sigma_x^2\right)\theta''+\frac{3}{4}f\theta'+vdxf''^2=x\left(f'\frac{\partial \theta}{\partial x}-\theta'\frac{\partial f}{\partial x}\right) \tag{16}$$

The boundary condition (13) now takes the following form:

$$\left.\begin{array}{ll} f(x,o)=f'(x,o)=0, & \theta(x,o)=1 \\ f'(x,\infty)=0, & \theta(x,\infty)=0 \end{array}\right\} \tag{17}$$

The rate of heat transfer in terms of the local Nusselt number, Nu_x and the local skin friction coefficient, C_{fx} take the following forms:

$$Nu_x(Gr/x)^{-1/4} = -\sqrt{1+\sigma_x^2}\,\theta'(x,o) \tag{18}$$

$$C_{fx}(Gr/x)^{1/4}/2=(1+\varepsilon)\sqrt{1+\sigma_x^2}\,f''(x,o) \tag{19}$$

3. Method of Solution

The nonlinear systems of partial differential equations are solved numerically with the help of implicit finite difference method known as the Keller box scheme developed by Keller (1978).

To apply the aforementioned method, equations (15) and (16) their boundary condition (17) are first converted into the following system of first order equations. For this purpose we introduce new dependent variables

$u(\xi,\eta)$, $v(\xi,\eta)$, $p(\xi,\eta)$ and $g(\xi,\eta)$ so that the transformed momentum and energy equations can be written as

$$f' = u \tag{20}$$

$$u' = v \tag{21}$$

$$g' = p \tag{22}$$

$$P_1 T v' + P_2 f\, v - P_3 u^2 + P_4 g - P_5 u + P_6 pv = \xi (u \frac{\partial u}{\partial \xi} - v \frac{\partial f}{\partial \xi}) \tag{23}$$

$$\frac{1}{\Pr} P_1 p' + P_2 f\ p + P_7 v^2 = \xi\ (u\ \frac{\partial g}{\partial \xi} - p\ \frac{\partial f}{\partial \xi}) \tag{24}$$

where $x = \xi$, $\theta = \mathrm{g}$ and

$$P_1 = \left(1+\sigma_x^2\right)\ ,\quad P_2 = \frac{3}{4},\quad P_3 = \frac{1}{2} + \frac{x\sigma_x \sigma_{xx}}{1+\sigma_x^2},\quad P_4 = \frac{1}{1+\sigma_x^2},\quad P_5 = \frac{Mx^{1/2}}{1+\sigma_x^2},$$
$$P_6 = \varepsilon\left(1+\sigma_x{}^2\right),\ P_7 = vd\ x \text{ and } \mathrm{T} = \left(1+\varepsilon\theta\right)$$

and the boundary conditions (17) are

$$\begin{aligned} &f(\xi,0) = 0,\ u(\xi,0) = 0, g\left(\xi,0\right) = 1 \\ &u(\xi,\infty) = 0,\ g\ (\xi,\infty) = 0 \end{aligned} \tag{25}$$

Now consider the net rectangle on the (ξ, η) plane shown in the Fig. 2 and denote the net points by

$$\begin{aligned} &\xi^0 = 0,\ \xi^n = \xi^{n-1} + k_n,\ n = 1,2,\cdots\cdots\cdots\cdots, N \\ &\eta_0 = 0,\ \eta_j = \eta_{j-1} + h_{j,}\quad j = 1,2,\cdots\cdots\cdots\cdots, J \end{aligned} \tag{26}$$

Here n and j are just sequence of numbers on the (ξ, η) plane, k_n and h_j are the variable mesh widths. Approximate the quantities f, u, v and p at the points (ξ^n,η_j) of the net by f_j^n, u_j^n, v_j^n, p_j^n which call net function. It is also employed that the notation P_j^n for the quantities midway between net points shown in Fig. 2 and for any net function as

$$\xi^{n-1/2} = \frac{1}{2}(\xi^n + \xi^{n-1}) \tag{27}$$

$$\eta_{j-1/2} = \frac{1}{2}(\eta_j + \eta_{j-1}) \tag{28}$$

$$g_j{}^{n-1/2} = \frac{1}{2}(g_j^n + g_j^{n-1}) \tag{29}$$

$$g_{j-1/2}^n = \frac{1}{2}(g_j^n + g_{j-1}^n) \tag{30}$$

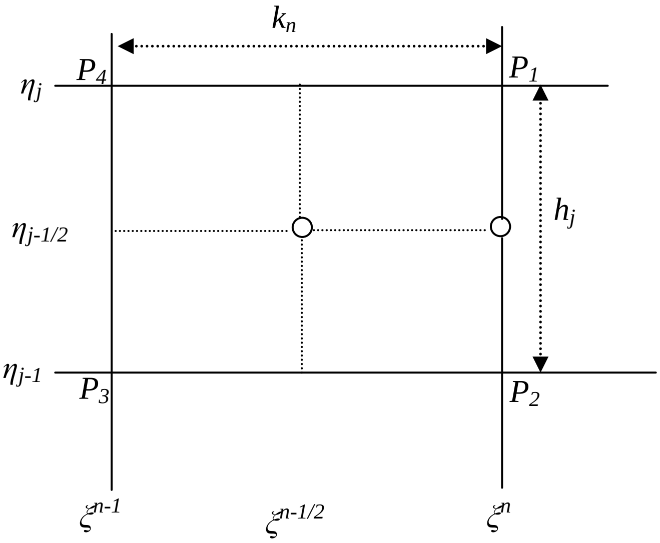

Fig. 2: Net rectangle of difference approximations for the Box scheme.

The finite difference approximations according to box method to the three first order ordinary differential equations (20) – (22) are written for the mid point $(\xi^n, \eta_{j-1/2})$ of the segment P_1P_2 shown in the Fig. 2 and the finite difference approximations to the two first order differential equations (23) and (24) are written for the mid point $(\xi^{n-1/2}, \eta_{j-1/2})$ of the rectangle $P_1P_2P_3P_4$. This procedure yields

$$\frac{f_j^n - f_{j-1}^n}{h_j} = u_{j-1/2}^n = \frac{u_{j-1}^n + u_j^n}{2} \tag{31}$$

$$\frac{u_j^n - u_{j-1}^n}{h_j} = v_{j-1/2}^n = \frac{v_{j-1}^n + v_j^n}{2} \tag{32}$$

$$\frac{g_j^n - g_{j-1}^n}{h_j} = p_{j-1/2}^n = \frac{p_{j-1}^n + p_j^n}{2} \tag{33}$$

$$\begin{aligned}&\frac{1}{2}(P_1T)^n_{j-1/2}\left(\frac{v^n_j - v^n_{j-1}}{h_j}\right)+\frac{1}{2}(P_1T)^{n-1}_{j-1/2}\left(\frac{v^{n-1}_j-v^{n-1}_{j-1}}{h_j}\right)+(P_2 fv)^{n-1/2}_{j-1/2}-(P_3u^2)^{n-1}_{j-1/2}+(P_4g)^{n-1}_{j-1/2}\\&-(P_5u)^{n-1}_{j-1/2}+(P_6pv)^{n-1}_{j-1/2} = \xi^{n-1/2}_{j-1/2}\left(u^{n-1/2}_{j-1/2}\frac{u^n_{j-1/2}-u^{n-1}_{j-1/2}}{k_n} - v^{n-1/2}_{j-1/2}\frac{f^n_{j-1/2}-f^{n-1}_{j-1/2}}{k_n}\right)\end{aligned} \tag{34}$$

$$\begin{aligned}&\frac{1}{2\Pr}\left\{(P_1)^n_{j-1/2}\right\}\left(\frac{p^n_j - p^n_{j-1}}{h_j}\right)+\frac{1}{2\Pr}\left\{(P_1)^{n-1}_{j-1/2}\right\}\left(\frac{p^{n-1}_j-p^{n-1}_{j-1}}{h_j}\right)+(P_2 fp)^{n-1/2}_{j-1/2}\\&+(P_7v^2)^{n-1/2}_{j-1/2} = \xi^{n-1/2}_{j-1/2}\left(u^{n-1/2}_{j-1/2}\frac{g^n_{j-1/2}-g^{n-1}_{j-1/2}}{k_n} - p^{n-1/2}_{j-1/2}\frac{f^n_{j-1/2}-f^{n-1}_{j-1/2}}{k_n}\right)\end{aligned} \tag{35}$$

The above equations are to be linearized by using Newton’s Quasi-linearization method. Then linear algebraic equations can be written in block matrix which form a coefficient matrix. The whole procedure, namely reduction to first order followed by central difference approximations, Newton's Quasi-linearization method and the block Thomas algorithm, is well known as the Keller-box method.

During the program test, the convergent criteria for the relative errors between two iterations are less 10^{-5}. A uniform grid of 201 points is used in x- direction with Δx = 0.05, while a non-uniform grid of 76 points lying between η = 0.0 and 10.017 is chosen. Grid points are concentrated towards the heated surface in order to improve resolution and the accuracy of the computed values of the surface shear stress and rate of heat transfer.

4. Results and Discussion

In the present study, numerical solutions are obtained in terms of the skin friction coefficient, Nusselt number, the streamlines as well as the isotherms for different values of the magnetic parameter M (= 0.0, 0.5, 1.0 and 2.0), viscous dissipation parameter vd (= 0.0, 1.0, 5.0, 10.0), variable viscosity variation parameter ε (= 0.0, 4.0, 8.0 and 15.0), the amplitude-to-length ratio of the wavy surface α = 0.3 and Prandtl number Pr = 0.73 which correspond to the air at 2100^0K.

It is seen that the effect of the magnetic parameter M leads to a decrease in the local skin friction coefficient C_{fx} and the local Nusselt number Nu_x in Figs. 3(a) and 3(b). This phenomenon can be easily understood from the fact that the magnetic parameter M increases the Lorentz force, which opposes the flow, therefore decrease the velocity and temperature gradient and hence the local skin friction coefficient C_{fx} and local Nusselt number Nu_x decrease. It is also observed that the skin friction coefficient C_{fx} and the local Nusselt number Nu_x decrease by approximately 25% and 9% respectively as M increases from 0.00 to 2.00.

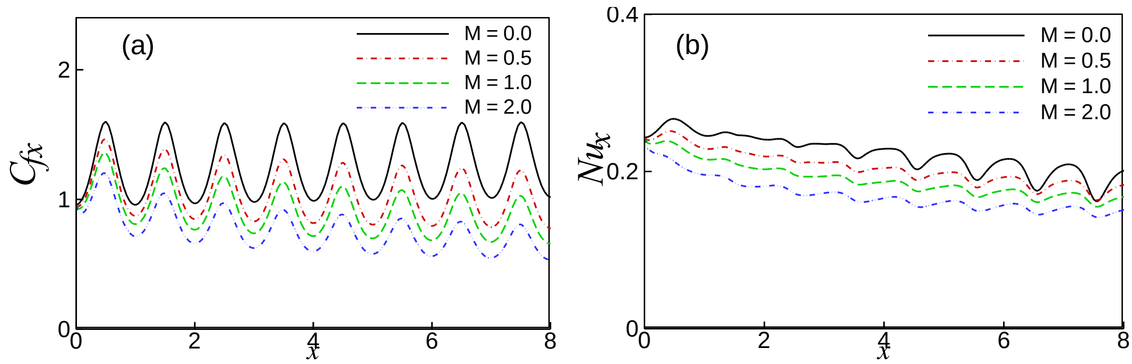

Fig. 3: Effect of M on (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x.

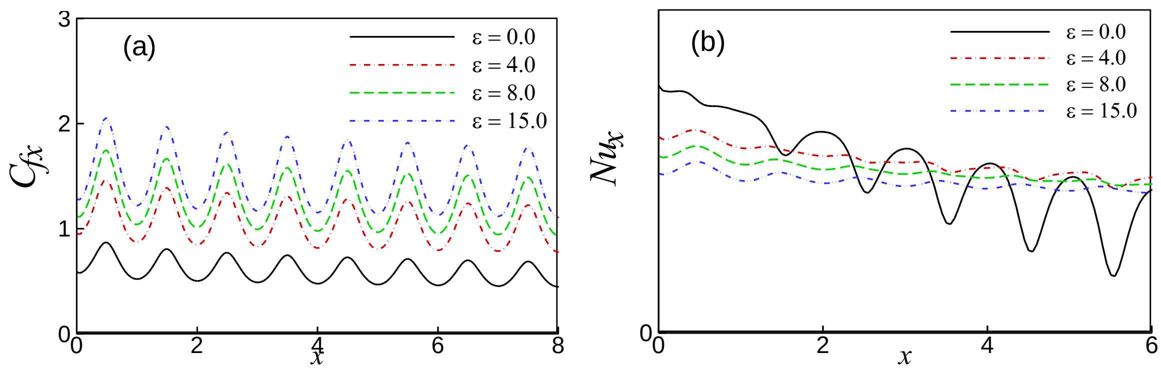

Fig. 4: Effect of ε on (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x.

The effect of the variable viscosity variation parameter on local skin friction coefficient C_{fx} and the local Nusselt number Nu_x are shown in Figs. 4(a)–(b) respectively while vd = 0.4, M = 0.5, α = 0.3 and Pr = 0.73. It is observed that an increase in the values of ε leads to enhance in the results of C_{fx}. When the variable viscosity variation parameter increases, then the temperature of the surface rises and decreases the temperature gradient. For this reason the local Nusselt number Nu_x decreases.

Figs. 5(a) and 5(b) display the results of the skin friction coefficient C_{fx} and the rate of heat transfer Nu_x against x for different values of viscous dissipation parameter vd while Prandtl number Pr = 0.73, ε = 4.0, M = 0.5 and α = 0.3. It is found that for the effect of viscous dissipation parameter in the fluid, the skin friction coefficient C_{fx} increases that is the frictional force at the wall enhances and the rate of heat transfer Nu_x reduces. It is evident that due to the viscous dissipation parameter vd, fluid temperature rises and temperature difference between solid wall and fluid become slower and consequently reduction rate of heat transfer happens.

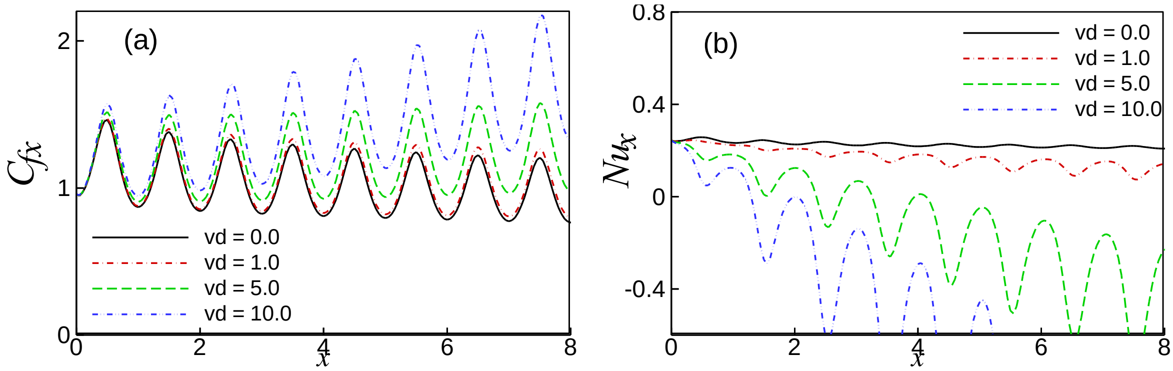

Fig. 5: Effect of vd on (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_{x}.

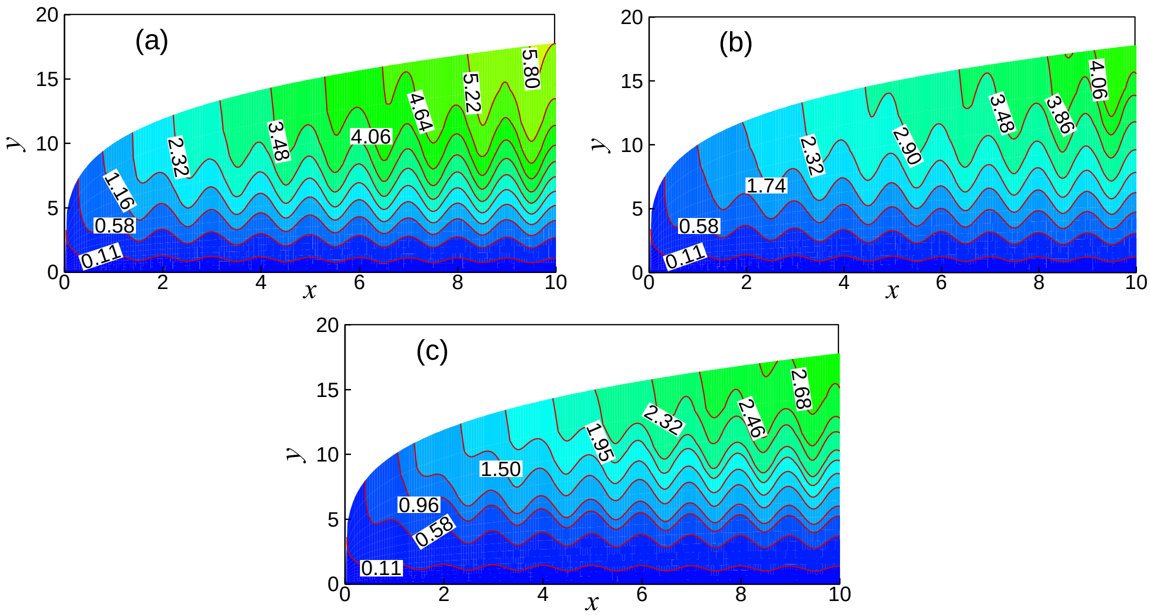

Fig. 6: Streamlines for (a) M = 0.5 (b) M = 1.0 (c) M = 2.0 while Pr = 0.73, ε = 4.0, vd = 0.4 and α = 0.3.

Figs. 6 and 7 illustrate the effect of the magnetic parameter M on the development of streamlines and isotherms respectively, which are plotted for α = 0.3, Pr = 0.73, ε = 4.0 and vd = 0.4. It is seen that when M = 0.5 the maximum values of stream function, that is, ψ_{max} within the computational domain is 5.80. From Fig. 6(b) for M = 1.0 the value of ψ_{max} is 4.06. From Fig. 6(c) for M = 2.0 the value of ψ_{max} is 2.68. It can be concluded that with the effect of the magnetic parameter M, the flow flux in the boundary layer decreases. From Figs. 7(a)–(c), it is noted that with the effect of M the thermal boundary layer thickness enhances significantly. Due to the effect of magnetic parameter M, the temperature within the boundary layer increases and the associate thermal boundary layer becomes thicker.

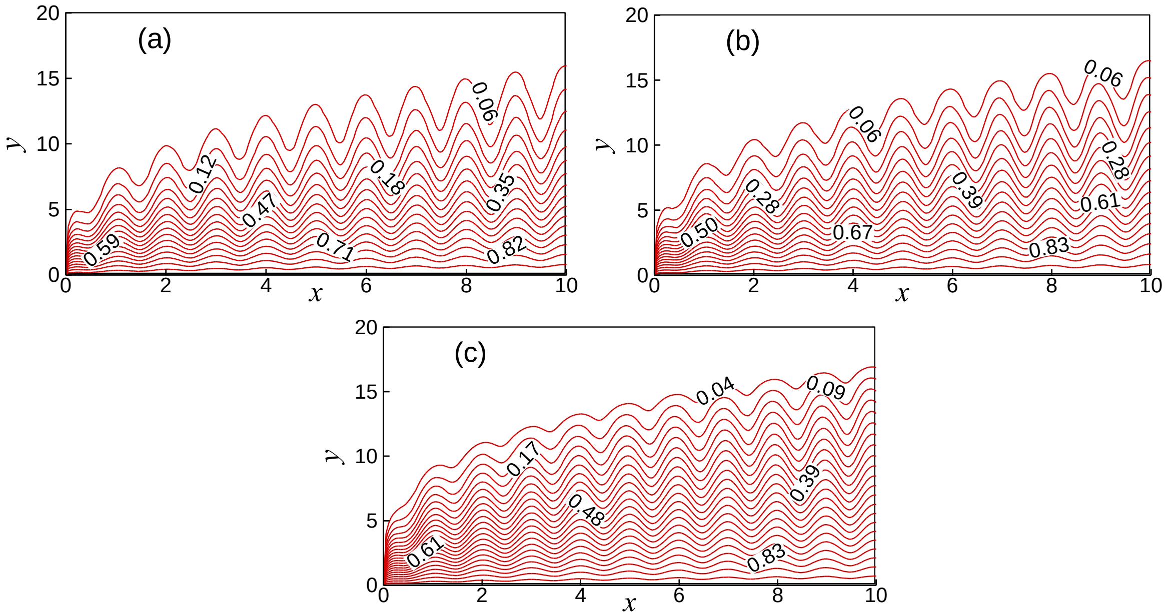

Fig. 7: Isotherms for (a) M = 0.5 (b) M = 1.0 (c) M = 2.0 while Pr = 0.73, ε = 4.0, vd = 0.4 and α = 0.3.

The effects of viscous dissipation parameter vd and variable viscosity variation parameter ε, on the development of streamlines which are displayed in Fig. 8 for α = 0.3, M = 0.5 and Prandtl number Pr = 0.73. The viscosity is independent of temperature with vd = 0.0 as shown in Fig. 8(a) and found that maximum value of stream function ψ_{max} is 6.52. Fig. 8(b) displays the results that an increasing values of vd, the velocity boundary layer thickness increases. In this case the maximum value of stream function ψ_{max} is 10.86. For increasing values of viscosity variation parameter ε the boundary layer becomes thinner and the maximum value of stream function ψ_{max} is 4.88 that is shown in Fig. 8(c). The combined effects of viscosity variation parameter ε and viscous dissipation parameter vd are shown in Fig. 8(d). Here the maximum value ψ_{max} is 6.09. From these figures it is observed that the value of stream function ψ becomes lower for larger values of viscosity variation parameter ε and becomes higher for viscous dissipation parameter vd.

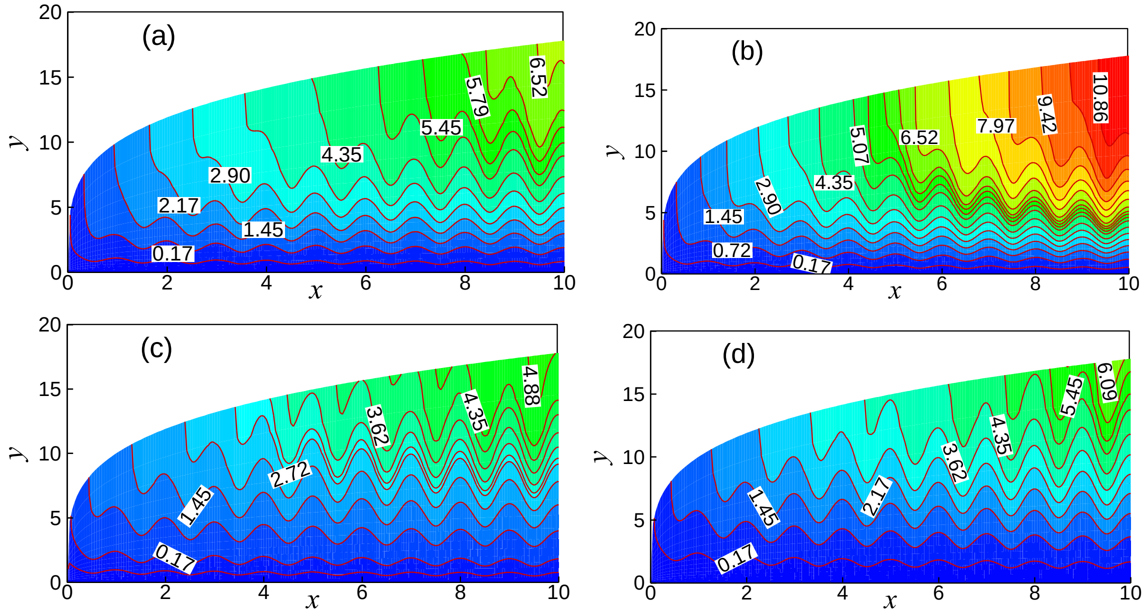

Fig. 8: Streamlines for (a) ε = 0.0, vd = 0.0 (b) ε = 0.0, vd = 5.0 (c) ε = 10.0, vd = 0.0 and (d) ε = 20.0, vd = 5.0 while α = 0.3, M = 0.5 and Pr = 0.73.

The variation of isotherms with viscosity variation parameter ε and viscous dissipation parameter vd for $\alpha = 0.3$, magnetic parameter $M = 0.5$ and Prandtl number Pr = 0.73 are shown in Fig. 9. We can say after observing the isotherms of this figure that temperature enhances within the boundary layer due to the higher values of variable viscosity variation parameter ε and viscous dissipation parameter vd.

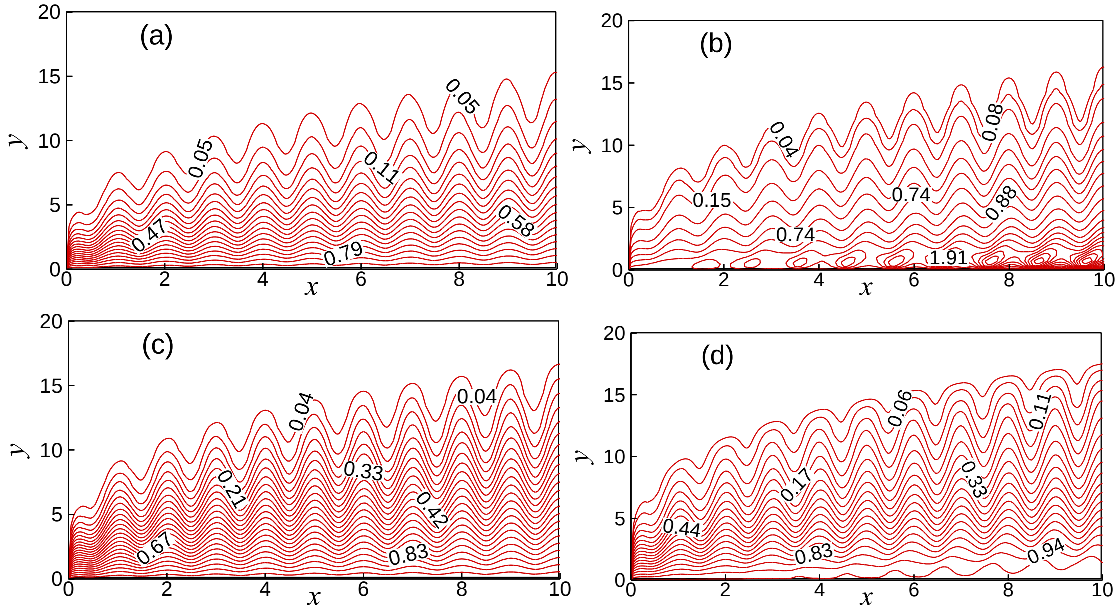

Fig. 9: Isotherms for (a) $\varepsilon = 0.0$, $vd = 0.0$ (b) $\varepsilon = 0.0$, $vd = 5.0$ (c) $\varepsilon = 10.0$, $vd = 0.0$ and (d) $\varepsilon = 20.0$, $vd = 5.0$ while $\alpha = 0.3$, $M = 0.5$ and Pr = 0.73.

Table 1: Comparison of the present numerical results of skin friction coefficient, $f''(x,0)$ and the heat transfer, -$\theta(x,0)$ with Hossain et al. (2002) for the variation of Prandtl number Pr while $vd = 0.0$, $\varepsilon = 0.0$, $M = 0.0$ and $\alpha = 0.1$.

Pr	$f''(x,0)$		$-\theta(x,0)$	
	Hossian et al. (2002)	Present work	Hossian et al. (2002)	Present work
1.0	0.908	0.90814	0.401	0.39914
10.0	0.591	0.59269	0.825	0.82663
25.0	0.485	0.48733	1.066	1.06847
50.0	0.485	0.41880	1.066	1.28351
100.0	0.352	0.35640	1.542	1.54198

A comparison of the present numerical results of the skin friction coefficient $f''(x,0)$ and the rate of heat transfer $-\theta'(x,0)$ with the results obtained by Hossain et al. (2002) is depicted in Table 1. Here, the magnetic parameter M, viscosity variation parameter ε and viscous dissipation parameter vd are ignored while different values of Prandtl number Pr = (1.0, 10, 25.0, 50.0 and 100.0) are chosen. From Table 1, it is clearly seen that the present results are excellent agreement with the solution of Hossain et al. (2002).

5. Conclusion

The effect of temperature dependent variable viscosity and viscous dissipation on magnetic field natural convection flow along a vertical wavy surface has been studied numerically. The present investigation can be concluded as follows:

- The skin friction coefficient increases for decreasing values of magnetic parameter M. For increasing values of viscosity variation parameter and viscous dissipation parameter the skin friction coefficient increase.
- The rate of heat transfer decrease for increasing values of magnetic parameter M, viscosity variation parameter and viscous dissipation parameter.
- The velocity of the fluid flow decreases and the temperature distribution of the fluid within the boundary layer significantly increases for increasing values of M.
- The velocity of the fluid flow decreases but velocity gradient increases for the effect of temperature dependent viscosity variation parameter ε. The velocity boundary layer becomes thinner for the effect of ε. On the other hand for the effect of ε the temperature increases and the thermal boundary layer becomes thicker.
- For increasing values of *vd*, the momentum and thermal boundary layer thickness enhanced.

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