



SLOSHING DYNAMICS IN SWAY EXCITED RECTANGULAR SCALED TANKS

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Abstract:

This paper presents the result obtained from a series of experiments conducted on different scaled rectangular tanks mounted on shake table, to study the scale effects of sloshing with sway excited motion. Three different tanks of model scales of 1:86, 1:57 and 1:43 are considered for present study. The tests are carried out for the aspect ratio (h/l , where h is liquid depth and l is the length of the tank) of 0.1625, 0.325, and 0.4875 which represents 25%, 50% and 75% of liquid fill levels, respectively. The sloshing oscillations occur in the longitudinal axis when subjected to sway excitations. Sloshing forces and sloshing oscillation time histories are measured using load cells and wave probes, respectively. The effect of excitation amplitudes and excitation frequencies on sloshing oscillations and sloshing force are presented herein. It is found that violent sloshing is experienced for 50% filled condition irrespective of scaled tanks, excitation amplitudes and excitation frequencies. The sloshing force is maximum in 1:43 scaled tank than other scaled sloshing tanks irrespective of the excitation frequency and amplitude for 50% fill level. Based on the experimental observations and analysis of results, it is concluded that proportionate volume of water and tank size decides the severity of sloshing in the partially filled moving tanks.

Keywords: Sloshing, sway, scale effects, rectangular tanks

1. Introduction

Motion of free liquid surface in partially filled containers due to external disturbances is defined as sloshing. For any partially filled tanks, dynamic behavior is significantly affected by the dynamic motion of the free surface liquid. Sloshing is experienced in various engineering applications such as liquid transporting trains and trucks, oil carrying ships, fuel tanks on space crafts and rockets, water oscillation in liquid storage tanks and reservoirs subjected to earthquake excitations. Oil transportation between countries in terms of both import and export is one of the major trades, supporting the economy of the country. Sloshing is a severe problem in the oil carrying ships. Violent oscillation of the oil inside the tank will lead to the instability and damage of the liquid cargo ships. Understanding the liquid sloshing phenomena will help the Engineers to incorporate the additional loads, in the design of tanks of liquid carrying ships and trucks. Studies have been carried out in the field of sloshing from 17th Century.

A detailed review of the existing literature reveals the importance of sloshing. Numerous studies were carried out to study the phenomena of sloshing experimentally, analytically and numerically. Liquid sloshing was initially investigated by Faraday (1831) which was followed by Benjamin and Ursell (1954). Moiseyev (1958) proposed a nonlinear theory to predict the sloshing behaviour in partly filled sway excited tank.

Faltinsen (1974) numerically simulated the nonlinear behaviour of sloshing under sway and roll motions. Nakayama and Washizu (1980) used a finite element based nonlinear approach to investigate the liquid motion and the resulting sloshing pressure in a container excited under forced oscillations. Series of experiments was conducted out by Scheffer and Fittschen (1984) to study the effects of wave approach angle and water depth on behavioral motion of ship. Waterhouse (1994) explicated the significance of aspect ratio in sloshing problem. Armenio and La Rocca (1996) developed mathematical models to analyse liquid sloshing in open rectangular tanks. Cariou and Casella (1999) examined the mathematical works for the sloshing simulations and emphasized the scope for further research work on impact loads and peak pressures. Kim (2001) studied the impact load on sloshing force using finite difference method in the two- and three-dimensional containers. Rognebakke and Faltinsen (2001) investigated the sloshing effects on the partially filled tanks subjected to regular beam sea waves. Celebi and Akyildiz

(2002) studied the nonlinear behaviour of liquid in partially filled rectangular tank using finite difference method by solving Navier-Stokes equation. Sames et al. (2002) studied the pressure responses due to sloshing in cylindrical and rectangular tanks. Frandsen (2004) studied the multiple resonance conditions developed in combined heave and sway excitations. Akyildiz and Unal (2005) compared the pressure distribution in a tank excited by roll motion for different fill levels. The nonlinear and linear behaviour of liquid sloshing dynamics in elastic containers and supported structures was presented by Ibrahim (2005). Graczyk et al. (2007) studied the sloshing induced pressure on the walls of the tank under random excitation and the structure response of the LNG tank subjected to vertical and horizontal accelerations. Liu and Lin (2008) studied the three-dimensional nonlinear sloshing behavior using numerical model (NEWTANK) and the numerical study was validated analytically. Virella et al. (2008) employed finite element tool (ABAQUS) to study the influence of non-linear behavior of sloshing on its model pressure and natural periods. Chen et al. (2009) calculated the impact pressure exerted by dynamic load due to sloshing in a partly filled tank. Khezzar et al. (2009) studied the sloshing phenomenon on a test platform (560 mm x 160 mm x 185 mm rectangular tank), subjected to impulsive impact. Panigrahy et al. (2009) observed the fluctuation in pressures exerted on the tank walls near the free surface and at the deeper surfaces. Nasar et al. (2010, 2012) investigated sloshing behaviour in a rectangular tank mounted on a barge and the barge responses under random excitations. Bogaert et al. (2010) conducted tests on full scale and large scale to study the hydro-elastic effects. Kim et al. (2012) conducted comparative study on model-scale tanks. The sloshing pressure at 1/50 scale tank at Seoul National University (SNU) was compared with the data's measured at MARINTEK facility. Xue et al. (2012) reported the influence of vertically arranged baffles on sloshing frequency with shake table tests. Hashemi et al. (2013) determined the dynamic sloshing effect in a 3-D rectangular tank analytically under earthquake ground motion. Mei-rong et al. (2014) compared the pressure and sloshing elevation by varying excitation frequencies for elastic tank with rigid tank. Kim et al. (2017) studied the scale effects of sloshing under roll and sway motions using three different scaled tanks. The results from the experiment were statistically analysed and a comparison of performance at three different fill levels (0.15*h*, 0.70*h* and 0.95*h*, where *h* is the height of tank) was also done. Sifting over the literatures, the dynamics of sloshing characteristics was varying with the fill levels hence the experimental and numerical studies have to be carried out for different fill levels. The present study focuses on to explore the sloshing effects in three different scaled sway excited tanks of rectangular ship tanks.

2. Experimental setup

The experimental study is carried out using shake table in the Department of Water Resources and Ocean Engineering at National Institute of Technology Karnataka (NITK), Surathkal. An experimental setup is designed and devised to measure the sloshing force based on ballast mass concept. The cross-section of the experimental setup is shown in Fig. 1. An indigenous make (Geotran) servo-hydraulic, single axis shake table of payload capacity 500 kgf is connected to a digital amplifier which is driven by a hydraulic actuator. The inertial forces experienced by the masses placed on the shake table platforms are measured by load cells. Capacitance type wave probes are used to capture sloshing motion of liquid inside the tank. The horizontal displacement of the shake table is measured using LVDT. Wave transformer (Scientech) is used to control excitation frequencies and amplitudes.

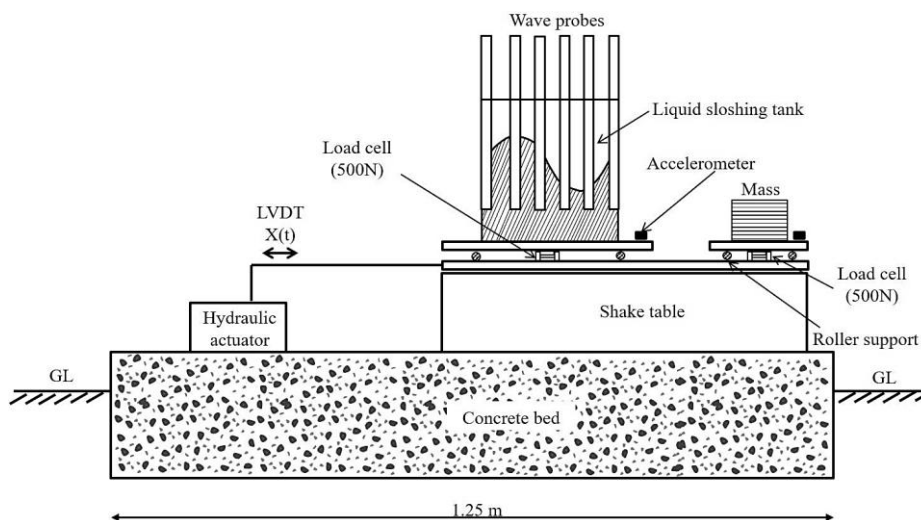


Fig. 1: Cross-section of the experimental setup.

4.1 Model tank details

Rectangular tanks of scale 1:86, 1:57 and 1:43 are fabricated using 12 mm thick acrylic sheets [Nasar et al. (2008)] and Table 1 specifies dimensions of rectangular tanks. The model tanks were selected based on the prototype of Liquid cargo carriers. The photographic view of the three-scaled tanks is shown in Fig. 2.

Table 1: Dimensions of the sloshing tanks

Sl. No.	Designation of tank	Dimensions in mm		
		Tank 1 (1:86)	Tank 2 (1:57)	Tank 3 (1:43)
1.	Length (l)	500	750	1000
2.	Width (b)	200	300	400
3.	Height (h)	325	487.5	650



Fig. 2: Photographic view of rectangular scaled tanks.

3. Methodology

The tank partially filled with liquid is mounted on the shake table and the table is harmonically excited in sway motion at different excitation amplitudes and frequencies. Experiments are conducted using Shake table for 25%, 50% and 75% fill levels. Natural frequencies for different fill levels (f_n) are calculated using the mathematical expression [Ibrahim (2005)] as follows:

$$f_n = \frac{1}{2\pi} \sqrt{\frac{n\pi g}{l} \tanh\left(\frac{n\pi h_s}{l}\right)}, \quad n = 1, 2, 3, \dots \quad (1)$$

Where, n is the surface mode number, h_s is the static water depth and l is the tank length.

The excitation frequencies for 1:86 scaled tank ranges from 0.4566 Hz to 2.794 Hz which covers up to fifth mode of sloshing excitation. Similarly, the excitation frequencies range between 0.4566 Hz to 2.813 Hz (f_5) and 0.4566 Hz to 1.9757 Hz (f_5) for 1:57 and 1:43 scaled tanks, respectively. Two different excitation amplitudes (a) of 4 mm and 8 mm are considered.

4. Results and Discussions

With the results obtained from the series of shake table experiments, the sloshing dynamics is explored with the following experimental parameters such as maximum free surface response (η_{max}), root mean square elevation (η_{rms}), maximum sloshing force (F'_{max}), and average of ten largest sloshing peaks (F'_{avg}), sloshing oscillation spectra and statistical analysis for three scaled tanks in sequence.

4.1 Maximum free surface response (η_{max})

The maximum free surface response gives the maximum sloshing run-up for given excitation amplitude and excitation frequency in the sway excited rectangular sloshing tank. The variation of normalized maximum free

response (η_{max}/a) for different frequency ratio (f/f_1) and different fill levels are projected in Fig. 3 for 1:86, 1:57 and 1:43 scaled tanks.

A comparison of experimental results with numerical results of Nasar et al. (2012) is performed. At odd sloshing frequencies, acceptable correlation between maximum free surface responses is observed. For all the scaled tanks, maximum free surface response is higher for 50% fill depth ($h_s/l = 0.325$) than the response obtained for 75% fill depth ($h_s/l = 0.4875$) and 25% fill level ($h_s/l = 0.1625$). It is also observed that the first mode of sloshing ($f = f_1$) is the critical mode for partially filled tank under sway excitation. The free surface response is in decreasing order with the wave excitations at $f = f_1, f_3$ and f_5 . On comparison of three different fill levels, the higher sloshing oscillations are observed in the order of 50%, 75% and 25% fill levels. Irrespective of the fill depths, the 1:43 scaled tank shows more sloshing response in comparison with 1:57 and 1:86 scaled tank.

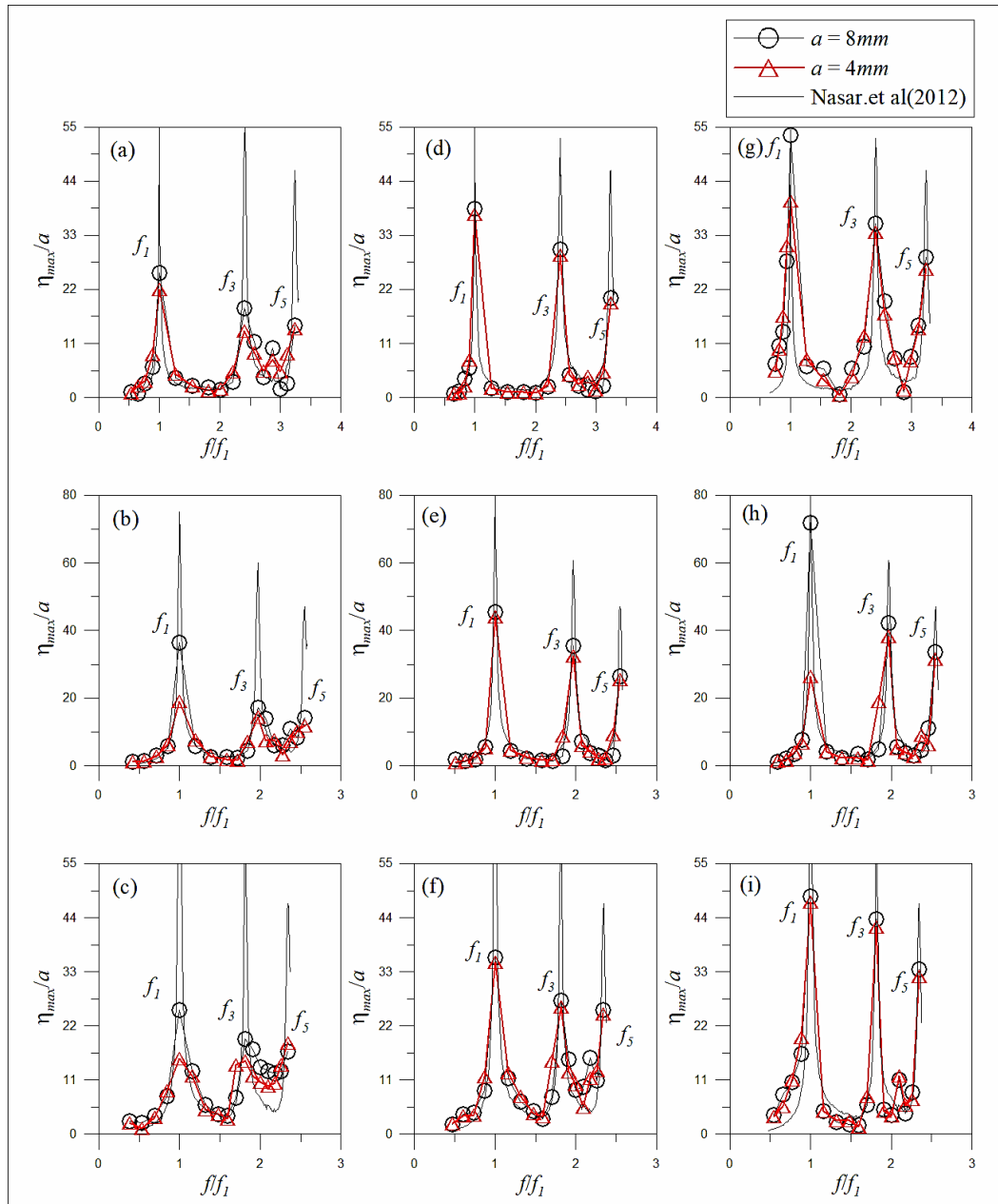


Fig. 3: Variation of η_{max}/a with frequency ratio (f/f_1) for, (a) $h_s/l = 0.1625$ for 1:86, (b) $h_s/l = 0.325$ for 1:86, (c) $h_s/l = 0.4875$ for 1:86, (d) $h_s/l = 0.1625$ for 1:57 (e) $h_s/l = 0.325$ for 1:57, (f) $h_s/l = 0.4875$ for 1:57, (g) $h_s/l = 0.1625$ for 1:43, (h) $h_s/l = 0.325$ for 1:43 and, (i) $h_s/l = 0.4875$ for 1:43 scale tank.

4.2 Root Mean Square Surface Elevation (η_{rms})

The variation of normalized Root Mean Square surface elevation (η_{rms}/a) with frequency ratio (f/f_1) is illustrated in Fig. 4 for different fill levels and different scaled tanks.

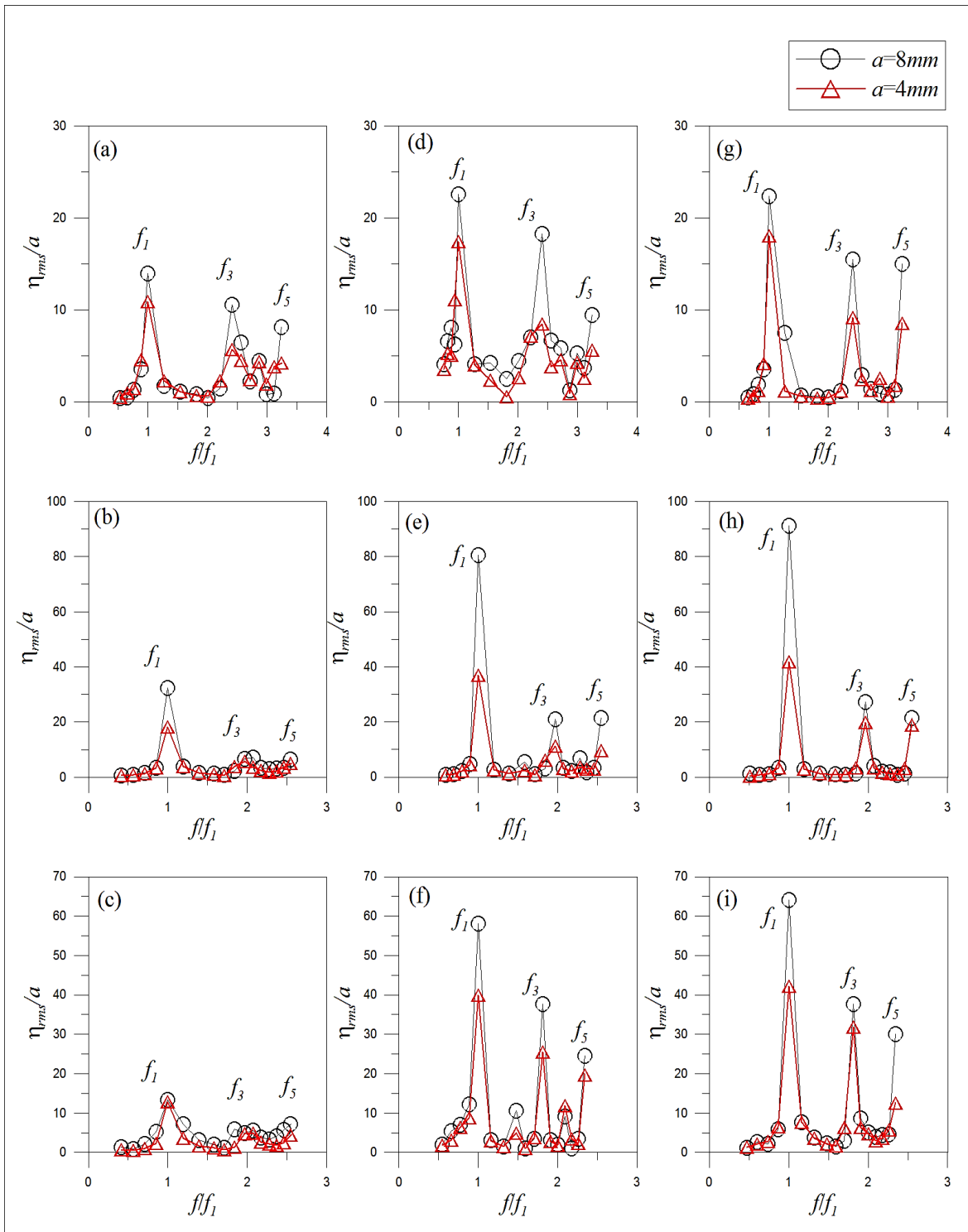


Fig. 4: Variation of η_{rms}/a with different frequency ratio (f/f_1) for, (a) $hs/l = 0.1625$ for 1:86, (b) $hs/l = 0.325$ for 1:86, (c) $hs/l = 0.4785$ for 1:86, (d) $hs/l = 0.1625$ for 1:57 (e) $hs/l = 0.325$ for 1:57, (f) $hs/l = 0.4785$ for 1:57, (g) $hs/l = 0.1625$ for 1:4.

By considering all the scaled tanks, normalized root mean square surface elevation is higher for 50% fill depth ($h_s/l = 0.325$) than the response obtained for 75% ($h_s/l = 0.4875$) and 25% ($h_s/l = 0.1625$) fill depth. Normalized root mean square surface elevation is observed in the decreasing order, $f = f_1 > f = f_3 > f = f_5$ i.e. at odd mode sloshing frequencies. Irrespective of the fill depths, the 1:43 scaled tank shows more sloshing response in comparison with 1:57 and 1:86 scaled tanks.

4.3 Sloshing dynamics

Sloshing dynamics is explained with the help of energy spectrum of sloshing oscillations for three different scaled (i.e., 1:86, 1:57 and 1:43) rectangular tanks at three fill depths (i.e. 25%, 50% and 75%). Odd modes contributions are dominating even modes and hence comparison of sloshing oscillation spectra of $f = f_1, f = f_3$ and $f = f_5$ is projected.

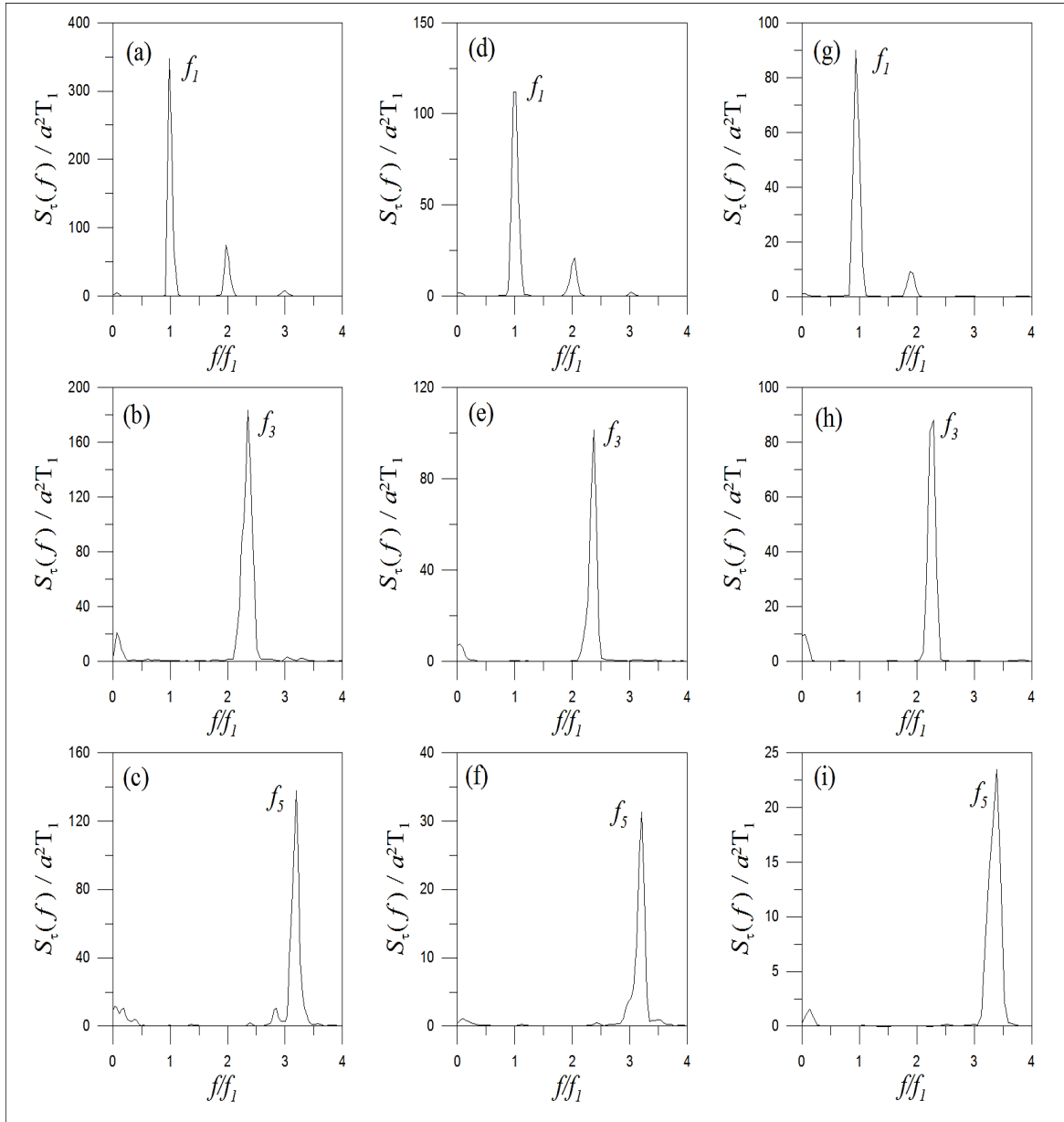


Fig. 5: Sloshing oscillations spectra for $h_s/l = 0.1625$ and excitation amplitude of 4 mm; (a) $f = f_1$ for 1:86 scale tank, (b) $f = f_3$ for 1:86 scale tank, (c) $f = f_5$ for 1:86 scale tank, (d) $f = f_1$ for 1:57 scale tank, (e) $f = f_3$ for 1:57 scale tank, (f) $f = f_5$.

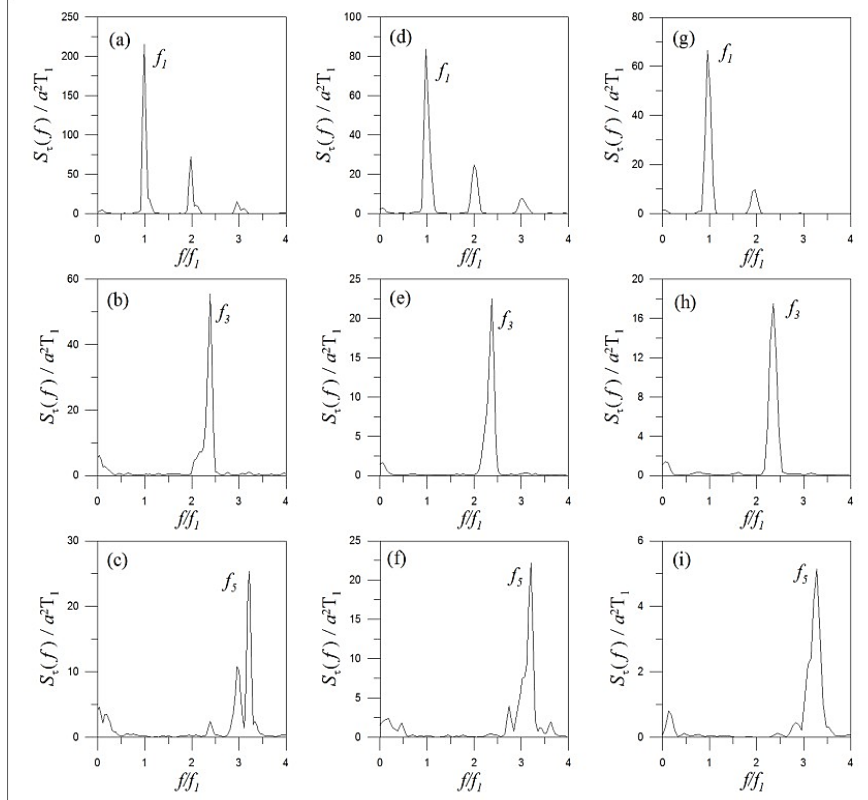


Fig. 6: Slushing oscillations spectra for $h_s/l = 0.325$ and excitation amplitude of 8 mm ; (a) $f = f_1$ for 1:86 scaletank, (b) $f = f_3$ for 1:86 scale tank, (c) $f = f_5$ for 1:86 scale tank, (d) $f = f_1$ for 1:57 scale tank, (e) $f = f_3$ for 1:57 scale tank, (f) $f = f_5$ for 1:57 scale tank, (g) $f = f_1$ for 1:43 scale tank, (h) $f = f_3$ for 1:43 scale tank, (i) $f = f_5$ for 1:43 scale tank.

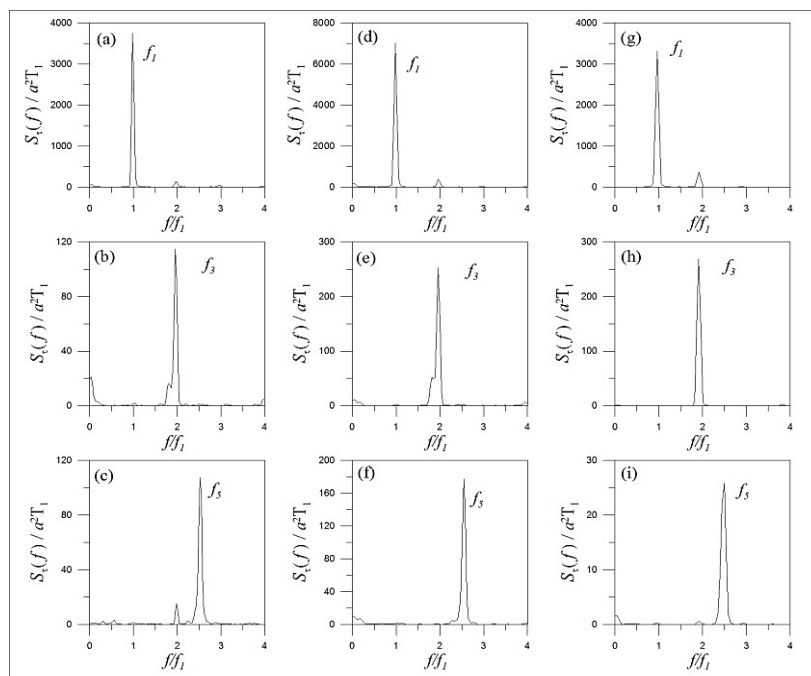


Fig. 7: Slushing oscillations spectra for $h_s/l = 0.325$ and excitation amplitude of 4 mm ; (a) $f = f_1$ for 1:86 scaletank, (b) $f = f_3$ for 1:86 scale tank, (c) $f = f_5$ for 1:86 scale tank, (d) $f = f_1$ for 1:57 scale tank, (e) $f = f_3$ for 1:57 scale tank, (f) $f = f_5$ for 1:57 scale tank, (g) $f = f_1$ for 1:43 scale tank, (h) $f = f_3$ for 1:43 scale tank, (i) $f = f_5$ for 1:43 scale tank.

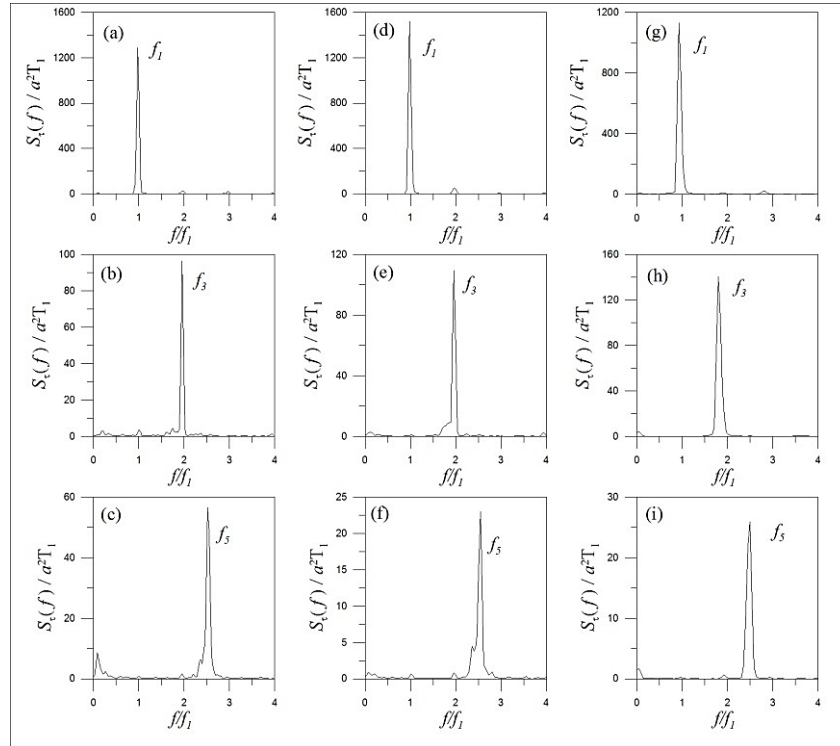


Fig. 8: Sloshing oscillations spectra for $h_s/l = 0.325$ and excitation amplitude of 8 mm ; (a) $f = f_1$ for 1:86 scaletank, (b) $f = f_3$ for 1:86 scale tank, (c) $f = f_5$ for 1:86 scale tank, (d) $f = f_1$ for 1:57 scale tank, (e) $f = f_3$ for 1:57 scale tank, (f) $f = f_5$ for 1:57 scale tank, (g) $f = f_1$ for 1:43 scale tank, (h) $f = f_3$ for 1:43 scale tank, (i) $f = f_5$ for 1:43 scale tank.

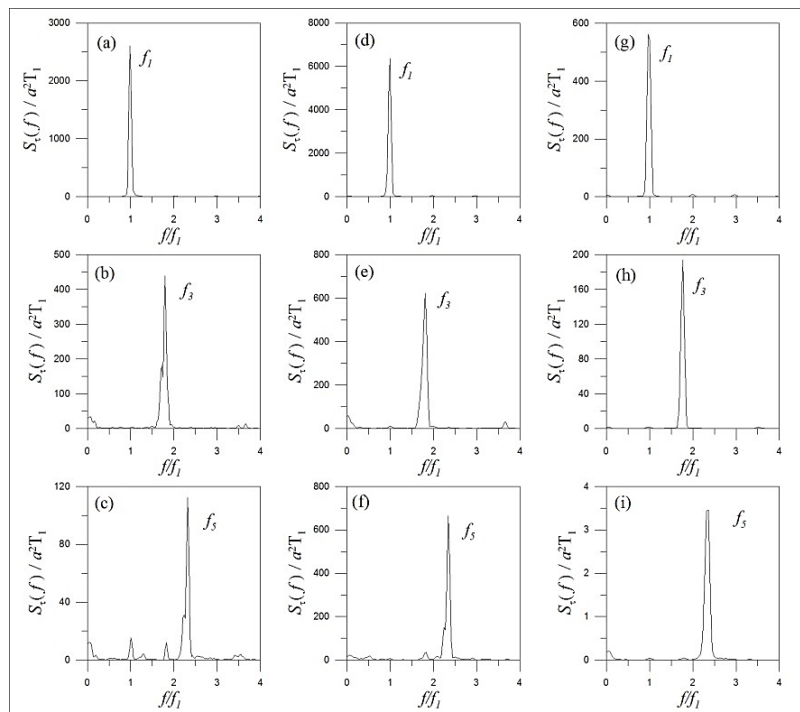


Fig. 9: Sloshing oscillations spectra for $h_s/l = 0.4875$ and excitation amplitude of 4 mm ; (a) $f = f_1$ for 1:86 scaletank, (b) $f = f_3$ for 1:86 scale tank, (c) $f = f_5$ for 1:86 scale tank, (d) $f = f_1$ for 1:57 scale tank, (e) $f = f_3$ for 1:57 scale tank, (f) $f = f_5$ for 1:57 scale tank, (g) $f = f_1$ for 1:43 scale tank, (h) $f = f_3$ for 1:43 scale tank, (i) $f = f_5$ for 1:43 scale tank.

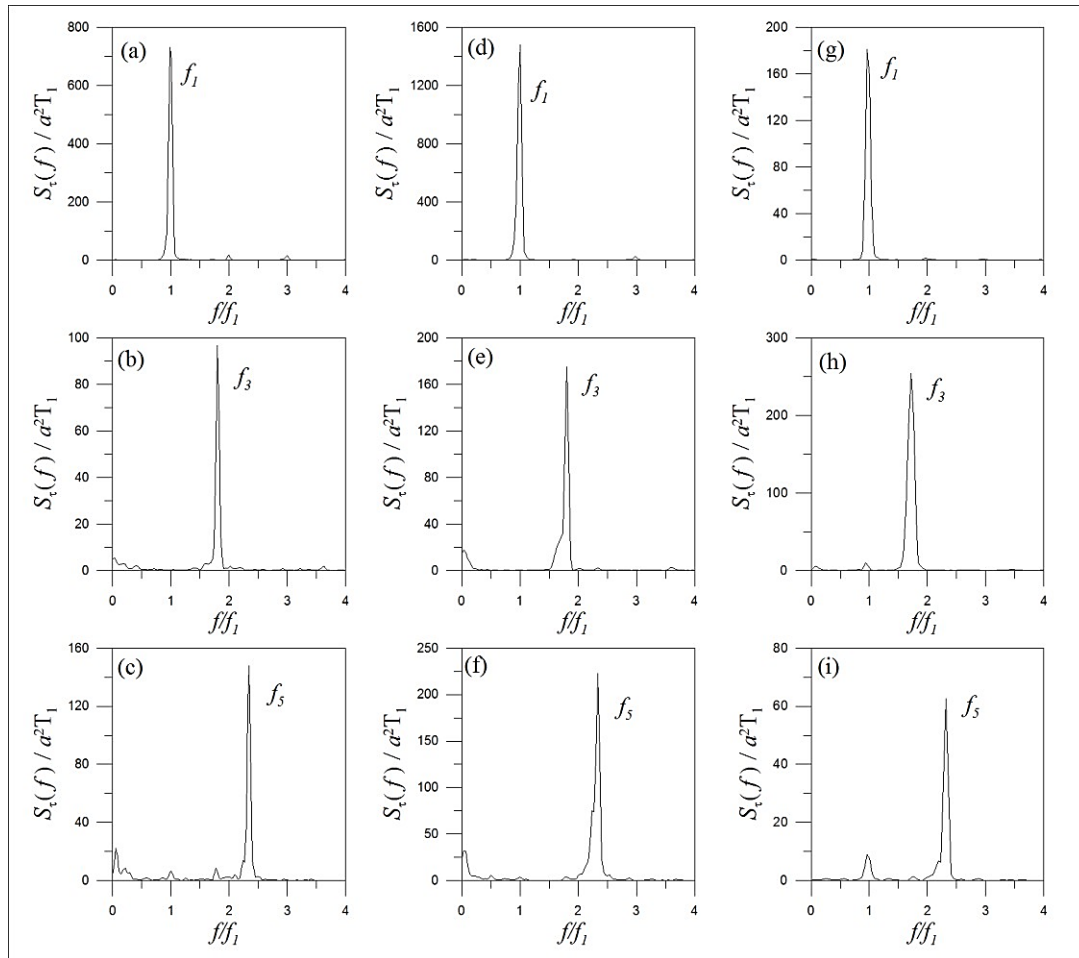


Fig. 10: Slushing oscillations spectra for $h_s/l = 0.4875$ and excitation amplitude of 8 mm; (a) $f = f_1$ for 1:86 scale tank, (b) $f = f_3$ for 1:86 scale tank, (c) $f = f_5$ for 1:86 scale tank, (d) $f = f_1$ for 1:57 scale tank, (e) $f = f_3$ for 1:57 scale tank, (f) $f = f_5$.

From Fig. 5 and Fig. 6, it is seen that there is an increase in energy concentration for 25% fill level ($h_s/l = 0.1625$) as the excitation amplitude increases from 4 mm to 8 mm. The energy concentration decreases with increase in sloshing mode for all the scaled tanks. When comparing the three tanks for $h_s/l = 0.1625$, the energy concentration in the scaled tank is in the order, 1:86 > 1:57 > 1:43. For 50% (Fig. 7 and Fig. 8) and 75% fill levels (Fig. 9 and Fig. 10), when comparing the three scaled tanks, the energy concentration is in the order, 1:57 > 1:86 > 1:43.

4.4 Slushing force

A proper estimation of sloshing force is the significant part of sloshing dynamics and it is the deciding parameter for the design and operation of any moving/fixed object which is carrying/holding a tank with partially filled liquid. Slushing force mainly depends on the frequency of the moving tank, excitation amplitude, geometry, size of sloshing tank and depth of liquid filled in the tank. By considering all above-mentioned factors, the sloshing force is measured using ballast mast concept for the shake table excitation frequencies.

The sloshing force is represented in normalized peak sloshing force (F'_{max}) and normalized average sloshing force (F'_{avg}) of ten largest peaks. The variation in the maximum sloshing force is normalized with the inertial force of liquid as it is treated as solid mass. The normalized peak sloshing force and normalized average sloshing force are given as follows, (Jamie, 2007):

$$F'_{max} = \frac{F_{max}}{m_w (2\pi f)^2 a} \tag{2}$$

$$F'_{avg} = \frac{F_{avg}}{m_w (2\pi f)^2 a} \tag{3}$$

Where, a is excitation amplitude, m_w is total mass of the fluid and f is excitation frequency.

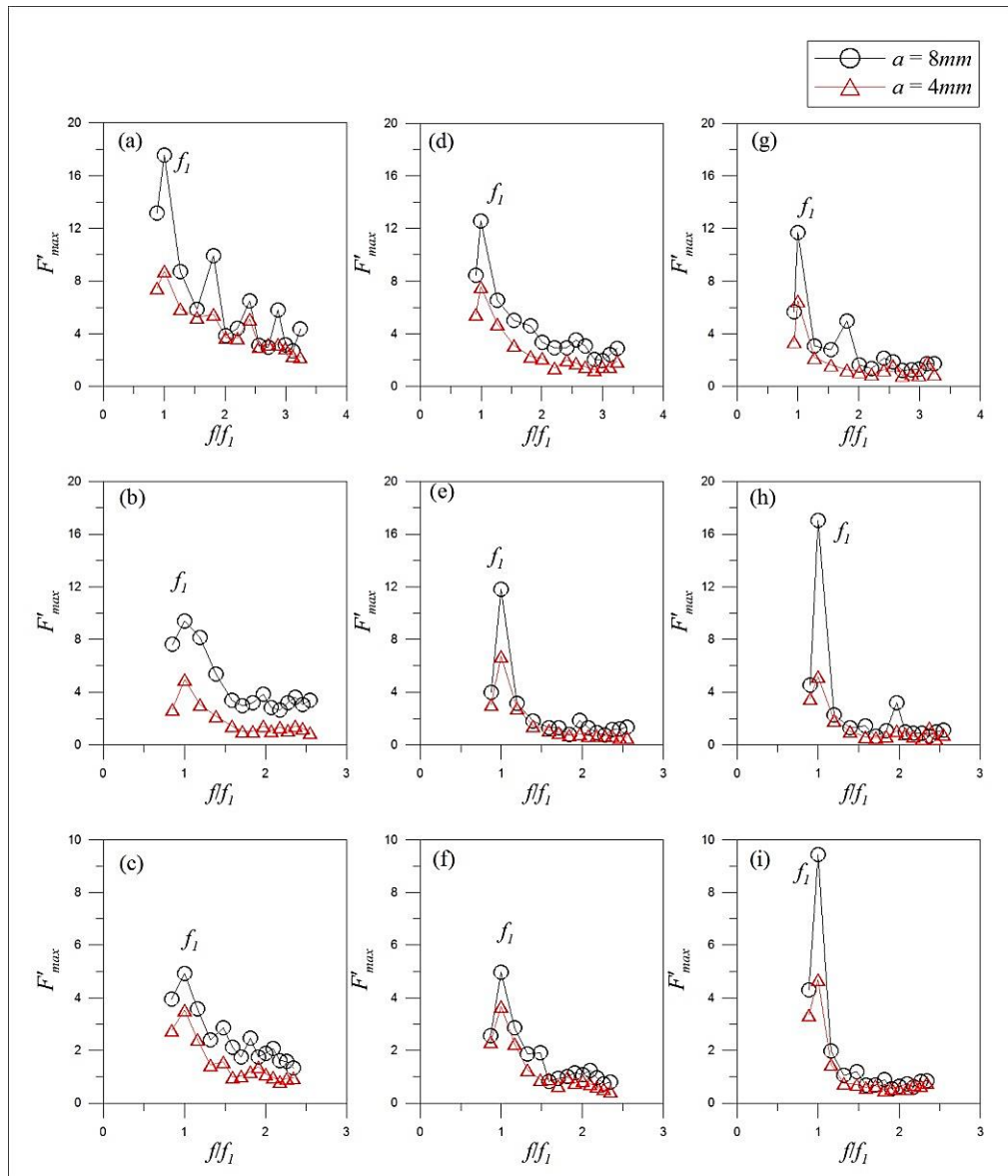


Fig. 11: Comparison of F'_{max} with frequencies ratio (f/f_1) for different scaled tanks (a) $h_s/l = 0.1625$ for 1:86, (b) $h_s/l = 0.325$ for 1:86, (c) $h_s/l = 0.4875$ for 1:86, (d) $h_s/l = 0.1625$ for 1:57, (e) $h_s/l = 0.325$ for 1:57, (f) $h_s/l = 0.4875$ for 1:57, (g) $h_s/l = 0.1625$ for 1:43, (h) $h_s/l = 0.325$ for 1:43 and, (i) $h_s/l = 0.4875$ for 1:43 scale tanks.

As the mode of frequency increases, F'_{max} and F'_{avg} decreases. Sloshing force is observed as high for the first mode of frequency compared with other modes of frequencies. Also, it is evident that the increase in excitation amplitude (from 4 mm to 8 mm) leads to increase the sloshing force. It is understood that 50% (i.e. $h_s/l = 0.325$) water depth experiences higher sloshing force than 25% (i.e. $h_s/l = 0.1625$) and 75% (i.e. $h_s/l = 0.4875$) water depths (Fig. 11 and Fig. 12). Sloshing force is high for smallest tank (1:86 scale tank) at low water depth (25% depth) irrespective of the excitation frequency and amplitude.

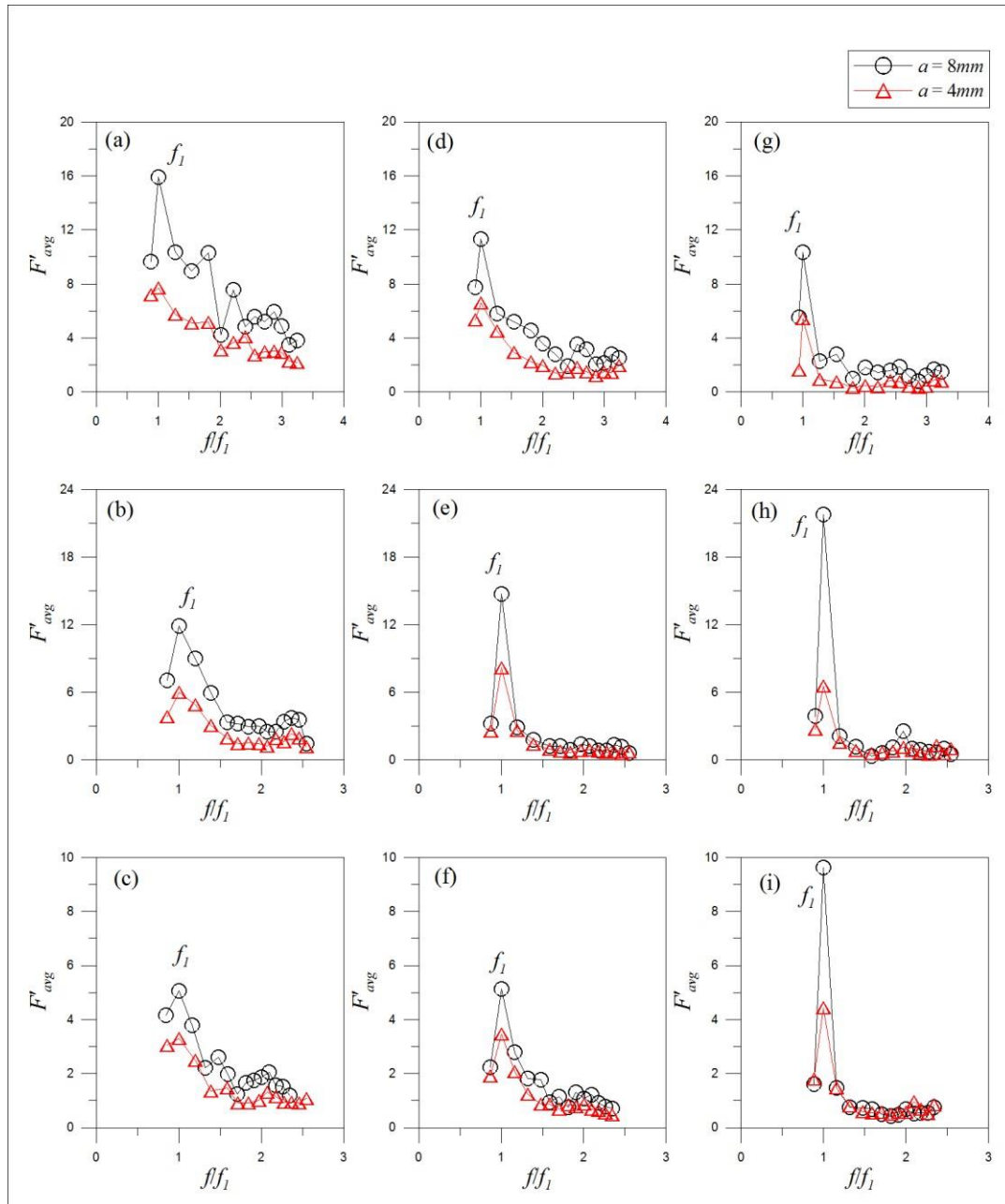


Fig. 12: Comparison of F'_{avg} with frequencies ratio (ff_1) for different scaled tanks (a) $h_s/l = 0.1625$ for 1:86, (b) $h_s/l = 0.325$ for 1:86, (c) $h_s/l = 0.4875$ for 1:86, (d) $h_s/l = 0.1625$ for 1:57, (e) $h_s/l = 0.325$ for 1:57, (f) $h_s/l = 0.4875$ for 1:57, (g) $h_s/l = 0.1625$ for 1:43, (h) $h_s/l = 0.325$ for 1:43 and, (i) $h_s/l = 0.4875$ for 1:43 scale tanks.

When the water depth is more than 50% of tank height (i.e. $h_s/l = 0.325$), then, sloshing force is maximum in the biggest tank (1:43 scale tank) than other scaled sloshing tanks irrespective of the excitation frequency and amplitude. From above observations, it is concluded that proportionate volume of water and tank size would decide the severity of sloshing in the partially filled moving tanks.

4.5 Statistical Study

Experimental results are analyzed statistically by three parameters Weibull distribution to study the probability of exceedance of forces at different fill depths (25%, 50% and 75%) in scaled rectangular tanks (1:86, 1:57 and 1:43). The cumulative probability distribution function (F(x)) of Weibull distribution is represented as,

$$F(x) = 1 - \exp\left(-\left[\frac{(x - \delta)}{\beta}\right]^\gamma\right) \tag{4}$$

Where β is the scale parameter ($\beta > 0$), γ is the shape parameter ($\gamma > 0$) and δ is the location parameter.

Method of moments of curve fitting is used to obtain the above-mentioned parameters. Probability of exceedance (S(x)) for Weibull distribution is represented as,

$$S(x) = 1 - F(x) \tag{5}$$

For all scaled tanks at three fill levels, semi-log graph gives the relation between probability of exceedance and corresponding force.

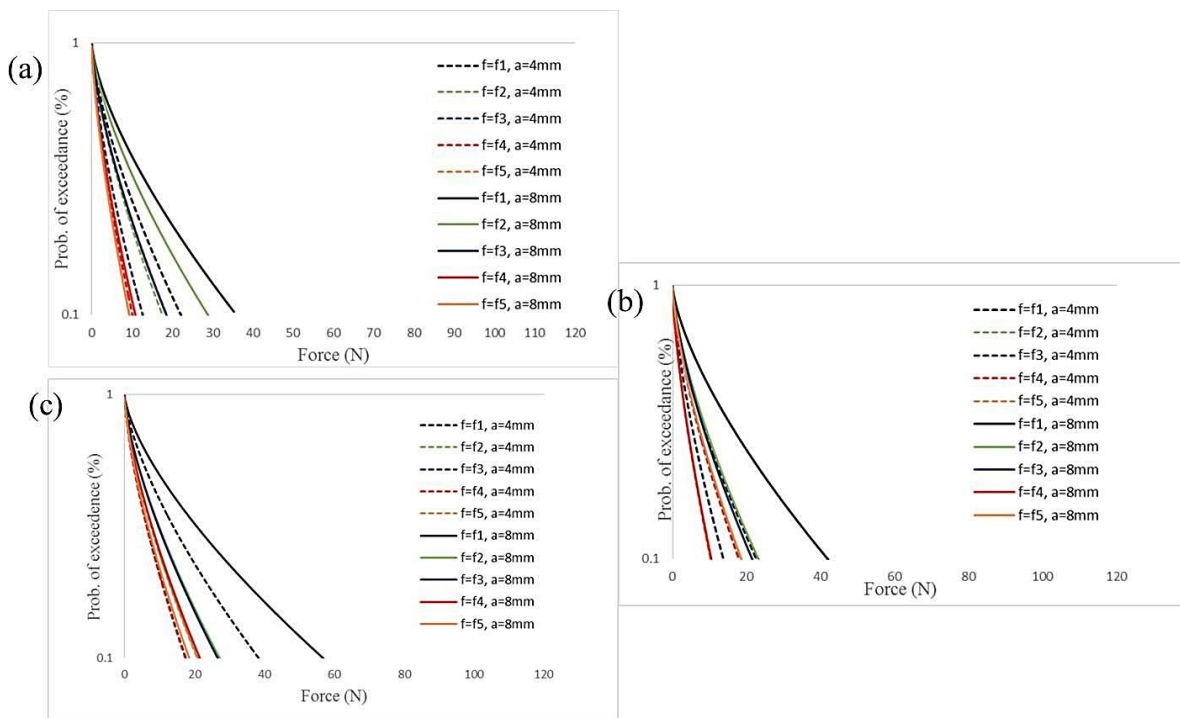


Fig. 13: Probability of exceedance of sloshing forces for $hs/l = 0.1625$ for tank scales, (a) 1:86, (b) 1:57 and, (c) 1:43

A right ward shift in the plots (Fig.13, Fig.14 and Fig.15,) is observed at same probability of exceedance when the excitation amplitude changes from 4 mm to 8 mm. This indicates that the sloshing force experienced by tank containment walls at 4 mm excitation amplitude is lesser compared to that at 8 mm excitation amplitude. At a specific probability of exceedance, among the five sloshing modes considered, the force exerted with first mode of oscillation is found higher compared to other four modes.

For different scaled tanks and at 25% fill depth, the 1:86 scaled tank experiences more force in comparison with 1:57 and 1:43 scaled tanks for both 4 mm and 8 mm excitation amplitudes.

At 50% and 75% fill depths, when tanks excite with 4 mm and 8 mm excitation amplitudes, the 1:43 scaled tank experiences more force compared to 1:57 and 1:86 scaled tank.

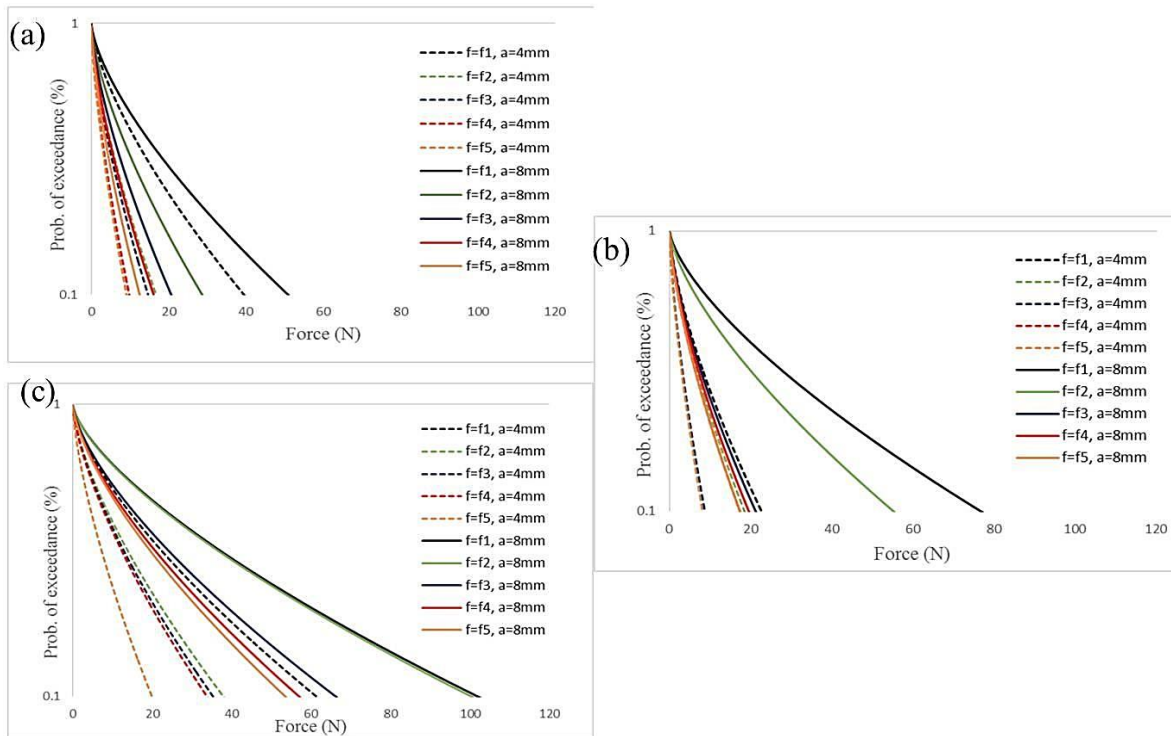


Fig. 14: Probability of exceedance of sloshing forces for $hs/l = 0.325$ for tank scales, (a) 1:86, (b) 1:57 and, (c) 1:43.

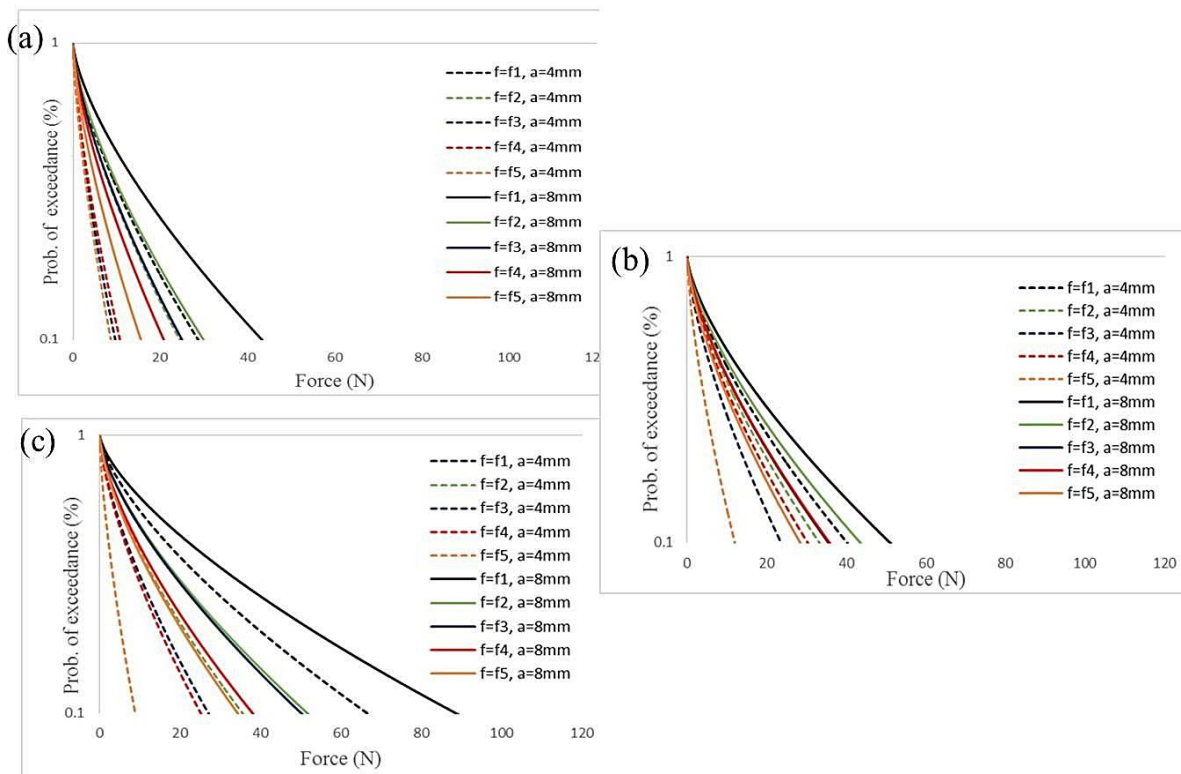


Fig. 15: Probability of exceedance of sloshing forces for $hs/l = 0.4875$ for tank scales, (a) 1:86, (b) 1:57 and, (c) 1:43.

5. Conclusions

The sloshing phenomena and its characteristics in rectangular tanks excited in harmonic sway motion are presented by using shake table experiments. Sloshing behaviour is reported for 1:86, 1:57 and 1:43 scaled tanks with three different fill levels. The key conclusions drawn from the experimental study are given below:

- An experimental setup is designed and devised for measuring sloshing force using ballast mass concept.
- It is concluded from the experiment that the critical mode of sloshing is the first mode of sloshing ($f = f_1$), for all fill depths (25%, 50% and 75%) and scale ratios (1:86, 1:57 and 1:43) considered.
- Normalised maximum free surface (η_{max}) response and normalised root mean square of sloshing elevation (η_{rms}) is observed in the order $f = f_1, f = f_3$ and $f = f_5$ i.e. at odd mode sloshing frequencies.
- Maximum energy (spectral energy) is concentrated for 50% fill level irrespective of the scaled tanks and excitation amplitudes. Energy concentration decreases in an order of 1:86, 1:57 & 1:43 for 25% fill level. For 50% and 75% fill level, energy concentration decreases in the order of 1:57, 1:86 & 1:43 irrespective of excitation amplitudes.
- Sloshing force (F'_{max} and F'_{avg}) is high for 1:86 scaled tank at 25% fill level for all excitation amplitudes and frequencies considered.
- When the fill level is more than 50% ($h_s/l = 0.325$), sloshing force is maximum in the 1:43 scaled tank than other scaled sloshing tanks irrespective of the excitation frequency and amplitude.
- From the experimental observations, it is concluded that proportionate volume of water and tank size decides the severity of sloshing in partially filled moving containers.
- It is also concluded that the 50% filled condition experiences violent sloshing irrespective of the scaling tanks, excitation amplitudes and excitation frequencies.

References

- Armenio, V. and La Rocca, M. (1996): On the analysis of sloshing of water in rectangular containers: Numerical study and experimental validation. *Ocean Engineering*, 23(8), 705–739. [https://doi.org/10.1016/0029-8018\(96\)84409-X](https://doi.org/10.1016/0029-8018(96)84409-X).
- Akyildiz, H. and Unal, E. (2005): Experimental investigation of pressure distribution on a rectangular tank due to the liquid sloshing. *Ocean Engineering*, 32(11–12), 1503–1516. <https://doi.org/10.1016/j.oceaneng.2004.11.006>.
- Benjamin, T. B. and Ursell, H. W. (1954): The stability of the plane free surface of a liquid in a vertical periodic motion. *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 225, 505–515.
- Bogaert, H., Léonard S., Brosset L., and Kaminski M.L. (2010): Sloshing and Scaling: Results from the Sloshel Project. *Proceedings of the Twentieth (2010) International Offshore and Polar Engineering Conference*, 88–97.
- Cariou, A. and Casella, G. (1999): Liquid sloshing in ship tanks: A comparative study of numerical simulation. *Marine Structures*, 12(3), 183–198. [https://doi.org/10.1016/S0951-8339\(99\)00026-X](https://doi.org/10.1016/S0951-8339(99)00026-X).
- Celebi, M. S. and Akyildiz, H. (2002): Nonlinear modeling of liquid sloshing in a moving rectangular tank. *Ocean Engineering*, 29(12), 1527–1553. [https://doi.org/10.1016/S0029-8018\(01\)00085-3](https://doi.org/10.1016/S0029-8018(01)00085-3)
- Chen, Y.G., Djidjeli, K., and Price W.G. (2009): Numerical simulation of liquid sloshing phenomena in partially filled containers. *Computers & Fluids*, 38, 830–842.
- Faltinsen, O. M. (1974): Nonlinear theory of sloshing in rectangular tanks. *Journal of Ship Research*. 18 (4), 224–241.
- Faraday, M. (1831): On the forms and states assumed by fluids in contact with vibrating elastic surfaces. *Phil. Trans. R. Soc. London*, 121, 39–346.
- Frandsen, J. B. (2004): Sloshing motions in excited tanks. *Journal of Computational Physics*, 196(1), 53–87.
- Graczyk, M., Moan, T. and Rognebakke, O. (2006): Probabilistic analysis of characteristic pressure for LNG Tanks. *Journal of Offshore Mechanics and Arctic Engineering*, 128(2), 133. <https://doi.org/10.1115/1.2185128>.
- Hashemi, S., Saadatpour, M. M., and Kianoush, M. R. (2013): Dynamic behavior of flexible rectangular fluid containers. *Thin-Walled Structures*. 66, 23–38.
- Ibrahim, R. A. (2005): *Liquid Sloshing Dynamics - Theory and Applications*, Cambridge University press, New York.
- Jamie, H., (2007): *The Effect of Screen Geometry on the Performance of a Tuned Liquid Damper*. M. Sc. thesis, Mc Master University, Hamilton, Ontario.
- Khezzar, L., Seibi, A. C., and Goharzadeh A. (2009): Water Sloshing in Rectangular Tanks – An Experimental Investigation and Numerical Simulation. *International Journal of Engineering (IJE)*, 3(2), 174–184.
- Kim, Y. (2001): Numerical simulation of sloshing flows with impact load. *Applied Ocean Research*, 23 (1), 53–62. [https://doi.org/10.1016/S0141-1187\(00\)00021-3](https://doi.org/10.1016/S0141-1187(00)00021-3).

- Kim, S., Kim, K., & Kim, Y. (2012): Comparative study on model-scale sloshing tests. *Journal of Marine Science & Technology*, 17, 47–58.
- Kim, S., Kim, Y. and Lee, J. (2017): Comparison of sloshing-induced pressure in different scale tanks. *Ships and offshore structures*, 12(2), 244-261.
- Liu, D. and Lin, P. (2008): A numerical study of three-dimensional liquid sloshing in tanks. *Ocean Engineering*, 36, 202-212.
- Mei-rong, J., Bing, R., Guo-yu, W. and Yong-xue, W. (2014): Laboratory investigation of the hydroelastic effect on liquid sloshing in rectangular tanks. *Journal of hydrodynamics*, 26(5), 751-761.
- Moiseyev, N. N. (1958): On the theory of nonlinear vibrations of a liquid of finite volume. *Applied Mathematics and Mechanics*, (PMM) 22(5), 612-621.
- Nakayama, T. and Washizu, K. (1980): Nonlinear analysis of liquid motion in a container subjected to forced pitching oscillation. *International Journal for Numerical Methods in Engineering*, 15(8), 1207–1220. <https://doi.org/10.1002/nme.1620150808>
- Nasar, T., Sannasiraj, S. A. and Sundar, V. (2008): Sloshing pressure variation in a barge carrying tank. *Ships and Offshore Structures*, 40(6), 185–203. <https://doi.org/10.1016/j.fluiddyn.2008.02.001>
- Nasar, T., Sannasiraj, S. A. and Sundar, V. (2010): Motion responses of barge carrying liquid tank. *Ocean Engineering*, 37(10), 935–946. <https://doi.org/10.1016/j.oceaneng.2010.03.006>
- Nasar, T., Sannasiraj, S. A. and Sundar, V. (2012): Liquid sloshing dynamics in a barge carrying container subjected to random wave excitation. *Journal of Naval Architecture and Marine Eng.*, 9(1), 43-65.
- Panigrahy, P. K., Saha, U. K. and Maity, D. (2009): Experimental studies on sloshing behavior due to horizontal movement of liquids in baffled tanks. *Ocean Engineering*, 36(3– 4), 213–222. <https://doi.org/10.1016/j.oceaneng.2008.11.002>
- Rognebakke, O.R. and Faltinsen, O.M. (2001): Effects of sloshing on ship motions. *Proceedings of the 16th Workshop on Water Waves and Floating Bodies*, Hiroshima, Japan.
- Scheffer, H. J., & Fittschen, T. (1984): Ship response to irregular oblique sea waves. *Symposium on Description and Modelling of Directional seas*, Technical University, Denmark, D-6-1-D-6-11.
- Sames, P. C., Marcouly, D., and Schellin, T. E. (2002): Sloshing in rectangular and cylindrical tanks. *Journal of Ship Research*, 46 (3), 186-200.
- Virella, J. C., Prato, C. A. and Godoy, L. A. (2008): Linear and nonlinear 2D finite element analysis of sloshing modes and pressures in rectangular tanks subject to horizontal harmonic motions. *Journal of Sound and Vibration*, 312(3), 442–460. <https://doi.org/10.1016/j.jsv.2007.07.088>
- Waterhouse, D. D. (1994): Resonant sloshing near critical depth. *Journal of Fluid Mechanics*, 281, 313-318.
- Xue, M. A., Zheng, J. and Lin, P. (2012): Numerical simulation of sloshing phenomena in cubic tank with multiple baffles. *Journal of Applied Mathematics*, Vol.2012. <https://doi.org/10.1155/2012/245702>.