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SIMILARITY SOLUTION OF STAGNATION – SPOT FLOW OF A MICROPOLAR FLUID ABOVE A FLAT EXPONENTIALLY ELONGATING PENETRABLE SURFACE WITH CONCENTRATION AND HEAT PRODUCTION/ABSORPTION

R. Parthiban[1], G. Palani[2*]

[1]Department of Mathematics, Government Arts College for Men, Affiliated to University of Madras, Chennai, India. Email: ramparthiban@gmail.com
[2]Department of Mathematics, Dr. Ambedkar Govt. Arts College, Affiliated to University of Madras, Chennai, India.
[*]Corresponding Author's E-mail: gpalani32@yahoo.co.in

Abstract:

The current study aims to explore the stagnation spot flow of a micropolar liquid about a plain linear exponentially expanding penetrable surface in the incidence of the chemical reaction and in-house heat production/absorption. Through similarity mapping, the mathematical modeling statements are reformed as ODEs and numerical results are found by shooting techniques. The impact of varying physical constants on momentum, micro-rotation, temperature, and concentration is demonstrated through graphs. The computed measures including shear, couple stress, and mass transfer with distinct measures of factors involved in this proposed problem are presented in a table. The presence of heat source increases the temperature of the fluid but has no impact on the velocity, angular velocity, and concentration.

Keywords: Boundary layer, concentration, contracting panel (shrinking sheet), heat spring/drop, micropolar fluid, stagnation spot flow, and suction/injection.

NOMENCLATURE

C_p	specific heat at constant pressure
C_f	shearing stress
f'	dimensionless fluid velocity
g'	dimensionless angular velocity
j	microinertia density
K_c	reaction level factor
l	reference length
L	length
$M_{\hat{x}}$	couple stress
m_w	surface couple stress
n	boundary value factor
$\hat{N}$	micro-rotation
$Nu_{\hat{x}}$	Nusselt number
Pr	Prandtl number
Q	heat source/sink factor
q_w	surface heat flux
$Re_{\hat{x}}$	local Reynolds number
S_c	Schmidt number
$Sh_{\hat{x}}$	Sherwood Number
s	suction/injection factor
$\hat{T}$	fluid temperature
$\hat{T}_¥$	free stream temperature
$\hat{u}$	velocity along the surface
$\hat{u}_e$	free stream velocity
$\hat{U}_1$	wall stretching factor
$\hat{U}_0$	free stream velocity factor
$\hat{v}$	velocity perpendicular to the surface
$\hat{v}_w(\hat{x})$	transpiration velocity
$\hat{x}$	direction along the surface
$\hat{y}$	direction perpendicular to the surface

Greek symbols

g	spiral gradient
d	slip factor
D	micropolar factor
e	shrinking factor
h	boundary layer length
θ	dimensionless temperature
u	kinematic viscosity
k	vortex viscosity
m	dynamic viscosity
r	fluid density
t_w	surface shear stress

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1. Introduction

Eringen's (1966) micropolar fluids are non-Newtonian fluids which consist of randomly oriented particles displaying microrotation and micro inertia phenomena that affects the dynamics of the fluid flow. Micropolar fluids with stretch can provide a mathematical model to represent blood with its deformable substructure like red cells and other particles in the plasma. Other examples of micropolar fluids are liquid crystals, polymeric fluids and fluids containing certain additives. Micropolar fluids play important application in the theory of lubrication, in particular to Journal Bearing and in the theory of porous media. The Eringen's theory has potential application in investigating flow of micro scale fluids and in the exploration of non-Newtonian fluid mechanics. The steady laminar boundary layer flow of an incompressible micropolar fluid past an infinite wedge using similarity solution was analysed by Nath [90] and shown that the couple stress and pressure gradient influence the velocity, microrotation profiles, and the surface shear stress. Attia (2003) studied the heat transfer in a steady laminar boundary layer flow of an incompressible micropolar fluid impinging on a permeable flat plate in a porous medium. The influence of the porosity andthe micropolar factor on both the velocity and heat transfer are deliberated. The heat transfer in a steady laminar stagnation spot boundary layer flow of an incompressible micropolar fluid colliding on a penetrable plane elongating surface with inhouse heat generation/absorption was numerically analyzed by Attia (2006). The influence of the micropolar factor, the stretching velocity, the inhouse heat parameter, and the Prandtl number on the flow and heat transfer were analyzed. Yacob and Ishak (2012) studied the steady 2D laminar stagnation spot flow of a micropolar fluid over an elongating/contracting sheet with convective boundary conditions. The rate of heat transfer and the skin friction at the surface decrease with increasing material parameters.

Some industrial operations include the cooling of continuous slips by drawing and stretching them amidst a quiescent fluid, for example the drawing of glass sheet. The properties of thefinished product depend solely on the rate at which the material is cooled. For electrically conducting fluids which subjected to magnetic field, the rate of cooling can be controlled to achieve the desired product. An application of such a process is in the purification of melted metals from non-metallic inclusions. The investigation of fluid flow over stretching sheets has been a popular research problem in the past few decades (see Mukhopadhyay (2013)). Such study has got many interesting industrial applications in processes like the extrusion of plastic sheets from a die (see Mukhopadhyay and Gorla (2012)), the manufacturing of glass by means of blowing, the cooling of metal plate in a big tub. The lateral velocity caused by the stretching surface disturbs the adjacent fluid, and the surface convective cooling (Mukhopadhyay et al. (2007) and Mahapatra et al. (2007)). Uninterrupted casting and spiraling of fibers are examples of flow involving elongating surface. Crane (1970) has obtained the analytical solution of an incompressible viscous flow over an elastic sheet moving with varying velocity. Heat transferal problem over an elongating surface with varying temperature levels was studied by Carragher and Crane (1982), Gupta and Gupta (1977). The effect of heat transferal over an elongating surface with constant suction was analysed by Chakrabarti and Gupta (1979). The analytical solution of heat transferal problem of a micropolar fluid over non-isothermal penetrable elongating sheet was studied by Hady (1996). The heat transferal problem over non-isothermal penetrable elongating sheet with suction or injection for micro-polar fluid was investigated using numerical methods by Hassanien and Gorla (1990). The analytical resolution of mixed convective current of micro-polar fluid on a nonlinear elongating plane sheet was given by Hayat et al. (2008). The heat transferal features along an elongating sheet with suction or blowing and having variable sheet temperature were studied by Chen and Char (1988). The investigation of Crane (1970) has been broadened for various aspects of flow and heat transfer characteristics for a linearly stretched surface by Datta et al. (1985), Vajravelu (1994), Mukhopadhyay et al. (2012), Ishak et al. (2008). The above problems can even be extended for a non-linearly stretching sheets and the case for a broader nonlinear power-law stretching sheet was analysed by Van Gorder and Vajravelu (2009) and they have established a unique solution along with some numerical results. Van Gorder and Vajravelu's problem with slip at the boundary was analysed by Mukhopadhyay (2013). The heat transferal features in an electricity conducting fluid above an elongating pane with variable temperature and heat production or immersion was analysed by Vajravelu and Rollins (1992). The mixed convective slip flow of a micro-polar liquid on a hot elongating surface having constant magnetic flux and heat production/immersion was investigated by Mahmoud and Waheed (2012).

Imposing surface suction of fluid with dominating viscous effects can reduce both the hydrodynamic and the thermal thickness of the boundary layer. This results in reducing both the fluid velocity and temperature. In a flow surface the skin friction factor is increased with increasing the suction for a given Reynolds number.

Boundary layer suction via slots near the tailing edge can increase lift and decrease drag of aerofoils operating at large incidence angles. Moreover, suction is used in vapor removal, in film boiling or nucleate boiling, fluid removal in single-phase flow through a porous heated surface. The thermal boundary layer thickness upsurges due to the blowing which results in reduction of heat transfer rate. Thus, to increase the maximum lift by maintaining laminar flow, to avoid transition, delaying separation and to reduce skin friction a suitable magnitude of suction or blowing can be used. The transport properties in the boundary layer region of an exponential elongating sheet were established by Magyari and Keller (1999) and for an exponentially porous elongating surface it was established by Elbashbeshy (2001). The thermal radiation influence in the boundary layer region formed on an exponentially elongating surface was analysed by Sajid and Hayat (2008) using HAM. The numerical study of the thermal radiation influence in the boundary layer region formed on an exponentially elongating sheet investigated by Bidin and Nazar (2009). The mixed convective flow on an exponentially elongating surface having magnetic field influence was investigated by Pal (2010). The magnetic effect on flow properties on an exponentially elongating surface was investigated by Ishak (2011). The influence of joule heating on steady two-dimensional flow of an incompressible micropolar fluid over a flat elongating/contracting sheet with the effects of first and second order slips and dissipative heat energy are analysed by Baitharu et al. (2021) using the Runge-Kutta method of fourth order with shooting technique. They found that the thermal buoyancy overpowers the inertia force and the second order slip is favorable for flow stability in both elongating and contracting surface.

In stagnation spot flow there is a spot on the surface in the flow domain with adjacent velocity being zero and having highest pressure as well as heat transfer. Stagnation-point flow has applications in production processes such as extracting polymer, production of paper, production of glass fiber and many others. The exact solution of stagnation spot flow on firm flat plane as well as axisymmetric plane were analysed by Hiemenz and Homann and these results can be found in the text book by Schlichting (1960). The equivalent temperature dispersal problem was analysed by Goldstein (1938). Stagnation spot flow on axially-symmetric plane for power law liquids was explored by Maiti (1965). Stagnation spot flow for power law liquids of non-Newtonian type investigated by Koneru and Manohar (1968). Heat transferal in stagnation spot flow near an elongating sheet was explored by Mahapatra and Gupta (2002). The viscous flow nearby a stagnation spot with the external flow having constant vorticity was analysed by Stuart (1959). The three-dimensional stagnation spot flow towards a moving plate with injection was studied by Paul (1974, 1976) and obtained the characteristics of the boundary layer. Stagnation spot flow near an elongating pane for a micropolar fluid was analysed by Nazar et al. (2004). The thermal radiation and viscous dissipation effects on an unsteady MHD mixed convection flow of a viscous incompressible fluid past a vertical porous plate, in the presence of variable wall heat flux and heat generation/absorption was analysed by Pandikunta et al. (2018) using perturbation technique and then solved numerically by using the shooting method. It is assumed that the free stream velocity follows an exponentially increasing or decreasing small perturbation law. The effects of the various parameters on the translation velocity, microrotation, temperature and the skin friction coefficient, couple stress coefficient at the wall are established. The heat and mass transfer of a steady laminar boundary layer flow of an electrically conducting fluid of second grade in a porous medium subject to a uniform magnetic field past a semi-infinite stretching sheet with power law surface temperature or power law surface heat flux was analysed by Baitharu et al. (2020) and it is found that the temperature distribution decreases with the increase in thermal radiation parameter in case of PST and PHF and he rate of mass transfer at the solid surface increases in the presence of magnetic field and decreases with heavier diffusing species. The heat and mass transfer of a steady laminar boundary layer flow of an electrically conducting fluid of second grade in a porous medium subject to a uniform magnetic field past a semi-infinite stretching sheet with power law surface temperature or power law surface heat flux was analysed by Baitharu et al. (2020). The variations in fluid velocity, fluid temperature and species concentration are presented graphically and the numerical values of skin friction, Nusselt number and Sherwood number are presented in tabular form for various values of flow parameters.

In several situations, the mass transfer of species happens together with chemical reactions. As chemical species be possibly produced or spent in a chemicalreaction, the chemical species flux need not have to be preservedin a volume element. The purpose of mass flux computation is to understand, design and control of system like vaporisation of water from a pond,in the distillation of alcohol, and the purification of blood in kidneys and liver. In industrial engineering, mass transfer operations include chemical components separation in a distillation process, and absorbers such as scrubbers or stripping. Coupled heat and mass transfer play role in industrial cooling towers where hot water and air flow together resulting in waterbeing cooled by expelling some of water vapour. First order compound reaction near a flat plate investigated by Chambre and Young (1958). First order identical compound reaction for unsteady flow over an upright plate having constant mass and heat transmission was investigated by Dekha et al. (1994). The chemical reaction on an unstable uphill motion over an isothermal plate was studied by Muthcumaraswamy and Ganesan (2001). The unstable convective mass and heat transmission over a semi-infinite upright porous non-stationary plate with heat production/immersion and chemical reaction was studied by Chamkha (2003). The effect of heat source and chemical reaction on MHD flow past a vertical plate subject to a constant motion with variable temperature and concentration was analysed by Rout et al. (2016) using Laplace transformation technique. The effects of various flow parameters on the flow dynamics are discussed. The steady two-dimensional MHD boundary layer flow of a Casson fluid over an exponentially stretching surface in the presence of thermal radiation and chemical reaction with stretching velocity, wall temperature and wall concentration are considered to be in the exponential forms was studied by Reddy (2016) by using similarity transformations and solving numerically using Runge-Kutta based shooting technique. The influence of various parameters on the fluid velocity, temperature, concentration, wall skin friction coefficient, the heat transfer coefficient and the Sherwood number have been computed and the results are presented graphically and discussed quantitatively. The heat and mass transfer in chemically reacting radiative Casson fluid flow over a slandering/flat stretching sheet in a slip flow regime with aligned magnetic field under the influence of non-uniform heat source/sink was analysed by Reddy et al. (2017) by using similarity transformations and solving numerically using Runge-Kutta based shooting technique. Also, the effects of various physical parameters on velocity, temperature and concentration fields presented using graphs.

Inspired by the above works the authors have taken the present study of investigating stagnation spot flow of an incompressible micropolar liquid on an exponentially elongating penetrable surface in the incidence of chemical reaction with internal heat production/immersion. Through similarity mapping, the mathematical modeling statements are reformed as ODE's and numerical results are found by shooting techniques with Runge-Kutta algorithm.

2. Details of the Flow Problem

The model is associated with 2-D steady stagnation spot micropolar liquid flow about a plain linear exponentially expanding penetrable surface in the incidence of chemical reaction and in-house heat production/immersion. The coordinate system of the flow is explained in Fig.1. The adjacent surface velocity is assumed to be $\hat{u}_w(\hat{x}) = \hat{U}_1 exp(\hat{x} / L)$ and the exterior flow velocity is $\hat{u}_e(\hat{x}) = \hat{U}_0 exp(\hat{x} / L)$ where $\hat{U}_0 > 0, \hat{U}_1 > 0$ constants. Here $\hat{x}$ is the coordinate taken along the flat surface. $\hat{C}_{¥}$,$\hat{T}_{¥}$ are respectively the concentration and temperature of the free stream. The impacts of microstructure, viscous dispersion as well as radiation are presumed to be naught also reduces the total spin $\hat{N}$ to micro rotation.

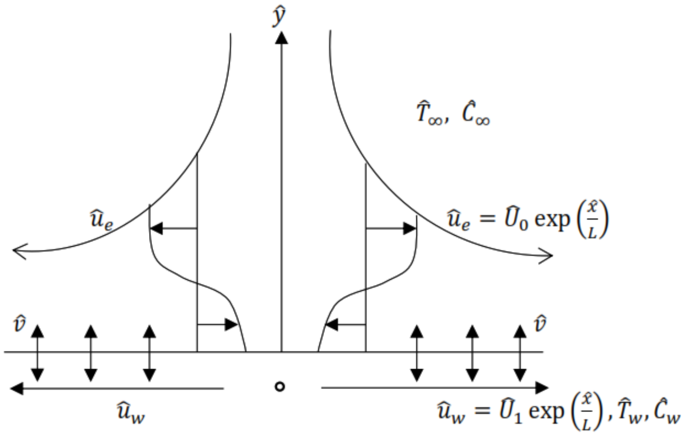

Fig. 1: Physical Model

With these Conjectures, the continuity, velocity, micro-rotation, concentration as well as energy equations are as follows:

$$\frac{\partial \hat{u}}{\partial \hat{x}}+\frac{\partial \hat{v}}{\partial \hat{y}}=0 \tag{1}$$

$$\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{u}}{\partial \hat{y}}=\hat{u}_e\frac{\partial \hat{u}_e}{\partial \hat{x}}+\left[\frac{\mu+\kappa}{\rho}\right]\frac{\partial^2 \hat{u}}{\partial \hat{y}^2}+\frac{\kappa}{\rho}\frac{\partial \hat{N}}{\partial \hat{y}} \tag{2}$$

$$\rho j\left[\hat{u}\frac{\partial \hat{N}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{N}}{\partial \hat{y}}\right]=\gamma\frac{\partial^2 \hat{N}}{\partial \hat{y}^2}-\kappa\left[2\hat{N}+\frac{\partial \hat{u}}{\partial \hat{y}}\right] \tag{3}$$

$$\hat{u}\frac{\partial \hat{T}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{T}}{\partial \hat{y}}=\frac{k}{\rho C_p}\frac{\partial^2 \hat{T}}{\partial \hat{y}^2}+\frac{Q_0}{\rho C_p}\left[\hat{T}-\hat{T}_\infty\right] \tag{4}$$

$$\hat{u}\frac{\partial \hat{C}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{C}}{\partial \hat{y}}=D\frac{\partial^2 \hat{C}}{\partial \hat{y}^2}-k_c\left[\hat{C}-\hat{C}_\infty\right] \tag{5}$$

$$\gamma=\left[\mu+\frac{\kappa}{2}\right]l=\mu\left[1+\frac{\Delta}{2}\right]l \tag{6}$$

Now $\hat{u},\ \hat{v}$ represents velocity elements in $\hat{x},\ \hat{y}$ directions respectively; κ-vortex viscosity; $\hat{N}$-micro-rotation; j-micro-inertia density; Q_0-volumetric heat generation / absorption; γ-spin gradient viscosity; $\Delta=\kappa/\mu$-micropolar factor and $l=\upsilon/\hat{U}_1$-reference length.

Related boundary settings:

$$\hat{u}=\hat{u}_w(\hat{x})=\hat{U}_1\exp(\hat{x}/L),\quad \hat{v}=\hat{v}_w(\hat{x}),\ \hat{T}=\hat{T}_w,\quad \hat{C}=\hat{C}_w,\ \hat{N}=-n\frac{\partial \hat{u}}{\partial \hat{y}}\ \ at\ \hat{y}=0 \tag{7}$$

$$\hat{u}=\hat{u}_e(\hat{x})=\hat{U}_0\exp(\hat{x}/L),\ \ \hat{N}\to 0,\ \hat{T}\to\hat{T}_\infty,\ \hat{C}\to\hat{C}_\infty\ \ as\ \hat{y}\to\infty \tag{8}$$

Here L-length, $\hat{v}_w(\hat{x})$ -transpiration velocity at the wall, $n(0\le n\le 1)$-boundary value factor. When $n=0$ *i.e.* $\hat{N}=0$ shows that the non-spiral state. (The micro-components in the concentrated flow particles could not rotate near the wall) and $n=1/2$ represents the anti-symmetric part of the stress tensor disappears and this implies a feeble concentration. Also $n=1$ exhibits the turbulent flows. We take the transpiration velocity at the wall as

$$\hat{v}=\hat{v}_w(\hat{x})=-\sqrt{\frac{\hat{U}_1\upsilon}{2L}}\exp\left[\frac{\hat{x}}{2L}\right]s$$

Where $s>0$ refers mass suction and $s<0$ refers mass injection.

Setforth the following similarity variables:

$$\eta=\sqrt{\frac{\hat{U}_1}{2\upsilon L}}exp\left[\frac{\hat{x}}{2L}\right]\hat{y},\ \ \psi=\sqrt{2\upsilon L\hat{U}_1}exp\left[\frac{\hat{x}}{2L}\right]f(\eta),\ \ \hat{N}=\sqrt{\frac{\hat{U}_1^{\,3}}{2\upsilon L}}exp\left[\frac{\hat{x}}{2L}\right]g(\eta),$$

$$\theta(\eta)=\frac{\hat{T}-\hat{T}_\infty}{\hat{T}_w-\hat{T}_\infty},\quad \varphi(\eta)=\frac{\hat{C}-\hat{C}_\infty}{\hat{C}_w-\hat{C}_\infty} \tag{9}$$

Equations (2.1) - (2.5) now reduce to:

$$\left[1+\Delta\right]f'''+ff''-2f'^2+\Delta g'+2\varepsilon^2=0 \tag{10}$$

$$\lambda g''+fg'-3f'g-2B\Delta\left[2g+f''\right]=0 \tag{11}$$

$$\theta''+Pr\,f\theta'+Pr\,Q\,\theta=0 \tag{12}$$

and $\varphi''+S_c f\varphi'-K_c S_c\,\varphi=0$ (13)

Conditions (7) and (8) become

$$f(0)=s,\ f'(0)=1,\ g(0)=-nf''(0),\ \theta(0)=1\ \ \&\ \ \varphi(0)=1 \text{ at } \eta=0 \tag{14}$$

$$f'(\eta) \to \varepsilon = \frac{\hat{U}_0}{\hat{U}_1},\ g(\eta) \to 0,\ \theta(\eta) \to 0\ \ \&\ \ \varphi(\eta) \to 0 \text{ as } \eta \to \infty \tag{15}$$

Here s -suction/injection factor, $Pr = \frac{\mu C_p}{k}$ -Prandtl number, $Q = \frac{2LQ_0}{\hat{U}_w \rho C_p}$ -heat increase/drop factor, $\varepsilon = \frac{\hat{U}_0}{\hat{U}_1}$ -velocity proportion factor, $S_c = \frac{\upsilon}{D}$ -Schmidt number, $K_c = \frac{2Lk_c}{\hat{U}_w}$ -reaction level factor and $\lambda = \frac{\upsilon}{\mu j}$ & $B = \frac{\upsilon L}{j\hat{U}_w}$ are dimensionless constants.

The physical factors shearing stress C_f ; $Sh_{\hat{x}}$ the Sherwood Number; Nusselt number $Nu_{\hat{x}}$ are given as:

$$C_f = \frac{2\tau_w}{\rho \hat{u}_w^2}, \qquad Sh_{\hat{x}} = \frac{\hat{x}M_w}{D\left[\hat{C}_w - \hat{C}_\infty\right]}, \qquad N\hat{u}_{\hat{x}} = \frac{\hat{x}q_w}{k\left[\hat{T}_\infty - \hat{T}_m\right]} \tag{16}$$

Surface shear stress t_w ; surface couple stress M_w ; surface heat q_w are given by

$$\tau_w = \left[[\mu + \kappa]\frac{\partial \hat{u}}{\partial \hat{y}} + \kappa \hat{N} \right]_{\hat{y}=0}, \qquad M_w = -D\left[\frac{\partial \hat{C}}{\partial \hat{y}}\right]_{\hat{y}=0}, \qquad q_w = \left[-k\frac{\partial \hat{T}}{\partial \hat{y}}\right]_{\hat{y}=0} \tag{17}$$

From equation (2.17) we get

$$Re_{\hat{x}}^{1/2} C_f = \sqrt{2}[1+(1-n)\Delta]f''(0), \qquad Re_{\hat{x}}^{-1/2} Sh_{\hat{x}} = -\sqrt{\frac{\hat{x}}{2L}}\varphi'(0), \qquad Re_{\hat{x}}^{-1/2} Nu_{\hat{x}} = -\sqrt{\frac{\hat{x}}{2L}}\theta'(0) \tag{18}$$

3. Numerical procedure for solution

The method of solution involves similarity transformation which reduces the partial differential equations into a non-linear ordinary differential equation. These non-linear ordinary differential equations have been solved by applying Runge-Kutta-Fehlberg forth-fifth order method (RKF45 Method) with the help of Shooting technique. This method is an adaptive, in the sense that it uses two separate calculations to determine if the step size is sufficiently small and we can modify the size of the step as appropriate.

$$f' = p, \quad p' = q \quad \& \quad q' = [1/(1+D)][-fq + 2p^2 - Dh - 2e^2] \tag{19}$$

$$g' = h \ \&\ h' = [1/l][-fh + 3pg + 2BD(2g+q)] \tag{20}$$

$$\theta' = z \ \&\ z' = -Pr\, fz - Pr\, Q\theta \tag{21}$$

$$f' = w \ \&\ w' = K_c S_c f - S_c f w \tag{22}$$

and the boundary settings

$$f(0) = s, \quad p(0) = 1, \quad g(0) = -nq(0), \quad \theta(0) = 1, \quad f(0) = 1 \tag{23}$$

So, we solve a group of nine first order differential equations (19) to (22). To solve them as an IVP, in addition to the four initial conditions (23) we need four initial values $q(0), h(0), z(0)$ & $w(0)$ i.e., $f''(0), g'(0), \theta'(0)$ & $f'(0)$ which are unknown. So initial estimates for $f''(0), g'(0), \theta'(0)$ & $f'(0)$ are used to find the numerical result. For the computation purpose it is compulsory to select an appropriate finite value, say η_∞ for $\eta \to \infty$ which guaranty the convergence. Then the computed values of $f'(0), g(0), \theta(0)$ & $f(0)$ at η_∞ are compared with the required values at $\eta = \eta_\infty$. If there is lack of accuracy in the numerical result then the guess values of $f''(0), g'(0), \theta'(0)$ & $f'(0)$ are revised to proceed as above to find a better output results. The process is continued until desired results are obtained.

4. Results and Discussion

The impact of Δ - micropolar factor, $\varepsilon = \hat{U}_0 / \hat{U}_1$ - velocity proportion factor and s - suction /injection factor on the boundary layer velocity is shown in Figs. 2 - 4. From Figs. 2 and 3 there are two different kinds of velocity boundary layer structures near the sheet have formed depending upon the value of the velocity ratio parameter $\varepsilon = \hat{U}_0 / \hat{U}_1$. It is apparent that the velocity increases as velocity ratio parameter ε increases. Also, the velocity boundary layer thickness increases as the micropolar parameter Δ increases as long as the free stream velocity is less than the stretching wall velocity (that is $U_e \le U_w$) but otherwise (that is $U_e > U_w$) a reverse trend is seen. From Fig. 4 it is clear that the velocity is small for large values of micropolar parameter Δ and this is due to the fact that in micropolar fluid flow the microrotation affects the flow field. Also, injection increases the velocity as compared to suction. Physically, $s > 0$ results in a decrease in the boundary layer thickness and so the velocity reduces. The parameter Q has no impact on the velocity as Q is uncoupled in the momentum equation.

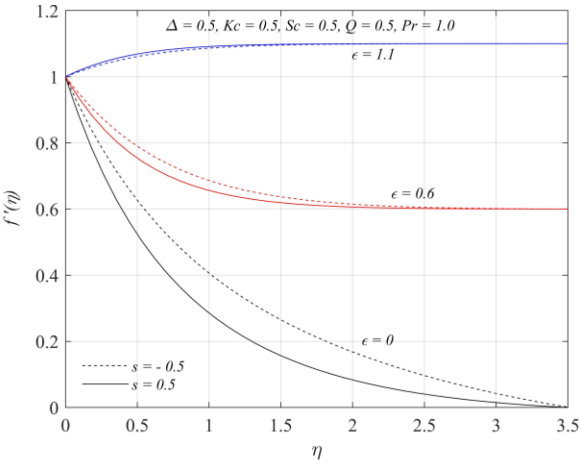

Fig. 2: Result for s and ϵ with $f'(\eta)$

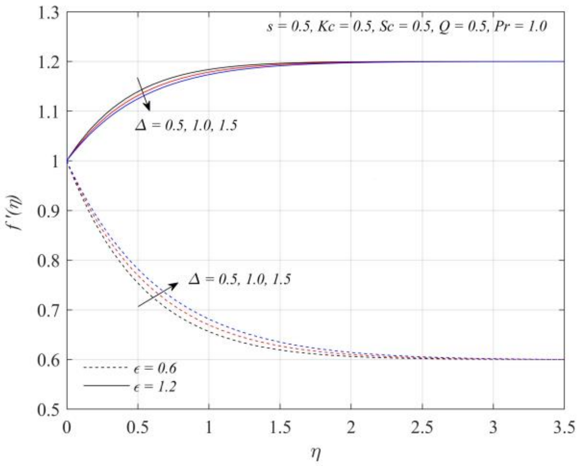

Fig. 3: Result for ϵ and Δ with $f'(\eta)$

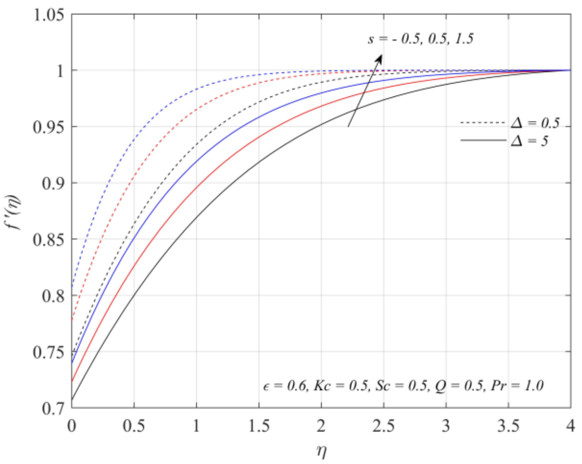

Fig. 4: Result for Δ and s with $f'(\eta)$

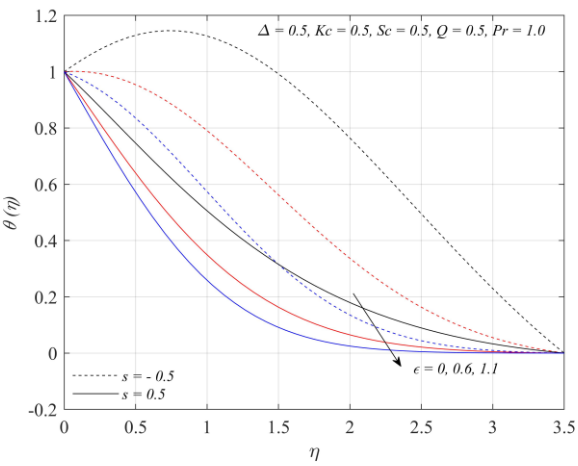

Fig. 5: Result for s and ϵ with $\theta(\eta)$

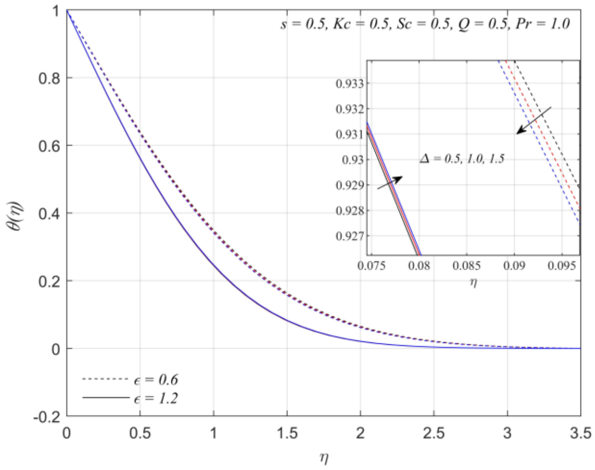

Fig. 6: Result for ϵ and Δ with $\theta(\eta)$

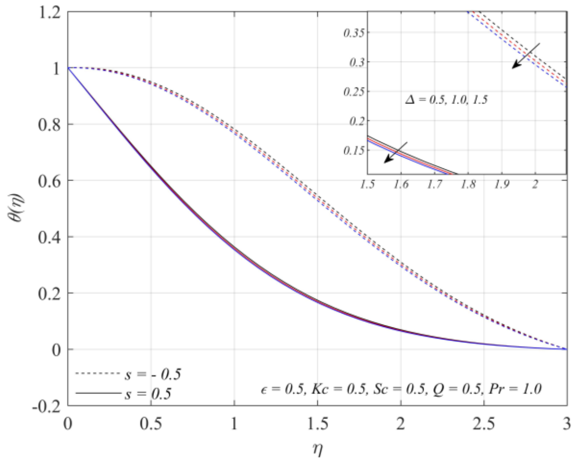

Fig. 7: Result for s and Δ with $\theta(\eta)$

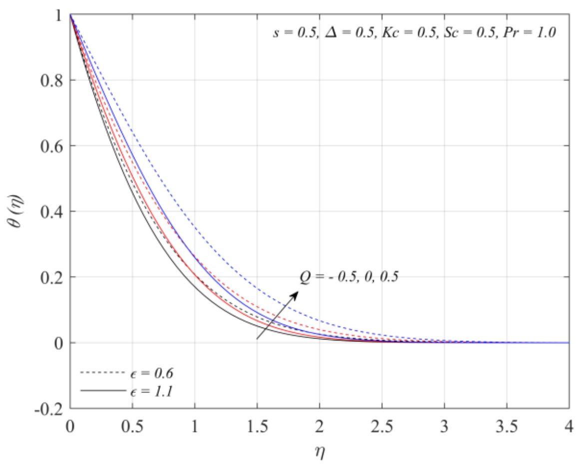

Fig. 8: Result for ϵ and Q with $\theta(\eta)$

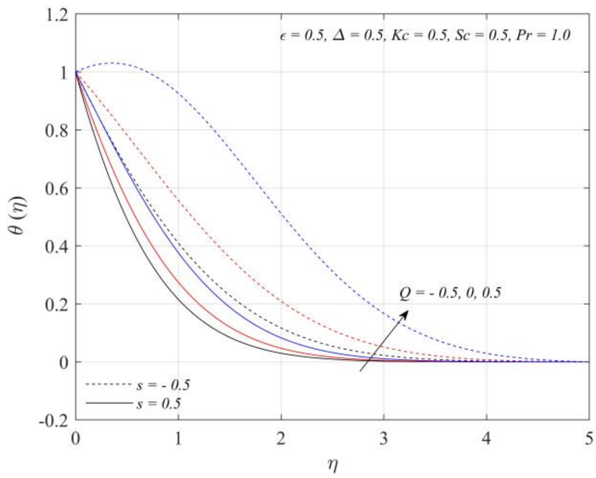

Fig. 9: Result for s and Q with $\theta(\eta)$

The impact of D , ε , s , Q - heat source or sink parameter, and Pr the Prandtl Number on the boundary layer temperature is shown in Figs. 5 - 10. Fig. 5 and Fig. 7 suggests that the temperature boundary layer thickness decreases as e raises and further the temperature for the injection case is higher than for suction case. From Fig. 6 it is evident that the temperature decreases marginally for increasing values of D when the free stream velocity is less than the elongating wall velocity but otherwise, a reverse trend is seen. From Fig. 8 the temperature profile increases as Q increases. Fig. 9 suggests that the temperature is less when $s > 0$ (for suction) compared to the case $s < 0$ (for blowing). We also observe that the temperature is high when $Q > 0$ (for heat source) when compared to the case $Q < 0$ (for heat sink). Also, as the Prandtl number Pr increases the temperature decreases is evident from Fig. 10. Physically, this means that the fluids with smaller Pr have higher thermal conductivity and larger thermal boundary layer and hence heat diffuses quicker from the sheet.

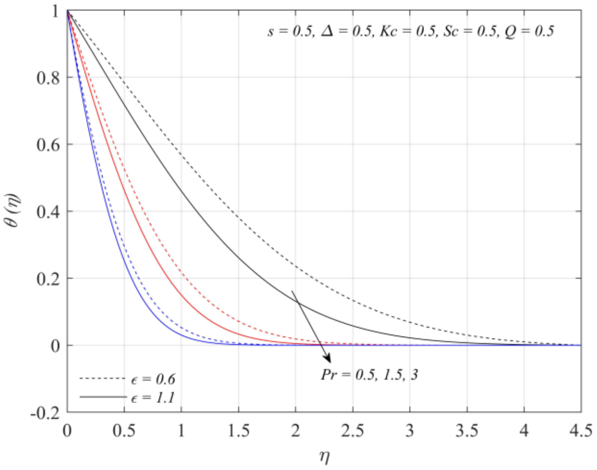

Fig. 10: Result for ϵ and Pr with $\theta(\eta)$

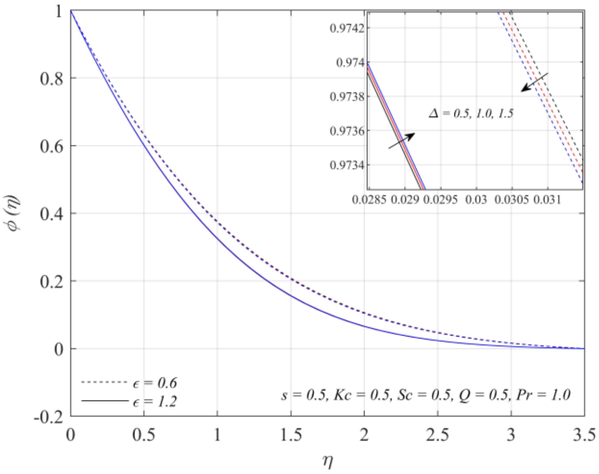

Fig. 11: Result for ϵ and Δ with $\phi(\eta)$

The impact of D , ε , s , K_c and S_c on the concentration profile is shown in Figs. 11 - 17. Fig. 11 suggests that the concentration drops marginally as ε raises and the free stream velocity is less than the elongating wall velocity but otherwise, a reverse trend is seen. From Fig. 12 the concentration in the fluid is less for the suction than for injection case and in both cases the concentration drops marginally as D increases. Fig. 13 the concentration declines as the velocity ration parameter $\varepsilon = \hat{U}_0 / \hat{U}_1$ increases. The heat source /sink parameter Q has no impact on concentration as Q is uncoupled in the concentration equation. From Fig. 14 & Fig. 15 it is apparent that the concentration trace declines for growing value of K_c , S_c and the rate is high for suction $(s > 0)$ than for blowing $(s < 0)$. For S_c the rate of decline is more. Fig. 16 & Fig. 17 it is apparent that the concentration trace declines for growing value of K_c , S_c and the rate is high for $\varepsilon = \hat{U}_0 / \hat{U}_1 < 1$ than for $\varepsilon = \hat{U}_0 / \hat{U}_1 > 1$. For S_c the rate of decline is more.

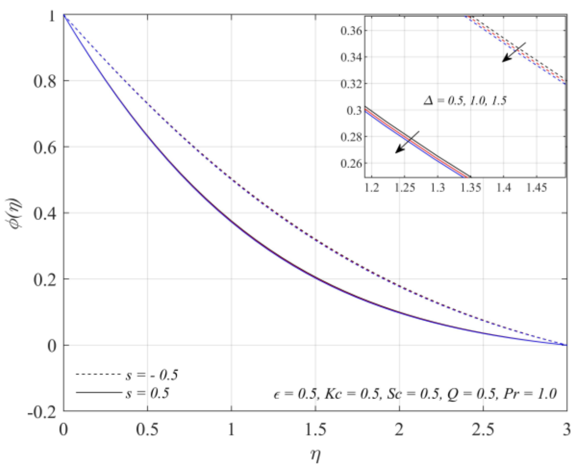

Fig. 12: Result for s and Δ with $\phi(\eta)$

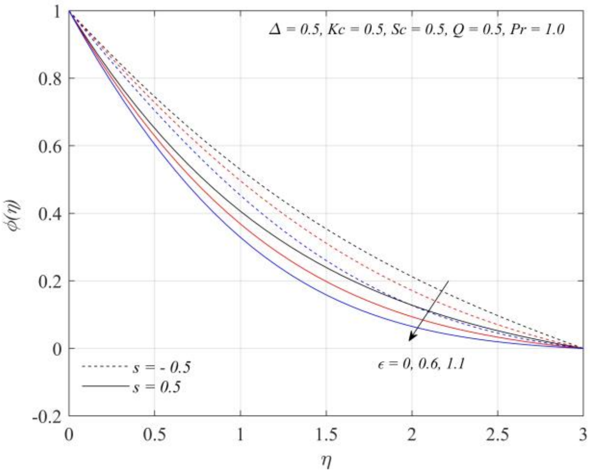

Fig. 13: Result for s and ϵ with $\phi(\eta)$

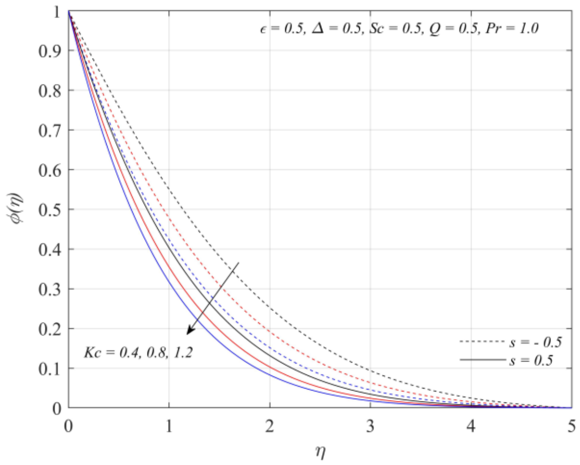

Fig. 14: Result for s and Kc with $\phi(\eta)$

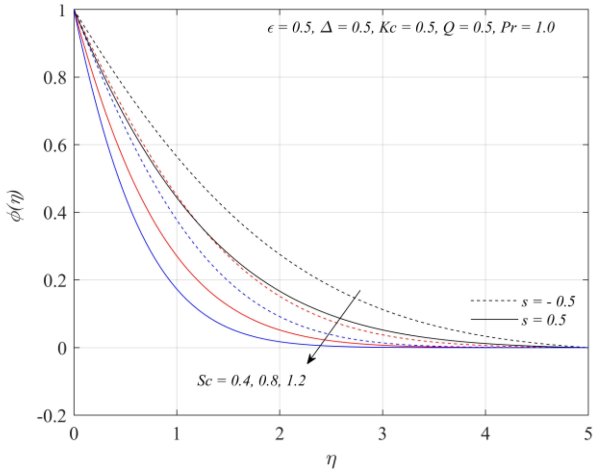

Fig. 15: Result for s and Sc with $\phi(\eta)$

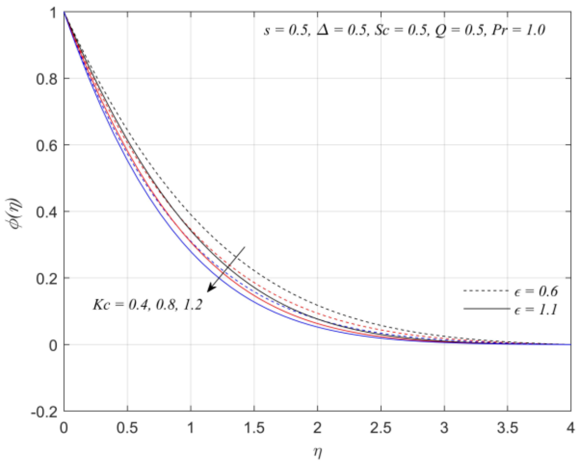

Fig. 16: Result for ϵ and Kc with $\phi(\eta)$

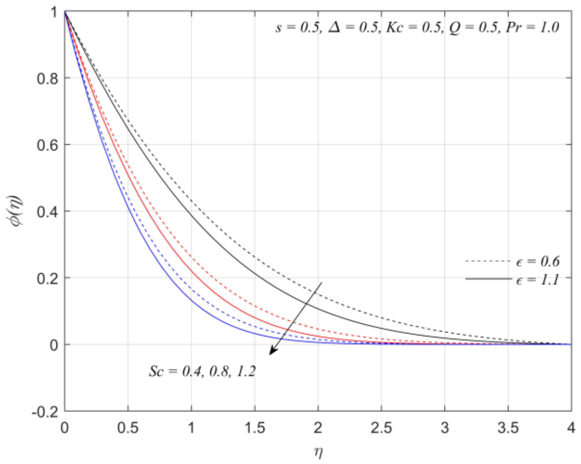

Fig. 17: Result for ϵ and Sc with $\phi(\eta)$

The bearing of D, e and s on boundary layer angular velocities are depicted in Figs. 18 – 20. Fig. 18 suggests that the angular velocity drops as $\varepsilon = \hat{U}_0 / \hat{U}_1$ raises. From Fig. 19 as D increases the angular velocity shows decreasing trend near the wall when $U_e < U_w$ but the trend reverses as we move away from the wall. But when $U_e > U_w$ a reverse trend is seen. From Fig. 20 we find that for suction $(s > 0)$ the angular velocity is high near the wall when compared to the blowing $(s < 0)$ but the trend reverses as we move away from the wall. Q has impact on angular velocity as it is uncoupled from the angular momentum equation.

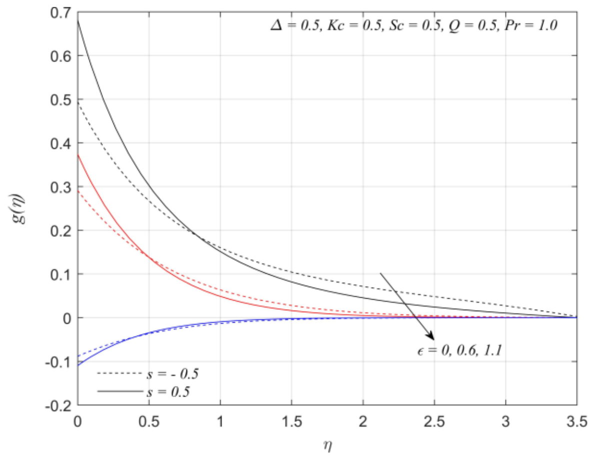

Fig. 18: Result for s and ϵ with $g(\eta)$

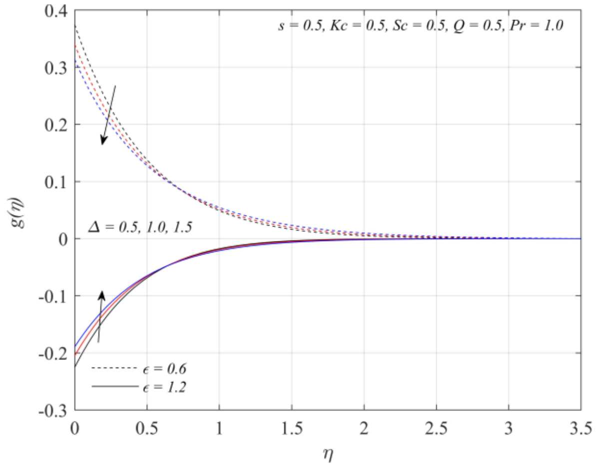

Fig. 19: Result for ϵ and Δ with $g(\eta)$

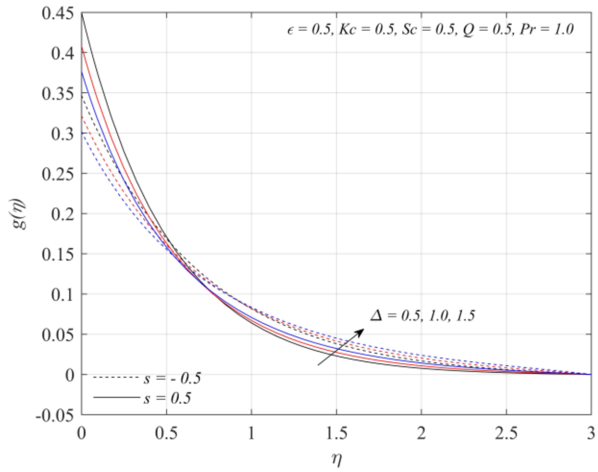

Fig. 20 Result for s and Δ with $g(\eta)$

Table 1: Variance of $f''(0)$, $-\theta'(0)$ & $-f'(0)$ for combination of parameter values at $n = 0.5$, $l = 0.5$ and $B = 0.5$

S. No	Δ	ε	s	Q	K_c	S_c	Pr	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1	0	0.5	0.5	0.5	0.5	0.5	1	- 1.0183	0.7969431	0.9130522
2	0.8	0.5	0.5	0.5	0.5	0.5	1	- 0.8700800	0.8091337	0.9171462
3	0.5	0.8	0.5	0.5	0.5	0.5	1	- 0.4083778	0.8687756	0.9375165
4	0.5	0.5	-0.5	0.5	0.5	0.5	1	- 0.7070088	0.1863649	0.6465085
5	0.5	0.5	0.5	-0.5	0.5	0.5	1	- 0.9176687	1.3183	0.9158340
6	0.5	0.5	0.5	0.5	0.8	0.5	1	- 0.9176687	0.8052377	0.9932506
7	0.5	0.5	0.5	0.5	0.5	0.8	1	- 0.9176687	0.8052377	1.1597
8	0.5	0.5	0.5	0.5	0.5	0.5	3.0	- 0.9176687	1.8647	0.9158340

5. Conclusion

The impact of D, e, s, Q, Pr, S_c and K_c on $f'(0), g(0), \theta(0)$ & $f(0)$ are observed as:

- The velocity, temperature and concentration are lesser for suction $(s > 0)$ when compared to the injection $(s < 0)$. The angular velocity is high near the wall for suction when compared to the blowing but the trend reverses as we move away from the wall.
- As micropolar factor D surges the velocity surges marginally when U_e £ U_w but a reverse trend is seen when $U_e > U_w$. The concentration and temperature profile decrease marginally when U_e £ U_w but a reverse trend is seen when $U_e > U_w$. As D increases the angular velocity shows decreasing trend near the wall but the trend reverses as we move away from the wall when U_e £ U_w but for $U_e > U_w$ the trend reverses.
- The concentration profile drops for surging values of K_c, S_c and the concentration is high for injection than for suction. The rate of concentration drop is high for S_c.
- The heat source / sink parameter Q has no impact on the velocity, concentration and the angular velocity. But as the Prandtl number Pr raises the temperature drops. The temperature is high for the case $Q > 0$ when compared to the case $Q < 0$.

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