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FINITE ELEMENT ANALYSIS OF FREE CONVECTION FLOW WITH MHD MICROPOLAR AND VISCOUS FLUIDS IN A VERTICAL CHANNEL WITH DISSIPATIVE EFFECTS

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Abstract:

In the present study, the vertical channel is divided into two regions one region is filled with Micropolar fluid and the other region is filled with viscous fluid. The channel is subjected to the transverse magnetic field. The coupled governing equations are solved subjected to the boundary conditions proposed by the previous authors using Finite Element Method (FEM). The results are compared with those computed by Kumar et al (2010). It is found that the increase in the magnetic field reduces the micro rotation (N) and velocity and enhances the temperature. Increase in the Eckert number (Ec), decreases the velocity, temperature and micro rotation. The Nusselt number and shear stress values are also analyzed.

Keywords: Micropolar fluid, viscous fluid, Vertical Channel, Viscous dissipation, MHD, FEM, Magnetic field, Free convection

Nomenclature:

U_1 :velocity in the region I
U_2 :velocity in the region II
U_0 :Average Velocity
T_1 :Temperature of the plate at $y=-h_1$
T_2 :Temperature of the plate at $y=h_2$
T_0 : Average temperature.
k_1 :Thermal conductivity in Region I
k_2 :Thermal conductivity in Region II
g :Acceleration due to gravity
n :Micro rotation parameter
K :Vertex viscosity
H_0 :Magnetic Field Intensity
Ec :Eckert number
Gr :Groshof number
R : Reynolds Number
N :Micro rotation number
h :channel width ratio,
m :viscosity ratio
b :Thermal expansion coefficient ratio
B_0 :Magnetic induction
Nu_1 :Nusselt number at $y=-1$
Nu_2 :Nusselt number at $y=1$
St_1 : shear stress at $y=-1$
St_2 : shear stress at $y=1$

<u>Greek letters:</u>

ρ_1 :density of fluid in Region I
ρ_2 :density of fluid in Region II
β_1 :Coefficient of Thermal expansion in Region I
β_2 :Coefficient of Thermal expansion in Region II
μ_1 :Viscosity of fluid in Region I
μ_2 :Viscosity of fluid in Region II
σ : electrical conductivity
γ :Spin gradient
γ_1 :Dynamic viscosity of Micropolar fluid
μ_e :Magnetic Field Permeability
α :Thermal conductivity ratio
ρ :density ratio
$\Pr$:Prandtl number

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1. Introduction

Micropolar fluids are non-Newtonian fluids with microstructures such as polymeric additives, colloidal suspensions, liquid crystals, etc. Eringen (1964) developed the theory of Micropolar fluids, in which the microscopic effects arising from the local structure and the micromotions of the fluids elements are taken into account, and extended it into the theory of thermo Micropolar fluids. In engineering, applications of Micropolar fluids include solidification of liquid crystals, lubricants, and colloidal suspensions, while in biology the animal blood may be modeled as a Micropolar fluid.

This study of viscous dissipation is applicable to polymer technology involving the stretching of plastic sheets. Many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. During the process of drawing, the strips are sometimes stretched. In such situations the rate of cooling has a great effect on the properties of the final product. By drawing them in a Micropolar fluid the rate of cooling may be controlled, thereby giving the final product some desired characteristics.
The application of electromagnetic fields in controlling the heat transfer as in aero dynamic heating leads to the study of MHD heat transfer. This MHD heat transfer has gained significance owing to recent advancement of space technology.

The Problems of Micropolar fluid flow between two vertical plates (channel) are of great technical interest. A lot of attention has been given by many researchers. Sastry and Rao (1982) have studied the effect of suction in the laminar flow of a Micropolar fluid in a channel, considering the poiseuille flow at the entry of the channel. Bhargava and Rani (1985) have examined the convective heat transfer in Micropolar fluid flow between parallel plates. Its extension to free and forced convection is an interesting area of research including liquid crystals, dilute solutions of polymer fluids and many types of suspensions, since in many configurations in the technology and nature, one continuously encounters masses of fluid rising freely in an extensive medium due to the buoyancy effects.

The subject of two-fluid flow and heat transfer has been extensively studied due to its importance in chemical and nuclear industries. The design of two-fluid heat transport system for space application requires knowledge of heat and mass transfer processes and fluid mechanics under reduced gravity conditions. Identification of the two-fluid flow region and determination of the pressure drop, void fraction, quality reaction and two-fluid heat transfer coefficient are of great importance for the design of two-fluid systems. Lohrasbi and Sahai (1988) studied two-phase MHD flow and heat transfer in a parallel plate channel with the fluid in one phase being electrically conducting. Malashetty and Leela (1992) have analyzed the Hartmann flow characteristics of two fluids in horizontal channel. The study of two-phase flow and heat transfer in an inclined channel has been studied by Malashetty and Umavathi (2000) and Malashetty et al (2001)

3. Mathematical Formulation

The two infinite parallel plates are placed at $Y_1 = -h_1$ and $Y_2 = -h_2$ along Y-direction as shown in Fig.1 and both plates are isothermal with different temperatures T_1 and T_2 respectively. The distance from ($-h_1$ to 0) represents region I and distance from (0 to h_2) represents region II where the first region is filled with Micropolar fluid and the second is with viscous fluid. The fluid flow in the channel is due to buoyancy forces. The transport properties of both fluids are assumed to be constant.

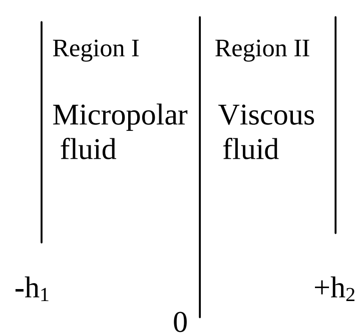

Fig. 1. Physical configuration

We consider the fluids to be incompressible and immiscible and the flow is steady laminar and fully developed.

The governing equations are

$$\frac{\partial U_1}{\partial Y}=0\ ,\ \frac{\partial U_2}{\partial Y}=0 \qquad \text{[Continuity Equations]} \qquad (1)$$

$$\rho_1=\rho_0[1-\beta_1(T_1-T_0)] \qquad (2)$$

$$\rho_2=\rho_0[1-\beta_2(T_2-T_0)] \qquad \text{[Equations of State]} \qquad (3)$$

$$\left(\mu_1+K\right)\frac{d^2U_1}{dY^2}+K\frac{dn}{dY}+\rho_1 g\beta_1\left(T_1-T_0\right)=\frac{\sigma B_0{}^2U_1}{\rho_1} \qquad (4)$$

$$\mu_2\frac{d^2U_2}{dY^2}+\rho_2 g\beta_2\left(T_2-T_0\right)=\frac{\sigma B_0{}^2U_2}{\rho_2} \qquad \text{[Equations of Momentum]} \qquad (5)$$

$$\gamma\frac{d^2n}{dY^2}-K\left[2n+\frac{dU_1}{dY}\right]=0 \qquad \text{[Conservation of angular momentum]} \qquad (6)$$

$$\frac{d^2T_1}{dY^2}+\frac{\mu_1}{k_1}\left(\frac{dU_1}{dY}\right)^2=0 \qquad (7)$$

$$\frac{d^2T_2}{dY^2}+\frac{\mu_2}{k_2}\left(\frac{dU_2}{dY}\right)^2=0 \qquad \text{[Equations of Energy]} \qquad (8)$$

To solve the above system of equations, we use the boundary and interface conditions proposed by T.Arimen et al (1973) as follows

$$U_1=0\ at\ Y=-h_1\quad ,\ U_2=0\ at\ Y=h_2$$

$$U_1\left(0\right)=U_2\left(0\right)$$

$$\left(\mu_1+K\right)\frac{dU_1}{dY}+Kn=\mu_2\frac{dU_2}{dY}\ ,\frac{dn}{dY}=0\ \ at\ Y=0\,,$$

$$n=0\ at\,Y=-h_2$$

For the corresponding temperature boundary conditions it is assumed that the temperature and heat flows are continuous at the interface.

$$T=T_1\ at\ Y=-h_1\ ,T=T_2\ at\ Y=h_2$$

$$T_1\left(0\right)=T_2\left(0\right)$$

$$k_1\frac{dT_1}{dY}=k_2\frac{dT_2}{dY}\ \ at\,Y=0$$

We assume that

$$\gamma=\left(\mu_1+\frac{K}{2}\right)j=\mu\left(1+\frac{K}{2}\right)j$$

and $T_2>T_1$

By introducing the following non dimensional variables,

$$y_1=\frac{Y_1}{h_1}\,,y_2=\frac{Y_2}{h_2}\,,u_1=\frac{U_1}{U_0}\,,u_2=\frac{U_2}{U_0}\,,\theta_1=\frac{T_1-T_0}{\Delta T}\,,\theta_2=\frac{T_2-T_0}{\Delta T}\,,N=\frac{h_1}{U_0}n\,,K'=\frac{\mu}{K}$$

The governing equations becomes

$$\frac{d^2u_1}{dy^2}+\frac{K'}{1+K'}\frac{dN}{dy}+\frac{1}{1+K'}\left[\frac{Gr}{R}\theta_1\right]-\frac{1}{1+K'}M^2u_1=0 \tag{9}$$

$$\frac{d^2N}{dy^2}-\frac{2K'}{2+K'}\left(2N+\frac{du_1}{dy}\right)=0 \tag{10}$$

$$\frac{d^2u_2}{dy^2}+\left[\frac{Gr}{R}\theta_2\right]\frac{bm}{h\rho}-\frac{M^2m}{h}u_2=0 \tag{11}$$

$$\frac{d^2\theta_1}{dy^2}+\Pr Ec\left(\frac{du_1}{dy}\right)^2=0 \tag{12}$$

$$\frac{d^2\theta_2}{dy^2}+\Pr Ec\frac{\alpha}{m}\left(\frac{du_2}{dy}\right)^2=0 \tag{13}$$

Where $Gr=\frac{g\beta_1\Delta T h_1^3}{\nu_1^2}$, $R=\frac{U_0h_1}{\nu_1}$,

$h=\frac{h_1}{h_2}$ (Channel width ratio), $m=\frac{\mu_1}{\mu_2}$ (viscosity ratio), $\alpha=\frac{k_1}{k_2}$, (Thermal conductivity ratio)

$\rho=\frac{\rho_1}{\rho_2}$ (Density ratio), $b=\frac{\beta_1}{\beta_2}$ (Thermal expansion coefficient ratio),

$M=\frac{\sigma\mu_e^2H_0^2h_1^2}{\mu_1}$, $\Pr=\frac{\mu_1 Cp}{k_1}$, $Ec=\frac{U_0^2}{Cp\ \Delta T}$

Subject to the boundary conditions:

$u_1(0)=u_2(0), u_2=0\ \ at\ \ y_2=1, u_1=0\ \ at\ \ y_1=-1$,

$\frac{du_1}{dy}+\frac{K'}{1+K'}N=\frac{1}{mh(1+K')}\frac{du_2}{dy}\quad at\ \ y=0$

$\frac{dN}{dy}=0\ \ at\ y=0\qquad N=0\ \ at\ \ y_1=-1$

$\theta_1=1\ \ at\ \ y_1=-1$, $\theta_2=0\ \ at\ \ y_2=1$, $\theta_1(0)=\theta_2(0)$

$$\frac{d\theta_1}{dy}=\frac{1}{h\alpha}\frac{d\theta_2}{dy}\quad at\ \ y=0 \tag{14}$$

3. Solution of the Problem:

The coupled governing equations are solved numerically using the regular Galerkin Finite Element method as given by Reddy (2005). For computational purpose each region is divided into 100 linear elements. Each element is 3 nodded.

The shape functions at each node of a typical i^{th} element are the Langrange's interpolation polynomials given by

$$S_i^1=\frac{\left(y-\left(\frac{2i-101}{100}\right)\right)\left(y-\left(\frac{2i-100}{100}\right)\right)}{\left(\frac{2i-102}{100}-\left(\frac{2i-101}{100}\right)\right)\left(\frac{2i-102}{100}-\left(\frac{2i-100}{100}\right)\right)}$$

$$S_i^2 = \frac{\left(y-\left(\frac{2i-102}{100}\right)\right)\left(y-\left(\frac{2i-100}{100}\right)\right)}{\left(\frac{2i-101}{100}-\left(\frac{2i-102}{100}\right)\right)\left(\frac{2i-101}{100}-\left(\frac{2i-100}{100}\right)\right)}$$

$$S_i^3 = \frac{\left(y-\frac{2i-101}{100}\right)\left(y-\left(\frac{2i-102}{100}\right)\right)}{\left(\frac{2i-100}{100}-\left(\frac{2i-102}{100}\right)\right)\left(\frac{2i-100}{100}-\left(\frac{2i-101}{100}\right)\right)}$$

The stiffness matrix equations corresponding to the governing equations (9) to (13) for i^{th} element are evaluated by using the following equations:

Region-I

$$\left[\int_\Omega \frac{dS_i^j}{dY}\frac{dS_i^p}{dY}dY + \frac{M^2}{1+K'}\int_\Omega S_i^p S_i^j\, dY\right]U_{1i}^j = -\left[\frac{K'}{1+K'}\int_\Omega \frac{dS_i^j}{dY}N_i^j S_i^j\, dY - \frac{Gr}{(1+K')R}\int_\Omega \theta_{1i}^j S_i^j\, S_i^p\, dY\right] + \left[S_i^j\frac{dU_{1i}^j}{dY} + \frac{K'}{1+K'}N_i^j S_i^j\right] \tag{15}$$

$$\left(S_i^p\frac{d\left(N_i^j S_i^j\right)}{dY} - \frac{2K'}{2+K'}S_i^p\left(U_{1i}^j S_i^j\right)\right) - \int_\Omega \frac{dS_i^j}{dY}\left(\frac{d\left(N_i^j S_i^j\right)}{dY}\right)dY - \frac{2K'}{2+K'}\left[2\int_\Omega\left(N_i^j S_i^j\right)S_i^p dY + \int_\Omega \frac{dS_i^j}{dY}\left(U_{1i}^j S_i^j\right)dY\right] = 0 \tag{16}$$

$$S_i^p\frac{d\left(\theta_{1i}^j S_i^j\right)}{dY} - \int_\Omega\left(\frac{dS_i^j}{dY}\frac{d\left(\theta_{1i}^j S_i^j\right)}{dY}\right)dY + \mathrm{P}r\, Ec\int_\Omega\left(\frac{dU_{1i}^j S_i^j}{dY}\right)^2 S_i^p\, dY = 0 \tag{17}$$

Region - II

$$S_i^p\frac{d\left(U_{2i}^j S_i^j\right)}{dY} - \int_\Omega\left(\frac{dS_i^j}{dY}\frac{d\left(U_{2i}^j S_i^j\right)}{dY}\right)dY + \frac{Gr}{R}\frac{bm}{h\rho}\int_\Omega\left(\theta_{2i}^j S_i^j\right)S_i^p\, dY - \frac{M^2 m}{h}\int_\Omega S_i^p\left(U_{2i}^j S_i^j\right)dY = 0 \tag{18}$$

$$S_i^p\frac{d\left(\theta_{2i}^j S_i^j\right)}{dY} - \int_\Omega\left(\frac{dS_i^j}{dY}\frac{d\left(\theta_{2i}^j S_i^j\right)}{dY}\right)dY + \Pr Ec\frac{\alpha}{m}\int_\Omega S_i^j\left(\frac{dU_{2i}^j S_i^j}{dY}\right)^2 dY = 0 \tag{19}$$

Where Ω is the typical element region $\left(\frac{2i-102}{100},\frac{2i-100}{100}\right)$

These coupled governing equations are solved iteratively subject to the boundary conditions given in (14) until the desired accuracy of 10^{-5} is attained. The Nusselt number and shear stress can be calculated at both walls by using the expressions

$$Nu_1 = \left[\frac{\partial\theta}{\partial Y}\right]_{Y=-1},\ Nu_2 = \left[\frac{\partial\theta}{\partial Y}\right]_{Y=1},\ St_1 = \left[\frac{\partial U_1}{\partial Y}\right]_{Y=-1},\ St_2 = \left[\frac{\partial U_2}{\partial Y}\right]_{Y=1}$$

4. Results and Discussion

Fig.2. shows the variation of velocity with Gr for R=1, H=5, Ec=0.001, K=1. From the figure it is seen that the velocity increases with Gr in both regions due to decreases in viscous force. The velocity is maximum in the absence of convection buoyancy force. The similar behavior has been observed in case of the micro rotation as shown in Fig. 3.

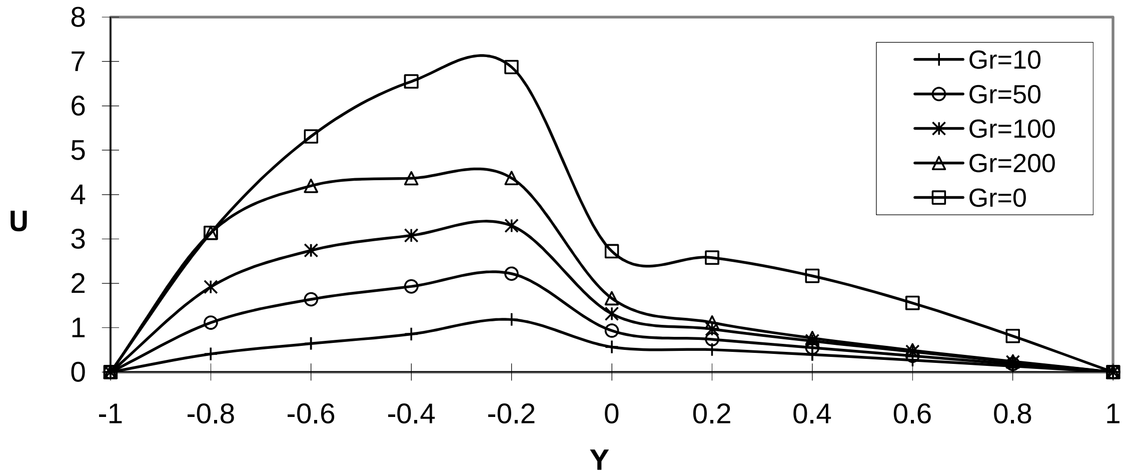

Fig. 2: Variations of velocity with different Gr (R=1, H=5, Ec=0.001, K=1)

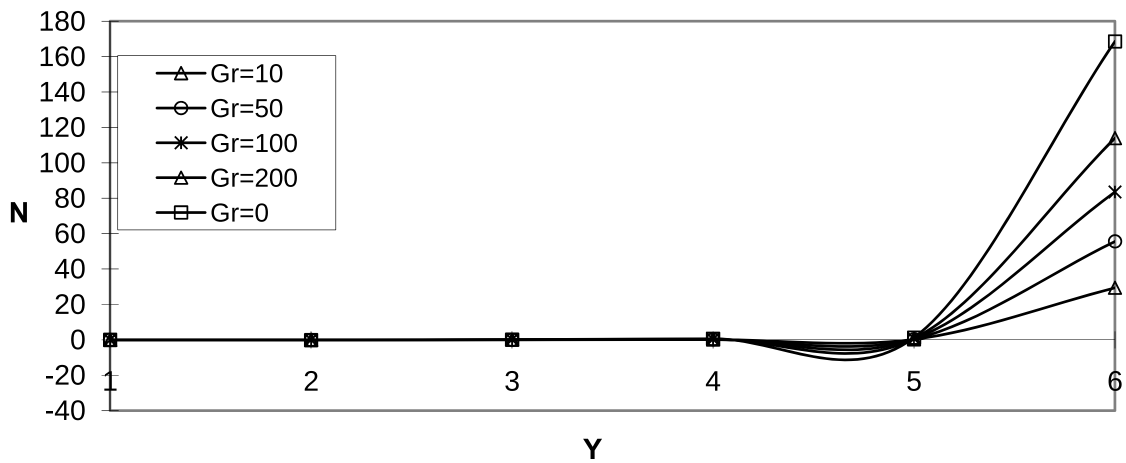

Fig. 3: Variations of N with different Gr (R=1, H=5, Ec=0.001, K=1)

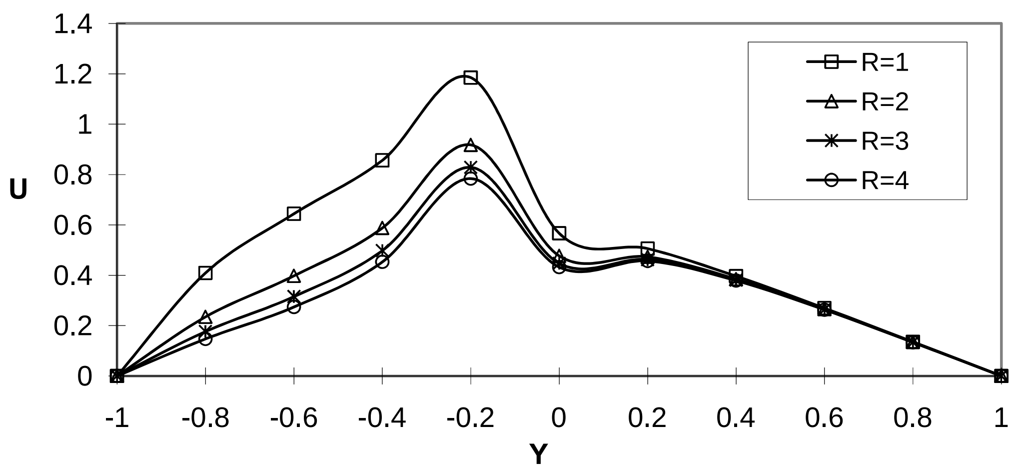

Fig .4: Variations of velocity with different R (Gr=10, H=5, Ec=0.001, K=1)

The variation of velocity with Reynold's number R is shown in Fig. 4 for Gr=10, H=5, Ec=0.001, K=1. It is observed that the velocity decreases with increase in the Reynolds number R as shown in Fig. 4 due to increase in inertia. The velocity reached its maximum in the Micropolar region. The Micro rotation also decreases with the increase in R as shown in Fig.5.

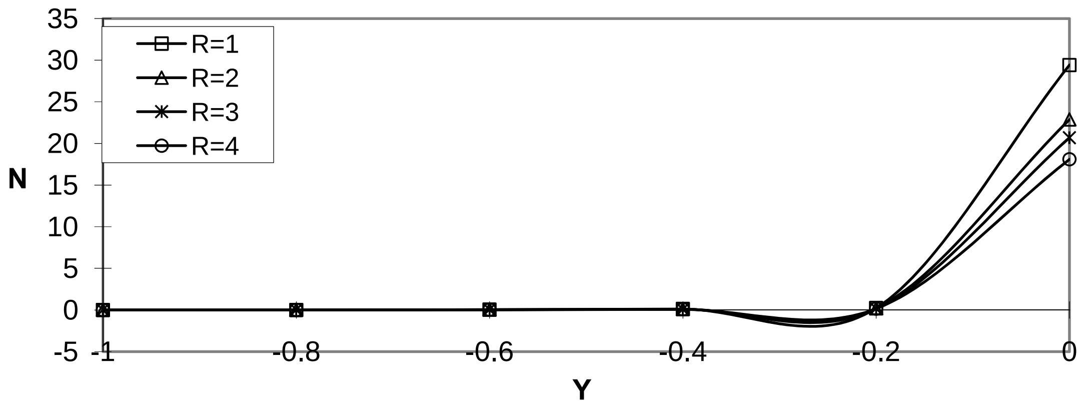

Fig.5: Variations of N with different R (Gr=10, H=5, Ec=0.001, K=1)

The variation of velocity is very small for higher values of R. The velocity (Fig. 6) and Micro rotation (Fig. 7) decreases with the increase in the magnetic field due to Lorenz force. The velocity is almost not affected by higher values of H. The absence of magnetic field enhances the velocity, Micro rotation and the profile is in good agreement with the study of Pop et al (2010).

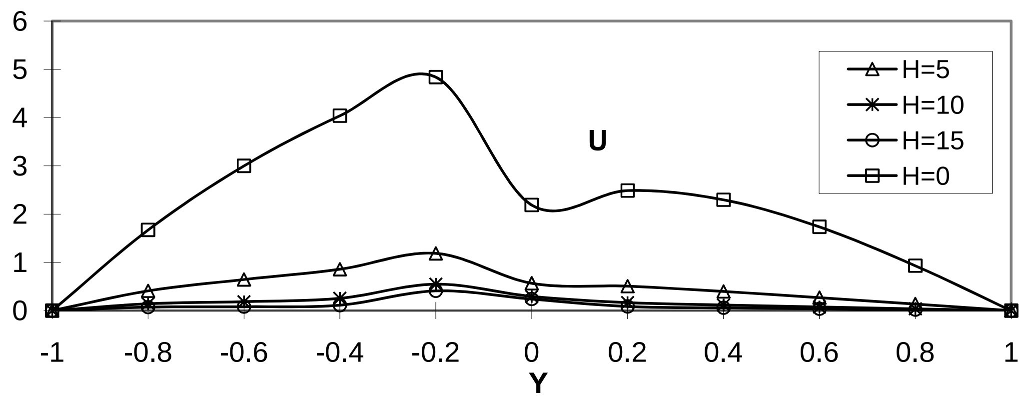

Fig. 6: Variations of velocity with different H (Gr=10, R=1, Ec=0.001, K=1)

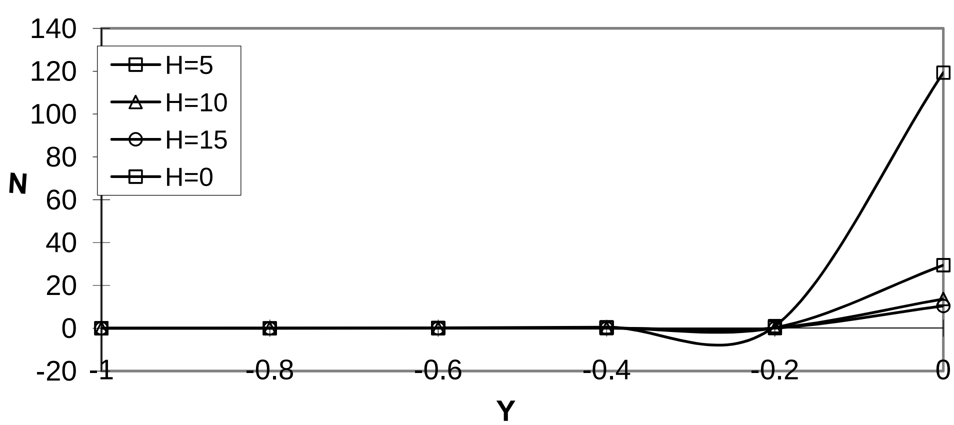

Fig. 7: Variations of N with different H (Gr=10, R=1, Ec=0.001, K=1)

The increase in the viscous dissipation (Ec) decreases the velocity (Fig. 8) and Micro rotation (Fig. 9). The absence of viscous dissipation increases the velocity, Micro rotation and decreases the temperature due to enthalpy. Fig.5, Fig.9 and Fig.10 for $\mathrm{Ec} = 0$ exhibits the identical behavior as given by Pop et al (2010). The increase in the viscous dissipation cools the fluid in both regions due to Enthalpy (Fig 10).

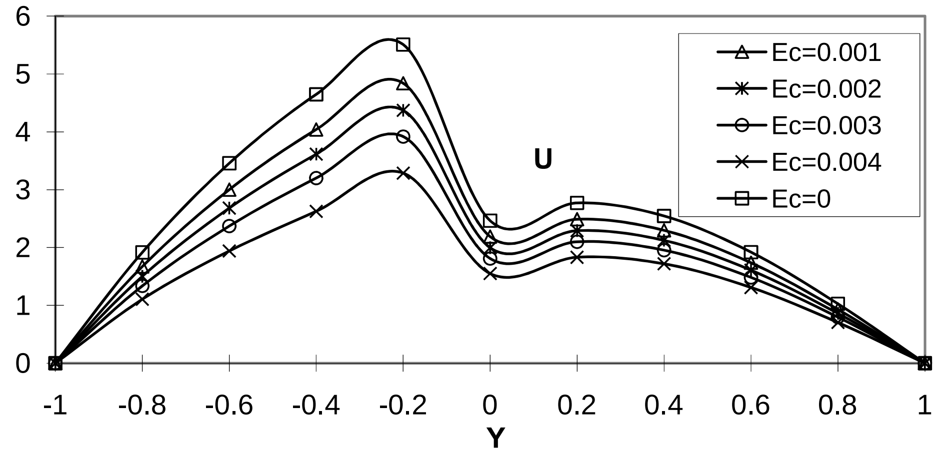

Fig. 8: Variations of velocity with different Ec (Gr=10, R=1, H=5, K=1)

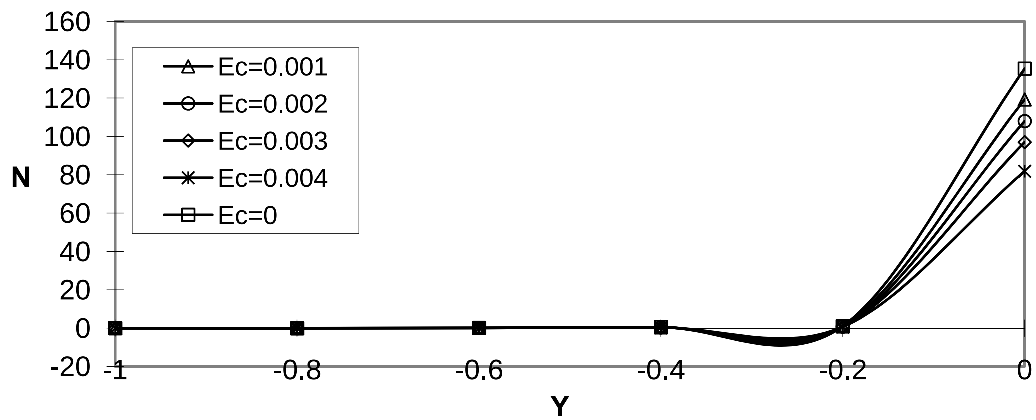

Fig. 9: Variations of N with different Ec (Gr=10, R=1, H=5, K=1)

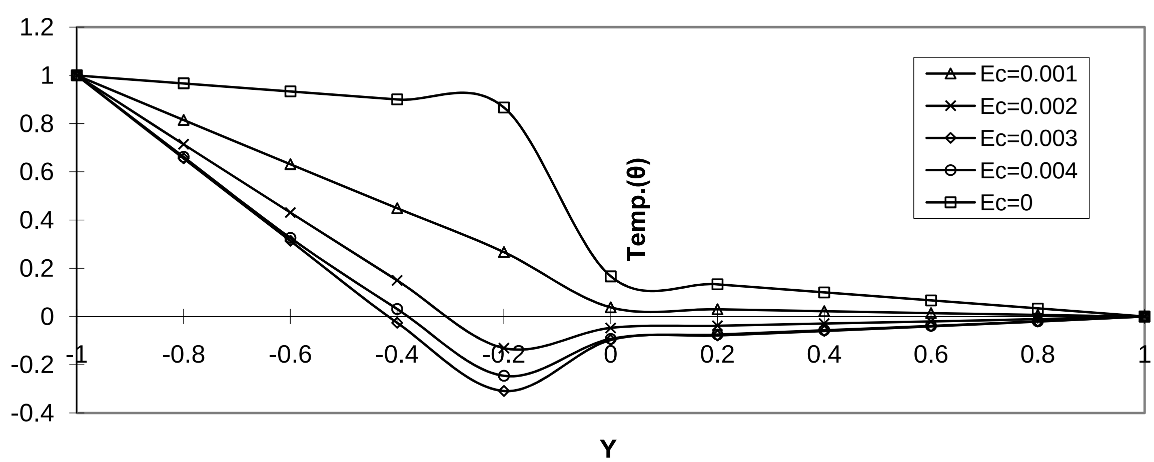

Fig. 10: Variations of Temperature with different Ec (Gr=10, R=1, H=5, K=1)

The increase in the material parameter K decreases the velocity in both regions (Fig 11). The variation of velocity in the viscous fluid region with K is very small. The increase in the material parameter K decreases the Microrotation (Fig 12). The results for K=1.5 are good agreement with Pop et al (2010).

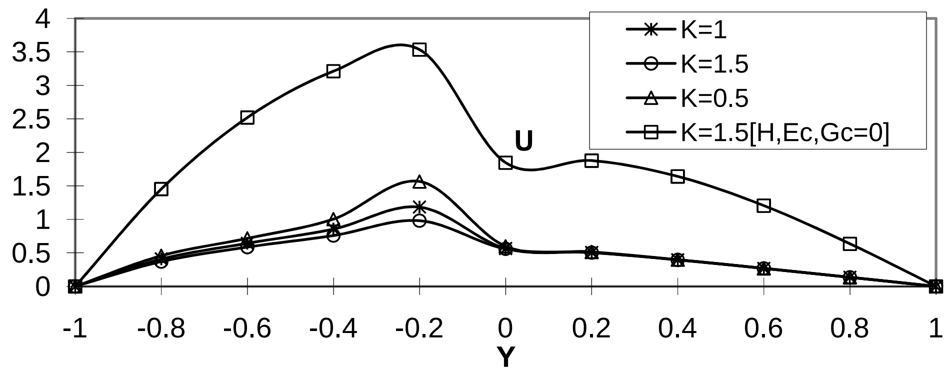

Fig. 11: Variations of velocity with different K (Gr=10, R=1, H=5, Ec=0.001)

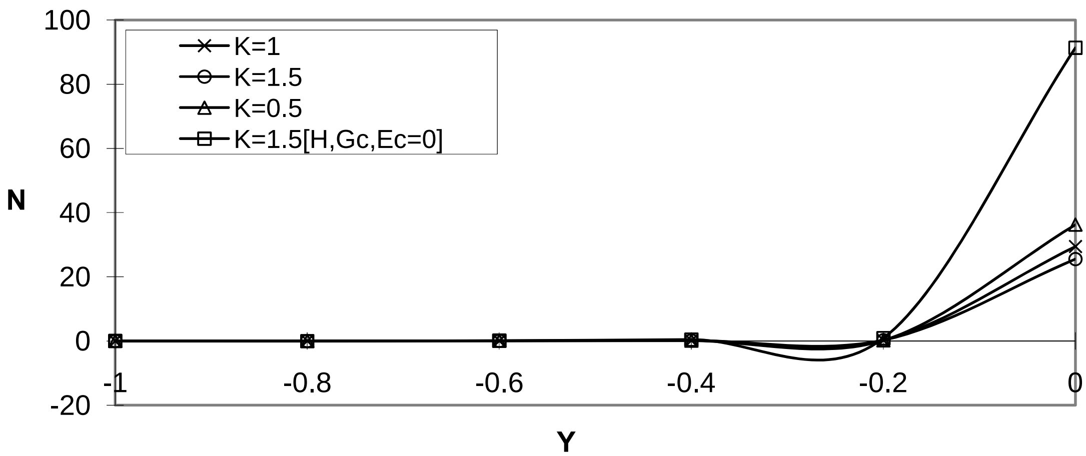

Fig 12-variations of N with different K(Gr=10, R=1, H=5, Ec=0.001)

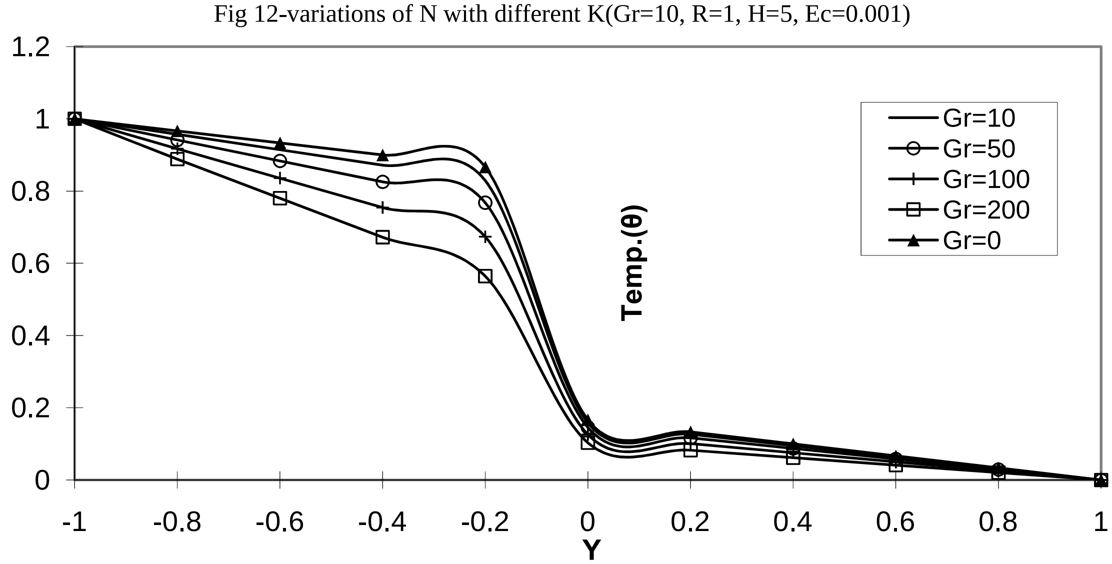

Fig 13-variations of Temperature with different Gr (R=1, H=5, Ec=0.001, K=1)

The increase in the convection Buoyancy force cools the fluid in both regions (Fig 13). The temperature increases rapidly with the Groshof number Gr due to buoyancy force. The absence of Gr gives the maximum temperature. The increase in R enhances the temperature (Fig 14) due to inertia.

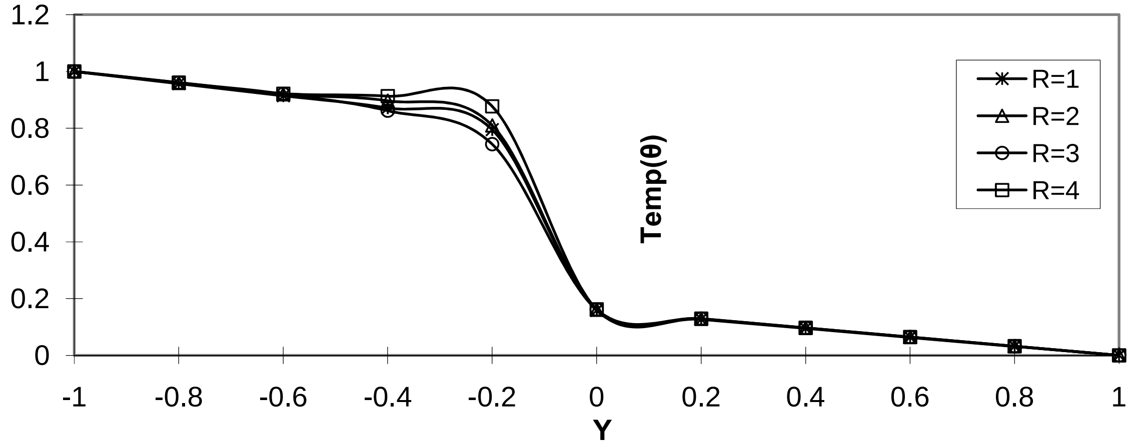

Fig 14-variations of Temperature with different R(Gr=10, H=5, Ec=0.001, K=1)

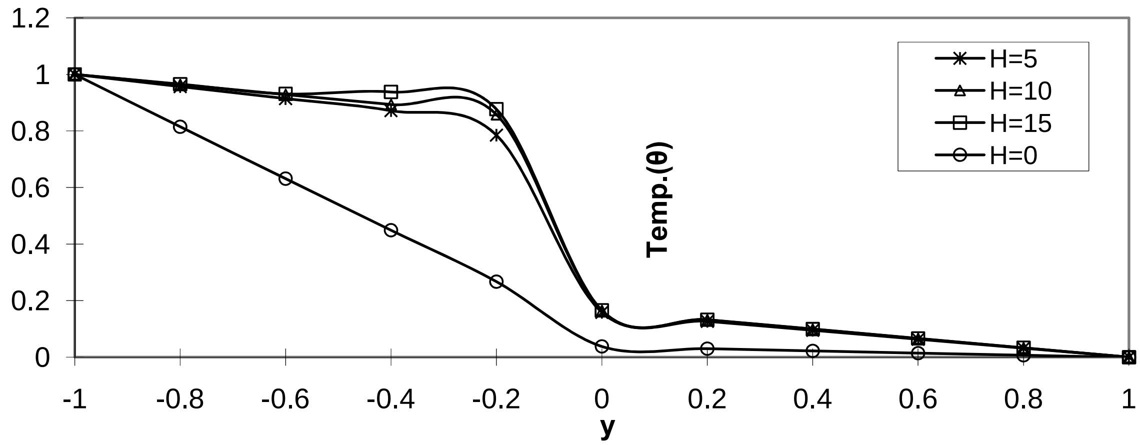

Fig 15-variations of Temperature with different H(Gr=10, R=1, Ec=0.001, K=1)

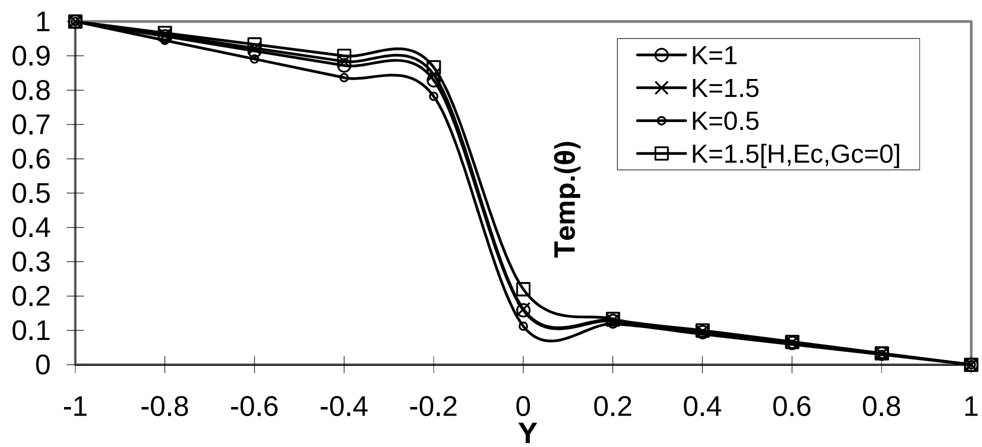

Fig 16-variations of Temperature with different K (Gr=10, R=1, H=5, Ec=0.001)

The increase in the magnetic field parameter (H) increases the temperature in both regions due to Lorentz force (Fig. 15). The absence of H varies temperature linearly. The variation in the temperature is very small for higher values of Ec. The increase in the material parameter enhances the temperature in both the regions (Fig. 16). The profiles of temperature are in good agreement with Pop et al (2010) for H=0 and Ec=0. As Gr, Ec, R increase the magnitude of rate of heat transfer on the boundary y=-h increases and on $y = h_2$ decreases due to the heat transfer from the plate $y = h_2$ to $y = -h_1$. As H, K increase the magnitude of rate of heat transfer on the boundary $y = -h_1$ decreases and on $y = h_2$ increases due to the heat conduction is within the plates.

5. Conclusion

Free convection flow with MHD micropolar and viscous fluids in a vertical channel with dissipative effects has been studied using Finite Element Method. From this study, following conclusion can be drawn:

a) The velocity is more in Micropolar region than viscous region.
b) The velocity attains its peak value near the interface in both regions.
c) The velocity increases with Gr & R and decreases with H, Ec & K.
d) The temperature is also more in Micropolar region than the viscous region.
e) The viscous dissipation enhances the heat transfer on the boundary $y = -h_1$ and depreciates the heat transfer on the boundary $y = h_2$. The reverse effect has been observed due to the magnetic field.
f) The viscous dissipation depreciates the stress on both boundaries. The reverse effect has been observed due to the magnetic field.

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