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THERMAL RADIATION EFFECT ON MHD NANOFLUID FLOW WITH VISCOUS DISSIPATION

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Abstract:

Recent advances in technology basically need enhanced thermal properties and this can be achieved by the use of nanoparticles. The inclusion of particle concentration in the base fluid augments the thermal conductivity which is useful in several areas of industries and engineering. Particularly, for the cooling of electronic devices, the drug delivery processes, peristaltic pumping processes, etc. The present investigation addresses the effect of thermal radiation on MHD nanofluid flow with viscous as well as Joule dissipation past over an expanding surface. The basic governing equations of the flow are converted into ordinary differential equations using stream function and similarity variable. The reduced nonlinear ODEs are solved numerically with MATLAB's built-in solver bvp4c. The parametric behavior on the flow phenomena is presented through graphs and described briefly. Some of the important findings are deployed as; Due to the higher conductivity of silver nanoparticles compared to copper, the temperature distribution in Ag-water nanofluid is greater than in Cu-water nanofluid. An increase in the thermal radiation parameter reduces the gap between the thermal boundary layers. The onset of Lorentz force increases the skin friction coefficient more rapidly in Cu-water nanofluid than in Ag-water nanofluid.

Keywords: Nanofluid; stretching sheet; viscous dissipation; Joule dissipation; thermal radiation.

NOMENCLATURE

x, y	dimensionless coordinates	Ec	Eckert number
u, v	dimensional x and y components of velocity (ms^{-1})	C_{fx}	the skin friction coefficient
T	temperature (K)	Nu_x	local Nusselt number
B_0	uniform magnetic field strength ($kgA^{-1}s^{-2}$)	f	dimensionless stream function
M	magnetic parameter		***Greek symbols***
C_p	specific heat at constant pressure ($Jkg^{-1}K^{-1}$)	α	fluid thermal diffusivity (m^2s^{-1})
T_w	wall temperature (K)	ρ	fluid density ($kg\ m^{-3}$)
T_∞	free stream temperature (K)	μ	dynamic viscosity ($kg\ m^{-1}s^{-1}$)
l	the characteristic length (m)	υ	kinematic viscosity ($m^2\ s^{-1}$)
g	dimensionless temperature	ϕ	nanoparticles volume fraction
k^*	the absorption coefficient	η	the similarity variable
A, b	constants	ψ	dimensional stream function ($m^2\ s^{-1}$)
k	thermal conductivity ($Wm^{-1}K^{-1}$)	σ	electrical conductivity ($s^3A^2kg^{-1}m^{-3}$)
Rd	thermal radiation parameter	β	thermal expansion coefficient (K^{-1})
Pr	Prandtl number		

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1. Introduction

The flow over a stretching surface is a significant phenomenon that finds wide range of applications in various engineering processes across multiple industries. These industries include glass production, wire drawing, extrusion, hot rolling, melt-spinning, manufacturing of plastic and rubber sheets, and cooling of large metallic plates in electrolyte baths, among others. In these applications, understanding and analyzing the fluid dynamics over a stretching surface is crucial for process optimization, product quality improvement, and overall efficiency enhancement. The research conducted by Sahoo and Dash (2012) investigates the heat and mass transfer characteristics of a convective boundary layer flow subjected to a magnetic field (MHD) over a stretching porous wall embedded in a porous medium. In their study, Kar *et al* (2014) conducted an analysis of the heat and mass transfer effects on a viscoelastic magnetohydrodynamic (MHD) flow over a stretching porous sheet. The flow considered in their investigation incorporates dissipative and radiative properties. Baitharu *et al* (2020) conducted a study on the heat and mass transfer effects in a porous medium for a second-grade magnetohydrodynamic (MHD) flow over a stretching sheet with radiative properties. Baithatu *et al* (2021) conducted a study to investigate the influence of Joule heating on the steady magnetohydrodynamic (MHD) convective flow of a micropolar fluid over a stretching or shrinking sheet with slip conditions. In general, oil, water, and ethylene glycol mixtures are commonly employed in heat transfer applications as base liquid. However, their effectiveness as heat transfer fluids is limited due to their low thermal conductivity, which adversely affects the heat transfer coefficient between the medium and the surface. As a result, a variety of techniques have been used to increase these fluids' thermal conductivity by suspending nano-, micro-, or larger-sized particle materials in liquids. Adding nanoscale particles to the base fluid is a novel way to enhance heat transmission. Due to the unique physical and chemical characteristics of materials with a nanometer-sized scale, nanotechnology has been extensively utilized in industry. Nanofluids are fluids with nanoscale particles added. Kameswaran *et al* (2012) have analyzed the effects of viscous dissipation as well as chemical reaction on hydromagnetic nanofluid flow due to a stretching or shrinking sheet. Some of their important findings are due to the higher thermal conductivity of silver than that of coper, the temperature variation in Ag-water is higher than that of Cu-water nanofluid. Further it is seen that due to high value of thermal radiation parameter, the thermal boundary layer thickness decreases which leads to thermal stability of the flow. Onset of Lorentz force enhances the skin friction coefficient more rapidly in case of Cu-water than that of Ag-water nanofluid which shows that Ag-water nanofluid reduces the skin-friction resulting stream line flow. In a numerical investigation by Khan and Pop (2010), the flow behavior of a nanofluid resulting from the stretching of a flat surface was examined. The study considered the influence of Brownian motion and thermophoresis within the nanofluid. Hamad (2011) conducted a study on the enhancement of convective flow and heat transfer in an incompressible viscous nanofluid past a vertically extending sheet by applying a magnetic field. It is investigated how various factors affect flow, heat transfer, Nusselt number, and skin friction coefficient. In a study conducted by Zaimi *et al* (2014), the flow and heat transfer characteristics over a stretching or shrinking sheet were investigated using Buongiorno's nanofluid model. The authors observed the presence of dual solutions within a specific range of suction and stretching/shrinking parameters. A study by Hafidzuddin *et al* (2016) investigated the flow and heat transfer characteristics over a permeable exponentially stretching or shrinking sheet with a generalized slip velocity, focusing on the boundary layer. In the presence of slip, Ghosh and Mukhopadhyay (2017) examined the viscous flow caused by a permeable sheet in nanofluid that is exponentially shrinking. Ghosh and Mukhopadhyay (2020) conducted a research study on the flow of a nanofluid over an exponentially porous and shrinking sheet, considering the presence of thermal and velocity slip conditions. The study specifically investigated the effects of silver (Ag) nanoparticles in two different base fluid types, namely water and kerosene oil. The investigation employed a single-phase fluid model for the nanofluid. The primary objective of the study was to understand the impacts of the silver nanoparticle dispersion on the flow characteristics when thermal and velocity slip conditions are present. By considering different base fluid types, the researchers aimed to examine the variations in the behavior of the nanofluid flow over the porous and shrinking sheet. They observed that as the volume fraction of nanoparticles increased, fluid velocity and temperature also increased.

Heat and mass transfer studies involving fluids with chemical reactions over a stretching sheet play a pivotal role in various industries, including metallurgy and chemical engineering. These studies have significant applications in various processes, including ‘food processing and polymer production’. Additionally, interactions between heat and mass transfer phenomena in the presence of chemical reactions have gained substantial attention in recent times due to their relevance in numerous industrial processes. These studies find practical applications in several areas. For instance, in the agricultural sector, they contribute to understanding the drying process, as well as the spatial variation of temperature and moisture across agricultural and fruit tree groves. Furthermore, they are relevant to evaluating the damage caused to crops due to freezing, as well as studying evaporation phenomena at the surface of water bodies and energy transfer in wet cooling towers. Another application is analyzing the flow in desert coolers, which are commonly used in arid regions. Pal and Mandal ((2014), (2011)) carried out research studies using numerical methods to investigate the mixed convection boundary layer flow of nanofluids in the vicinity of a stagnation point over a permeable stretching or shrinking sheet. The study considered various physical phenomena, including viscous dissipation, heat source sink, thermal radiation, and chemical reaction.

The constant utilization of thermal radiation in linear form increases the thermal efficiency of the nanofluid (Gul *et al* (2020)). In reality, heat radiation in linear form functions as intended in nanofluids when the volume fraction of nanoparticles is limited to 5%. Even the researchers (Anwar *et al* (2021), Alaidrous and Eid (2020), Arfa *et al* (2021), Mabood *et al* (2020)) employed the nonlinear stress linear thermal radiation term in non-Newtonian fluids. There is extremely little nonlinear thermal radiation in the case of non-Newtonian fluids (Hady *et al* (2012), Shankar and Gorfie (2014), Madhu and Kishan (2015), Uddin *et al* (2015)). For the Darcy-Forchheimer flow of hybrid nanofluids under velocity slip circumstances, thermal stratification, and nonlinear thermal radiation, Haider *et al* (2021) provide a numerical solution. Mahanthesh *et al* (2018) the mixed convective three-dimensional slip flow of water-based nanofluids with temperature jump boundary condition. They observed that the velocity and thermal slip boundary condition showed a significant effect on momentum and thermal boundary layer thickness at the wall. Also, the presence of nanoparticles stabilizes the thermal boundary layer growth. Sekhar *et al* (2018) studied the velocity, thermal, and mass slips on chemically reacting magnetohydrodynamic boundary layer flow over a radiated stretching surface in the presence of suction. They observed that the slips have the propensity to control boundary layer flow. Velocity, temperature, and concentration show the increasing nature before and after a point, as the velocity slip increases and the momentum, thermal, and concentration boundary layer thickness become thinner for velocity slip. Tarakaramu *et al* (2021) investigated the effects of non-linear thermal radiation and Joule heating on MHD three-dimensional visco-elastic nanofluid flow due to a surface stretching in lateral directions. They observed that the momentum of the visco-elastic nanofluid is better than that of a viscous fluid and the model plays a significant role in the field of manufacturing and engineering applications. Berrehal *et al* (2022) have examined the effects of spherical and non-spherical (cylinder, disk, platelets, etc.) shapes of silver (Ag) nanoparticles on heat transfer enhancement and inherent irreversibility in hydromagnetic water base nano-liquid flow over a convectively heated stretching sheet with heat generation/absorption. This study revealed that cylindrical shape Ag nanoparticles generate high entropy and fluid friction irreversibility, whereas disk shape Ag nanoparticles exhibit high transfer enhancement rate. Dharmaiah *et al* (2022) explored MHD micropolar nanofluid flow across a permeable stretching plane under non-linear radiation. The novelty of the study is the implication of heat generation with activation energy, Neild's condition and Second order momentum slip of micropolar nanoparticles on the sheet. The salient fluid flow characteristics of three distributions are expressed figuratively for multifarious selected germane parameters and it has applications in materials processing.

1.1 Objective

As per the literature mentioned above, the current study focuses on the influence of radiative heat on the MHD nanofluid flow with the combined effect of viscous and Joule dissipation over an expanding surface. It aims to further enhancement of these factors influence the flow and thermal properties of nanofluids. However, the study based upon the comparative analysis between Ag-water and Cu-water nanofluids.

1.2 Novelty of the investigation:

The novelty of the proposed study renders several key aspects;

- The advanced thermal conductivity of Ag nanoparticles over Cu nanoparticles to achieve enhanced temperature distribution in nanofluids.
- The proposed investigation includes the effects of thermal radiation combined with viscous dissipation, and Joule heating on MHD nanofluid flow.
- The present analysis provides detailed parametric behaviour, highlighting the role of various factors affecting the flow phenomena.
- Thermal conductivities of nanoparticles affect temperature distribution shows a greater comparison between Ag-water and Cu-water nanofluids.
- The analysis is critical for the various applications in industries and engineering, likely cooling of electronic devices and drug delivery processes.

2. Formulation and Solution of the Problem

Let's think about the continuous laminar flow of an incompressible nanofluid across a stretching sheet in two dimensions. The slit from which the sheet is pulled is where the system's origin is located. The continuous stretching surface's direction is treated as the x-axis in this coordinate system. The y-axis is measured perpendicular to the sheet's surface. The physical model of the problem is shown in Fig. 1. The fluid is a water-based nanofluid made up of copper and silver nanoparticles of two distinct sorts. Table 1 enlists the nanofluid's thermophysical characteristics. There have been some presumptions made. They are

(i) The induced magnetic field is of minor significance when compared to the applied magnetic field.

(ii) The absence of slip between the base fluid and nanoparticles is attributed to their thermal equilibrium.

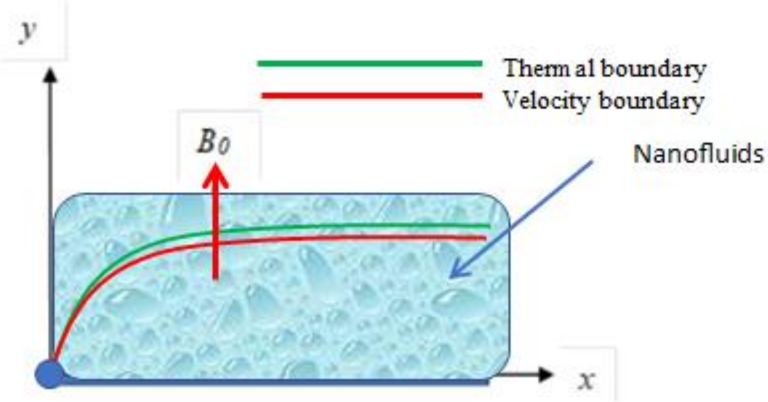

Fig. 1: Flow geometry

The basic governing equations for conservation of mass, momentum, and energy of the nanofluid flow are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_{nf}B_0^{\ 2}}{\rho_{nf}}u, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial y}+\frac{\sigma_{nf}B_0^2}{(\rho C_p)_{nf}}u^2, \tag{3}$$

The boundary conditions for Eqs. (1)-(3) are assumed in the form:

$$\left.\begin{array}{l} y=0: u=u_w=bx, v=0, T=T_w=T_\infty+A\left(\dfrac{x}{l}\right)^2, \\ y\to\infty: u\to 0, T\to T_\infty \end{array}\right\} \tag{4}$$

Using Rosseland approximation for radiation, we can write

$$q_r=-\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}, \tag{5}$$

Additionally, we make the assumption that the temperature difference within the flow allows for the expansion of T^4 in the Taylor series. Subsequently, by expanding T^4 about T_∞ and neglecting higher-order terms, the expression simplifies to $T^4 \equiv 4T_\infty^{\ 3}T-3T_\infty^{\ 4}$. Therefore, the Eq. (3) is simplified to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2-\frac{16\sigma T_\infty^{\ 3}}{3k^*(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial y^2}+\frac{\sigma_{nf}B_0^2u^2}{(\rho C_p)_{nf}} \tag{6}$$

Table 1: Thermophysical properties of water and copper and silver nanoparticles.

	Properties			
	ρ (kg/m^3)	C_p (J/kg K)	k (W/mK)	β (K^{-1})
Pure water	997.1	4179	0.613	2.1 x 10^{-4}
Cu	8933	385	401	1.67 x 10^{-5}
Ag	10500	235	429	1.89 x 10^{-5}

The thermal diffusivity $\left(\alpha_{nf}\right)$, density $\left(\rho_{nf}\right)$, dynamic viscosity $\left(\mu_{nf}\right)$, and the heat capacitance $\left((\rho C_p)_{nf}\right)$ of the nanofluid are given by

$$\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \tag{7}$$

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \tag{8}$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{9}$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s, \tag{10}$$

The Maxwell-Garnett model (Garnett, 1904) is used to approximate the thermal conductivity of nanofluids containing spherical nanoparticles. Therefore,

$$k_{nf} = k_f\left[\frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}\right], \tag{11}$$

Similarly, the electrically conductivity model is described as

$$\sigma_{nf} = \sigma_f\left[\frac{\sigma_s + 2\sigma_f - 2\phi(\sigma_f - \sigma_s)}{\sigma_s + 2\sigma_f + \phi(\sigma_f - \sigma_s)}\right], \tag{12}$$

In Eqs. (7)-(12), the subscripts *nf*, *f*, and *s* represent the thermophysical properties of the nanofluid, base fluid, and nano-solid particles, respectively.

Let us introduce the stream function $\psi(x, y)$ such that

$$u = \frac{\partial\psi}{\partial y}, u = -\frac{\partial\psi}{\partial x}, \tag{13}$$

then the equation (1) is identically satisfied.

Let us introduce the following similarity variables

$$\left.\begin{array}{l} u = bxf'(\eta), v = (b\upsilon_f)^{1/2} f(\eta), T = T_\infty + (T_w - T_\infty)g(\eta) \\ \eta = \left(\dfrac{b}{\upsilon_f}\right)^{1/2}, \psi = (\upsilon_f b)^{1/2} x f(\eta) \end{array}\right\} \tag{14}$$

With the help of Eqs. (7)-(12) and (14), Eqs. (2) and (6) are reduced into the following equations:

$$f''' - \phi_1\left[f'^2 - ff'' + \frac{\phi_5}{\phi_2}Mf'\right] = 0 \tag{15}$$

$$\left(\phi_4 + Rd\right)g'' - Pr\phi_3\left[2f'g - fg' - Ec(f'')^2 - EcM(f')^2\right] = 0 \tag{16}$$

The corresponding boundary conditions are

$$\left.\begin{array}{l} \eta = 0: f = 0, f' = 1, g = 1 \\ \eta \to \infty: f' \to 0, g \to 0, \end{array}\right\} \tag{17}$$

The non-dimensional parameters presented in Eqs. (15) and (16) are provided as follows:

$$M = \frac{\sigma B_0^{\,2}}{\rho_f b}, \Pr = \frac{\upsilon_f(\rho C_p)_f}{k_f}, Ec = \frac{u_w^{\,2}}{(C_p)(T_w - T_\infty)}, Rd = \frac{16\sigma T_\infty^{\,3}}{3k^* k_f}, \tag{18}$$

where

$$\left.\begin{aligned}&\phi_1=(1-\phi)^{2.5}\left[1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right)\right],\phi_2=1-\phi+\phi\left(\frac{\rho_s}{\rho_f}\right),\phi_3=1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f},\\&\phi_4=\frac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)},\phi_5=\frac{\sigma_s+2\sigma_f-2\phi(\sigma_f-\sigma_s)}{\sigma_s+2\sigma_f+\phi(\sigma_f-\sigma_s)}\end{aligned}\right\} \tag{19}$$

At the surface of the wall, the shearing stress τ_w is given by

$$\tau_w=-\mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}=-\frac{1}{(1-\phi)^{2.5}}\sqrt{\upsilon_f b^3 x}\,f''(0), \tag{20}$$

The skin friction coefficient is characterized as

$$C_f=\frac{2\tau_w}{\rho u_w{}^2} \tag{21}$$

With the help of Eq. (20), Eq. (21) becomes

$$C_f(1-\phi)^{2.5}\sqrt{Re_x}=-2f''(0). \tag{22}$$

The heat transfer rate at the surface of the wall is given by

$$q_w=-k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}+\left(q_r\right)_{y=0} \tag{23}$$

The Nusselt number is defined as

$$Nu_x=\frac{xq_w}{k_f(T_w-T_\infty)} \tag{24}$$

The dimensionless wall heat transfer rate is given as

$$Nu_x=-\left(\frac{k_{nf}}{k_f}+Rd\right)g'(0) \tag{25}$$

In all the above, Re_x represents the local Reynolds number and is defined as

$$Re_x=\frac{xu_w}{\upsilon_f} \tag{26}$$

Table 2: Comparison of wall temperature gradient $\left(-g'(0)\right)$

ϕ	*Ec*	Pr	Kameswaran *et al* (2012)		Present results (*Rd* = 0)	
			Cu-water	Ag-water	Cu-water	Ag-water
0.1	0.3	0.7	1.08852	1.05732	1.0885234	1.0573214
0.1	0.3	1.0	1.33333	1.27613	1.3333312	1.2761325
0.1	0.3	3.0	2.50973	2.41245	2.5097302	2.4124506

3. Results and Discussion

An electrically conducting MHD nanofluid flow for the influence of thermal radiation with the combined effect of viscous and Joule dissipation over an expanding surface is analyzed in this discussion. The proposed investigation considered with two types of nanofluids namely Cu-water and Ag-water. The thermophysical models for the viscosity, density, conductivity etc. for the nanofluids plays important role in enhancing the flow properties. Further, the thermal attributes of the solid particles and the water is depicted through Table 1 collected from the experimental results presented in literatures. The numerical result presented in Table 2 shows a comparative analysis between the earlier work of Kameswaran *et al* (2012) in particular case considering the case of base fluid water and found to be in good agreement. The effects of important pertinent flow parameters on velocity, temperature, concentration are analyzed through graphs. The surface drag, wall heat and mass transfer rates are analyzed through tables.

3.1 Effect of flow parameters on velocity profile

Fig. 2 and Fig. 3 show the effect of nanoparticle volume fraction (ϕ) and magnetic parameter (M) on the velocity field respectively. Particle concentration is one of the important factors that are useful in enhancing the viscosity and thermal conductivity as well as the other thermophysical properties used in this problem. Volume fraction is generally defined as the amount of solid particle present in the total volume of the liquid presented in percentage (%). Fig. 2 shows the range of the volume fraction within the range of 0.3% to 0.7%. The observation reveals that the fluid velocity decreases with the decrease in nanoparticle volume fraction irrespective to the type of nanofluid considered. The comparative analysis shows that the Cu-water nanofluid have greater impact in enhancing the fluid velocity than that of Ag-water. The fact is that the density of Cu nanoparticle is lesser than that of Ag and maximum agglomeration of the Ag nanoparticles occurs near the surface. Fig. 3 provides the role of magnetic parameter affecting the fluid velocity for both the case of Ag and Cu-water nanofluids. The interaction of magnetic field on the flow phenomena generates a resistive force and this force particularly called Lorentz force. The behaviour of the Lorentz force is to resists the fluid motion therefore, the increase in magnetic parameter retards the fluid velocity. This resulted in the bounding surface thickness of the velocity attenuates significantly. From both the figures, it is clear that the velocity of Cu-water nanofluid is more than that of Ag-water nanofluid.

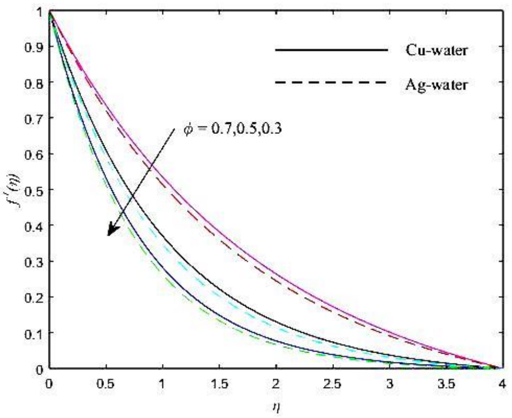

Fig. 2: Influence of the nanoparticle volume fraction ϕ on the velocity.

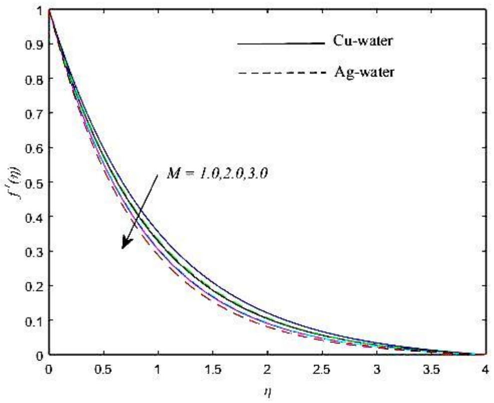

Fig. 3: Influence of the magnetic parameter M on the velocity.

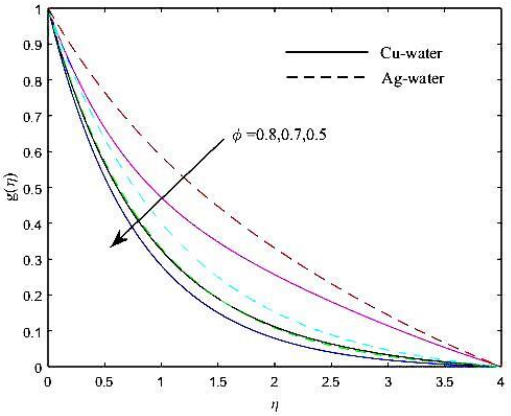

Fig. 4: Influence of the nanoparticle volume fraction ϕ on the temperature profile.

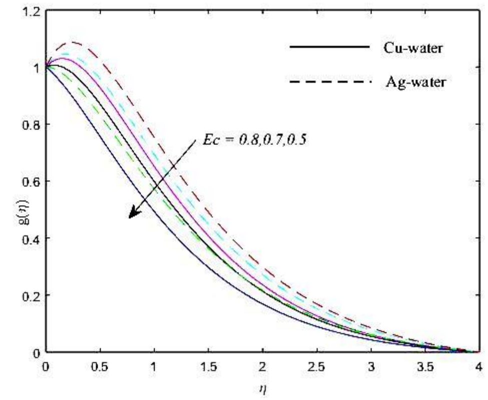

Fig. 5: Influence of the Eckert number Ec on the temperature profile.

3.2 Effect of flow parameters on temperature profile

Figs. 4-7 show the effect of nanoparticle volume fraction (ϕ), Eckert number (Ec), thermal radiation parameter (Rd) and Prandtl number (Pr) on the temperature profile respectively. It is noticed that the temperature of the flow field decreases with the decrease of nanoparticle volume fraction as well as Eckert number whereas with the increase in thermal radiation parameter as well as Prandtl number, the temperature of the flow field decreases for both Cu-water and Ag-water nanofluid. In the same vein we also noticed that the temperature distribution in Ag-water nanofluid is higher than that of Cu-water nanofluid. This is due to the conductivity of silver is more than that of coper. The increase in the value of thermal radiation parameter decreases the gap between the thermal boundary layers. For higher Prandtl number nanofluids, the kinematic viscosity dominates the thermal diffusivity as a result of which the temperature of the flow field is decreasing for both the nanofluids.

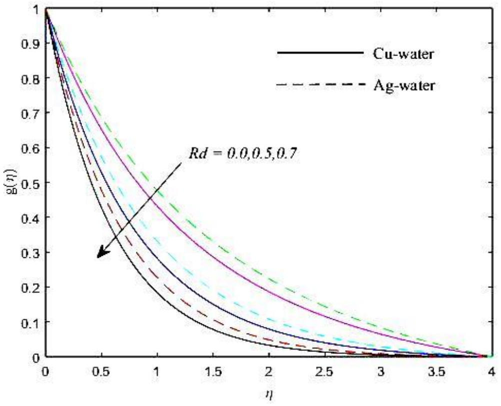

Fig. 6: Influence of the thermal radiation parameter Rd on the temperature profile

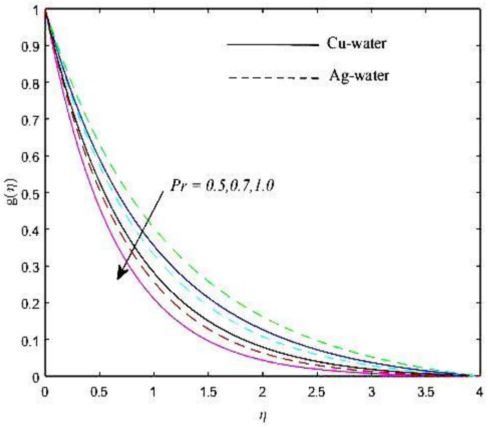

Fig. 7: Influence of the Prandtl number Pr on the temperature profile

Table 3: Effect of M and ϕ on Skin-friction

M	ϕ	$-2f''(0)$	
		Cu-water	Ag-water
0	0.3	2.3614	2.5190
	0.5	1.8871	2.0273
	0.7	1.1820	1.2663
1	0.3	2.6837	2.8238
	0.5	2.0620	2.1917
	0.7	1.2559	2.3363
2	0.3	2.9728	3.1000
	0.5	2.2247	2.3458
	0.7	1.3268	1.4038

Table 3 shows the effect of magnetic parameter (M) and nanoparticle volume fraction (ϕ) on skin friction coefficient. It is observed that the skin friction coefficient increases with the increase in magnetic parameter whereas the reverse effect is noticed with the increase in nanoparticle volume fraction for both the fluids. After observing minutely, it is clear that onset of Lorentz force enhances the skin friction coefficient more rapidly for Cu-water nanofluid than that of Ag-water nanofluid.

Table 4 shows the effect of thermal radiation parameter (*Rd*), nanoparticle volume fraction (ϕ), Eckert number (*Ec*) and Prandtl number (*Pr*) on Nusselt number. It is observed that the rate of heat transfer at the surface of the wall increases with the increase in thermal radiation parameter and Prandtl number but the reverse effect is observed in the case of increase in nanoparticle volume fraction and Eckert number.

Table 4: Effect of *Rd*, ϕ , *Ec* and *Pr* on Nusselt number

Rd	ϕ	*Ec*	*Pr*	$-g'(0)$	
				Cu-water	Ag-water
0.0	0.1	0.3	0.5	0.6999	0.6754
	0.2	0.3	0.5	0.6542	0.6097
	0.1	0.5	0.5	0.6298	0.6022
	0.1	0.3	0.7	0.8474	0.8162
0.5	0.1	0.3	0.5	1.0432	1.0041
	0.3	0.3	0.5	0.8849	0.7786
	0.1	0.7	0.5	0.7864	0.7349
	0.1	0.3	1.0	1.5513	1.4941

4. Conclusions

The effect thermal radiation on magnetohydrodynamics nanofluid flow in presence of viscous dissipation and chemical reaction has been studied. The above study brings out the following conclusion:

i. Onset of Lorentz force decelerates the velocity of both the nanofluids.
ii. Since silver has a higher conductivity compared to copper, the temperature distribution within Ag-water nanofluid surpasses that of Cu-water nanofluid.
iii. As the thermal radiation parameter increases, the separation between the thermal boundary layers diminishes.
iv. The thickness of the concentration boundary layer expands in both nanofluid varieties as the nanoparticle volume fraction rises
v. Heavier diffusing species exert a more pronounced inhibiting impact on the concentration distribution within the flow field.
vi. Onset of Lorentz force enhances the skin friction coefficient more rapidly for Cu-water than that of Ag-water nanofluid.
vii. The mass transfer rate decreases as the temperature difference dominates the concentration with the enhancement of Soret number.

The present analysis neither provides the parametric behavior of the pertinent factors involved in the flow phenomena but also has several aspects in recent scenario. In particular, several production processes in industries where cooling is one of the important criteria to get the better size and shape of the final product. For the cooling of electronic devises, CPU, etc. the nanofluids are useful in avoiding overheating. Further, in biomedical, the drug delivery processes, hyperthermia treatment, peristaltic pumping processes it utilizes greatly. In an advance one can use hybrid nanofluid flow instead of nanofluid to get a comparative study with the present one by implementing several other body forces in the flow phenomena.

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