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GEOMETRICAL EFFECTS OF PITTING CORROSION ON RELIABILITY OF UNSTIFFENED SHIP PANEL

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Abstract:

This research evaluates the reliability of pitted unstiffened ship panels in marine environments. The influence of real pit geometry at the deck and bottom positions of a capsized bulk carrier has been investigated considering both with and without buckling effects. The study explores whether the pit shape observed in hold frame of bulk carrier (ellipsoidal pits) have a significant impact on the reliability index than the pit shape observed in the inner bottom of tankers (conical pits). The effects of structural location, load condition, and buckling effects are explicitly considered to identify potential vulnerabilities arising from geometric influences. Although pitting corrosion has been extensively studied by many researchers, limited research has addressed the reliability implications of actual pit shapes observed from field inspections of ship structures. The results demonstrate that the combined effects of pit geometry, panel location, and buckling behavior significantly influence the time-dependent reliability of ship panels over time. The study provides insights into how real pit geometries influence the time-dependent reliability of ship panels under varying load conditions and locations, aiding structural design optimization and enhancing predictive models for assessing the safety and integrity of marine structures.

Keywords: Time dependent reliability analysis, pitting corrosion, pit geometry, strength reduction, plate buckling

NOMENCLATURE

A_0	Initial/Intact cross-sectional area of panel	k_w	A factor to combine the wave-dynamic composite moments with the still water moment
A_P	Smallest cross-sectional area due to surface pits	k_d	A factor to combine the wave-induced and dynamic moments
$A(t)$	Time-variant minimum cross-sectional area of panel	f_{ps}	Coefficient of strength assessment
B	Moulded breadth of the ship	f_{sw}	Distribution factor along the length of the ship
C	Wave coefficient	n	Navigation coefficient
C_B	Block coefficient of the ship	r_0	Radius of a corrosion pit at X_0
D	Flexural Rigidity	t	Thickness of panel
D_m	Damage	x_r	Model uncertainty factor
E	Modulus of Elasticity	z_0	Depth of a corrosion pit at X_0
L	Length of the ship	***Greek Symbols***	
M_d	Slamming induced bending moment	β	Reliability Index
$M_{wv-h-mid}$	Vertical wave bending moment at midship due to hogging condition	δ_0	Ratio of pit depth to radius
$M_{wv-s-mid}$	Vertical wave bending moment at midship due to sagging condition	μ	Mean value of random variable
$R_u(t)$	Time dependent strength reduction factor	ν	Poisson's ratio
Z	Section modulus	σ	Standard Deviation of random variable
M_{sw}	Stillwater Bending Moment	σ^2	Variance of random variable
W_0	Shape function	σ_0	Average yield stress in compression
M_{wv}	Wave-induced Bending Moment	σ_L	Induced Stress
X	Location of point under consideration	σ_{cr}	Critical buckling stress
X_0	Location of a pit center	σ_u	Ultimate strength

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a	Length of unstiffened panel	σ_y	Yield strength
b	Width of unstiffened panel	τ	Pit depth progression
f_M	A coefficient, depends on type of ship	ϕ	Cumulative Distribution Function

1. Introduction

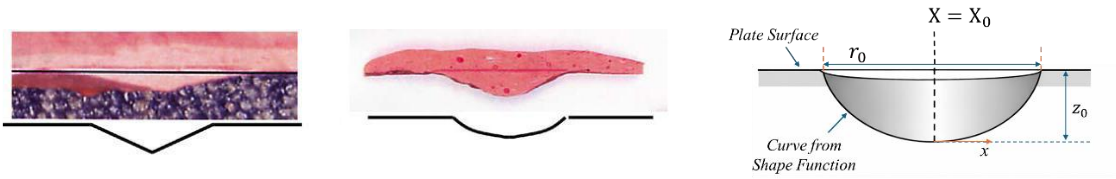
Corrosion is the degradation of materials due to the natural tendency for most of the metals to react with their environment. Statistics on ship hulls indicate that corrosion is the most serious cause that is responsible for the failure of most of the ships and ageing is the main life-limiting factor (Emi et al., 1993). Nouri and Khedmati (2020) concluded that due to pitting corrosion, the reduction in ultimate strength is more pronounced in aged ships, with a relative reduction in ultimate strength ranging from 16% to 32% for ships aged 5 to 25 years, regardless of whether they are in hogging or sagging conditions. Marine environment is remarkably corrosive to ships and other steel structures. All kinds of corrosion are responsible for significant compromise in the integrity of steel structures. General and pitting corrosions are considered the most prevalent forms of corrosion among the basic four types of corrosion (i.e., general corrosion, pitting corrosion, grooving corrosion, and edge corrosion (DNV, 2016). The homogenous loss of plate thickness is referred to as the general corrosion. Because of pitting corrosion, metal deteriorates locally in the forms of pits with the effect of strength and deformability reduction. These reductions are very crucial for the integrity of marine structures (Islam and Sumi, 2011a). Chemistry has no role to play when the effect of corrosion on structural integrity is discussed; rather, it mostly has a geometric influence (Oka et al., 1990). The damage of a structure due to pitting results from a reduction in cross-sectional area, and this damage intensifies with increasing pit depth and the formation of multiple pits in the same cross-section. The shape of the pit also directly affects the extent of damage. Consideration of actual pit shape is thus of great importance to study the effect of pitting corrosion.

The reliability assessment evaluates a structure's ability to perform with the desired performance under the targeted environmental or loading conditions i.e., the probability of survival. Estimation of probability of failure is the main technique of reliability analysis. The deterministic rule-based designs fail to ensure efficiency and safety when uncertainty arises. Consideration of uncertainties is the main merit of reliability analysis. In case of failure probability of an unstiffened ship panel under marine environmental load due to pitting corrosion, uncertainties involve due to random initiation of pit location and random characteristics of material properties and environmental loads (Han et al., 2019). If the reliability of a structure depends on time, then it is called time dependent reliability and so is the case in this study as time is a dominating variable in progression of a pit, i.e., damage increases with the ageing of structure. Hence a time dependent reliability analysis is usually discussed when we bring the topic about pitting corrosion. Therefore, to prevent the failure of ships due to corrosion and ensuring an efficient and safe design of any structure, measuring time dependent reliability can be an alternate solution.

The research gap identified in this study stems from the fact that limited reliability-based research was conducted considering the real pit shape geometry that was observed in hold frames of bulk carriers (conical) and inner bottom plates of tankers (ellipsoidal) in the marine environment. This study aims to fill this gap by performing a time-dependent reliability analysis of unstiffened ship panels considering the variability of these pit shapes under vertical bending loads in a marine environment. Uzun and Aktaş (2021) concluded that buckling of corroded steel members is a critical factor to consider as it affects the structural resistance and consequently greatly influences the probability of failure (P_f) of the structure. For effectively studying the buckling behavior due to in-plane loads in marine environment the unstiffened panels are considered (Das and Zhang, 2003). Therefore, consideration of the buckling effects is one of the most important aspects of this study. The environmental load considered in this study is comprised of still water bending moment and wave induced bending moment in hogging condition whereas slamming induced dynamic moment is also considered in sagging condition. First the ultimate strength of panel is considered as average yield strength with the effect of buckling mode and then disregarding the effect of buckling. The reduction in strength with incremental damage due to pit has been assessed by assuming the linearity of material properties till yielding in case of without buckling analysis whereas an empirical formula proposed by Paik et al. (2003) has been used for the case with buckling analysis. The dimension of the unstiffened panel is 2160 mm × 880 mm × 16 mm and it has been considered that the panel is corroded by two different pit shapes (conical and ellipsoidal). The corrosion model used was suggested by Yamamoto and Ikegami (1998). To perform time dependent reliability analysis, the limit state function and the associated random variables associated with marine environment are obtained and then the First-order Reliability Method (FORM) has been used. The vessel under investigation in this study is the capesized single side skin bulk carrier (ISSC-2000) which has been frequently used as a reference ship in various research studies and the change in reliability index due to real pit geometry (conical and ellipsoidal), and varying load conditions including with or without buckling has been investigated at bottom and deck location for the vessel. The findings of this study are significant for maritime domain as this study provides an in-depth understanding of unstiffened ship panels' behavior under varying load conditions by quantifying failure probability, thereby contribute to the structural design optimization and maintenance of vessel. Furthermore, this study evaluates the level of vulnerability of unstiffened ship panels influenced by real pit geometry and structural location with consideration of buckling effects. This will enhance structural safety and longevity through prediction of failure with respect to time through comparison of target reliability index. This helps naval architects to make informed decisions about structural failure, design optimization, corrosion management, and safety protocols. Therefore, the study contributes to the development of safer and more reliable ships, ultimately enhancing the resilience of marine industry.

2. Pit Geometry in Marine Environment

Nakai et al. (2005, 2004b, 2004a) discovered pit shape with conical cross section in the hold frames of bulk carriers and pit shape with ellipsoidal cross section in the tanker's inner bottom plates. Fig. 1 shows the cross-sectional geometry of the actual pit shapes which were typically observed in their studies. They have conducted series of investigations on real geometric shape of pitting corrosion that was seen on the hold frame of bulk carriers. On the basis of the minimal cross-section of the corroded surface, Paik et al. (2003) developed an empirical formula for predicting the ultimate compressive strength and shear strength of pitted steel plates. The formula was derived using both experimental and numerical analysis on marine plates exposed to pitting corrosion.



(a) (b) (c)

Fig. 1: Cross sectional view of corrosion pit: (a) circular cone shape pit; (b) ellipsoidal shape pit; (c) illustration of shape function parameters

Islam and Sumi (2011b) demonstrated pitting corrosion's effects on the strength and deformability of rectangular steel plates under uniaxial tension and pure bending from the viewpoint of geometrical variation in corrosion pits. They considered the actual pit shapes found on hold frames of bulk carriers and bottom structure of a tanker and used the following two different shape functions for simulating real pit shape:

Conical Shape

$$W_0(X - X_0, r_0, \delta_0) = \delta_0\left(\max\left\{0, r_0 - \sqrt{(x - x_0)^2 + (y - y_0)^2}\right\}\right) \quad (1)$$

Ellipsoidal Shape

$$W_0^2(X - X_0, r_0, \delta_0) = \delta_0^2\left(\max\left\{0, r_0^2 - \{(x - x_0)^2 + (y - y_0)^2\}\right\}\right) \quad (2)$$

The parameters are mentioned in the list of nomenclature. The parameter δ_0 is the ratio of pit depth to radius (z_0/r_0). This ratio is a random variable with normal distribution. The shape functions defined in Equations (1) and (2) represent the local thickness reduction caused by individual corrosion pits. In the conical pit model, the thickness reduction varies linearly with the radial distance from the pit center, resulting in a triangular cross-sectional profile. In contrast, the ellipsoidal pit model produces a smoothly varying curved profile, which leads to a more concentrated material loss near the pit center. These functions enable realistic simulation of pit geometries reported from field observations of ship structures.

The damage data adopted in the present study originate from the investigations of Islam and Sumi (2011b), where actual conical and ellipsoidal pit geometries observed in ship structures were examined. In those studies, the effects of pit geometry on damage progression were quantified statistically through Monte Carlo Simulation. The resulting damage-versus-degree-of-pitting (DOP) relationships form the basis of the corrosion model used in the present research. The present study does not repeat the statistical simulation used to generate those damage data to avoid duplicity; rather, the previously established damage data were utilized as input for the subsequent reliability analysis. Having established the pit geometries and the associated damage characteristics, the methodology used for reliability assessment is presented in the following section.

3. Method of Reliability Analysis

In structural design, reliability plays an important role as it considers the uncertainties associated with loads, material properties and environmental conditions. Structural reliability analysis is essential to ensure that any structure remains safe even when subjected to unfavorable external forces. The reliability of a system is characterized by its limit state function, which serves as a threshold differentiating acceptable and unacceptable performance of the structure. During reliability analysis, various design variables are treated as random variables with probability distributions and the probability of failure of the structural is evaluated in terms of Reliability Index (β). Several methodologies, including First Order Reliability Method (FORM), Second Order Reliability Method (SORM) and Monte Carlo Simulation (MCS) have been employed in various domains to assess the reliability or probability of failure of structures (Fiessler et al., 1979; Hasofer and Lind, 1974). Choi et al. (2021) have analyzed time-dependent reliability of a corroded plate-stiffened prismatic pressure vessel. Investigation has been carried out for the effect of general and pitting corrosion on the outer shell by observing the change in reliability index. They have followed Melchers general and pitting corrosion model for their research and their limit state function was developed based on response surface methodology. Han et al (2019) reported that the reliability of ship plates is compromised by corrosion, which can lead to material loss and reduced load-carrying capacity. They have developed a cellular automata based (CAs-based) pitting corrosion model for cylindrical shape and hence performed time-variant reliability analysis of steel plates in marine environments. Several Ship Structure Committee (SSC) reports have published to emphasize the importance of probabilistic methods in evaluating ship structure reliability. SSC-363 demonstrated the need for probabilistic stress analysis, while SSC-398 introduced limit state functions and target reliability indices for assessing failure modes such as buckling and yielding. SSC-475 focused on ultimate strength and buckling behaviour. Collectively, these reports provide a unified, reliability-based design and assessment framework. Das and Zhang (2003) provided detailed guideline on reliability analysis of marine structures considering various failure modes other than fatigue failure. It is mentioned that the time-dependent reliability of unstiffened panel is influenced by the correlation between environmental loads and material strength, necessitating models that incorporate these dependencies to accurately predict failure probabilities.

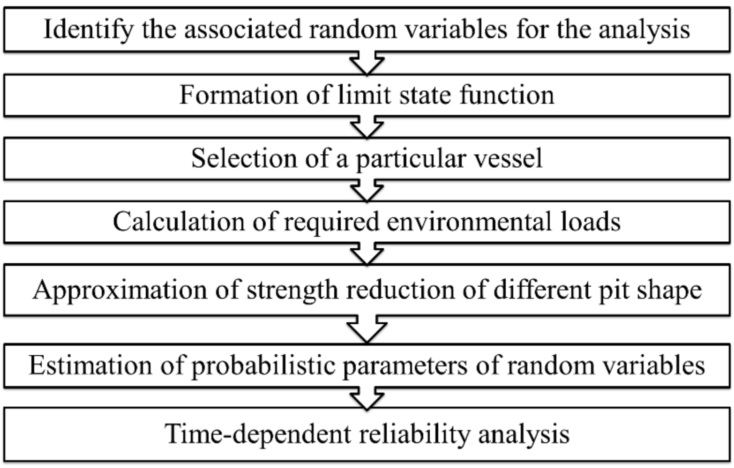


Fig. 2: Flow chart of time-dependents reliability analysis

The reliability assessment performed in this study is based on the interaction between corrosion-induced degradation, structural resistance, and environmental loading. Figure 2 shows the process of structural reliability analysis that has been followed in this study. First several variables associated with the analysis have been identified and then they have been estimated by following appropriate guidelines as discussed in the subsequent sections. These quantities are then incorporated into the limit state functions developed for the target ship panel, from which the time-dependent reliability indices are evaluated using the First-Order Reliability Method (FORM). For the calculation purpose, ISSC-2000 bulk carrier was selected as it satisfies the requirement of this study. Bending loads from marine environment has been estimated for the two furthest possible locations from the neutral axis of a ship section and those are deck and bottom of the selected vessel.

The damage of marine pitted steel plate was previously assessed by Islam and Sumi (2011b) considering the real pit shape (conical and ellipsoidal) geometries. The published damage progression data obtained from that work are adopted in the present study and incorporated into the reliability framework through the time-dependent strength reduction factor and minimum cross-sectional area. The variation in cross section has been accommodated by considering the same ratio of damage in the extended part. Unlike the previous studies, which focused on damage characterization, strength reduction, and deformability of pitted plates; the present work evaluates the time-dependent reliability of unstiffened ship panels subjected to environmental loads using the First-Order Reliability Method (FORM), considering both with-buckling and without-buckling conditions. The probabilistic parameters that involve the uncertainties have been estimated following the guideline of Das and Zhang (2003). The final step is performing time-dependent reliability analysis for both the conical and ellipsoidal pit shape using the limit state function and probabilistic particulars estimated in the previous steps. The first order reliability method (FORM) is used for the analysis as it could provide a balance between accuracy and computational efficiency. The application of the reliability assessment requires the estimation of structural resistance, environmental loading, and their associated uncertainties, which are described in the subsequent sections. Step by step procedure of the flow chart illustrated in Fig. 2 is discussed in the following subsections.

3.1 Limit state function (LSF)

The LSF represents the likelihood that the marine system will not reach the specified limit state within the designated reference timeframe as it acts as boundary between desired and undesired performance of the structure (Chandrasekaran, 2015). A general limit state function for reliability analysis is given by,

$$G(R, Q) = R - Q \tag{3}$$

where, G is the limit state function, R is a random variable function representing the resistance (strength) and Q is a random variable representing the load (response). The limit state function G (R, S) characterizes the structural conditions and defines two domains of safety in relation to the limit surface as follows:

$G(R, Q) > 0$ is the safe domain

$G(R, Q) = 0$ on the limit surface

$G(R, Q) < 0$ in the unsafe domain

In this study, reliability analysis of an unstiffened ship panel is performed considering corrosion by two different types pit shapes at two different crucial locations (bottom and deck). The panel is considered as subjected to compressive stress induced predominantly by vertical still-water bending moment and wave induced bending moments. The performance function g associated with a corroded unstiffened ship panel in compression can be expressed as:

With effect of buckling mode

$$g = x_r . R_u(t) . \sigma_u - \sigma_L \tag{4}$$

Without effect of buckling mode

$$g = x_r . \sigma_y . A(t) - A_0 \sigma_L \tag{5}$$

σ_L is the induced stress and defined by, (Das and Zhang, 2003)

$$\sigma_L = \frac{M_{sw} + k_w M_{wv}}{SM} \qquad \text{(fog hogging condition)}$$

$$\sigma_L = \frac{-M_{sw} + k_w (M_{wv} + k_d M_d)}{SM} \qquad \text{(for sagging condition)}$$

here, x_r is the model uncertainty factor, σ_u is the ultimate strength, R_u is the Strength Reduction Factor, M_{sw} is still water bending moment, k_w is a factor to combine the wave-dynamic composite moments with the still water moment, M_{wv} is wave induced bending moment, SM is the appropriate section modulus depending on desired location, k_d is a factor to combine the wave-induced and dynamic moments. M_d represents the slamming-induced moment which is sagging in nature and only need to be considered in sagging condition. When buckling effect is disregarded (Equation 5), σ_y is the material yield strength, $A(t)$ is the time-variant minimum cross-sectional area and A_0 is the initial cross-sectional area (880×16 mm) of the ship panel.

3.2 First order reliability method (FORM)

The first-order reliability method (FORM) is a generalized form of the advanced first-order second-moment method (AFOSM), initially introduced by Hasofer and Lind (1974). This study uses the First Order Reliability Method (FORM) as this method considers different types of probability distribution functions and the correlation structure of the associated random variables which was a limitation of the previous methods. Moreover, this method provides a balance between accuracy and computational efficiency than other methods for simple structure like unstiffened ship panel. The procedure is as follows (all the random variables in this study are considered independent):

1. The basic random variables are transformed into standard normal variables using the cumulative distribution function (CDF) of each variable:

   $$F_{X_i}(x_i) = \Phi(u_i) \qquad [i = 1,2,3 \dots . N] \tag{6}$$

   The transformation from physical space to standard normal space is given by:

   $$u_i = \Phi^{-1}\left[F_{X_i}(x_i)\right] = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \tag{7}$$

   where, X_i is the value in physical space, μ_{X_i} is the mean, σ_{X_i} is the standard deviation. This transformation normalizes the random variables so that they have, Mean 1 and Standard Deviation 0.

2. The failure surface (limit state) based on the associated random variables of the problem is defined as: $g(X) = g\{F_X^{-1}[(u)]\} = \acute{g}(U) = 0$

3. Most probable failure point (MPFP) is optimized by linearization of failure surface. The coordinates of the optimized MPFP are determined as: $u^* = \beta\alpha^*$. Where β is the reliability index that represents the shortest distance between origin of standardized surface to the failure surface $g(U) = 0$.

4. Finally, the probability of failure is approximated by $p_f = \phi(-\beta)$. Here, Φ is the cumulative distribution function of the standard normal distribution.

The reliability calculations were performed using a custom developed code (Python) implementing the First-Order Reliability Method (FORM). The program incorporates the limit state formulations, probabilistic descriptions of the random variables, environmental load model, and the age-dependent corrosion damage and strength-reduction relationships described in the preceding sections. The reliability index was evaluated for each structural age, pit geometry, structural location, and loading condition considered in the analysis.

3.3 Target reliability index

Conducting a reliability analysis is required when designing a structure to ensure that it can be deemed reliable. To carry out this analysis, a reference or target value is needed. The target reliability index (β_t) indicates the necessary reliability level to ensure safety and serviceability. The target value of reliability index is suggested by various standards and regulations. Specific reliability indices have been recommended for different failure modes in ship structures by Ship Structure Committee (1997). Table 3.2 presents the suggested target reliability index for various failure modes as suggested by them.

Table 1: Recommended Target Reliability Indices (Failure Probabilities)

Failure Mode	**Commercial Ships**	**Naval Ships**
Primary (Initial Yield)	5.0 (2.9×10^{-7})	6.0 (1.0×10^{-9})
Primary (Ultimate)	3.5 (2.3×10^{-4})	4.0 (3.2×10^{-5})
Secondary	2.5 (6.2×10^{-3})	3.0 (1.4×10^{-3})
Tertiary	2.0 (2.3×10^{-2})	2.5 (6.2×10^{-3})

4. Bending Load in Marine Environment

In this study, both hogging and sagging in head sea condition with unrestricted navigation is considered for conducting the reliability analysis. According to Veritas (2022), maximum permissible still water bending moment, M_{sw} is estimated as follows:

$$M_{sw} = f\left(190\frac{1+n}{2}C.L^2.B(C_B+0.7)10^{-3} - M_{wv-h-mid}\right) \quad \text{[for Hogging]} \tag{8}$$

$$M_{sw} = -f\left(190\frac{1+n}{2}\,C.L^2.B(C_B+0.7)10^{-3} + M_{wv-s-mid}\right) \quad \text{[for Sagging]} \tag{9}$$

and, maximum permissible wave-induced bending moment, M_{wv} is estimated as follows:

$$M_{wv-h} = 190\,f\,f_{ps}\,n\,C\,L^2\,B\,C_B\,10^{-3} \quad \text{[hogging]} \tag{10}$$

$$M_{wv-s} = -110\,f\,f_{ps}\,n\,C\,L^2\,B\,(C_B+0.7)10^{-3} \quad \text{[sagging]} \tag{11}$$

where, f is a distribution factor along the ship length (which is 1.00 for midship as for the case of this study), n is the navigation coefficient (equal to 1.00, for unrestricted navigation) C is a wave coefficient, L, B and C_B are ship length, moulded breadth and block coefficient respectively, f_{ps} is the coefficient for strength assessments which is a function of applicable design load (equal to 1.00, for extreme sea design load). $M_{wv-h-mid}$ and $M_{wv-s-mid}$ represents vertical wave bending moment at amidship in hogging and sagging condition respectively. The wave coefficient C can be estimated by:

$$C = (118 - 0.36L)\frac{L}{1000} \quad \text{for } 65\text{ m} \le L \le 90\text{m} \tag{12}$$

$$= 10.75 - \left(\frac{300-L}{100}\right)^{1.5} \quad \text{for } 90\text{ m} \le L \le 300\text{ m}$$

$$= 10.75 \quad \text{for } 300\text{ m} \le L \le 350\text{ m}$$

$$= 10.75 - \left(\frac{L-350}{150}\right)^{1.5} \quad \text{for } L > 350\text{ m}$$

5. Strength of Unstiffened Ship Panel

In the failure analysis of a ship structure, three different modes are usually considered besides fatigue failure (Das and Zhang, 2003). The tertiary mode of failure i.e., with or without the effect of buckling in failure has been considered in this study. The plating of the structure of a ship is divided into relatively small rectangular panels, by means of beams (or longitudinals) and girders (or transverses) (Muckle, 1967). These relatively small rectangular panels between the stiffeners are known as unstiffened ship panels.

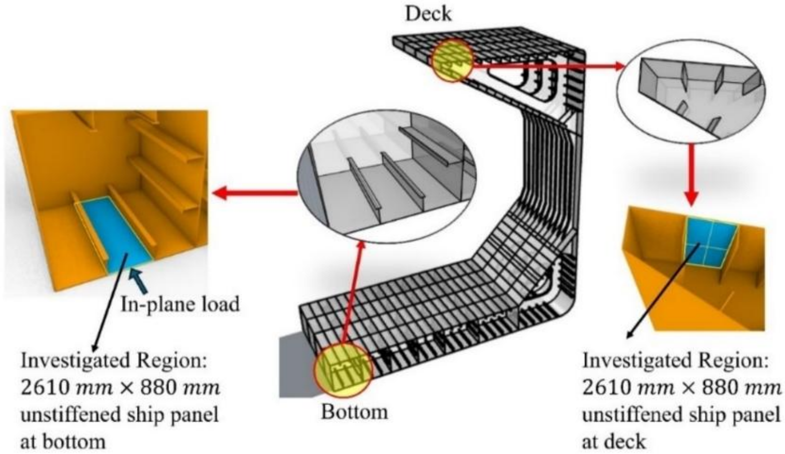


Fig. 3: Schematics for the unstiffened ship panel investigated at midship

In unstiffened panels, the stress acting on the panel arises from the global hull girder bending in form of compressive stress rather than from local bending of the panel itself. Vertical still-water and wave-induced bending moments cause the ship hull girder to bend globally, producing longitudinal normal stresses that vary linearly over the ship depth. Panels located at the deck and bottom, being at the extremities from the neutral axis, experience these stresses primarily as in-plane axial compressive stresses (as illustrated in Fig. 4) under hogging or sagging conditions. Therefore, to investigate the effects of pit geometry on reliability of pitted ship panel, the time-variant reliability analysis has been performed in this study where loads are considered at the bottom and deck of a capsized bulk carrier. The vessel under investigation in this study is the capsized single side skin bulk carrier which has been frequently used as a reference ship in various research studies. This vessel has been analyzed in previous reports by the International Ship Structure Committee (ISSC) in 2000 and 2012. The unstiffened panels under investigation at deck and bottom location of ISSC 2000 bulk carrier at this study are presented in Fig. 3, while Table 2 provided a summary of the main particulars and dimensions. The bulk carrier is stiffened with longitudinal stiffeners with the exception in part of side shells that located between the wing tank and the hopper tank where the vessel is transversely stiffened (Özgüç and Barltrop, 2008).

Table 2: Hull sectional properties of the model ship (ISSC-2000)

Items	**Values**
LBP (m)	285.0
Breadth (m)	50.0
Depth (m)	26.9
Draft (m)	17.0
Block coefficient	0.836
Speed (knots)	15.5
DWT	180,000
Cross-sectional area (m^2)	5.582
Neutral axis above the base line (m)	11.024
I (m^4)	
Vertical	682.110
Horizontal	1,796.763
Z (m^3)	
Deck	42.965
Bottom	61.872

Unstiffened panels have some constraints at their edges (by longitudinals and transverses), which is provided by the torsional rigidity of the stiffening members. Although these stiffeners provide some restraint at the plate edges because of their torsional rigidity, the restraint is generally weak. When the deck or bottom buckles, the beams twist, generating only a small edge-constraining moment. As a result, the panel's edge is only lightly restrained, and its buckling behavior is similar to that of a simply supported plate.

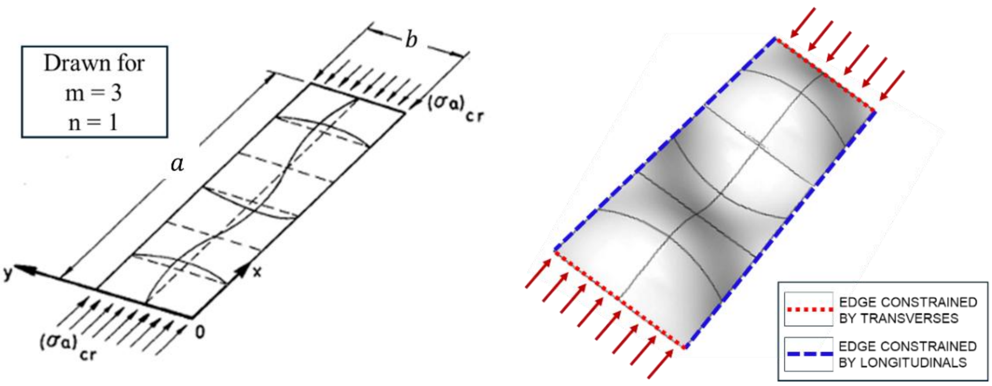


Fig. 4: Form of unstiffened ship panel for critical buckling stress

The critical buckling stress, σ_{cr} for the panel is given by,

$$\sigma_{cr} = \frac{\pi^2 D}{b^2 t}\left(m\frac{b}{a} + \frac{n^2}{m}\frac{a}{b}\right)^2 \tag{13}$$

where, a is the length and b is the breadth of the unstiffened panel and the direction of load being applied is in the direction of the length , flexural rigidity, $D = \frac{Et^3}{12(1-\nu^2)}$, E is the Young's Modulus, t is the thickness of unstiffened ship panel and ν is the poison's ratio. If the least value of the critical buckling stress is required, and this will be seen with n = 1 and minimum value of m = a/b. The equation means that the panel will buckle into m half waves in the length and n half waves in the breadth (illustrated in Fig. 4).

The unstiffened panel used for the study is SM490A steel plate (Islam and Sumi, 2011a), with a yield strength of 365 MPa and Young's Modulus 206.9 GPa. Its composition includes carbon (0.16%), manganese (1.43%), silicon (0.36%), phosphorus (0.017%) and sulfur (0.005%). It is considered that, the unstiffened panel have a length of 2610 mm, breadth of 880 mm and thickness of 16 mm at both deck and bottom location for effectively compare the reliability indices. From Equation (11), half waves m in the direction of length is 3 with a flexural rigidity of 77606.45 N-m. This gives us the critical buckling stress for unstiffened ship panel under investigation as 247 MPa. Therefore, the unstiffened panel will buckle if it faces an in-plane critical stress of 247 MPa. Although 247 MPa is below the yield strength of 365 MPa for the steel, this is expected because buckling is a stability failure that can occur at stresses lower than the yield stress, especially in slender, unstiffened panels.

The ultimate strength (σ_u) of the unstiffened ship panel was formulated by following SSC 398 report where the influence of critical buckling stress has taken into consideration. The strength of an unstiffened plate subjected to in-plane load acting on the plate width can be estimated by:

a. For $\frac{a}{b} \geq 1.0$

$$\frac{\sigma_u}{\sigma_0} = \begin{cases} = \left(\frac{\sigma_{cr}}{\sigma_0}\right)^{\frac{1}{2}} & if\ \beta \geq 3.5 \\ = \frac{2.25}{\beta} - \frac{1.25}{\beta^2} & if\ 1.0 \leq \beta < 3.5 \\ = 1.00 & if\ \beta < 1.0 \end{cases} \tag{14}$$

where, $\frac{\sigma_{cr}}{\sigma_0} = \frac{4\pi^2}{12(1-\nu^2)} \cdot \frac{1}{\beta^2} = \frac{3.612}{\beta^2}$

b. For $\frac{a}{b} < 1.0$

$$\frac{\sigma_u}{\sigma_0} = \alpha C_u + 0.08(1-\alpha)\left(1 + \frac{1}{\beta^2}\right)^2 \leq 1.0 \tag{15}$$

where, $C_u = \begin{cases} = \left(\frac{4\pi^2}{12(1-\nu^2)} \cdot \frac{1}{\beta^2}\right)^{\frac{1}{2}} & if\ \beta \geq 3.5 \\ = \frac{2.25}{\beta} - \frac{1.25}{\beta^2} & if\ 1.0 \leq \beta < 3.5 \\ = 1.00 & if\ \beta < 1.0 \end{cases}$

Here, σ_0 is the average yield stress in compression, $\beta = \frac{b}{t}\sqrt{\frac{\sigma}{E}}$ represents the slenderness ratio of the unstiffened ship panel with σ as the normal stress and $\alpha = a/b$ is the aspect ratio. The average yield strength in compression is considered to be 365 MPa for SM490A steel.

6. Statistical Characteristics of Associated Variables

Consideration of uncertainties is the main key factor behind the reliability concept as said before. Statistical technique is the best tool to quantify, analyze, and manage uncertainties. The level of uncertainty in the magnitude of any variable can be represented by coefficient of variance (COV) and probability distribution which tells about the likelihood of different outcome of an uncertain variable. The uncertain variables that are associated with this analysis are of different types like material properties, environmental loads, geometry of structure (section modulus) etc.

Probability density functions (PDFs) are used to represent the statistical variability of uncertain load and resistance parameters. These PDFs describe the likelihood of different values of bending moments, material strength, and section properties occurring during the service life of the ship, and they form the basis for the reliability analysis. The reliability analysis is performed by combining the probabilistic descriptions of resistance and load effects through the defined limit state function. At each time step, the reduction in resistance due to pitting corrosion is accounted for, while the load effects are described by their respective PDFs. The reliability index is then evaluated using the First-Order Reliability Method (FORM), which identifies the most probable failure point in the standard normal space. The statistical characteristics of the strengths and loads have been schematically represented by

PDF and CDF in Fig. 5 in terms of related mean, COV and distribution. The x-axis represents the variable, while the left y-axis plots the PDF and the right y-axis shows the CDF, which represents the probability of exceedance of a given value.

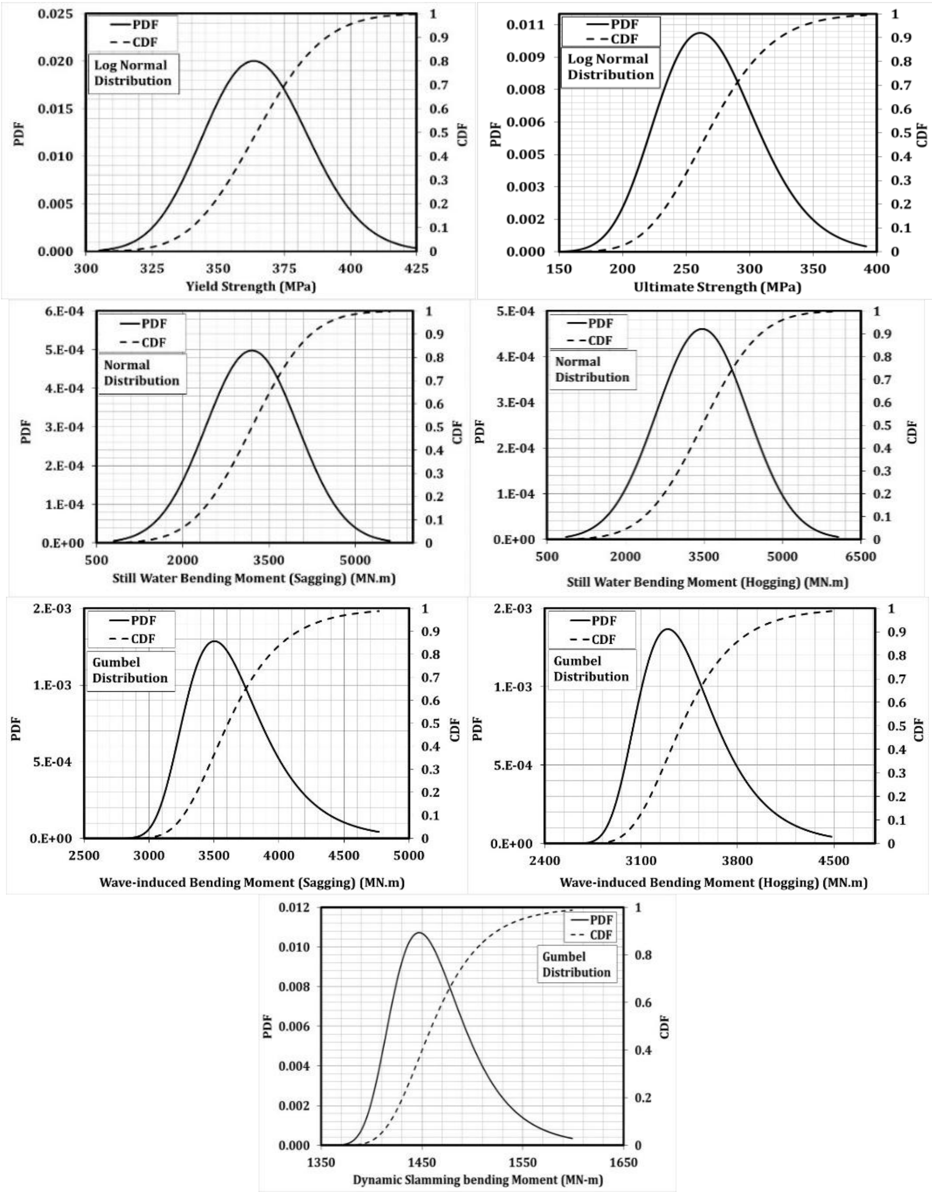


Fig. 5: Statistical characteristics of variables associated with reliability analysis

In the current analysis, the maximum permissible values of different types of bending loads in marine environment for the reference ship have been estimated by following Equations (8) to (12). Then, the mean value, COV and probability distribution for associated variables are determined by following guidelines mentioned in book by Das and Zhang (2003). The suggested distribution and COV value for associated random variables with reliability analysis are summarized in Table 3. Reference values for the still water moments are estimated for maximum permissible condition. For commercial ships, it is set at 60% of the maximum permissible calculated value (Mansour et al, 1994; Ship Structure Committee, 1997). The mean value of the wave-induced bending moment is to be taken as the 50% probability-of-exceedance value of the maximum permissible bending moment. The slamming-induced bending moment is categorized as a sagging moment and is only considered when evaluating the sagging loading scenario. For fuller-formed commercial vessels, it is recommended to use 20% of the maximum wave-induced bending moment (M_{wv}) as mean value of dynamic slamming bending moment. The steel material considered in this study is SM490A having yield strength 365.0 MPa. SSC-368 suggests to consider 15% larger value for these strengths in order to correct for inherent conservation in calculated strengths i.e., minimum yield strength concept in failure criterion. In the current analysis, the suggested increment in strengths has not been considered as the material strength (of SM490A steel) of intact structure is experimentally measured in laboratory. Rather a model uncertainty factor (x_r) has been considered to adhere for the uncertainties involved with the material. The variation in loads of static part is mainly due to change in cargo and ballast over time. The same in the dynamic part is due to uncertainties in the sea model, uncertainties in the ship linear response and uncertainties in the correlation with the full scale measurement (Das and Zhang, 2003). The summary of statistical characteristics of uncertain variables that are involved in this analysis is shown in Table 3.

Table 3: Summary of variables associated with reliability analysis

Variables		Mean	COV	Distribution
Model uncertainty factor (x_r)		1.0	0.15	Normal
Yield Strength (σ_y)		365 MPa	0.05	Log-normal
Ultimate Strength (σ_u)		270 MPa	0.15	Log-normal
Still water bending moment (M_{sw})				
	for sagging condition	3201 MPa	0.25	Normal
	for hogging condition	3465 MPa	0.25	Normal
Wave-induced bending moment (M_{wv})				
	for sagging condition	3668 MPa	0.10	Gumbel
	for hogging condition	3448 MPa	0.10	Gumbel
Section Modulus (SM)				
	deck	42.965 m^3	0.04	Normal
	bottom	61.872 m^3	0.04	Normal
Wave-dynamic composite moments and still water moment combination factor (k_w)		1.0	0.05	Normal
Wave-induced and dynamic moments combination factor (k_d)		0.7	0.15	Normal
Dynamic Slamming bending Moment (M_d)		1467 MPa	0.03	Gumbel

7. Strength Reduction Considering Real Pit Geometry

Paik et al. (2003) derived the following empirical formula for predicting the reduced ultimate strength for compressive load and shear load basing on damage (D_m) where damage is the ratio of maximum diminution in cross sectional area due to surface pit to intact cross-sectional area:

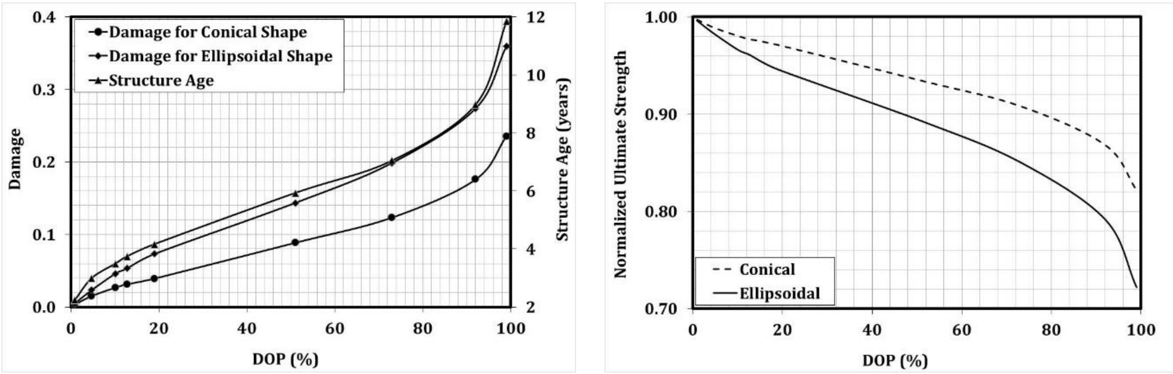
$$R_u = (1 - D_m)^{0.73} \quad (16)$$

Here, R_u is the ultimate strength of pitted plates normalized by that of an intact plate. Ahmmad and Sumi (2010) validate Equation (16) for tensile strength reduction due to conical pit whereas Islam and Sumi (2011a) proves the applicability of the same for ellipsoidal pit. Yamamoto and Ikegami proposed a corrosion model (used in this research) by dividing the total life of a pitted structure into three different life spans and all these lives are random variables (Table 4).

Table 4: Statistical characteristics of variables associated with corrosion model

Life Span	Distribution
Life of paint coating	Log-normal
Transition time between active and progressive pitting	Exponential
Progression of pit depth	Log-normal

Corrosion is a time-dependent phenomenon, where damage progressively accumulates with structure age. To quantify the extent of surface damage caused by pitting corrosion, the affected area is measured through the pitting intensity, commonly expressed as the degree of pitting (DOP). DOP is defined as the ratio of the pitted surface area to the total surface area of the structural component and therefore increases with time as corrosion progresses. Figure 6 is based on the damage progression data adopted from Islam and Sumi (2011b) and the corresponding strength-reduction relationship obtained using the formulation of Paik et al. (2003). As shown in Fig. 6(a), the progression of DOP with structural age exhibits a distinct nonlinear trend. During the early service period (0–3 years), DOP increases slowly, indicating pit initiation. This is followed by an accelerated growth phase (3–6 years), during which damage accumulates rapidly. At advanced ages, DOP approaches a near-saturation level (≈90–99%), corresponding to severe damage due to pitting. In the current study, two types of pit shapes (conical and ellipsoidal) with the same level of pitting (DOP) as well as same spatial pit distribution have been considered. The change in damage with structure age for various pit shapes and strength reduction for same spatial pit distribution & same corrosion model are shown in Figs. 6(a)-6(b). Accordingly, the damage data derived from the DOP % (structure age) for each pit shape is used as the basis for all subsequent strength reduction and reliability analyses.



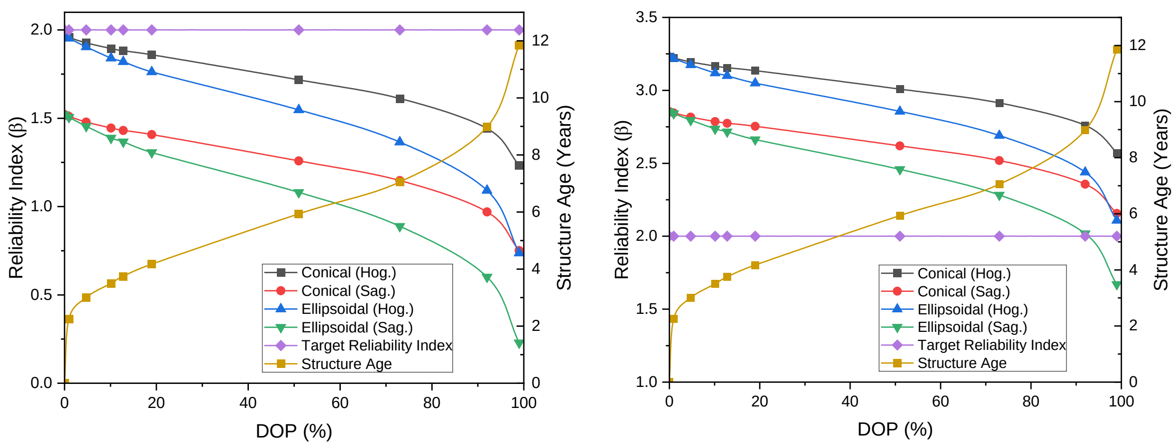
(a) (b)

Fig. 6: Geometrical effect of pitting corrosion on (a) damage (b) strength reduction

This comparison of strength reduction in Figure 6(b) highlights the greater impact of ellipsoidal pits on structural strength, attributed to their higher amount of material degradation during corrosion. As the reliability analysis directly depends on the available structural resistance, these differences in damage and strength reduction are expected to influence the reliability indices presented in the subsequent section. At initial DOP levels, the difference is minimal. But as DOP approaches 50% and beyond, the reduction in strength is accelerated more significantly for ellipsoidal pits compared to conical pits. The persistent higher damage caused by ellipsoidal pits highlights the necessity of assessing structural vulnerability from the reliability viewpoint.

8. Effect of Pit Geometry on Reliability Index

The study explores whether the pit shape observed in the hold frames of bulk carriers (conical) have a significant impact on the reliability index than the pit shape observed in the inner bottom plates of tankers by other researchers, considering changes in structural location, load conditions and buckling effects, while identifying potential vulnerabilities arising due to these factors.



(a) (b)

Fig. 7: Pit shape effect on reliability of unstiffened ship panel (with buckling effect) (a) at deck, (b) at bottom

The variation in reliability due to geometrical effects of two types of pit shape (conical and ellipsoidal) on ship unstiffened panel with buckling effect for both the hogging and sagging moment at deck is shown in Fig. 7(a) and at bottom is shown in Fig. 7(b). Under sagging moments, the reliability index exhibits consistently lower values compared to hogging moments for both pit geometries. This highlights the increased structural demand under sagging moments. The slamming induced bending moment plays the vital role behind this severity of sagging condition.

Fig. 8 compares the decrease in reliability index (β) for the real pit shapes effect on unstiffened ship panel between deck and bottom considering sagging moment. It is observed that unstiffened ship panel at deck is less reliable than the bottom and most severe case will arise for the ellipsoidal pit shape. At DOP 0% (age 0 years), the reliability index at the bottom plate under sagging is 2.85 for both conical and ellipsoidal pits, meeting the target reliability index of 2.0 and demonstrating acceptable structural safety. However, at the deck plate, the reliability index for both conical and ellipsoidal pits is 1.52. This vulnerability of deck arises due to the higher distance from the neutral axis as can be seen in Table 2. This emphasizes the necessity for structural design modifications at the deck to account for tensile stresses and the location's distance from the neutral axis.

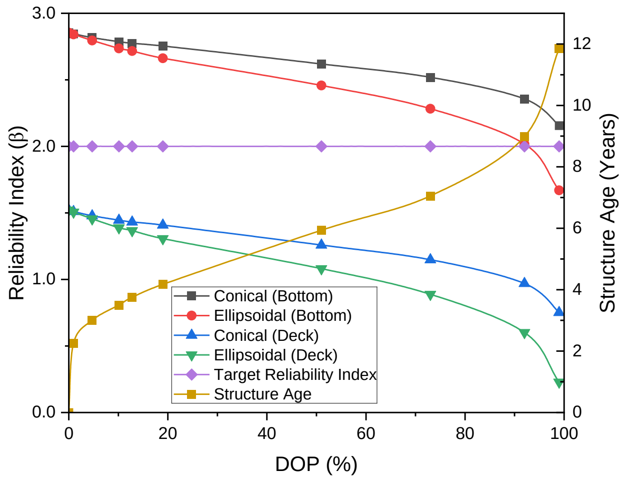


Fig. 8: Comparison of pit shape effect between deck and bottom for sag. moment (with buckling effect)

As the structure ages and DOP increases, a clear divergence in reliability indices becomes evident. By DOP 90% (Age ~9 years), the bottom plate reliability index under sagging for conical pits remains at 2.35, just above the target reliability index, while the ellipsoidal pit reliability index drops to 2.01, nearing the threshold. At DOP 99% (Age ~12 years), with conical pits reaching 2.16 and ellipsoidal pits dropping sharply to 1.67, ellipsoidal shape pit at the bottom plate fall below the target reliability index. This indicates an immediate need for maintenance as it approaches the latter stages of its operational lifespan. While the bottom plate maintains an acceptable level of safety for a longer duration, the deck plate exhibits significant deficiencies early in its operational life, especially for ellipsoidal pits. This reinforces the need for targeted design modifications at the deck and periodic inspections and maintenance to ensure continued structural integrity and compliance with safety standards.

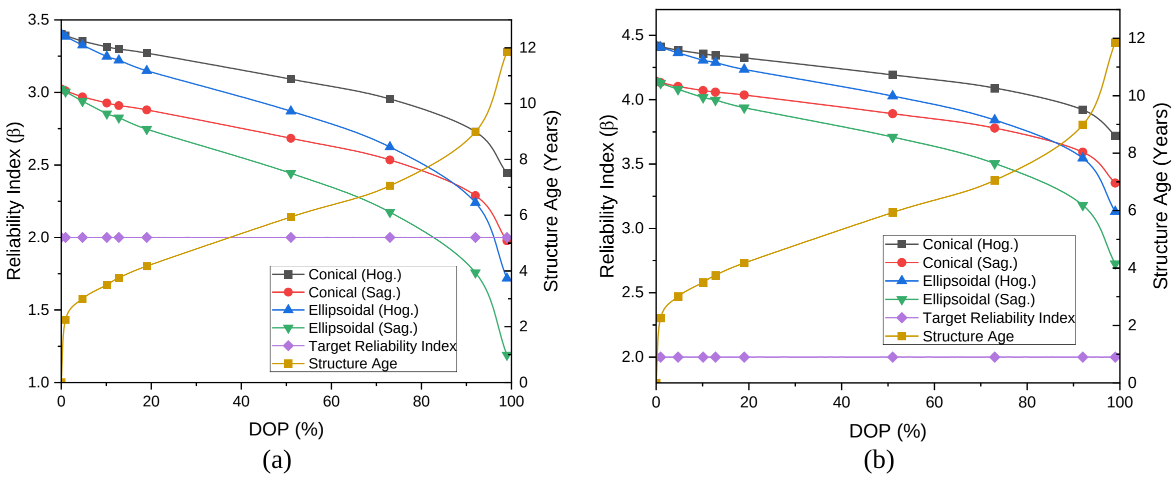


Fig. 9: Pit shape effect on reliability of unstiffened ship panel (without buckling effect) (a) at deck, (b) at bottom

Figs. 9(a)-9(b) schematically represent the condition of reliability with the real pit shape effect where now the failure of unstiffened ship panel is considered without buckling effect. Reliability index values are higher in these figures than Figs. 8(a)-8(b). For without buckling effects, we can also see the same trend that under the sagging moments, the reliability index exhibits consistently lower values compared to hogging moments for both pit geometries.

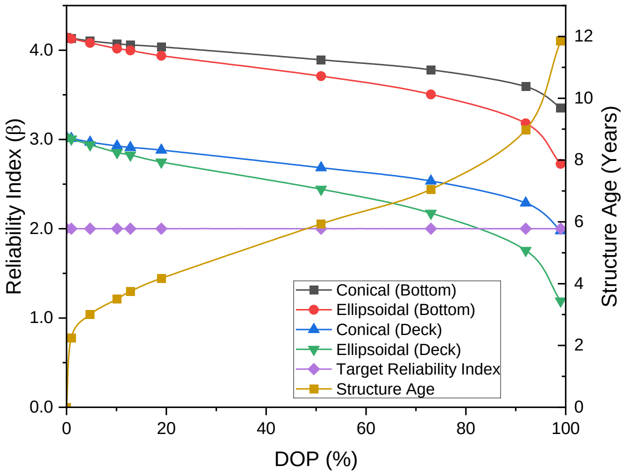


Fig. 10: Comparison of pit shape effect between deck and bottom for sag. moment (without buckling effect)

A comparison between pit shape effects between deck and bottom for sagging moment without buckling effect is also illustrated in Fig. 10. Without buckling the bottom plate maintains an acceptable safety margin across the operational lifespan. This is a desirable result as the buckling effect is disregarded in the latter case. At DOP 84% (Age ~8 years), the bottom plate reliability index for both pits remain comfortably above β_t, while the ellipsoidal pit reliability index drops to 2.01. At DOP 99% (Age ~12 years), the bottom plate under sagging maintains reliability indices of 3.35 for conical pits and 2.73 for ellipsoidal pits, both of which exceed the target reliability index. Conversely, the deck plate reliability index drops to 1.98 for conical pits and 1.19 for ellipsoidal pits. In these cases, it is also observed that deck location is more vulnerable with the ellipsoidal pit effect. The reliability index for ellipsoidal pits at the deck plate fall well below the recommended threshold, indicating the need for maintenance and design improvement at this structural location if it is corroded with ellipsoidal pit shape.

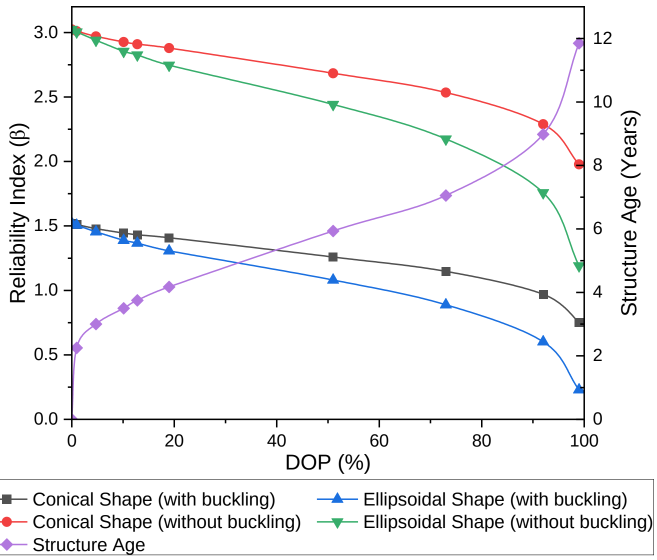


Fig. 11: Effect of buckling load on steel pitted plate with the variation in pit shape

The analysis result for buckling effect is visible in Fig. 11. Without initiation of pitting the reliability index is 47% less when buckling effect is considered in failure mode. The reduction is 58% in the case of conical pit and 75% in case of ellipsoidal pit at DOP 100%. For all the cases ellipsoidal pit shape shows more vulnerability. This is reasonable as the material degrades more due to ellipsoidal shape than the conical shape even for the case of same pit spatial distribution with the same rate of change in pit diameter. The strength formulation adopted in this study follows the approach originally proposed by Paik et al. (2003), in which the degradation of panel strength is expressed primarily as a function of the minimum effective cross-sectional area and overall damage severity, rather than edge sharpness. This results in a more pronounced reduction of effective stiffness and cross-sectional properties of ellipsoidal pit shape.

9. Validation of the study

In reliability analysis, the focus is on assessing the probability of failure using a probabilistic framework that accounts for uncertainties in material properties, pit geometries, and loading conditions. In terms of validation, this approach differs from experimental validation, which typically involves deterministic testing of specific samples under controlled scenarios. This study utilizes empirical models, such as empirical strength reduction formula proposed by Paik et al. (2003), probabilistic corrosion model proposed by Yamamoto and Ikegami (1998), stochastic modeling of random variables and advanced probabilistic method like FORM to evaluate the reliability of unstiffened ship panels. While experimental validation provides valuable insights for certain studies, its direct applicability is limited for this research, as the focus lies on stochastic modeling and probabilistic evaluation.

However, a comparative literature validation can support the credibility of reliability analysis method and considerations used for the scope of this study. For literature validation of methodology and results of the current study, the work by Han et al. (2019) based on cellular automata (CA) algorithm of corrosion modeling can be referred. Their study focused on a corroded unstiffened plate of a crude carrier ship subjected to cylindrical shaped corrosion, using a section modulus of 66.3 m^3 at bottom position, under sagging condition, without considering the buckling effect. The comparison between the initial results of their work and this work is presented in Fig. 12.

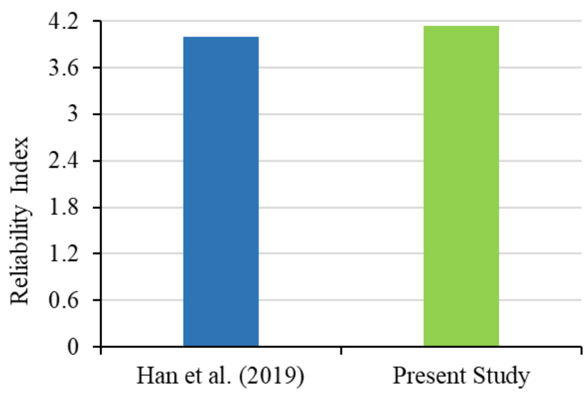


Fig. 12: Comparison of Reliability Index at 0% DOP

The initial reliability index (at year 0) under sagging condition was observed to be similar to this study. The latter stages cannot be compared as they are using different pit geometry (cylindrical) and corrosion model. This study shows a slightly higher initial reliability index due to differences in section modulus, material properties, and magnitude of the load. This similarity at the initial stage (DOP 0%) serves as a literature validation for the methodology and results of this research, even though this study provides additional insights by incorporating buckling effects, location and different pit geometries (conical and ellipsoidal).

10. Conclusion

In this study the effect of conical pit shape and ellipsoidal pit shape on time variant reliability of pitted unstiffened ship panel has been analyzed by using First Order Reliability Method. The concluding remarks from this study are as follows:

a) Material degradation as well as strength reduction depends on pit shape and so consideration of real pit shape is an essential part in reliability analysis of ship structure;

b) Ellipsoidal pit shape shows more vulnerability than the conical pit shape as the material degrades more due to ellipsoidal shape even for the case of same pit spatial distribution with the same rate of change in pit diameter. At DOP 99% with buckling effects considered, the reliability index at the bottom location decreased from 2.16 for conical pits to 1.67 for ellipsoidal pits, while at the deck location it decreased from 2.15 to 0.23. These results demonstrate that the actual pit geometry has a significant influence on the predicted safety margin of the structure;

c) Still water bending moment and wave induced bending moment take part in total moment of hogging condition whereas an extra component namely slamming induced moment takes part in total moment of sagging condition making the sagging moment more vulnerable form reliability point of view;

d) Reliability of unstiffened pitted ship panel depends on the location with respect to the location of neutral axis especially when bending load is concerned. At the intact condition with buckling effects considered, the reliability index at the deck was approximately 1.52 compared with 2.85 at the bottom location, indicating substantially greater vulnerability of structural members located farther from the neutral axis under vertical bending loads;

e) Buckling mode severely affect the ultimate strength of an unstiffened ship panel which in turn has remarkable effect on magnitude of reliability index. The inclusion of buckling reduced the reliability index by approximately 47% at the intact condition and by up to 75% for highly corroded plates with ellipsoidal pits. This highlights the necessity of incorporating buckling behaviour in reliability-based assessment of marine structures;

f) Reliability index comparison with the recommended target value is essential in reliability analysis to identify level of vulnerability arises from various structural location and identify areas requiring immediate attention.

Among all investigated cases, the combination of deck location, sagging loading condition, ellipsoidal pit geometry, and buckling effects produced the lowest reliability indices and therefore represents the governing failure scenario for the considered vessel. Overall, the study demonstrates that the reliability of pitted unstiffened ship panels is governed not only by the degree of corrosion but also by the combined effects of pit geometry, structural location, loading condition, and buckling behaviour. By aligning design and maintenance strategies with these insights, stakeholders in the maritime industry can ensure the safety, longevity, and efficiency of marine structures, even in challenging environments.

CRediT Author Statement

S. M. Usham: Methodology, Formal Analysis, Investigation, Visualization, Validation, Writing – Original draft preparation **M. R. Islam:** Supervision, Conceptualization, Investigation, Writing – Reviewing and Editing, **M. T. Ali:** Investigation, Writing – Reviewing and Editing

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