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HEAT GENERATION EFFECTS ON MHD BOUNDARY LAYER FLOW OF A MOVING VERTICAL PLATE WITH SUCTION

B. Rushi Kumar¹ and K. Gangadhar²

¹Fluid Dynamics Division, School of Advanced Sciences, Vellore Institute of Technology, Vellore, T.N, India– 632014, Email: <u>rushibkumar@gmail.com</u>

²Department ²Department of Mathematics, Acharya Nagarjuna University, Ongole Campus, Ongole, A.P, India – 523001, Email: <u>kgangadharmaths@gmail.com</u>

Abstract:

This work is focused on the numerical solution of a two-dimensional, steady, incompressible electrically conducting, laminar free convection boundary layer flow of a continuously moving vertical porous plate in the presence of a transverse magnetic field and heat generation. The basic equations governing the flow are in the form of partial differential equations and have been reduced to a set of non-linear ordinary differential equations by applying suitable similarity transformations. The problem is solved numerically using shooting techniques with the forth order Runge-Kutta method. The physical behavior of different parameters for velocity, temperature and concentration has been examined graphically and analyzed quantitatively.

Keywords: MHD, free convection, heat generation, suction, moving vertical plate.

NOMENCLATURE

а	stratification rate of the gradient of ambient temperature profiles	и, v	dimensionless velocity components		
b	stratification rate of the gradient of ambient concentration profiles	F_w	dimensionless suction velocity		
В	constant	C_{f}	local skin friction coefficient		
f	dimensionless stream function	c_{n}	specific heat at constant pressure		
8	acceleration due to gravity	Greek symbols			
Gr	local temperature Grashof number	β_r	coefficient of thermal expansions		
Gc	local concentration Grashof number	β_{c}	coefficient of concentration expansions		
B_0	magnetic induction	η^{c}	dimensionless similarity variable		
Q_0	volumetric rate of heat generation/absorption	$f(\eta)$	dimensionless stream function		
\overline{D}_{m}	mass diffusivity	$\theta(\eta)$	dimensionless temperature of the fluid		
M	magnetic parameter	$\phi(\eta)$	dimensionless concentration of the fluid		
Q	heat source/sink parameter	μ, ν	dynamic and kinematic viscosities		
Sc	Schmidt number	ho	density of the fluid		
Pr	Prandtl number	σ	electrical conductivity		
Т	temperature	$ au_{}$	shearing stress		
С	concentration	$\psi^{"}$	stream function		
<i>C</i>	free stream concentration	α	thermal diffusivity		

1. Introduction

Boundary layer behavior over a moving continuous solid surface is an important type of flow occurring in a number of engineering processes. To be more specific, heat treated materials traveling between a feed roll and a wind-up roll, aerodynamic extrusion of plastic sheets, glass fiber and paper production, cooling of an infinite

metallic plate in a cooling path, manufacturing of polymeric sheets are examples for practical applications of continuous moving flat surfaces. Since the pioneering work of Sakiadis (1961) various aspects of the problem have been investigated by many authors. Mass transfer analysis at the stretched sheet were found in the studies by Erickson et al. (1966) and relevant experimental results were reported by Tsou et al. (1967) regarding several aspects for the flow and heat transfer boundary layer problems in a continuously moving sheet. Crane (1970) and Grubka (1985) have analyzed the stretching problem with constant surface temperature, while Soundalgekar (1974) investigated the Stokes problem for a viscoelastic fluid wall temperature and heat flux. Raptis and Singh (1985) studied flow past an impulsively started vertical plate in a porous medium by a finite difference method. The fluid considered in that paper is an optically dense viscous incompressible fluid of linearly varying temperature dependent viscosity. Ambethkar(2008) studied numerical solutions of heat and mass transfer effects of an unsteady MHD free convective flow past an infinite vertical plate with constant suction.

Alam and Rahman (2006) studied the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. They also investigated MHD free convective heat and mass transfer flow past an inclined surface with heat generation. Salem (2006) discussed coupled heat and mass transfer in Darcy-Forchheimer mixed convection from a vertical flat plate embedded in a fluid saturated porous medium under the effects of radiation and viscous dissipation. Alam and Rahman (2008) analyzed the effects of chemical reaction and thermophoresis on MHD mixed convective heat and mass transfer flow along an inclined plate in the presence of heat generation/absorption with viscous dissipation and joule heating. Paresh Vyas and Ashutosh Ranjan (2010) discussed the dissipative MHD boundary- layer flow in a porous medium over a sheet stretching nonlinearly in the presence of radiation. Muthuraj and Srinivas(2009) studied the influence of magnetic field and wall slip conditions on steady flow between parallel flat wall and a long wavy wall with Soret effect.

The heat source/sink effects in thermal convection are significant where there may exist high temperature differences between the surface (e.g. space craft body) and the ambient fluid. Heat generation is also important in the context of exothermic or endothermic chemical reaction. Tania et al (2010) has investigated the effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Furthermore, Moalem (1976) studied the effect of temperature dependent heat sources taking place in electrically heating on the heat transfer within a porous medium. Vajravelu and Nayfeh (1992) reported on the hydro magnetic convection at a cone and a wedge in the presence of temperature dependent heat generation or absorption effects. Moreover, Chamkha (1999) studied the effect of heat generation or absorption on hydro magnetic three-dimensional free convection flow over a vertical stretching surface.

Gupta et al (1977) studied heat and mass transfer on a stretching sheet with suction or blowing. Ibrahim and Makinde (2010) have investigated the effects of chemically reacting MHD boundary layer flow of heat and mass transfer over a moving vertical plate with suction. This paper aims to find numerical solutions of the coupled equations that govern the flow by using shooting technique with the forth order Range-Kutta method. In the problem formulation, the continuity, momentum, energy and concentration equations are reduced to some parameter problem by introducing suitable transformation variables. Pertinent results with respect to embedded parameters are displayed graphically for the velocity, temperature and concentration profiles and were discussed quantitatively. The local skin-friction coefficient and the heat and mass transfer results are obtained for representative values of the important parameters.

2. Mathematical Formulation

Consider a two-dimensional free convective flow on the steady incompressible laminar MHD heat and mass transfer characteristics of a linearly started porous vertical plate, the velocity of the fluid far away from the plate surface is assumed zero for a quiescent state fluid. The flow configurations are linear. All the fluid properties are assumed to be constant except for the density variations in the buoyancy force term of the linear momentum equation. The magnetic Reynolds number is assumed to be small, so that the induced magnetic field is neglected. The Hall effects, the viscous dissipation and the joule heating terms are also neglected. The flow configuration is shown in Fig. A. Then under Boussinesq's approximations, the governing boundary-layer equations that are based on the balance laws of mass, linear momentum, energy and concentration species for this investigation can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u + g\beta_r(T - T_\infty) + g\beta_c(C - C_\infty)$$
(2)



Fig. A: Physical model and coordinate system

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty)$$
(3)

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2}$$
(4).

The boundary conditions at the plate surface and for into the cold fluid may be written as

$$v = V, \quad u = Bx, \quad T = T_w = T_\infty + ax, \quad C = C_w = C_\infty + bx \quad \text{at } y = 0$$

 $u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty$ (5)

The non-dimensional governing equations and boundary conditions can be obtained from Equations (1) - (5) using the following non-dimensional quantities

$$\eta = y \sqrt{\frac{B}{\upsilon}}, \quad \psi = x \sqrt{\upsilon B} f(\eta), \quad \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad \phi(\eta) = \frac{C - C_{\infty}}{C_w - C_{\infty}}, \quad M = \frac{\sigma B_0^2}{\rho B},$$
$$Gr = \frac{g \beta_r (T_w - T_{\infty})}{x B^2}, \quad Gc = \frac{g \beta_c (C_w - C_{\infty})}{x B^2}, \quad Pr = \frac{\upsilon}{\alpha}, \quad Sc = \frac{\upsilon}{D_m}, \quad Q = \frac{Q_0}{\rho c_p B}, \quad F_w = \frac{V}{\sqrt{Bv}}$$
(6)

The velocity components u and v are respectively obtained as follows:

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$

with this new set of independent and dependent variables defined by Equation (6), the partial differential equations (2) to (4) are transformed in to local similarity equations as follows:

$$f''' + ff'' - f'(f' + M) + Gr\theta + Gc \phi = 0$$
(8)

$$\theta'' + \Pr f \theta' - \Pr f' \theta + \Pr Q \theta = 0 \tag{9}$$

$$\phi'' + Scf \phi' - Scf' \phi = 0 \tag{10}$$

The corresponding boundary conditions (5) then take the following form

$$f'(0) = 1, f(0) = -Fw, \theta(0) = 1, \phi(0) = 1$$
⁽¹¹⁾

$$f'(\infty) = 0, \theta(\infty) = \phi(\infty) = 0 \tag{12}$$

where prime denotes partial differentiation with respect to η .

Skin-friction: Skin-friction coefficient at the moving vertical plate is given by

$$C_{f} = \frac{\tau_{w}}{\rho c (cv)^{\frac{1}{2}}} = xf''(0)$$

Where $\tau_w = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)_{y=0}$ is the shear stress at the moving vertical plate.

3. Numerical Method of Solution

The set of coupled non-linear governing boundary layer equations (8)-(10) together with the boundary conditions (11&12) are solved numerically by using Runge-Kutta fourth order technique along with shooting method. First of all, higher order non-linear differential Equations (8)-(10) are converted into simultaneous linear differential equations of first order and they are further transformed into initial value problem by applying the shooting technique (Alam et al. (2006)). In a shooting method, the missing (unspecified) initial condition at the initial point of the interval is assumed, and the differential equation is then integrated numerically as an initial value problem to the terminal point. The accuracy of the assumed missing initial condition is then checked by comparing the calculated value of the dependent variable at the terminal point with its given value there. If a difference exists, another value of the missing initial condition must be assumed and the process is repeated. This process is continued until the agreement between the calculated and the given condition at the terminal point is within the specified degree of accuracy. For this type of iterative approach, one naturally inquires whether or not there is a systematic way of finding each succeeding (assumed) value of the missing initial condition.

Table 1: Variation of $f''(0)$,	$\theta'(0)$ and $\phi'(0)$ at the	plate with Gr , Gc , M ,	F_{w} , Sc for Pr=0.72, Q=0.

Gr GC			Б	~	Ibrahim and Makinde (2010)			Present work		
0/	00	М	F_{w}	Sc	<i>f</i> "(0)	$-\theta'(0)$	-¢'(0)	<i>f</i> "(0)	$-\theta'(0)$	-¢'(0)
0.1	0.1	0.1	0.1	0.62	0.888971	0.7965511	0.7253292	0.894849	0.801534	0.735005
0.5	0.1	0.1	0.1	0.62	0.695974	0.8379008	0.7658018	0.702698	0.839303	0.770631
1.0	0.1	0.1	0.1	0.62	0.475058	0.8752835	0.8020042	0.481438	0.875411	0.804742
0.1	0.5	0.1	0.1	0.62	0.686927	0.8421370	0.7701717	0.694923	0.842284	0.773517
0.1	1.0	0.1	0.1	0.62	0.457723	0.8818619	0.8087332	0.465707	0.88064	0.809833
0.1	0.1	1.0	0.1	0.62	1.264488	0.7089150	0.6400051	1.2681	0.726867	0.665555
0.1	0.1	3.0	0.1	0.62	1.868158	0.5825119	0.5204793	1.87106	0.625405	0.573521
0.1	0.1	0.1	1.0	0.62	0.570663	0.5601256	0.5271504	0.573969	0.563931	0.534166
0.1	0.1	0.1	3.0	0.62	0.275153	0.2955702	0.2902427	0.006165	0.013604	013586
0.1	0.1	0.1	0.1	0.78	0.893454	0.7936791	0.8339779	0.898284	0.899983	0.838183
0.1	0.1	0.1	0.1	2.62	0.912307	0.7847840	1.6504511	0.916155	0.793118	1.64847

The resultant initial value problem is solved by employing Runge-Kutta fourth order technique. The step size $\Delta \eta = 0.05$ is used to obtain the numerical solution with decimal place accuracy as the criterion of convergence. From the process of numerical computation, the skin-friction coefficient, the Nusselt number and the Sherwood number, which are respectively proportional to $f''(0), -\theta'(0)$ and $-\phi'(0)$, are also sorted out and their numerical values are presented in a tabular form.

4. Comparison with Previous Work

A comparison of the plate surface temperature $\theta(0)$ and the local heat transfer rate at the plate surface $\theta'(0)$ in the absence of both magnetic field and buoyancy forces for various values of M with those of Ibrahim and Makinde and found them in excellent agreement as shown in Table 1.

Table 2: Variation of f''(0), $\theta'(0)$ and $\phi'(0)$ at the plate with Q for $Gr = Gc = M = F_w = 0$, Sc = 0.62, Pr=0.72.

Q	F "(0)	$-\theta'(0)$	-¢'(0)
0.1	0.894849	0.801534	0.735005
0.5	0.881538	0.459877	0.740885
1.0	0.981438	1.54751	0.678902
1.5	0.920881	0.626364	0.716893
2.0	0.899371	0.0302118	0.726039
3.0	0.727373	6 10174	0.77133

From Table 2, it is important to note that the local skin friction together with the local heat and mass transfer rate at the moving plate surface increases with increasing intensity of buoyancy forces (Gr, Gc), the Schmidt number (Sc), However, an increase in the magnetic field (M), magnitude of fluid suction (Fw), heat source/sink parameter (Q) causes a decrease in both skin friction and surface heat transfer rate and an increase in the surface mass transfer rate.



Fig. 1: Variation of velocity with M



Fig. 2: Variation of velocity with Gr

5. Results and Discussion

The governing equations (8)-(10) subject to the boundary conditions (11)-(12) are integrated as described in section 3. The Prandtl number was taken to be Pr=0.72 which corresponds to air, the value of Schmidt number (Sc) were chosen to be Sc=0.24,0.62, 0.78,2.62, representing diffusing chemical species of most common interest in air like H_2 , H_2O , NH_3 and Propel Benzene respectively. Paying attention on positive value of the Heat generation effects on MHD boundary layer flow of a moving vertical plate... 157

buoyancy parameters that is, local temperature Grashof number Gr > 0 and local concentration Grashof number Gc > 0. The effects of various parameters on velocity profiles in the boundary layer are shown in Figs. 1-6. It is noticed from Figs. 1-6, that the velocity is higher near the moving vertical plate surface and decrease to its zero value far away from the moving vertical plate surface satisfying the far field boundary condition for all parameter values.



Fig. 3: Variation of velocity with Gc



Fig. 4: Variation of velocity with Fw



Fig. 5: Variation of velocity with Sc

Fig. 6: Variation of velocity with Q

In Fig. 1 the effect of increasing the magnetic field strength on the momentum boundary layer thickness is illustrated. It is now a well established fact that the magnetic field presents a damping effect on the velocity field by creating drag force that opposes the fluid motion, causing the velocity to decease. However, in this case an increase in the M only slightly slows down the motion of the fluid away from the moving vertical plate surface towards the free stream velocity, while the fluid velocity near the moving vertical plate surface decreases. Figs. 2, 3, 4 & 6 depict the variation of the boundary-layer velocity with the buoyancy forces parameters (Gr, Gc), magnitude of fluid suction (Fw) and heat source/sink parameter (Q). In both cases an upward acceleration of the fluid in the vicinity of the vertical wall is observed with increasing intensity of buoyancy forces. Further downstream of the fluid motion decelerates to the free stream velocity. Fig. 5 shows that a slight decrease in the fluid velocity with an increase in the Schmidt number (Sc).

In general the fluid temperature attains its maximum value at the moving vertical plate surface and decreases exponentially to the free stream zero value away from the plate satisfying the boundary condition. This is observed in Figs. 7-12. From these figures, it is interesting to note that the thermal boundary layer thickness

decreases with an increase in the intensity of the buoyancy forces (Gr, Gc). Moreover, the fluid temperature increases with an increase in the Schmidt number (Sc), Magnetic field (M), heat source/sink parameter (Q) and magnitude of fluid suction (Fw) leading to an increase in thermal boundary layer thickness.



Fig. 7: Variation of temperature θ with M



Fig. 9: Variation of temperature θ with Gc



Fig. 11: Variation of temperature θ with Sc



Fig. 8: Variation of temperature θ with Gr



Fig. 10: Variation of temperature θ with Fw



Fig. 12: Variation of temperature θ with Q



Fig. 13: Variation of concentration ϕ with M



Fig. 15: Variation of concentration ϕ with Gc



Fig. 17: Variation of concentration ϕ with Sc



Fig. 14: Variation of concentration ϕ with Gr



Fig. 16: Variation of concentration ϕ with Fw



Fig. 18: Variation of concentration ϕ with Q

Figs. 13-18 illustrate chemical species concentration profiles against span wise coordinate η for varying values physical parameters in the boundary layer. The species concentration is highest at the moving vertical plate surface and decrease to zero far away from the moving vertical plate satisfying the boundary condition. From these figures, it is important to reveal that the concentration boundary layer thickness decreases with an increase

in, the buoyancy forces (Gr, Gc), Schmidt number (Sc), heat source/sink parameter (Q). Moreover, the fluid concentration increases with an increase in the magnetic field (M) and magnitude of fluid suction (Fw) leading to an increase in thermal boundary layer thickness.

7. Conclusion

In this paper the effect of heat generation on MHD boundary layer flow of a moving vertical flat plate with suction have been studied numerically. Shooting method along with fourth order Runge-Kutta algorithm is employed to integrate the equations governing the flow. Comparison with previously published work is performed and excellent argument has been observed. From the present numerical investigation, following conclusions may be drawn:

- For increased value of magnetic parameter, the velocity profile decreases but the temperature and concentration profile increases slightly.
- The thermal and concentration boundary layer thickness decreases with an increase in the intensity of the buoyancy forces Gr and Gc.
- The fluid temperature and concentration increases leading to an increase in thermal boundary layer thickness.
- An increase in wall suction increases the boundary layer thickness and decreases the skin friction.
- In general, the presence of the heat generation term in the energy equation yields an augment in the fluid's temperature
- The velocity, concentration distribution within the boundary layer decreases with the increase in values of the Schmidt number.

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