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CONJUGATE EFFECTS OF TEMPERATURE SENSITIVE VISCOSITY AND THERMAL CONDUCTIVITY ON FREE CONVECTION FLOW ALONG A WAVY SURFACE WITH HEAT GENERATION

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Abstract:

The effects of internal heat generation on natural convection flow with temperature dependent variable viscosity along a uniformly heated vertical wavy surface have been investigated. The governing boundary layer equations are first transformed into a non-dimensional form using suitable set of dimensionless variables. The resulting nonlinear system of partial differential equations are mapped into the domain of a vertical flat plate and then solved numerically employing the implicit finite difference method, known as Keller-box scheme. Numerical results of the surface shear stress in terms of skin friction coefficient and the rate of heat transfer in terms of local Nusselt number, the stream lines as well as the isotherms are shown graphically for a selection of parameters set consisting of viscosity variation parameter ε, thermal conductivity parameter γ, heat generation parameter Q and Prandtl number Pr. Numerical results of the local skin friction coefficient and the rate of heat transfer for different values are presented in tabular form and graphically.

Keywords: Natural convection, internal heat generation, uniform surface temperature, wavy surface, thermal conductivity, variable viscosity.

NOMENCLATURE

C_{fx}	Local skin friction coefficient
C_p	Specific heat at constant pressure [J/kg/K]
f	Dimensionless stream function
g	Acceleration due to gravity [m/s^2]
Gr	Grashof number
k	Thermal conductivity [W/m/K]
k_∞	Thermal conductivity of the ambient fluid [W/m/K]
L	Characteristic length associated with the wavy surface [m]
$\bar{n}$	Unit normal to the surface
Nu_x	Local Nusselt number
P	Pressure of the fluid [N/m^2]
Pr	Prandtl number
H	Dimensionless similarity variable
θ	Dimensionless temperature function
ψ	Stream function [m^2/s]
μ	Viscosity of the fluid [kg/m/s]
μ_∞	Viscosity of the ambient fluid
Q	Heat generation parameter
Q_0	Heat generation constant
q_w	Heat flux at the surface [W/m^2]
T	Temperature of the fluid in the boundary layer [K]
T_w	Temperature at the surface [K]
T_∞	Temperature of the ambient fluid [K]
u, v	Dimensionless velocity components along the (x, y) axes [m/s]
x, y	Axis in the direction along and normal to the tangent of the surface

Greek symbols

α	Amplitude of the surface waves
β	Volumetric coefficient of thermal expansion [K^{-1}]
ε	Variable viscosity
ν	Kinematic viscosity [m^2/s]
ρ	Density of the fluid [kg/m^3]
σ_0	Electrical conductivity
τ_w	Shearing stress

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1. Introduction

It is necessary to study the heat transfer from an irregular surface because irregular surfaces are often present in many applications such as flat-plate solar collectors, heat exchanger etc. It is often encountered in heat transfer devices to enhance heat transfer. The natural convection heat transfer from an isothermal vertical wavy surface was first studied by Yao (1983) using an extended Prantdl's transposition theorem and a finite-difference scheme. He proposed a simple transformation to study the natural convection heat transfer from isothermal vertical wavy surfaces, such as sinusoidal surface.

Moulic and Yao (1989) also investigated mixed convection heat transfer along a vertical wavy surface showing that the force convection component of the heat transfer contains two harmonics. The amplitude of the first harmonic is proportional to the amplitude of the wavy surface and the natural convection component is a second harmonic, with a frequency twice that of the wavy surface. Alam et al. (1997) have also studied the problem of free convection from a wavy vertical surface in presence of a transverse magnetic field. Combined effects of thermal and mass diffusion on the natural convection flow of a viscous incompressible fluid along a vertical wavy surface have been investigated by Hossain and Rees (1999). Wang and Chen (2001) investigated transient force and free convection along a vertical wavy surface in micropolar fluid. Hossain et al. (2002) have studied the problem of natural convection of fluid with temperature dependent viscosity along a heated vertical wavy surface. Natural and mixed convection heat and mass transfer along a vertical wavy surface have been investigated by Jang et al. (2003, 2004). Recently, Molla et al. (2004) have studied natural convection flow along a vertical wavy surface with uniform surface temperature in presence of heat generation/absorption. Tashtoush and Al-Odat (2004) investigated magnetic field effect on heat and fluid flow over a wavy surface with a variable heat flux. Natural convection along a vertical complex wavy surface (2006) has been studied by Yao and found that the total heat-transfer rates for a complex surface are greater than that of a flat plate and enhancement of heat-transfer rate depend on the ratio of amplitude and wavelength of a surface. Parveen and Alim (2011) and Parveen and Alim (2012) investigated Joule heating effect on Magnetohydrodynamic natural convection flow along a vertical wavy surface. They have studied the effects of Joule heating and temperature dependent viscosity on MHD flow along wavy surface and shown that local skin friction coefficient and the rate of heat transfer reduces significantly for higher values magnetic field strength and Joule heating effect. Alim et al. (2011) have studied the effects of temperature dependent thermal conductivity on natural convection flow along a vertical wavy surface with heat generation. They found that the frictional force at the wall enhances and rate of heat transfer reduces when internal heat generation is higher whereas the local rate of heat transfer and skin friction both rise up near the leading edge due to higher values of thermal conductivity. Hossain et al. (2000) investigated the flow of viscous incompressible fluid with temperature dependent viscosity and thermal conductivity past a permeable wedge with uniform surface heat flux.

The present study is to incorporate the idea of the effects of temperature sensitive viscosity and thermal conductivity on free convection flow along a wavy surface with heat generation. Numerical results of the streamlines and isotherms, local skin friction coefficient and the rate of heat transfer are shown graphically and some selected results have been presented in tabular form and then discussed.

2. Method of Solution

The governing boundary layer equations are transformed into a non-dimensional form and the resulting non-linear system of partial differential equations is reduced to local non-similar partial differential forms by adopting appropriate transformations. The transformed boundary layer equations are solved numerically using in-house FORTRAN code based on Keller box method described by Keller (1978) and later by Cebeci and Bradshaw (1984) along with Newton's linearization approximation.

3. Formulation of The Problem

It is assumed that the surface temperature of the vertical wavy surface T_w is uniform, where $T_w > T_\infty$, T_∞ is the ambient temperature of the fluid. The boundary layer analysis outlined below allows $\bar{\sigma}(\bar{x})$ being arbitrary, but our detailed numerical work assumed that the surface exhibits sinusoidal deformations. The wavy surface may be described by

$$\bar{y}_w = \bar{\sigma}(\bar{x}) = \alpha \sin\left(\frac{n\pi\bar{x}}{L}\right) \qquad (1)$$

where L is the characteristic length associated with the wavy surface.

The geometry of the wavy surface and the two-dimensional cartesian coordinate system are shown in Figure 1.

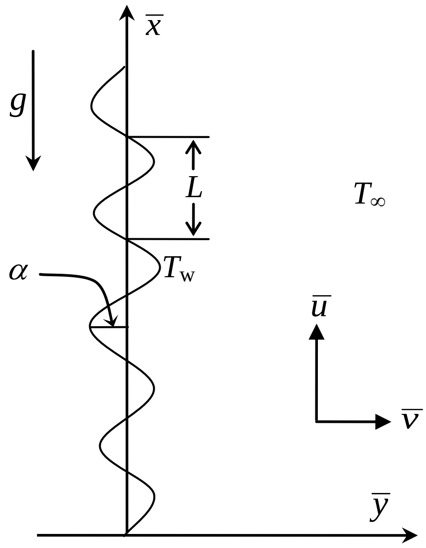

Fig.1: Physical model and coordinate system

Under the Boussinesq and boundary layer approximations, the governing equations for continuity, momentum and energy take the following forms:

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{2}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{1}{\rho}\nabla\cdot(\mu\nabla\bar{u}) + g\beta(T - T_\infty) \tag{3}$$

$$\bar{u}\frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{v}}{\partial \bar{y}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{y}} + \frac{1}{\rho}\nabla\cdot(\mu\nabla\bar{u}) \tag{4}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{k}{\rho C_p}\nabla^2 T + \frac{Q_0(T - T_\infty)}{\rho C_p} \tag{5}$$

where $(\bar{u}, \bar{v})$ are the velocity components parallel to $(\bar{x}, \bar{y})$, g is the acceleration due to gravity, $\bar{p}$ is the dimensional pressure of the fluid, ρ is the density, β is the coefficient of thermal expansion, $\mu(T)$ is the dynamic viscosity, $k(T)$ is the thermal conductivity of the fluid in the boundary layer region depending on the fluid temperature $T, \nu\ (= \mu/\rho)$ is the kinematics viscosity and C_p is the specific heat due to constant pressure.

The boundary conditions relevant to the above problem are

$$\begin{aligned} &\bar{u} = 0,\ \bar{v} = 0,\ T = T_w \quad at \quad \bar{y} = \bar{y}_w = \bar{\sigma}(\bar{x}) \\ &\bar{u} = 0,\ T = T_\infty,\ \bar{p} = p_\infty\, as \quad \bar{y} \to \infty \end{aligned} \tag{6}$$

where p_∞ is the pressure of fluid outside the boundary layer.

The variable viscosity chosen in this study that is introduced by Charraudeau (1975) and used by Hossain et al. (2002) as follows:

$$\mu = \mu_\infty[1 + \varepsilon^*(T - T_\infty)] \quad \text{and} \quad k = k_\infty[1 + \gamma^*(T - T_\infty)] \tag{7}$$

where μ_∞ is the viscosity of the ambient fluid, k_∞ is the thermal conductivity of the ambient fluid and

$\varepsilon^{*}=\frac{1}{\mu_f}\left(\frac{\partial\mu}{\partial T}\right)_f$ is a constant evaluated at the film temperature of the flow $T_f=1/2(T_w+T_\infty)$.

The above equations are further non-dimensionalised using the new variables:

$$x=\frac{\bar{x}}{L},\quad y=\frac{\bar{y}-\bar{\sigma}}{L}Gr^{\frac{1}{4}},\ p=\frac{L^2}{\rho\nu^2}Gr^{-1}\bar{p}$$
$$u=\frac{\rho L}{\mu}Gr^{-1/2}\bar{u},\quad v=\frac{\rho L}{\mu}Gr^{-1/4}(\bar{v}-\sigma_x\bar{u}) \tag{8}$$
$$\sigma_x=\frac{d\bar{\sigma}}{d\bar{x}}=\frac{d\sigma}{dx},Gr=\frac{g\beta(T_w-T_\infty)}{\nu^2}L^3,\theta=\frac{T-T_\infty}{T_w-T_\infty}$$

where L is the characteristic length associated with the wavy surface, θ is the non-dimensional temperature function and (u, v) are the dimensionless velocity components parallel to (x, y), Gr is the Grashof number.

Introducing the above dimensionless dependent and independent variables into Equations (2)–(5) lead to the following non-dimensional equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{9}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+Gr^{1/4}\sigma_x\frac{\partial p}{\partial y}+\theta+\left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}+\varepsilon\left(1+\sigma_x^2\right)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} \tag{10}$$

$$\sigma_x\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-Gr^{1/4}\frac{\partial p}{\partial y}-\sigma_{xx}u^2\sigma_x\left(1+\sigma_x^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}+\varepsilon\sigma_x\left(1+\sigma_x^2\right)\frac{\partial\theta}{\partial y}\frac{\partial u}{\partial y} \tag{11}$$

$$u\frac{\partial\theta}{\partial x}+v\frac{\partial\theta}{\partial y}=\frac{1}{\Pr}\left(1+\sigma_x^2\right)(1+\gamma\theta)\frac{\partial^2\theta}{\partial y^2}+\frac{1}{\Pr}\left(1+\sigma_x^2\right)\gamma\left(\frac{\partial\theta}{\partial y}\right)^2+Q\theta \tag{12}$$

where $\Pr=\frac{C_p\mu}{k_\infty}$ is the Prandtl number, $Q=\frac{Q_0L^2}{\mu c_p Gr^{1/2}}$ is the heat generation parameter and

$\varepsilon=\varepsilon^{*}(T_w-T_\infty)$ is the thermal conductivity variation parameter.

It can easily be seen that the convection induced by the wavy surface is described by Equations (9)–(12). We further notice that, Equation (11) indicates that the pressure gradient along the y-direction. For the present problem this pressure gradient ($\partial p/\partial x=0$) is zero. Equation (11) further shows that $Gr^{1/4}\partial p/\partial y$ is $O(1)$ and is determined by the left-hand side of this equation. Thus, the elimination of $\partial p/\partial y$ from Equations (10) and (11) leads to the following equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\left(1+\sigma_x{}^2\right)(1+\varepsilon\theta)\frac{\partial^2 u}{\partial y^2}-\frac{\sigma_x\sigma_{xx}}{1+\sigma_x{}^2}u^2+\varepsilon\left(1+\sigma_x{}^2\right)\frac{\partial u}{\partial y}\frac{\partial\theta}{\partial y}+\frac{1}{1+\sigma_x{}^2}\theta \tag{13}$$

The corresponding boundary conditions for the present problem then turn into

$$\left.\begin{aligned}&u=v=0,\ \theta=1\ \text{at}\ \ y=0\\&u=v=0,\ \theta=0\ \text{as}\ \ y\to\infty\end{aligned}\right\} \tag{14}$$

Now we introduce the following transformations to reduce the governing equations to a convenient form:

$$\psi=x^{3/4}f(x,\eta),\quad \eta=yx^{-1/4},\quad \theta=\theta(x,\eta) \tag{15}$$

where $f(\eta)$ is the dimensionless stream function, η is the pseudo similarity variable and ψ is the stream function that satisfies the equation (9) and is defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{16}$$

Introducing the transformation Equation (15) into Equations (13) and (12) the following system of non linear equations are obtained:

$$\left(1+\sigma_x{}^2\right)(1+\varepsilon\theta) f''' + \frac{3}{4} ff'' + \varepsilon\left(1+\sigma_x{}^2\right) f''\theta' - \left(\frac{1}{2} + \frac{x\sigma_x\sigma_{xx}}{1+\sigma_x^2}\right) f'^2 + \frac{1}{1+\sigma_x{}^2}\theta = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{17}$$

$$\frac{1}{\Pr}\left(1+\sigma_x{}^2\right)\theta'' + \frac{3}{4} f\theta' + Q\theta x^{\frac{1}{2}} = x\left(f'\frac{\partial \theta}{\partial x} - \theta'\frac{\partial f}{\partial x}\right) \tag{18}$$

The boundary conditions (14) now take the following form:

$$\left.\begin{array}{ll} f(x,o) = f'(x,o) = 0, & \theta(x,o) = 1 \\ f'(x,\infty) = 0, & \theta(x,\infty) = 0 \end{array}\right\} \tag{19}$$

where prime denote the differentiation with respect to η.

The quantities of physical interest are the shearing stress τ_w in terms of the skin-friction coefficients C_{fx} and the rate of heat transfer in terms of Nusselt number Nu_x which can be written as:

$$C_{fx} = \frac{2\tau_w}{\rho U^2} \text{ and } Nu_x = \frac{q_w x}{k_\infty (T_w - T_\infty)} \tag{20}$$

$$\text{where } \tau_w = (\mu \quad \bar{n}.\nabla\bar{u})_{y=0} \text{ and } q_w = -k(\bar{n}.\nabla T)_{y=0} \tag{21}$$

Using the transformations (15) into Eq. (20), the local skin friction coefficient C_{fx} and the rate of heat transfer in terms of the local Nusselt number Nu_x takes the following form:

$$C_{fx}(Gr/x)^{1/4}/2 = (1+\varepsilon)\sqrt{1+\sigma_x^2}\; f''(x,o) \tag{22}$$

$$Nu_x (Gr/x)^{-1/4} = -(1+\gamma)\sqrt{1+\sigma_x^2}\,\theta'(x,o) \tag{23}$$

Finally, it should be mentioned that for the computational purpose the period of oscillations in the waviness of this surface has been considered to be π .

4. Results and Discussion

Numerical values of local shearing stress and the rate of heat transfer are calculated from Equations (22) and (23) in terms of the skin-friction coefficient C_{fx} and Nusselt number Nu_x respectively for a wide range of the values of axial distance x starting from the leading edge for different values of the relevant parameters such as conductivity parameter γ, the viscosity variation parameter ε, heat generation parameter Q and Prandtl number Pr and these are shown in tabular form in Table 1 and graphically in Figures 2-7. It is observed from Table 1 that as the Prandtl number Pr increases, both skin friction coefficient C_{fx} and the rate of heat transfer Nu_x decrease but skin friction coefficient C_{fx} and the rate of heat transfer Nu_x enhances for higher values of conductivity variation parameter γ with $Q = 0.1$ whereas C_{fx} and Nu_x behave quite different with larger values of γ when with $Q = 0.6$. In this case both skin friction and the rate of heat transfer decrease with the larger values of γ this is due to the combined effects of γ and Q. It is also noted further that frictional force at the wall, which is characterized by C_{fx} , enhances and the rate of heat transfer Nu_x reduces for higher values of heat generation parameter Q , which is physically realizable as fluid temperature rise and temperature difference of solid surface and fluid reduce for higher values of heat generation parameter Q and eventually follows the result just mentioned.

Table 1: Skin friction coefficient C_{fx} and the local rate of heat transfer Nu_x when x = 3.0 for the variation of Prandtl number Pr, heat generation parameter Q and conductivity variation parameter γ with ε = 2.0 and α = 0.2.

Pr	Q = 0.1, γ = 1.0		Q = 0.1, γ = 5.0	
	C_{fx}	Nu_x	C_{fx}	Nu_x
0.72	1.26877	0.22751	1.45292	0.48649
2.0	1.09461	0.21168	1.26688	0.58468
5.0	0.96342	0.05399	1.10855	0.53211
7.0	0.92394	-0.07286	1.05586	0.43993
9.0	0.89802	-0.20609	1.01892	0.33040
Pr	Q = 0.6, γ = 1.0		Q = 0.6, γ = 5.0	
	C_{fx}	Nu_x	C_{fx}	Nu_x
0.72	2.00746	-1.26376	1.93249	-1.54543
2.0	2.16103	-3.20544	1.92987	-3.49289
5.0	2.58944	-8.49525	2.02643	-8.02480
7.0	2.87554	-12.67246	2.10952	-11.21048
9.0	3.15765	-17.37374	2.19488	-14.53360

The effects of temperature dependent thermal conductivity parameter γ (= 0.0 and 6.0) and viscosity variation parameter ε (= 0.0 and 0.5) on the development of streamlines are displayed in Figure 2 for the amplitude of the wavy surface α = 0.3 and Prandtl number Pr = 0.72. For γ = 0.0 and ε = 0.0, the thermal conductivity and viscosity are independent of temperature as shown in Fig. 2(a) and found that ψ_{max} , the maximum value of stream function ψ is 15.517 which occurs away from the leading edge. From Figures 2(a) and (b) we found no significant change in the boundary layer thickness due to the increase in the values of ε but ψ_{max} changes slightly and becomes 16.467. Figures 2(c) and (d) show that ψ_{max} reduces from 24.122 to 20.556 as ε changes from 0.0 to 5.0 with the fixed value of γ = 6.0. It is also observed from the figures 2(a) to 2(d) that ψ_{max} increases for higher values of γ but no significant changes have been found in the boundary layer thickness. The effects of temperature dependent thermal conductivity parameter γ and viscosity variation parameter ε on the isotherms are displayed in Figures 3(a)- 3(d) for the amplitude of the wavy surface α = 0.3 and Prandtl number Pr = 0.72. It is observed from these figures that the temperature within the boundary layer decrease for higher values of thermal conductivity variation parameter γ and the temperature rise for larger values of viscosity variation parameter ε, no significant changes have been found in the thermal boundary layer thickness.

The variation of the local skin friction coefficient C_{fx} and local rate of heat transfer Nu_x for different values of Prandtl number Pr for Q = 0.4, ε = 2.0, γ = 1.0 and α = 0.3 are shown in Figures 4(a) and 4(b) while α = 0.2. It is observed from these figures that the skin friction coefficient C_{fx} decreases till the position of x = 1.09590 and from that position of x the skin friction coefficient C_f cross the sides and then the skin friction coefficient C_f increases and the rate of heat transfer Nu_x decreases for increasing values of the Prandtl number Pr. Figures 5(a) and 5(b) display the results on the skin friction coefficient C_{fx} and the rate of heat transfer Nu_x for different values of viscosity variation parameter ε while Pr = 0.72. Q = 0.4, γ = 1.0 and α = 0.3. It is observed from these figures that the skin friction coefficient C_{fx} increases and the rate of heat transfer Nu_x decreases for larger values of viscosity variation parameter ε . The effects for different values of thermal conductivity parameter γ, the skin friction coefficient C_{fx} and the rate of heat transfer Nu_x while Prandtl number Pr = 0.72, heat generation parameter Q = 0.4, viscosity variation parameter ε = 2.0 and amplitude of the wavy surface α = 0.3 are shown in Figures 6(a) and 6(b). It has been seen from Figure 6(a) that as γ increases, the skin friction coefficient C_{fx} increases up to the certain position of x and cross the sides in between x = 2.50172 and x = 3.01250 from that position of x skin friction coefficient C_{fx} decreases for increasing values of thermal conductivity parameter γ. In Figure 6(b) is shown that the rate of heat transfer Nu_x increases up to the certain position of x from that position of x the rate of heat transfer Nu_x decreases for increasing values of thermal conductivity parameter γ.

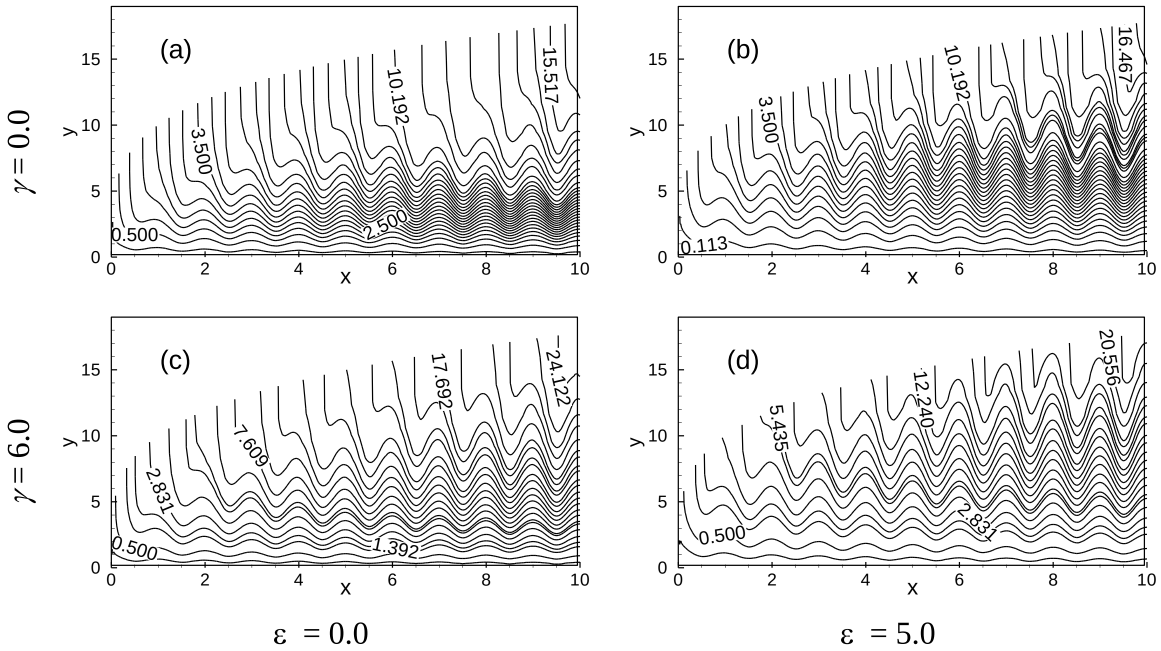

Figure 2: Streamlines for (a) ε = 0.0, γ = 0.0, (b) ε = 5.0, γ = 0.0, (c) ε = 0.0, γ = 6.0 and (d) ε = 5.0, γ = 6.0 while Pr = 0.72, Q = 0.4 and α = 0.3.

Figures 7(a) and 7(b) display the results on the skin friction coefficient C_{fx} and the rate of heat transfer Nu_x for different values of heat generation parameter Q with ε = 1.0, α = 0.3 , γ = 1.0 and Pr = 0.72. It is observed from these figures that the skin friction coefficient C_{fx} increases and the rate of heat transfer Nu_x decrease gradually from zero value at lower stagnation point along the *x*-direction for increasing values of heat generation parameter Q(= 0.0, 0.2, 0.4, 0.7, 0.9).

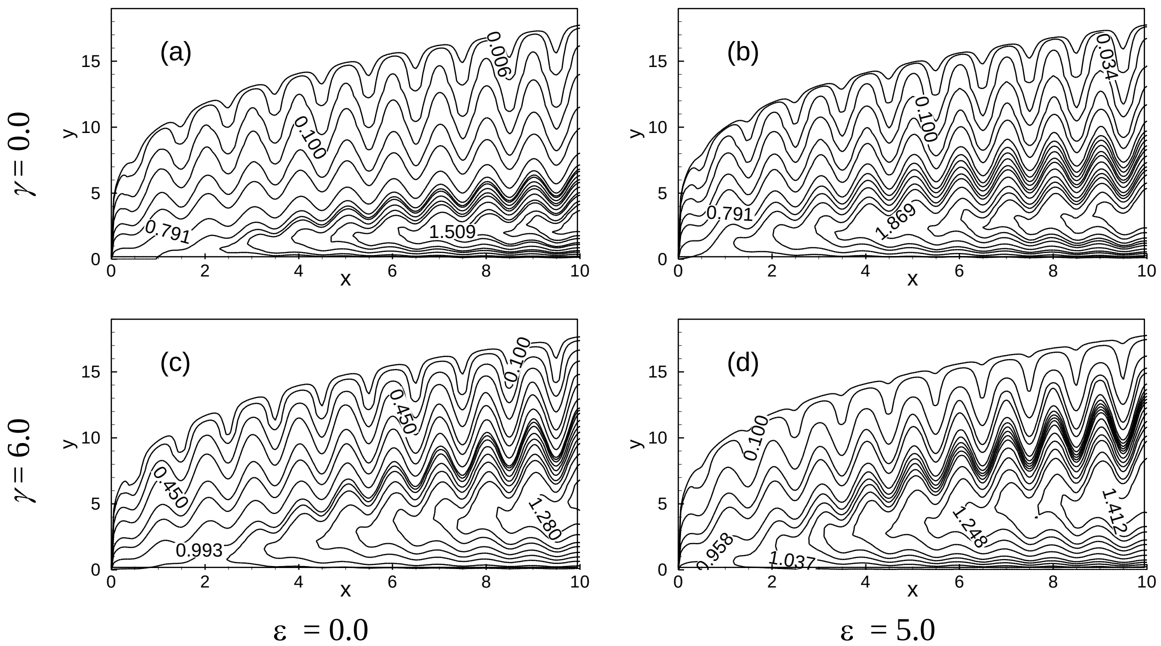

Figure 3: Isotherms for (a) ε = 0.0, γ = 0.0, (b) ε = 5.0, γ = 0.0, (c) ε = 0.0, γ = 6.0 and (d) ε = 5.0, γ = 6.0 while Pr = 0.72, Q = 0.4 and α = 0.3.

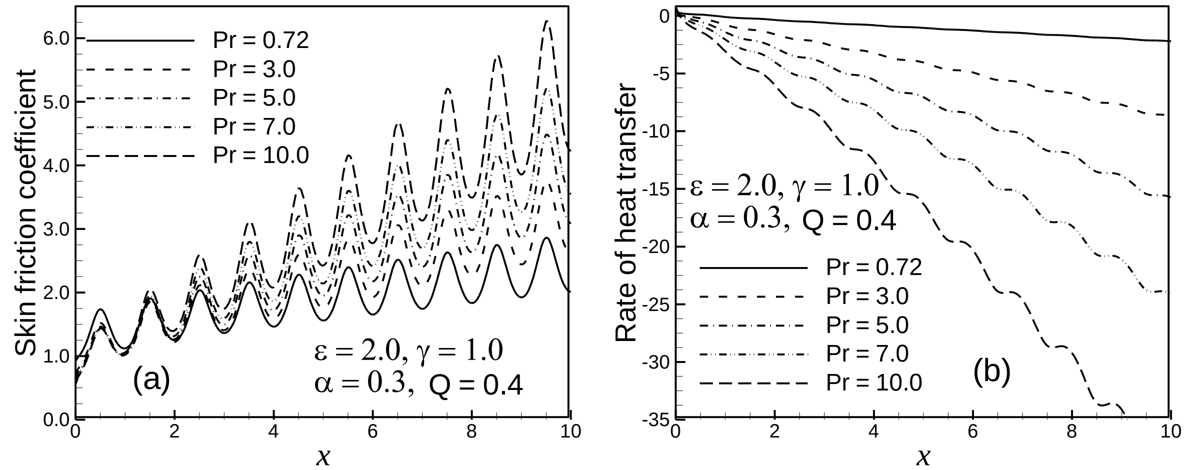

Figure 4: Variation of (a) skin-friction coefficient C_{fx} and (b) rate of heat transfer Nu_x against x for different values of Prandtl number Pr with Q = 0.4, ε = 2.0, γ = 1.0 and α = 0.3.

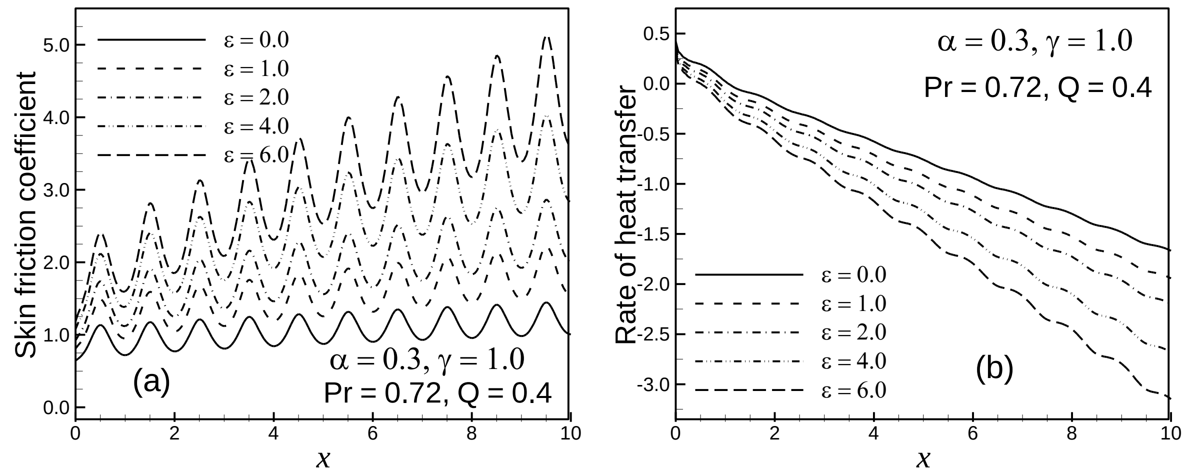

Figure 5: Variation of (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x against x for different values of ε with Pr = 0.72. Q = 0.4, γ = 1.0 and α = 0.3.

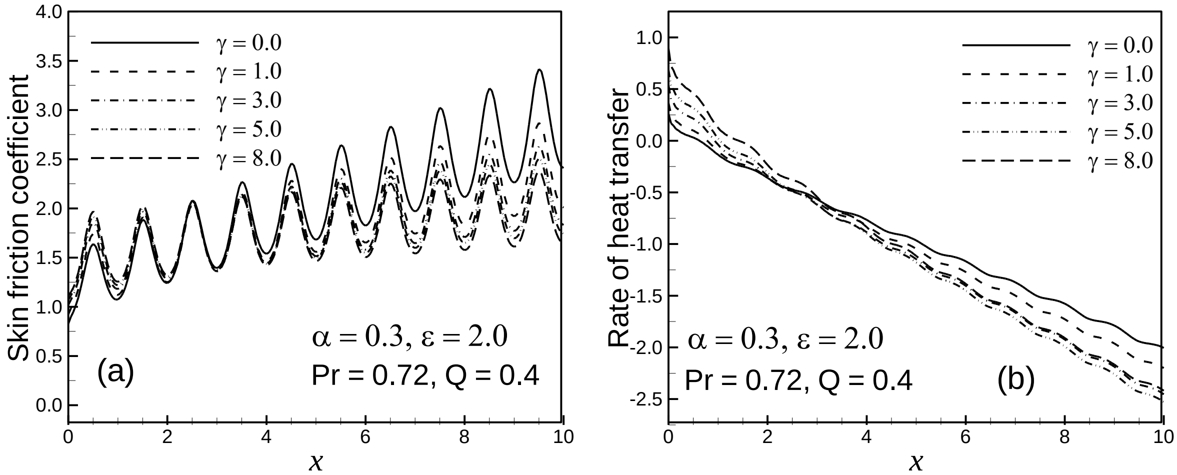

Figure 6: Variation of (a) skin friction coefficient C_{fx} and (b) rate of heat transfer Nu_x against x for different values of γ with Pr = 0.72. Q = 0.4, ε = 2.0 and α = 0.3.

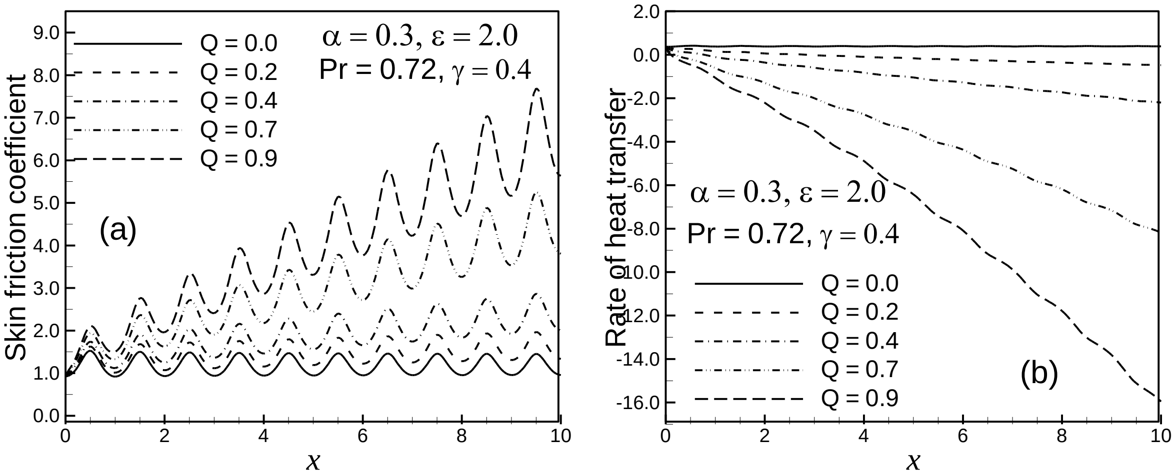

Figure 7: Variation of (a) skin-friction coefficient C_{fx} and (b) rate of heat transfer Nu_x against x for different values of Q with $\varepsilon = 1.0$, $\alpha = 0.3$, $\gamma = 1.0$ and Pr = 0.72.

5. Conclusion

The effects of temperature dependent thermal conductivity variation parameter γ, viscosity variation parameter ε, heat generation parameter Q and Prandtl number Pr on momentum and heat transfer have been studied in detail. From the present investigation the following conclusions may be drawn:

The skin friction reduces and the rate of heat transfer enhances significantly for higher values of the Prandtl number Pr, over the whole boundary. The frictional force at the wall enhances for the higher values of heat generation parameter Q, the viscosity variation parameter ε over the whole boundary layer but the rate of heat transfer reduces significantly. The skin friction and the rate of heat transfer both rise near the leading edge due to the thermal conductivity parameter γ increase.

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