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Role of $f(R,T)$ Gravity in Bianchi Type-V Dark Energy Model with Electromagnetic Field based on Lyra Geometry

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Abstract

In this paper, a Bianchi type V Dark energy cosmological model scenario is studied with the electromagnetic field in Lyra based on $f(R,T)$ gravity. The source of the magnetic field is along the x axis. Field equations found in Lyra geometry with electromagnetic field $(F_{23} \neq 0)$ in $f(R,T)$ gravity, by choosing $f(\tilde{R},T) = f_1(\tilde{R}) + f_2(T)$ with $f_1(\tilde{R}) = \mu\tilde{R}$ and $f_2(T) = \mu T$. A hybrid scale factor discusses the physical and dynamical aspects of $f(R,T)$ gravity.

Keywords: Lyra geometry; Bianchi type-V; Dark energy; ΛCDM model; $f(R,T)$ gravity; Electromagnetic field.

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1. Introduction

Bianchi type-V universes are natural generalizations of open FRW models that eventually become isotropic and homogeneous. They play an essential role in understanding galaxy formation in the early Universe [1]. There are substantial theoretical arguments for an anisotropic phase in the evolution of the Universe.

Numerous high-redshift supernovae discoveries demonstrate that the early Universe is expanding faster than previously anticipated [2-5]. In addition to these tests, observations such as cosmic microwave background (CMB) radiations and large-scale structure [6-8] suggest that the Universe is expanding quickly. Because of their capacity to explain the observed faster expansion of the Universe, modified gravity theories have sparked much attention in recent years. The essential ideas among them are $f(R), f(G), f(T)$ And $f(R,T)$ gravity [9-22].

The most prevailing significant theory in the current cosmological model of the Universe, examined with the FRW model and Bianchi type models, is $f(R,T)$. The Lagrangian is a function of Ricci scalar R. The trace of the stress-energy tensor T. Many authors [23-33] have recently studied the dark energy model in $f(R,T)$ gravity based on different contexts so far. Recently, Basumatary and Dewri [34] have reviewed the dark

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energy model with a particular form of scale factor in Sen-Dunn's theory of gravitation based on Bianchi type VI_0. The magnetic field [35-37] found in both galactic and intergalactic space has played a significant role in understanding the distribution of energy in the Universe. Numerous authors have explored the influence of magnetic fields on cosmological development in diverse circumstances, both theoretically and practically [38-46]; Melvin [47] has also revealed that matter was strongly ionized and seamlessly associated with the field during the development of the Universe.

Einstein introduced the cosmological constant in his general relativity field equations in 1917. However, his field equations do not acknowledge static solutions without the cosmological factor. After discovering and explaining the red-shift of galaxies, Einstein regretted the introduction of the cosmological constant [48]. Weyl [49] anticipated a further general theory that geometrically described gravitation and electromagnetism. Lyra [50] used a gauge function to introduce a structureless manifold to geometrize gravitation and electromagnetism. Similarly, Sen and his group [51,52] proposed a new scalar-tensor theory of gravitation and developed an analog of the EFE using Lyra geometry. Further, Halford [53] has pointed out that under the standard general relativistic interpretation, the constant vector displacement field (ϕ_i) in Lyra's geometry serves as a cosmological constant. As a result, Halford exhibits that the scalar-tensor approach based on Lyra's geometry predicts the same effects as Einstein's theory within the observable constraints.

Recently, Bhardwaj *et al.* [54] have explored dark energy models in f (R, T) gravity. Also, Singh and Devi [55] investigate the locally rotationally symmetric (LRS) Bianchi type-I cosmological models in $f(R,T)$ gravity with Hybrid Expansion Law.

2. Overview of $f(R,T)$ Gravity and the Field Equations

Bianchi type-V space-time is

$$ds^2 = -dt^2 + A^2dx^2 + e^{-2mx}(B^2dy^2 + C^2dz^2) \tag{1}$$

where A, B, C are functions of time t alone and m is a constant.

The action of $f(R,T)$ gravity using Lyra geometry [56,57] is given by

$$S = \frac{1}{16\pi G}\int f(\tilde{R}\,,T)\sqrt{-g}\; d^4x + \int L_m\sqrt{-g}\; d^4x \tag{2}$$

in which the function of Ricci scalar R, the trace of the energy-momentum tensor, and Lagrangian matter density are indicated by $\tilde{R}\,, T$ and L_m Respectively.

Also, the function of Ricci scalar R together with the Riemannian curvature [57] is

$$\tilde{R} = R + 3\nabla_i\phi^i + \frac{3}{2}\,\phi^i\phi_i \tag{3}$$

and the stress-energy tensor T_{ij} of the matter is

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta\sqrt{-g}L_m}{\delta g^{ij}} \tag{4}$$

where its trace $T = g^{ij}T_{ij}$

Here, the matter Lagrangian L_m depends only on the metric tensor component (g_{ij}) rather than its derivatives. So that it reduces to

$$T_{ij} = g_{ij}L_m - 2\frac{\partial L_m}{\partial g^{ij}} \tag{5}$$

Now, by varying the action S in eq. (2) for the metric tensor g_{ij}, the gravitational field equations of $f(\tilde{R},T)$ gravity takes the form as

$$f_{\tilde{R}}(\tilde{R},T)R_{ij} - \frac{1}{2}f(\tilde{R},T)g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j)f_{\tilde{R}}(\tilde{R},T)$$
$$= \frac{8\pi G}{c^2}T_{ij} - f_T(\tilde{R},T)T_{ij} - f_T(\tilde{R},T)\ominus_{ij} \tag{6}$$

where,

$$\ominus_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} \tag{7}$$

Such that $f_{\tilde{R}}(\tilde{R},T) = \frac{\partial f(\tilde{R},T)}{\partial \tilde{R}}, f_T(\tilde{R},T) = \frac{\partial f(\tilde{R},T)}{\partial T}$ and ∇_i indicates covariant derivative in $f(\tilde{R},T)$ gravity model.

It is essential to reveal the physical nature of the matter field through $\ominus_{ij}$ is used to form the field equations of $f(\tilde{R},T)$ gravity. The critical point of the case is that the model depicts diverse natures by considering different frames of Harko *et al*. [56]. So, in this study, it is assumed as

$$f(\tilde{R},T) = f_1(\tilde{R}) + f_2(T) \tag{8}$$

The energy-momentum tensor with the electromagnetic field considered as

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij} + E_{ij} \tag{9}$$

Here, ρ and p denote the energy density and thermodynamic pressure of the matter, where (E_{ij}) represent the electromagnetic fields of the source and is given as

$$E_{ij} = \frac{1}{4}\left(F_{j\alpha}F^{i\alpha}g^{\alpha\beta} - \frac{1}{4}g_{ij}F^{\alpha\beta}F_{\alpha\beta}\right) \tag{10}$$

such that the Maxwell equation satisfies the relation

$$F_{ij,\alpha} + F_{j\alpha,i} + F_{\alpha i,j} = 0 \text{ and } \left[F^{ij}\left(\sqrt{-g}\right)\right]_{,j} = 0 \tag{11}$$

In the co-moving coordinate system, the magnetic field is in the direction of x axis so that F_{23} is the only non-vanishing component of the model i.e. $F_{23} = K =$ constant. Also $u^i = (0,0,0,1)$ is the four-velocity vector in a co-moving coordinate system satisfies the condition $u_iu^i = -1$.

The non-vanishing components of the source of E_{ij} for the given line element are as follows:

$$E_1^1 = -E_2^2 = -E_3^3 = E_4^4 = \frac{K^2}{8\pi B^2C^2} \tag{12}$$

Now by contracting eq. (6), it reduces to

$$f_1{}'(\tilde{R},T) - \frac{1}{2}f_1(\tilde{R})g_{ij} + (g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j)f_1{}'(\tilde{R})$$
$$= -\frac{8\pi G}{c^2}T_{ij} + f_2{}'(T)T_{ij} + \left[f_2{}'(T)\,p + \frac{1}{2}\,f_2(T)\right]g_{ij} \tag{13}$$

For a perfect fluid matter source, the field equations of $f(\tilde{R},T)$ gravity obtained by choosing $f_1(\tilde{R}) = \mu\,\tilde{R}$ and $f_2(T) = \mu T$, where μ is an arbitrary constant, as given by

$$\tilde{R}_{ij}-\frac{1}{2}\tilde{R}\,g_{ij}=-\left(\frac{8\pi G-\mu c^2}{c^2}\right)T_{ij}+\left[p+\frac{1}{2}T\right]g_{ij} \tag{14}$$

Now, applying eq. (3) in (14), the field equations for Lyra Geometry [57] are obtained as

$$R_{ij}-\frac{1}{2}Rg_{ij}+\frac{3}{2}\phi_i\phi_j-\frac{3}{4}g_{ij}\phi_i\phi^j=-h\,T_{ij}+\left[p+\frac{1}{2}T\right]g_{ij} \tag{15}$$

where $\phi_i=(0,0,0,\beta\,(t))$ is a time like displacement field vector and $h=\left(\frac{8\pi G-\mu C^2}{\mu C^2}\right)=$ constant, and the symbols have their usual meaning as in Riemannian Geometry.

For the line element (1), to eq. (9), the Einstein Field Equations (15), reduces to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}}{B}\frac{\dot{C}}{C}-\frac{m^2}{A^2}+\frac{3}{4}\beta^2=-hp+\left(\frac{\rho-p}{2}\right)+\frac{hK^2}{8\pi B^2C^2} \tag{16}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{C}\dot{A}}{CA}-\frac{m^2}{A^2}+\frac{3}{4}\beta^2=-hp+\left(\frac{\rho-p}{2}\right)-\frac{h\,K^2}{8\pi B^2C^2} \tag{17}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{m^2}{A^2}+\frac{3}{4}\beta^2=-hp+\left(\frac{\rho-p}{2}\right)-\frac{h\,K^2}{8\pi B^2C^2} \tag{18}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}-\frac{3m^2}{A^2}-\frac{3}{4}\beta^2=h\rho+\left(\frac{\rho-p}{2}\right)+\frac{h\,K^2}{8\pi B^2C^2} \tag{19}$$

$$2\frac{\dot{A}}{A}-\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0 \tag{20}$$

The energy conservation equation $T^{i}_{i;j}=0$ takes the form

$$\dot{\rho}+\frac{3}{2}\beta\dot{\beta}+\left[(\rho+p)+\frac{3}{2}\,\beta^2\right]\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=0 \tag{21}$$

3. Solutions and the Physical Behavior of the Model in $f(\widetilde{R},T)$ Gravity

Case-I: Presence of magnetic field: In solving the above-filed equations (16) - (20), the following physical parameters are vital, and these parameters are defined as follows:
The spatial volume (V) and the scale factor $a(t)$ is given by

$$V=a^3=ABC \tag{22}$$

The generalized mean Hubble parameter (H) defined as

$$H=\frac{1}{3}(H_1+H_2+H_3) \tag{23}$$

where $H_1=\frac{\dot{A}}{A}, H_2=\frac{\dot{B}}{B}\ and\ H_3=\frac{\dot{C}}{C}$ are the directional Hubble's parameters along $x, y,$ and z axes, respectively.
Eqs. (22) and (23) give

$$H=\frac{\dot{a}}{a}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \tag{24}$$

The scalar expansion (θ), shear expansion (σ^2) and the anisotropy parameter (Δ) are defined as

$$\theta=3H=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C} \tag{25}$$

$$\sigma^2=\frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2+\left(\frac{\dot{B}}{B}\right)^2+\left(\frac{\dot{C}}{C}\right)^2-\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}\dot{C}}{BC}-\frac{\dot{C}\dot{A}}{CA}\right] \tag{26}$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{27}$$

Eq. (20) gives

$$A^2 = BC \tag{28}$$

In the Einstein field equations (16) - (20), there are five highly non-linear differential equations with six unknown variables, namely A, B, C, p, ρ, β. Thus, to establish these six variables, let us consider the power-law relation

$$C = B^n \tag{29}$$

where $n \neq 1$ is a positive constant that preserves the anisotropy of the space-time [58].
Now, a hybrid form of scale factor [59] is

$$a(t) = t^l \xi^{nt} \tag{30}$$

Here, l, n, ξ are positive constants such that ξ lies between 2 and 3 $(i.e. 2 \leq \xi \leq 3)$ where, for $\xi = 2.718$, the eq. (30) reduces to the hybrid scale factor, and it is crucial to construct cosmic transit from the early age of deceleration to late time acceleration [58].
Eqs. (22), (28), (29), and (30) together yields

$$A = t^l \xi^{nt} \tag{31}$$

$$B = (t^l \xi^{nt})^{\frac{2}{n+1}} \tag{32}$$

and, $$C = (t^l \xi^{nt})^{\frac{2n}{n+1}} \tag{33}$$

Then the line element (1) reduces to

$$ds^2 = -dt^2 + (t^l \xi^{nt})^2 dx^2 + e^{-2mx}\left[(t^l \xi^{nt})^{\frac{4}{n-1}} dy^2 + (t^l \xi^{nt})^{\frac{4n}{n-1}} dz^2\right] \tag{34}$$

From eq. (22), the volume of the model reduces to

$$V = a^3 = (t^l \xi^{nt})^3 \tag{35}$$

The other dynamical parameters are (Eqs. (24) - (27)) as follows:

$$H = n \ln \xi + \frac{l}{t} \tag{36}$$

$$\theta = 3n \ln \xi + \frac{3l}{t} \tag{37}$$

$$\sigma^2 = \left(\frac{n-1}{n+1}\right)^2 \left(n \ln \xi + \frac{l}{t}\right)^2 \tag{38}$$

$$A_m = \frac{2}{3}\left(\frac{n-1}{n+1}\right)^2 \tag{39}$$

The Deceleration Parameter (DP) ($q = -\frac{a\,\ddot{a}}{\dot{a}^2}$), obtained from eq. (31) as

$$q = -1 + \frac{\frac{l}{t^2}}{\left(n \ln \xi + \frac{l}{t}\right)^2} \tag{40}$$

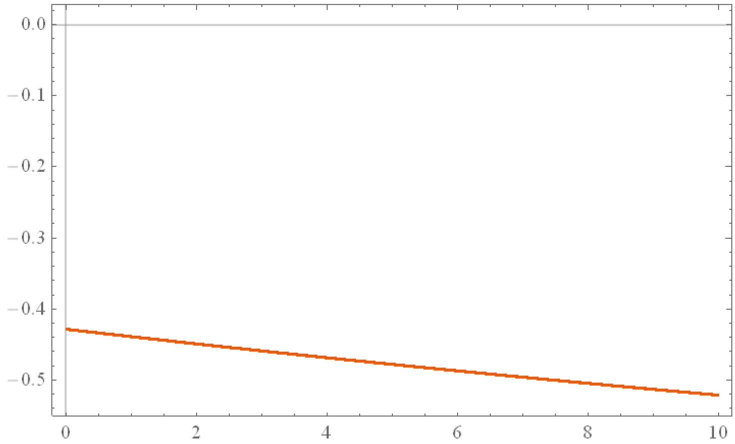

Fig. 1. Deceleration Parameter (q) vs. time (t) for $l = 1.75, n = 0.02, \xi = 2.25$.

Eqs. (16)-(18) and then applying in eq. (19), yields

$$K_1\rho = -6\left(n\ \ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} + \left(\frac{4m^2}{\left(t^l\xi^{nt}\right)^2}\right) \tag{41}$$

where $(K_1 = h\gamma - h - 1 + \gamma)$ is constant. Choosing the parameters $\gamma = -0.96$ and $h = 2$ to find the deterministic solution and draw up the model's behavior. These values are crucial in determining the model's behavior when utilizing the hybrid form of the scale factor.

In addition, another condition $p = \gamma\rho$ used to derive the dark energy model, and by applying this condition; the pressure obtained as

$$K_1 p = -6\gamma\left(n\ln\xi + \frac{l}{t}\right)^2 + 2\gamma\,\frac{l}{t^2} + \left(\frac{4m^2\gamma}{\left(t^l\xi^{nt}\right)^2}\right) \tag{42}$$

Adding Eqs. (16) - (18), the displacement field vector $(\beta(t))$ is obtain as

$$(1-\gamma)\frac{3}{4}\,\beta^2 = -3\left(n\ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} - 3\gamma\left(n\ln\xi + \frac{l}{t}\right)^2 + \frac{m^2(1+3\gamma)}{\left(t^l\xi^{nt}\right)^2} - \frac{K_2 h(1-\gamma)}{\left(t^l\xi^{nt}\right)^4} - \frac{3K_3}{K_1}\left(n\ln\xi + \frac{l}{t}\right)^2 - \frac{K_3}{K_1}\left(\frac{n}{\xi} - \frac{l}{t^2}\right) + \frac{2K_3}{K_1}\,\frac{m^2}{\left(t^l\xi^{nt}\right)^2} - (1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi + \frac{l}{t}\right)^2 \tag{43}$$

Where $K_2 = \frac{K^2}{24\pi}$ and $K_3 = (1+\gamma)(1-\gamma)$ are both positive constant and depict the variation of the graph by considering the choice of $K_1 = -5.88, K_2 = 0.16\ and\ K_3 = 0.078$, being $K = 3.5$.

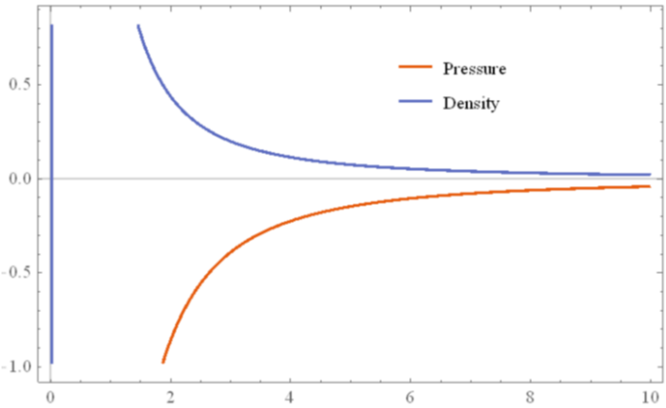

Fig. 2. Pressure (p) and density (ρ) vs. time (t) for $n = 0.02,\ l = 1.75, \xi = 2.25, h = 2, \gamma = -1.96, m = 0.09$

From Eqs. (41) and (42), the trace $(T = \rho - 3p)$ and the function of Ricci Scalar tensor $(\tilde{R})$ and the Riemannian curvature R of $f(\tilde{R}, T)$ gravity with electromagnetic field based on Lyra geometry are obtained as

$$T = \left(\frac{1-3\gamma}{K_1}\right)\left[-6\left(n\ \ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} + \frac{4m^2}{\left(t^l\xi^{nt}\right)^2}\right] \tag{44}$$

$$R = \frac{6m^2}{\left(t^l\xi^{nt}\right)^2} - 6\left(n\ \ln\xi + \frac{l}{t}\right)^2 - 6\left(\frac{n}{\xi} - \frac{l}{t^2}\right) - 8K_4\left(n\ \ln\xi + \frac{l}{t}\right)^2 \tag{45}$$

And,

$$\begin{aligned}\tilde{R} = {} & \frac{6m^2}{\left(t^l\xi^{nt}\right)^2} - 6\left(n\ \ln\xi + \frac{l}{t}\right)^2 - 6\left(\frac{n}{\xi} - \frac{l}{t^2}\right) - 8K_4\left(n\ \ln\xi + \frac{l}{t}\right)^2 + \\ & \left(\frac{2}{1-\gamma}\right)W_2\left[\frac{3l}{t^2}\left(n\ln\xi + \frac{l}{t^2}\right)\left(1 + 2\gamma + \frac{2K_3}{K_1}\right) + \frac{2l}{t^2}(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi + \frac{l}{t^2}\right) - \right. \\ & \frac{2l}{t^3}\left(2 + \frac{K_3}{K_1}\right) - 2m^2(1+3\gamma)\left(t^l\xi^{nt}\right)^{-2}\left(n\ln\xi + \frac{l}{t}\right) + 4hK_2(1-\gamma)\left(t^l\xi^{nt}\right)^{-4}\left(n\ln\xi + \right. \\ & \left.\left.\frac{l}{t}\right) - \frac{4K_3}{K_1}\ m^2\left(t^l\xi^{nt}\right)^{-2}\left(n\ln\xi + \frac{l}{t}\right)\right] + 9W_1\left(n\ln\xi + \frac{l}{t}\right) + \frac{2}{1-\gamma}\left[-3\left(1 + \gamma + \right.\right. \\ & \left.\frac{K_3}{K_1}\right)\left(n\ \ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} + \frac{m^2(1+3\gamma)}{\left(t^l\xi^{nt}\right)^2} - \frac{K_2h(1-\gamma)}{\left(t^l\xi^{nt}\right)^4} - \frac{K_3}{K_1}\left(\frac{n}{\xi} - \frac{l}{t^2}\right) + \frac{2K_3}{K_1}\frac{m^2}{\left(t^l\xi^{nt}\right)^2} - \\ & \left.(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi + \frac{l}{t}\right)^2\right]\end{aligned} \tag{46}$$

where $K_4 = \frac{n^2+n+1}{(n+1)^2}$ is a positive constant and keeping $n = 0.02$ to describe the model's behavior, provided $n \neq 1$. W_1 and W_2 are given by

$$W_1 = \frac{2}{\sqrt{3(1-\gamma)}}\left[-3\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}-3\gamma\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{m^2(1+3\gamma)}{\left(t^l\xi^{nt}\right)^2}-\frac{K_2h(1-\gamma)}{\left(t^l\xi^{nt}\right)^4}-\right.$$
$$\left.\frac{3K_3}{K_1}\left(n\ln\xi+\frac{l}{t}\right)^2-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\frac{m^2}{\left(t^l\xi^{nt}\right)^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t}\right)^2\right]^{\frac{1}{2}} \quad (47)$$

And

$$W_2 = \frac{\sqrt{3(1-\gamma)}}{2}\left[-3\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}-3\gamma\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{m^2(1+3\gamma)}{\left(t^l\xi^{nt}\right)^2}-\frac{K_2h(1-\gamma)}{\left(t^l\xi^{nt}\right)^4}-\right.$$
$$\left.\frac{3K_3}{K_1}\left(n\ln\xi+\frac{l}{t}\right)^2-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\frac{m^2}{\left(t^l\xi^{nt}\right)^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t}\right)^2\right]^{\frac{-1}{2}} \quad (48)$$

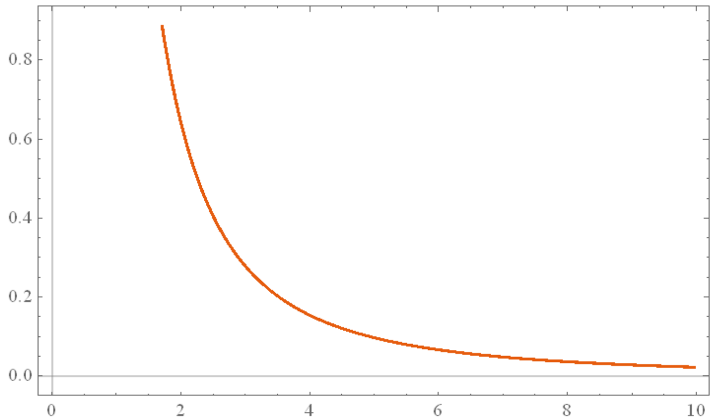

Fig. 3. Displacement field vector (β^2) vs. time (t) for $n=0.02,\ l=1.75, \xi=2.25, h=2, \gamma=-1.96,\ m=0.09$.

Here, the dark energy model using the frame of Harko *et al.* [56] as $f(\tilde{R},T)=\mu\tilde{R}+\mu T$, is considered. Eqs. (44) and (46) give

$$\frac{1}{\mu}f(\tilde{R},T)=\frac{6m^2}{\left(t^l\xi^{nt}\right)^2}-6\left(n\ \ln\xi+\frac{l}{t}\right)^2-6\left(\frac{n}{\xi}-\frac{l}{t^2}\right)-8K_4\left(n\ \ln\xi+\frac{l}{t}\right)^2+$$
$$\left(\frac{2}{1-\gamma}\right)W_2\left[\frac{3l}{t^2}\left(n\ln\xi+\frac{l}{t^2}\right)\left(1+2\gamma+\frac{2K_3}{K_1}\right)+\frac{2l}{t^2}(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)-\right.$$
$$\frac{2l}{t^3}\left(2+\frac{K_3}{K_1}\right)-2m^2(1+3\gamma)(t^l\xi^{nt})^{-2}\left(n\ln\xi+\frac{l}{t}\right)+4hK_2(1-\gamma)(t^l\xi^{nt})^{-4}\left(n\ln\xi+\right.$$
$$\left.\left.\frac{l}{t}\right)-\frac{4K_3}{K_1}m^2(t^l\xi^{nt})^{-2}\left(n\ln\xi+\frac{l}{t}\right)\right]+9W_1\left(n\ln\xi+\frac{l}{t}\right)+\frac{2}{1-\gamma}\left[-3\left(1+\gamma+\right.\right.$$
$$\left.\frac{K_3}{K_1}\right)\left(n\ \ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{m^2(1+3\gamma)}{\left(t^l\xi^{nt}\right)^2}-\frac{K_2h(1-\gamma)}{\left(t^l\xi^{nt}\right)^4}-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\frac{m^2}{\left(t^l\xi^{nt}\right)^2}-$$
$$\left.(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t}\right)^2\right]+\left(\frac{1-3\gamma}{K_1}\right)\left[-6\left(n\ \ln\xi+\frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{4m^2}{\left(t^l\xi^{nt}\right)^2}\right] \quad (49)$$

Case-II: In the absence of a magnetic field, i.e. $F_{23} = K = 0$

In this case, the pressure (p), density (ρ), and the displacement vector $(\beta(t))$ is

$$K_1 p = -6\gamma\left(n\ln\xi + \frac{l}{t}\right)^2 + 2\gamma\,\frac{l}{t^2} + \frac{4m^2\gamma}{\left(t^l\xi^{nt}\right)^2} \tag{50}$$

$$K_1\rho = -6\left(n\,\ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} + \frac{4m^2}{\left(t^l\xi^{nt}\right)^2} \tag{51}$$

and,

$$(1-\gamma)\frac{3}{4}\,\beta^2 = -3\left(n\ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} - 3\gamma\left(n\ln\xi + \frac{l}{t}\right)^2 + \frac{m^2(1+3\gamma)}{\left(t^l\xi^{nt}\right)^2}$$
$$-\frac{3K_3}{K_1}\left(n\ln\xi + \frac{l}{t}\right)^2 - \frac{K_3}{K_1}\left(\frac{n}{\xi} - \frac{l}{t^2}\right) + \frac{2K_3}{K_1}\,\frac{m^2}{\left(t^l\xi^{nt}\right)^2} - (1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi + \frac{l}{t}\right)^2 \tag{52}$$

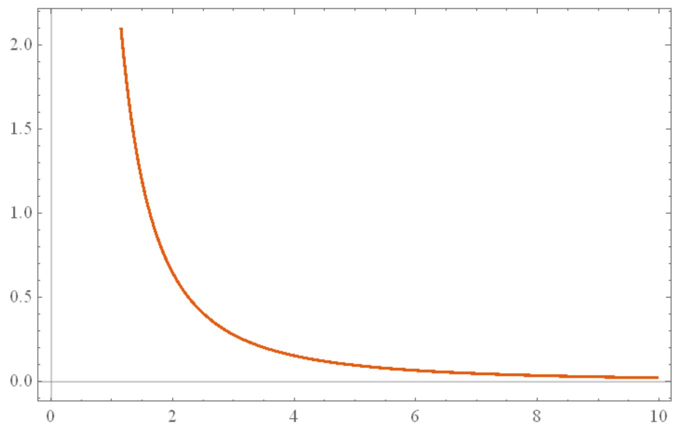

Fig. 4. Displacement field vector (β^2) vs. time (t) (absence of a magnetic field)

In the same way, the trace (T), the function of the Ricci Scalar Tensor $(\tilde{R})$, Riemannian curvature R and the $f(\tilde{R}, T)$ gravity obtained as follows:

$$T = \left(\frac{1-3\gamma}{K_1}\right)\left[-6\left(n\,\ln\xi + \frac{l}{t}\right)^2 + \frac{2l}{t^2} + \frac{4m^2}{\left(t^l\xi^{nt}\right)^2}\right] \tag{53}$$

$$R = \frac{6m^2}{\left(t^l\xi^{nt}\right)^2} - 6\left(n\,\ln\xi + \frac{l}{t}\right)^2 - 2\left(\frac{n}{\xi} - \frac{l}{t^2}\right) - 4K_4\left(n\,\ln\xi + \frac{l}{t}\right)^2 \tag{54}$$

and,

$$\tilde{R} = \frac{6m^2}{(t^l\xi^{nt})^2} - 6\left(n\,\ln\xi + \frac{l}{t}\right)^2 - 6\left(\frac{n}{\xi} - \frac{l}{t^2}\right) - 8K_4\left(n\,\ln\xi + \frac{l}{t}\right)^2$$
$$\left(\frac{2}{1-\gamma}\right)\mathrm{W}_3\left[\frac{3l}{t^2}\left(n\ln\xi + \frac{l}{t^2}\right)\left(1 + 2\gamma + \frac{2K_3}{K_1}\right) + \frac{2l}{t^2}(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi + \frac{l}{t^2}\right) -\right.$$
$$\left.\frac{2l}{t^3}\left(2 + \frac{K_3}{K_1}\right) - 2m^2(1+3\gamma)(t^l\xi^{nt})^{-2}\left(n\ln\xi + \frac{l}{t}\right) - \frac{4K_3}{K_1}\,m^2(t^l\xi^{nt})^{-2}\left(n\ln\xi + \frac{l}{t}\right)\right]$$

$$+9W_4\left(n\ln\xi+\frac{l}{t}\right)+\frac{2}{1-\gamma}\left[-3\left(1+\gamma+\frac{K_3}{K_1}\right)\left(n\ \ln\xi+\ \frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{m^2(1+3\gamma)}{\left(t^l\xi^{nt}\right)^2}-\right.$$
$$\left.\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\frac{m^2}{\left(t^l\xi^{nt}\right)^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t}\right)^2\right] \tag{55}$$

and,

$$\frac{1}{\mu}f\left(\tilde{R},T\right)=\frac{6m^2}{\left(t^l\xi^{nt}\right)^2}-6\left(n\ \ln\xi+\ \frac{l}{t}\right)^2-6\ \left(\frac{n}{\xi}-\frac{l}{t^2}\right)-8K_4\left(n\ \ln\xi+\ \frac{l}{t}\right)^2+$$
$$\left(\frac{2}{1-\gamma}\right)W_3\ \left[\frac{3l}{t^2}\left(n\ln\xi+\ \frac{l}{t^2}\right)\left(1+2\gamma+\frac{2K_3}{K_1}\right)+\frac{2l}{t^2}(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t^2}\right)-\right.$$
$$\left.\frac{2l}{t^3}\ \left(2+\frac{K_3}{K_1}\right)-2m^2(1+3\gamma)(t^l\xi^{nt})^{-2}\left(n\ln\xi+\frac{l}{t}\right)-\frac{4K_3}{K_1}\ m^2(t^l\xi^{nt})^{-2}\left(n\ln\xi+\frac{l}{t}\right)\right]+$$
$$9W_4\left(n\ln\xi+\frac{l}{t}\right)+\frac{2}{1-\gamma}\left[-3\left(1+\gamma+\frac{K_3}{K_1}\right)\left(n\ \ln\xi+\ \frac{l}{t}\right)^2+\frac{2l}{t^2}+\frac{m^2(1+3\gamma)}{\left(t^l\xi^{nt}\right)^2}-\right.$$
$$\left.\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\frac{m^2}{\left(t^l\xi^{nt}\right)^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t}\right)^2\right]\ +\left(\frac{1-3\gamma}{K_1}\right)\left[-6\left(n\ \ln\xi+\right.\right.$$
$$\left.\left.\frac{l}{t}\right)^2+\ \frac{2l}{t^2}+\left(\frac{4m^2}{\left(t^l\xi^{nt}\right)^2}\right)\right] \tag{56}$$

where,

$$W_3=\ \frac{2}{\sqrt{3(1-\gamma)}}\ \left[-3\left(n\ln\xi+\frac{l}{t}\right)^2+\ \frac{2l}{t^2}-3\gamma\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{m^2(1+3\gamma)}{\left(t^l\xi^{nt}\right)^2}\ -\frac{3K_3}{K_1}\ \left(n\ln\xi+\right.\right.$$
$$\left.\left.\frac{l}{t}\right)^2-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\ \frac{m^2}{\left(t^l\xi^{nt}\right)^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t}\right)^2\ \right]^{\frac{1}{2}} \tag{57}$$

and

$$W_4=\ \frac{\sqrt{3(1-\gamma)}}{2}\ \left[-3\left(n\ln\xi+\frac{l}{t}\right)^2+\ \frac{2l}{t^2}-3\gamma\left(n\ln\xi+\frac{l}{t}\right)^2+\frac{m^2(1+3\gamma)}{\left(t^l\xi^{nt}\right)^2}-\frac{3K_3}{K_1}\ \left(n\ln\xi+\right.\right.$$
$$\left.\left.\frac{l}{t}\right)^2-\frac{K_3}{K_1}\left(\frac{n}{\xi}-\frac{l}{t^2}\right)+\frac{2K_3}{K_1}\ \frac{m^2}{\left(t^l\xi^{nt}\right)^2}-(1-\gamma)\left(\frac{n-1}{n+1}\right)^2\left(n\ln\xi+\frac{l}{t}\right)^2\ \right]^{\frac{-1}{2}} \tag{58}$$

The state finder parameters defined as

$$r=\frac{\ddot{a}}{a\,\dot{H}}=1+3\,\frac{\dot{H}}{H^2}+\frac{\ddot{H}}{H^3} \tag{59}$$

and, $s=\frac{r-1}{3\left(q-\frac{1}{2}\right)}$ (60)

which are obtained from (31), (36), and (40) as follows:

$$r=1+\frac{3}{n\xi\ (\ln n)^2}-\frac{3l}{n^2t^2(\ln\xi)^2}+\frac{2l}{t^3\left(n\ \ln\xi+\frac{l}{t}\right)^3} \tag{61}$$

and $$s=\frac{\left[\frac{3}{n\xi\,(\ln\xi)^2}-\frac{3l}{n^2t^2(\ln\xi)^2}+\frac{2l}{t^3\left(n\ \ln\xi+\frac{l}{t}\right)^3}\right]}{3\left[-\frac{3}{2}+\ \frac{\frac{l}{t^2}}{\left(n\ \ln\xi+\frac{l}{t}\right)^2}\right]} \tag{62}$$

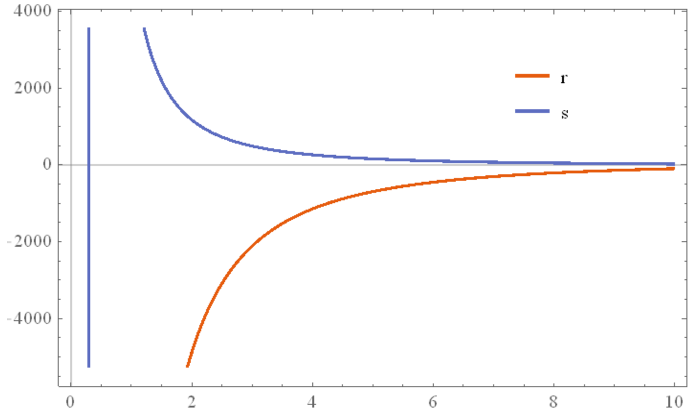

Fig. 5. Statefinder parameters "r" and "s" vs. cosmic time (t), for $n = 0.02, l = 1.75, \xi = 2.25$.

4. Conclusion

In this paper, the field equations in Lyra based $f(\tilde{R}, T)$ gravity solved with $F_{23} \neq 0$ by using $a(t) = t^l \xi^{nt}$. The absence of an electromagnetic field does not affect much in the present model. Also, at the initial moment $t = 0$, all the parameters like R, $\beta, \tilde{R}, T, p, \rho,$ and $f(\tilde{R}, T)$ tends to infinity. The beginning is the initial singularity with infinite pressure and density. When t→ ∞; $\beta, R, \tilde{R}$, T, p, ρ, and $f\left(\tilde{R}, T\right)$ tends to be a constant value. From the eq. (35), it is seen that the universe approaches an infinitely large volume as time increases in both scenarios of the $f(\tilde{R}, T)$ gravity model. The behavior of DP is $q < 0$ for $t \to \infty$. The orientation of the energy density and pressure are positive and negative directions in each case with an increase in time (t). As per these observations of p and q, the present model has dark energy with the accelerated expansion of the Universe with an electromagnetic field. This result fits the recent observational data (LSS, CMB, SNe Ia). The displacement field vector $\beta^2(t)$ positively gradually decreases with an increase in cosmic time (t). The ΛCDM model does not evolve in this model. Eqs. (38) and (39) show that this model of $f(\tilde{R}, T)$ gravity is not free from shear scalar, and the model's behavior is totally anisotropic provided $n \neq 1$.

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