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Perfect Fluid Coupled String Universe in *f(R,T)* Gravity

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Abstract

In this paper, we have investigated the Bianchi type VI_0 space-time in the presence of string of clouds coupled with perfect fluid within the context of *f(R,T)* gravity. The exact solutions to the field equations have been obtained by considering the expansion scalar Θ is proportional to the shear scalar σ and the exponential form of an average scale factor. Some physical and kinematical parameters of the constructed model have been discussed and presented graphically, and it is interesting to note that the resultant model resembles the recent observational data. Also, we have discussed the energy conditions for a perfect fluid-coupled string cosmological model.

Keywords: Bianchi type VI_0 space-time; Perfect fluid; Cloud string; *f(R,T)* Gravity.

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1. Introduction

It is well understood that the present universe is undergoing a late-time cosmic accelerated expansion that is confirmed by various high redshift supernovae experiments [1-3]. This accelerated expansion is thought to be driven by the fluid known as dark energy. The origin of which is still a mystery in modern cosmology, which is being quietly held due to the universe's negative pressure. Many theoretical and experimental development has been done to explore hidden mistrial aspects of the universe. In recent years, various eminent scientists have devoted their work to studying these unknown features of the universe by developing different cosmological models in various gravity theories. Amongst these modified gravity theories like *f(R)*, *f(T)*, *f(R,T)*, *f(G)*, *f(Q)*, and many more, which are the modifications of general relativity are providing satisfactory solutions to cosmological problems. In this context, Chirde *et al*. [4] investigated the LRS Bianchi-type I metric with the source as barotropic perfect fluid and cosmic string in the framework of *f(T)* gravity using three different functional forms of *f(T)* gravity. Matsuo [5] proposed a fluid model of self-gravitating strings and conferred his assumption that black holes turn into strings around the end of black hole evaporation. Later on, Bhoyar *et al*. [6] studied Kantowaski-Sachs cosmological model with viscous cosmic string in the

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quadratic form of teleparallel gravity for a particular choice of *f(T)* formalism. Aditya and Reddy [7] examined the dynamics of the perfect fluid cosmological model in the presence of a massive scalar field in *f (R,T)* gravity. They studied the recent scenario of accelerating expansion of the universe by computing energy conditions. Similarly, Gore *et al*. [8], Poonia *et al*. [9], and Lambat and Pund [10] studied string as well as perfect fluid models in different gravitation theories.

Much interest has been developed in studying string fluid cosmological models since string fluids are believed to play an important role during the early evolution and late-time accelerated expansion of the universe. Rani *et al*. [11] studied accelerating Bianchi type III perfect fluid string cosmological model within the context of *f(R,T)* gravity. Ram and Chandel [12] studied the dynamics of a magnetized string universe by assuming the hybrid expansion law. Using the Bianchi-type I metric, Gaikwad *et al*. [13] analyzed the massive string magnetized barotropic perfect fluid cosmological model. Bali *et al*. [14] have investigated a massive string magnetized barotropic perfect fluid cosmological model in general relativity using the Bianchi type I metric. Pawar *et al.* [15,16] examined the perfect fluid string and viscous fluid string cosmological models and obtained the deterministic solutions to field equations within the context of *f(T)* gravity. Katore *et al*. [17] inspected a massive string-magnetized perfect fluid universe in the bimetric theory of gravitations. Zia *et al*. [18] have obtained the transit dark energy model with perfect fluid using generalized hybrid expansion low in the frame of reference to *f(R,T)* gravity. The string of clouds in the presence of perfect fluid and decaying vacuum energy density Λ have been analyzed by Pradhan *et al*. [19]. Tripathi *et al*. [20] investigated the inhomogeneous Bianchi type I cosmological model for stiff perfect fluid distribution. Bali and Pareek [21] studied magnetized string cosmology for perfect fluid distribution using Bianchi type III space-time.

Motivated by the situations discussed above in this paper, we have considered Bianchi type VI_0 space-time to construct the string cosmological model for perfect fluid distribution within the context of *f(R,T)* gravity. This paper is divided into several sections: Sec. 2 deals with methodology, elementary definitions, and equations of motion in context to *f(R,T)* gravity. In Sec. 3, considering Bianchi type VI_0 space-time, we have obtained the corresponding field equations. In Sec. 4, we obtained the deterministic solutions to highly non-linear field equations, calculated the different physical and kinematical quantities along with energy conditions, and presented them graphically. Lastly, in Sec. 5, we have concluded the investigations.

2. Methodology and Brief Review of *f(R,T)* Gravity

The $f(R,T)$ gravity theory is the modification or generalization of Einstein's General Theory of Relativity, which has been proposed by Harko *et al*. [22]. In this gravity theory, the field equations are derived from variational, Einstein’s- Hilbert action principle, which is given by

$$S = \frac{1}{16\pi}\int \sqrt{-g}\, f(R,T)\, d^4 x + \int \sqrt{-g}\, L_m d^4 x, \quad (1)$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R, the trace T of the stress-energy tensor of the matter T_{ij} and L_m is the matter of Lagrangian density. The stress-energy tensor T_{ij} for matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}L_m)}{\delta g^{ij}}, \quad (2)$$

and its trace is given by $T = g^{ij}T_{ij}$.
Assuming that the Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} rather than its derivative, so (2) leads to

$$T_{ij} = g_{ij}L_m - 2\frac{\partial(L_m)}{\partial g^{ij}}. \quad (3)$$

By varying the action S in (1) with respect to the metric tensor components g^{ij}, the field equations of $f(R,T)$ gravity theory are obtained as

$$f_R(R,T)\, R_{ij} - \frac{1}{2}f(R,T)\, g_{ij} + f_R(R,T)\big(g_{ij}\,\Box - \nabla_i\nabla_j\big) = 8\pi\, T_{ij} - f_T(R,T)\, T_{ij} - f_T(R,T)\Theta_{ij}, \quad (4)$$

with $\Theta_{ij} = g^{lm}\left(\frac{\delta T_{lm}}{\delta g^{ij}}\right)$, which follows from the relation $\delta\left(\frac{g^{lm}T_{lm}}{\delta g^{ij}}\right) = T_{ij} + \Theta_{ij}$, and $\Box = \nabla^i\nabla_i$, $f_R(R,T) = \frac{\partial f(R,T)}{\partial R}$, $f_T(R,T) = \frac{\partial f(R,T)}{\partial T}$, and ∇_i are the covariant derivatives. The contraction of (4) yields

$$f_R(R,T)R + 3\Box f_R(R,T) - 2f(R,T) = \big(8\pi - f_T(R,T)\big)T - f_T(R,T)\Theta, \quad (5)$$

with $\Theta = g^{ij}\Theta_{ij}$. Eliminating $\Box f_R(R,T)$ from (4) and (5) we get,

$$f_R(R,T)\left(R_{ij} - \frac{1}{3}Rg_{ij}\right) + \frac{1}{6}f(R,T)\, g_{ij} = F_1 + F_2, \quad (6)$$

where

$$F_1 = \big(8\pi - f_T(R,T)\big)\left(T_{ij} - \frac{1}{3}Tg_{ij}\right),$$

$$F_2 = -f_T(R,T)\Theta_{ij} + \frac{1}{3}f_T(R,T)\Theta\, g_{ij} + \nabla_i\nabla_j f_R(R,T).$$

From (2) we have

$$\frac{\delta T_{ij}}{\delta g^{lm}} = \left(\frac{\delta g_{ij}}{\delta g^{lm}} + \frac{1}{2}g_{ij}g_{lm}\right)L_m - 2\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} - \frac{1}{2}g_{ij}T_{lm}. \quad (7)$$

Using the relation $\frac{\delta_{ij}}{\delta g^{lm}} = -g_{i\gamma}g_{j\sigma}\delta_{lm}^{\gamma\sigma}$ with $\delta_{lm}^{\gamma\sigma} = \frac{\delta T^{\gamma\sigma}}{\delta g^{lm}}$, Θ_{ij} is obtained as

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{lm}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}}. \quad (8)$$

We now consider the matter as perfect fluid, and thus the stress-energy tensor of the matter Lagrangian is given by

$$T_{ij} = (\rho + p)\, u_i\, u_j - pg_{ij}, \quad (9)$$

where ρ and p are the respective energy density and pressure, $u^i = (1,0,0,0)$ are the co-moving coordinates for four velocities satisfying the conditions $u^i u_i = 1$, and $u^i \nabla_j u_i = 0$. Using (8), we have obtained the expression for the variation of stress energy of perfect fluid as

$$\Theta_{ij} = -2T_{ij} - p g_{ij}. \tag{10}$$

In this paper, we consider the cosmological consequences of the model proposed by Harko *et al*. [22] as

$$f(R,T) = R + 2f(T), \tag{11}$$

where $f(T)$is an arbitrary function of the trace of the stress-energy tensor of matter. Combining (10) and (11) the field equation of $f(R,T)$ gravity from (4) leads to

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2f'(T) T_{ij} + [2pf'(T) + f(T)] g_{ij}. \tag{12}$$

where the overhead prime denotes the differentiation with respect to the argument.

3. Metric and Field Equations

We consider the Bianchi type-VI_0 metric as

$$ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2x} dy^2 - C^2 e^{-2x} dz^2, \tag{13}$$

where the scale factors $A, B,$ and C are functions of cosmic time *t* only.
The energy-momentum tensor for a string of clouds with perfect fluid distribution is given as

$$T_\mu^\nu = (\rho + p) u_\mu u^\nu - p \delta_\mu^\nu - \lambda x_\mu x^\nu, \tag{14}$$

in which u^μ denotes a four-velocity vector and x^μ denotes a space-like unit vector of the cloud string satisfying the conditions, $u^\mu u_\mu = 1 = -x^\mu x_\mu$ and $u^\mu x_\nu = 0$, for $\mu \neq \nu$ and ρ is the proper energy density of the particle, p is the isotropic pressure, λ is the strings tension density.
In a co-moving coordinate system, we have

$$u^\mu = (0,0,0,1),\ x^\mu = (A^{-1}, 0,0,0), \tag{15}$$

If the configuration of particle density is indicated by ρ_p , then we assume

$$\rho = \rho_p + \lambda. \tag{16}$$

The energy condition leads to $\rho \geq 0$ and $\rho_p \geq 0$, leaving the sign of λ unrestricted.
Using the co-moving coordinate system by choosing the function given by Harko *et al*. [22] as

$$f(T) = \mu T, \tag{17}$$

where μ is a constant. Now assuming the co-moving coordinate system, we obtained the field equations for Bianchi type-VI_0 space-time (13) from (12) in the framework of $f(R,T)$ gravity as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{1}{A^2} = 8\pi(p - \lambda) + \mu(3p - 3\lambda - \rho), \tag{18}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{A^2}=8\pi p+\mu(3p-\lambda-\rho), \tag{19}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{1}{A^2}=8\pi p+\mu(3p-\lambda-\rho), \tag{20}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{1}{A^2}=-8\pi\rho+\mu(p-\lambda-3\rho), \tag{21}$$

$$\frac{\dot{B}}{B}-\frac{\dot{C}}{C}=0, \tag{22}$$

where the overhead dot (.) denotes the derivative with respect to cosmic time t. Here we have five highly non-linear differential field equations with six unknowns, namely, $A, B, C, p, \lambda,$ and ρ.

We find some kinematical space-time quantities, as follows:

The average scale factor a and the spatial volume V are respectively defined as

$$a=\sqrt[3]{ABC},\ \ V=a^3. \tag{23}$$

Also, the volumetric expansion rate of the universe is described by the generalized mean Hubble's parameter (H) given by

$$H=\frac{1}{3}\sum_{i=1}^{3}H_i=\frac{1}{3}(H_1+H_2+H_3), \tag{24}$$

in which $H_1=\frac{\dot{A}}{A}$, $H_2=\frac{\dot{B}}{B}$, and $H_3=\frac{\dot{C}}{C}$ denotes the directional Hubble parameters.

Using (23) and (24), we have obtained the expansion scalar Θ, the mean anisotropy parameter Δ, the shear scalar σ^2, and the deceleration parameter q, respectively as

$$\Theta=\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}=3H, \tag{25}$$

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2, \tag{26}$$

$$\sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-\Theta^2\right), \tag{27}$$

$$q=-\frac{a\ddot{a}}{\dot{a}^2}=-1+\frac{d}{dt}\left(\frac{1}{H}\right). \tag{28}$$

4. Solutions of Field Equations

From (22) we get

$$B=\kappa C \tag{29}$$

where κ is an integrating constant but without loss of generality, we consider $\kappa=1$.

Now using (29) and subtracting (18) from (19) we get

$$(8\pi+2\mu)\lambda=\frac{\ddot{A}}{A}-\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}^2}{B^2}-\frac{2}{A^2}. \tag{30}$$

Using (29) and subtracting (21) from (20) we get

$$(8\pi+2\mu)(p+\rho)=\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}-\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{B}^2}{B^2}. \tag{31}$$

For the deterministic solutions, we consider the expansion scalar Θ is proportional to the shear scalar σ which lead to the following analytic relation

$$A = B^{\zeta}, \tag{32}$$

where ζ is a constant.

We consider the average scale factor as given by Barrow [23] as

$$a = exp(Kt^{f}), \tag{33}$$

where $K > 0$ and $0 < f < 1$.

From (23), (29), and (32) we get the value of potential metric functions as

$$A = e^{\frac{3\zeta Kt^{f}}{\zeta+2}},\ B = C = e^{\frac{3Kt^{f}}{\zeta+2}}. \tag{34}$$

Using (34) in (13) we get

$$ds^2 = dt^2 - e^{\frac{6\zeta Kt^{f}}{\zeta+2}}dx^2 - e^{\frac{2\left(3Kt^{f}+x\zeta+2x\right)}{\zeta+2}}dy^2 - e^{\frac{2\left(3Kt^{f}-x\zeta-2x\right)}{\zeta+2}}dz^2, \tag{35}$$

In the following, we have determined the spatial volume V, the mean Hubble's parameter H, the expansion scalar Θ, the mean anisotropy parameter Δ, the shear scalar σ^2, and the deceleration parameter q, respectively as

$$V = e^{3Kt^{f}}, \tag{36}$$

$$H = Kft^{f-1}, \tag{37}$$

$$\Theta = 3Kft^{f-1}, \tag{38}$$

$$\Delta = \frac{2(\zeta-1)^2}{(\zeta+2)^2}, \tag{39}$$

$$\sigma^2 = \frac{3K^2f^2t^{2f-2}(\zeta-1)^2}{(\zeta+2)^2}, \tag{40}$$

$$q = -1 + \frac{(1-f)}{Kft^{f}}. \tag{41}$$

The Ratio

$$\frac{\sigma^2}{\Theta^2} = \frac{1}{3}\frac{(\zeta-1)^2}{(\zeta+2)^2}. \tag{42}$$

for $\zeta \neq 1, -2$.

From the above parameters, it is observed that the volume is an exponential function that is free from an initial singularity. And it is seen from Figs. 1 and 2 that Hubble's parameter is a diminishing function of cosmic time and vanishes when $t \to \infty$ showing the model in (35) is expanding in nature, and the rate of expansion is large in the beginning but gets slower through time while the deceleration parameter decreases in negative and approaches to -1 when $t \to \infty$, that determines the universe is in accelerated phase which is best suited with recent cosmological observations [1,2] that shows accelerating phase of the universe i.e. $-1 \leq q < 0$. Also, the ratio in (42) shows that the discussed model doesn't approach isotropy.

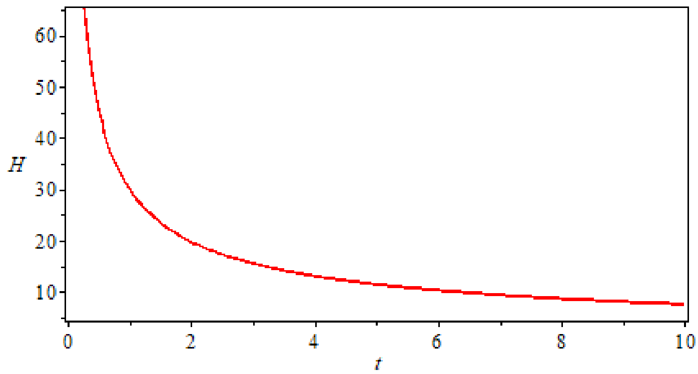

Fig. 1. Variation of H Vs. t for $f = 0.4$, and $K = 75$.

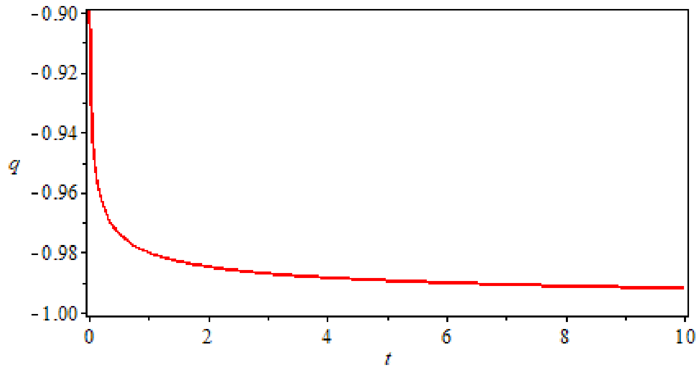

Fig. 2. Variation of q Vs. t for $f = 0.4$, and $K = 75$.

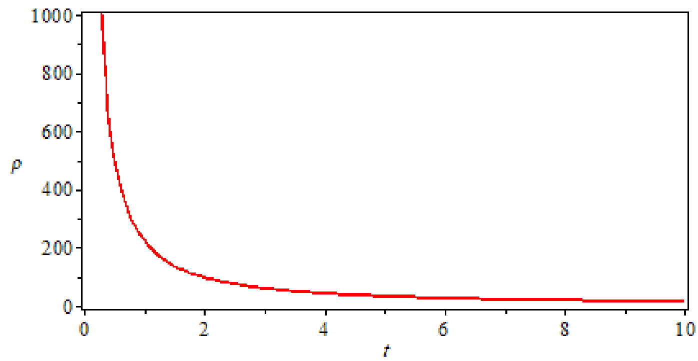

Fig. 3. Variation of ρ Vs. t for $f = 0.4$, $K = 75$, $\zeta = 2.1874$, $\mu = 2.7$, and $\delta = 3$.

Now we assume the equation of state for the string cloud model as

$$\rho = \delta\lambda \tag{43}$$

where δ is constant.
From (30) we have obtained the energy density and the tension density as

$$\rho = \delta\lambda = \frac{3\delta K f t^f(\zeta-1)(3Kft^f+f-1)-2\delta(\zeta+2)t^2 e^{\frac{-3\zeta K t^f}{\zeta+2}}}{(8\pi+2\mu)(\zeta+2)t^2}. \quad (44)$$

The particle density is obtained as

$$\rho_p = \frac{3(\delta-1)Kft^f(\zeta-1)(3Kft^f+f-1)-2(\delta-1)(\zeta+2)t^2 e^{\frac{-3\zeta K t^f}{\zeta+2}}}{(8\pi+2\mu)(\zeta+2)t^2}. \quad (45)$$

From (31) using (44) the pressure is obtained as

$$p = \frac{3Kft^f\{3Kft^f(1-\zeta)[(\delta-1)\zeta+2\delta]-(\zeta+2)(f-1)[(\delta-1)\zeta-\delta-1]\}+2\delta(\zeta+2)^2t^2 e^{\frac{-3\zeta K t^f}{\zeta+2}}}{(8\pi+2\mu)(\zeta+2)^2t^2}. \quad (46)$$

The physical behavior of energy density is depicted in Fig. 3, which demonstrates that the energy density is decreasing function of cosmic time t, and with an increasing infinitely large time, the energy density vanishes. Whereas from Fig. 4, it can be seen that the pressure increases in negative to become null and void when $t \to \infty$.

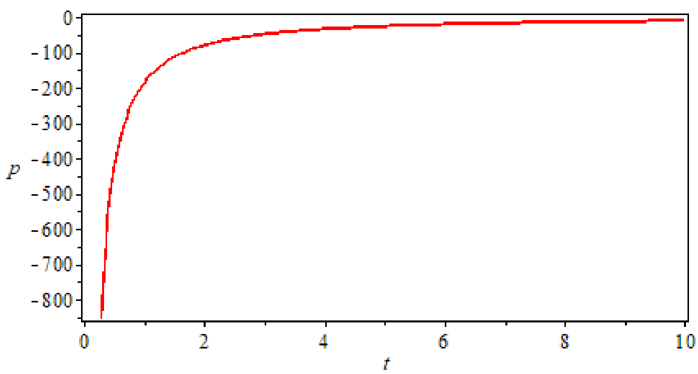

Fig. 4. Variation of p Vs. t for $f = 0.4$, $K = 75$, $\zeta = 2.1874$, $\mu = 2.7$, and $\delta = 3$.

4.1. *The energy conditions*

The energy conditions are respectively defined as [24]

(i) Null Energy Condition: $\rho + p \geq 0$
(ii) Weak Energy Condition: $\rho \geq 0, \rho + p \geq 0$
(iii) Strong Energy Condition: $\rho + 3p \geq 0$
(iv) Dominant Energy Condition: $\rho \pm p \geq 0, \rho \geq 0$

Using (31), (44), and (46) we can express the conditions as

$$\rho + p = \frac{3Kft^f\{3Kft^f\zeta(\zeta-1)+(\zeta+1)(\zeta+2)(f-1)\}}{(8\pi+2\mu)(\zeta+2)^2t^2}. \quad (47)$$

$$\rho - p = \frac{3Kft^f\{3Kft^f(\zeta-1)[(2\delta-1)\zeta+4\delta]+(\zeta+2)(f-1)[(2\delta-1)\zeta-2\delta-1]\}-4\delta(\zeta+2)^2t^2e^{\frac{-3\zeta Kt^f}{\zeta+2}}}{(8\pi+2\mu)(\zeta+2)^2t^2}. \quad (48)$$

$$\rho + 3p = \frac{3Kft^f\{3Kft^f(1-\zeta)[(2\delta-3)\zeta+4\delta]-(\zeta+2)(f-1)[(2\delta-3)\zeta-2\delta-3]\}+4\delta(\zeta+2)^2t^2e^{\frac{-3\zeta Kt^f}{\zeta+2}}}{(8\pi+2\mu)(\zeta+2)^2t^2}. \quad (49)$$

The graphical behavior of energy conditions has been expressed in Fig. 5. The energy behavior of the model changes dynamically with the evolution of cosmic time. The NEC, WEC, and DEC are satisfied for the derived model (35). On the other hand, SEC is violated throughout the evolution providing the anti-gravitational effect, known to be the dark energy, resulting in the cosmic accelerated expansion. This is in good agreement with [25,26].

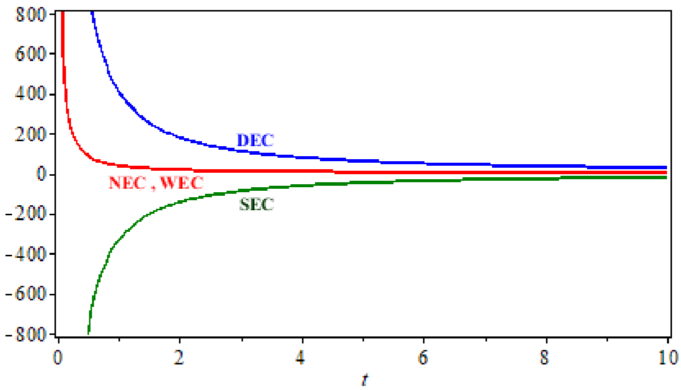

Fig. 5. Variation of Energy Conditions Vs. t for $f = 0.4$, $K = 75$, $\zeta = 2.1874$, $\mu = 2.7$, and $\delta = 3$.

5. Conclusion

In this paper, the authors have presented the Bianchi type VI_0 space-time in the presence of string of clouds coupled with perfect fluid within the context of *f(R,T)* gravity. To obtain the exact solutions to the highly non-linear differential field equations, we have considered the expansion scalar Θ is proportional to the shear scalar σ and the average scale factor as presented by Barrow [23]. The constructed model is free from an initial singularity and behaves like the present universe, which is accelerating as well as expanding, which is well suited to the recent observations. Also, it is observed that the model is anisotropic. The energy density is a diminishing function of cosmic time, whereas the pressure rises in negative, and both vanish at an infinitely large time. We have also discussed the energy conditions for the derived model, and it is observed that the NEC, WEC, and DEC get satisfied; however, the SEC is violated throughout the evolution resulting in the cosmic accelerated expansion, which is in good agreement with the recent observations [25,26].

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