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Anisotropic Bianchi type VI_0 Space-Time with Barotropic Fluid in Sáez - Ballester Theory of Gravitation

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Abstract

In this paper, we have discussed the exact solution of Einstein's field equations for anisotropic Bianchi type VI_0 cosmological model in the framework of the scalar-tensor theory of gravitation given by Sáez–Ballester for barotropic fluid distribution. To obtain an exact solution, we have assumed that expansion (θ) is proportional to shear (σ), which leads to $A = B^k$, where k is a constant and $A,\ B$ are metric potentials and also assumed barotropic condition $p = \gamma\rho;\ (0 \leq \gamma \leq 1)$ where p being isotropic pressure and ρ is matter density. Some physical and geometrical properties of the model are also discussed.

Keywords: Cosmological model; Bianchi type VI_0; Sáez-Ballester theory.

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1. Introduction

Numerous scholars have shown keen interest in investigating cosmological models proposed by Sáez and Ballester's [1] theory due to its efficacy in elucidating the universe's early stages. Various alternative theories have been postulated to Einstein's model in an effort to illuminate the fundamental nature of the cosmos during its initial evolution. The proposition by Sáez and Ballester delineates the behavior of weak fields as well as the coupling of a dimensionless scalar field, offering insights into the problem of missing matter in a non-flat FRW universe. Piementel [2] explained the significance of scalar-tensor theories through the presentation of novel vacuum solutions within the Brans-Dicke theory. Under the paradigm of the Sáez and Ballester framework, different researchers, such as Mohanty *et al*. [3], Reddy *et al*. [4], Adhav *et al*. [5], Katore *et al*. [6], Pradhan *et al*. [7], Rao *et al*. [8], Hasmani *et al*. [9], Mishra and Chand [10], Vinutha and Venkatavasavi [11] have studied many cosmological models.

By utilizing homogeneous and isotropic models offered by FRW (Friedmann-Robertson-Walker) line elements, the universe's current state can be effectively depicted. Bianchi models I to IX, characterized by spatial homogeneity and anisotropy, are presently

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under examination to enhance the comprehension of the universe's early history. Given that Bianchi type VI_0 Space-times are an easy generalization of Bianchi type-I space-time models, which are anisotropic extensions of FRW models featuring zero curvature and hold particular significance. Baro *et al*. [12] have conducted a study on anisotropic models within Lyra geometry. Barrow [13] proposed that Bianchi type VI_0 models could undergo isotropization in specific instances, providing improved explanations for certain cosmological issues. The exploration of Bianchi type VI_0cosmological models have been pursued by Ellis and MacCallum [14], Collins [15], Dunn and Tupper [16], Roy and Singh [17], Roy *et al*. [18], Ribeiro and Sanyal [19], Ram [20], Tikekar and Patel [21], Bali *et al*. [22-24].

In a recent study, Tyagi *et al*. [25] have discussed VI_0 model incorporating a magnetic field. Bali and Poonia [26], Bali and Kumari [27], and Goyal [28] have investigated the inflationary scenario of Bianchi type VI_0 cosmological models. Investigations by Ram *et al*. [29] and Lambat *et al*. [30] have centered on Bianchi type VI_0 models within Lyra geometry. Ugale and Deshmukh [31] have investigated VI_0 model within modified f(R, T) gravity, while Basumatay and Dewri [32] have explored VI_0 models within the Sen-Dunn theory of gravitation.

In this paper, investigation of a Bianchi type VI_0 model with barotropic fluid distribution is conducted within the framework of the Sáez and Ballester theory. Various physical and geometrical characteristics of the model are discussed in terms of cosmic time.

2. The Metric and Field Equations

Bianchi Type VI_0 metric in the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{-2mx}dy^2 + C^2e^{2mx}dz^2 \tag{1}$$

where $A, B,\ C$are functions of cosmic time 't'.
The field equations by Sáez-Ballester for combined scalar and tensor fields are

$$G_{ij} - \omega\phi^n\left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,a}\phi^{,a}\right) = -T_{ij} \tag{2}$$

where the scalar field ϕ satisfies the following conditions

$$2\phi^n\phi_{,i}^{;i} + n\phi^{n-1}\phi_{,a}\phi^{,a} = 0$$

and

$$G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij} \tag{2}$$

The energy momentum tensor for a perfect fluid distribution is given by

$$T_{ij} = (\rho + p)u_iu_j + pg_{ij} \tag{3}$$

and

$$g_{ij}u^iu^j = -1 \tag{4}$$

where ρ is the energy density of the cosmic matter, p is the isotropic pressure, and u^i is the four-velocity vector.

The field equations can be written as

$$\frac{B_{44}}{B}+\frac{C_{44}}{C}+\frac{B_4C_4}{BC}+\frac{m^2}{A^2}-\frac{\omega}{2}\phi^n\phi_4^2=-p \quad (5)$$

$$\frac{A_{44}}{A}+\frac{C_{44}}{C}+\frac{A_4C_4}{AC}-\frac{m^2}{A^2}-\frac{\omega}{2}\phi^n\phi_4^2=-p \quad (6)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{m^2}{A^2}-\frac{\omega}{2}\phi^n\phi_4^2=-p \quad (7)$$

$$\frac{A_4B_4}{AB}+\frac{A_4C_4}{AC}+\frac{B_4C_4}{BC}-\frac{m^2}{A^2}+\frac{\omega}{2}\phi^n\phi_4^2=\rho \quad (8)$$

$$\frac{B_4}{B}-\frac{C_4}{C}=0 \quad (9)$$

$$\phi_{44}+\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)\phi_4+\frac{n}{2}\frac{\phi_4^2}{\phi}=0 \quad (10)$$

$$\left(\frac{A_4}{A}+\frac{B_4}{B}+\frac{C_4}{C}\right)=-\frac{\rho_4}{\rho+p} \quad (11)$$

where the subscript '4' denotes the ordinary differentiation with respect to 't'.
On integrating the equation (9), we have
$B=LC$, L is the constant of integration.
Taking $L=1$, we get

$$B=C \quad (12)$$

Using (12), equations (5)-(11) lead to

$$2\frac{B_{44}}{B}+\frac{B_4^2}{B^2}+\frac{m^2}{A^2}-\frac{\omega}{2}\phi^n\phi_4^2=-p \quad (13)$$

$$\frac{A_{44}}{A}+\frac{B_{44}}{B}+\frac{A_4B_4}{AB}-\frac{m^2}{A^2}-\frac{\omega}{2}\phi^n\phi_4^2=-p \quad (14)$$

$$2\frac{A_4B_4}{AB}+\frac{B_4^2}{B^2}-\frac{m^2}{A^2}+\frac{\omega}{2}\phi^n\phi_4^2=\rho \quad (15)$$

$$\phi_{44}+\left(\frac{A_4}{A}+2\frac{B_4}{B}\right)\phi_4+\frac{n}{2}\frac{\phi_4^2}{\phi}=0 \quad (16)$$

$$\left(\frac{A_4}{A}+2\frac{B_4}{B}\right)=-\frac{\rho_4}{\rho+p} \quad (17)$$

3. Solution of Field Equations

We have assumed that expansion θ is proportional to shear σ $(\theta \propto \sigma)$, which leads to

$$A=B^K \quad (18)$$

where K is a constant and A, B are functions of t.
Also, we assume barotropic condition as

$$p=\gamma\rho \quad (19)$$

where $0\le\gamma\le 1$, p is isotropic pressure and ρ is matter density.
From equations (16) and (18), we have

$$\phi_4^2\phi^n = \frac{k_1}{B^{4+2k}} \tag{20}$$

Where, k_1 is constant of integration.
By using (17), (18) and (19), we have

$$\rho = B^{-(K+2)(1+\gamma)} \tag{21}$$

From equations (13), (15), (18) and (21), we get

$$2B_{44} + 2(K+1)\frac{B_4^2}{B} = \frac{(1-\gamma)}{B^{\{(K+2)(1+\gamma)-1\}}} \tag{22}$$

$$\text{Let } B_4 = f(B) \tag{23}$$

Equation (22) leads to

$$\frac{df^2}{dB} + 2\frac{(K+1)}{B}f^2 = \frac{(1-\gamma)}{B^{\{(K+2)(1+\gamma)-1\}}} \tag{24}$$

On solving (24), we have

$$B_4^2 = \frac{1}{(K+2)B^{\{K(1+\gamma)+2\gamma\}}} + \frac{k_2}{B^{2(K+1)}} \tag{25}$$

Take $k_2 = 0$ (constant of integration)
Equation (25) leads to

$$B = \left\{\frac{(K+2)(1+\gamma)}{2}\left(\frac{t}{\sqrt{K+2}} + k_3\right)\right\}^{\frac{2}{(K+2)(1+\gamma)}} \tag{26}$$

where k_3 is constant of integration.
The line element (1) is given by

$$ds^2 = -dt^2 + \left\{\frac{(K+2)(1+\gamma)}{2}\left(\frac{t}{\sqrt{K+2}} + k_3\right)\right\}^{\frac{4K}{(K+2)(1+\gamma)}} dx^2$$

$$+ \left\{\frac{(K+2)(1+\gamma)}{2}\left(\frac{t}{\sqrt{K+2}} + k_3\right)\right\}^{\frac{4}{(K+2)(1+\gamma)}} (e^{-2mx}dy^2 + e^{2mx}dz^2) \tag{27}$$

4. Physical and Geometrical Aspects

Some physical and geometrical properties of metric (27) are given as

$$\text{Matter density } \rho = \frac{4}{\left\{(K+2)(1+\gamma)\left(\frac{t}{\sqrt{K+2}} + k_3\right)\right\}^2} \tag{28}$$

$$\text{Isotropic pressure } p = \frac{4\gamma}{\left\{(K+2)(1+\gamma)\left(\frac{t}{\sqrt{K+2}} + k_3\right)\right\}^2} \tag{29}$$

From equation (20), the scalar function ϕ is obtained as

$$\phi = \left[\left(\frac{n+2}{2}\right)k_1\sqrt{K+2}\,\frac{(\gamma+1)}{(\gamma-1)}\left\{\frac{(K+2)(1+\gamma)}{2}\right\}^{\frac{-2}{(1+\gamma)}}\left(\frac{t}{\sqrt{K+2}} + k_3\right)^{\frac{(\gamma-1)}{(\gamma+1)}} + \left(\frac{n+2}{2}\right)k_4\right]^{\frac{2}{(n+2)}} \tag{30}$$

Where k_4 is a constant of integration.
Expansion θ is given by

$$\theta = \frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}$$

$$\theta = \frac{2}{\sqrt{K+2}(1+\gamma)\left(\frac{t}{\sqrt{K+2}}+k_3\right)} \tag{31}$$

Hubble parameter $H = \frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)$

$$H = \frac{2}{3\sqrt{K+2}(1+\gamma)\left(\frac{t}{\sqrt{K+2}}+k_3\right)} \tag{32}$$

Spatial volume $V = ABC$

$$V = \left[\frac{(K+2)(1+\gamma)}{2}\left\{\frac{t}{\sqrt{K+2}} + k_3\right\}\right]^{\frac{2}{(1+\gamma)}} \tag{33}$$

Deceleration parameter $q = -1 + \frac{d}{dt}\left[\frac{1}{3}\left(\frac{A_4}{A} + \frac{B_4}{B} + \frac{C_4}{C}\right)\right]$

$$q = -1 - \frac{2}{3(K+2)(1+\gamma)\left(\frac{t}{\sqrt{K+2}}+k_3\right)^2} \tag{34}$$

Shear scalar $\sigma^2 = \frac{1}{2}\left(\frac{A_4^2}{A^2} + \frac{B_4^2}{B^2} + \frac{C_4^2}{C^2}\right) - \frac{\theta^2}{6}$

$$\sigma = \frac{(K-1)}{\sqrt{3}}\left[\frac{2}{(K+2)\sqrt{(K+2)}(1+\gamma)\left(\frac{t}{\sqrt{K+2}}+k_3\right)}\right] \tag{35}$$

$$\sigma = \frac{(K-1)}{\sqrt{3}(K+2)}\theta$$

or $\sigma \propto \theta$

5. Graphical Representations

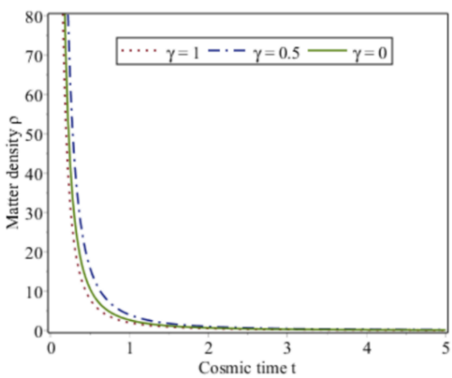

Fig. 1. Graph between metter density (ρ) and cosmic time (t) for K=1 and k_3=0.

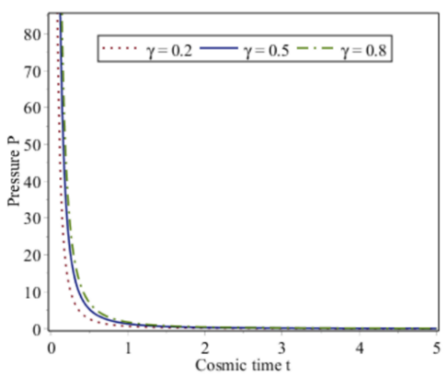

Fig. 2. Graph between isotropic pressure (P) and cosmic time (t) for K=1 and k_3=0.

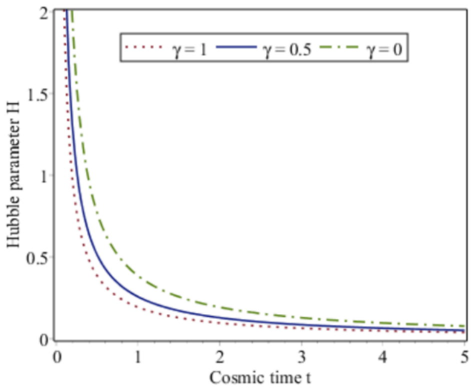

Fig. 3. Graph between Hubble parameter (H) and cosmic time (t) for K=1 and k_3=0.

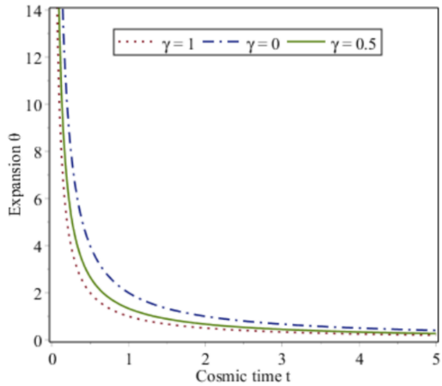

Fig. 4. Graph between expansion (θ) and cosmic time (t) for K=1 and k_3=0.

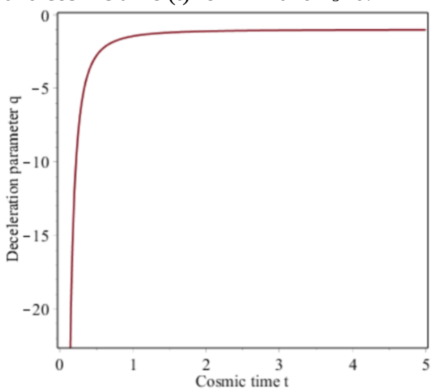

Fig. 5. Graph between deceleration parameter (q) and cosmic time (t) for K=1 and k_3=0.

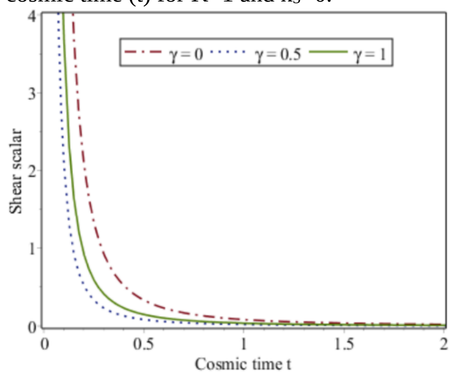

Fig. 6. Graph between shear scalar (σ^2) and cosmic time (t) for K=2 and k_3=0.

6. Special Cases

Case I: When $\gamma = 0$ (Dust Universe)
From equation (24)

$$\frac{df^2}{dB} + 2\frac{(K+1)}{B} f^2 = \frac{(1-\gamma)}{B^{K+1}}$$

$$B_4^2 = \frac{1}{(K+2)B^K} + \frac{k_5}{B^{2(K+1)}} \tag{36}$$

where k_5 is constant of integration.
or

$$B = \left[(K+2)\left\{\left(\frac{t}{2} + k_6\right)^2 - k_5\right\}\right]^{\frac{1}{(K+2)}} \tag{37}$$

where k_6 is constant of integration.
The line element (1) is given by

$$ds^2 = -dt^2 + \left[(K+2)\left\{\left(\frac{t}{2}+k_6\right)^2 - k_5\right\}\right]^{\frac{2K}{(K+2)}} dx^2$$

$$+\left[(K+2)\left\{\left(\frac{t}{2}+k_6\right)^2 - k_5\right\}\right]^{\frac{2}{(K+2)}} (e^{-2mx}dy^2 + e^{2mx}dz^2) \quad (38)$$

Matter density $\rho = \frac{1}{(K+2)\left\{\left(\frac{t}{2}+k_6\right)^2 - k_5\right\}}$ (39)

Scalar function ϕ is obtained as

$$\phi = \left[\left(\frac{n+2}{2}\right)\sqrt{\frac{k_1}{k_5}}\frac{1}{(k+2)} ln\left\{\frac{\left(\frac{t}{2}+k_6\right)-\sqrt{k_5}}{\left(\frac{t}{2}+k_6\right)+\sqrt{k_5}}\right\} + \left(\frac{n+2}{2}\right)k_9\right]^{\frac{2}{(n+2)}} \quad (40)$$

Where k_9 is a constant of integration.

Expansion $\theta = \frac{\left(\frac{t}{2}+k_6\right)}{\left\{\left(\frac{t}{2}+k_6\right)^2 - k_5\right\}}$ (41)

Hubble parameter $H = \frac{\left(\frac{t}{2}+k_6\right)}{3\left\{\left(\frac{t}{2}+k_6\right)^2 - k_5\right\}}$ (42)

Spatial volume $V = (K+2)\left\{\left(\frac{t}{2}+k_6\right)^2 - k_5\right\}$ (43)

Deceleration parameter $q = -1 - \frac{\left\{\left(\frac{t}{2}+k_6\right)^2 + k_5\right\}}{6\left\{\left(\frac{t}{2}+k_6\right)^2 - k_5\right\}^2}$ (44)

Shear scalar $\sigma = \frac{(K-1)}{\sqrt{3}(K+2)}\theta$

Case II: When $\gamma = 1$ (Stiff or 'Zel' Devich Universe)
From equation (24)

$$\frac{df^2}{dB} + 2\frac{(K+1)}{B}f^2 = 0$$

$$B_4 = \frac{k_7}{B^{(K+1)}}$$

where k_7 is constant of integration.
or

$$B = [(K+2)\{k_7t + k_8\}]^{\frac{1}{(K+2)}} \quad (45)$$

where k_8 is constant of integration.
The line element (1) is given by

$$ds^2 = -dt^2 + [(K+2)\{k_7t + k_8\}]^{\frac{2K}{(K+2)}}dx^2 + [(K+2)\{k_7t +$$

$$k_8\}]^{\frac{2}{(K+2)}}(e^{-2mx}dy^2 + e^{2mx}dz^2) \quad (46)$$

Matter density $\rho = \frac{1}{[(K+2)\{k_7 t+k_8\}]^2}$ (47)

scalar function $\phi = \left[\left(\frac{n+2}{2}\right)\frac{\sqrt{k_1}}{k_7(k+2)} ln\{k_7 t + k_8\} + \left(\frac{n+2}{2}\right) k_{10}\right]^{\frac{2}{(n+2)}}$ (48)

where k_{10} is a constant of integration.

Expansion $\theta = \frac{k_7}{(k_7 t+k_8)}$ (49)

Hubble parameter $H = \frac{k_7}{3(k_7 t+k_8)}$ (50)

Spatial volume $V = (K+2)(k_7 t + k_8)$ (51)

Deceleration parameter $q = -1 - \frac{k_7^2}{3(k_7 t+k_8)^2}$ (52)

Shear scalar $\sigma \propto \theta$

7. Conclusion

Here, we have explored anisotropic and spatially homogeneous Bianchi type VI_0 cosmological model with barotropic fluid distribution in Sáez – Ballester theory. The model (28) has a singularity at $t = -k_3\sqrt{(k+2)}$ and has no initial singularity. Similarly, matter density and scalar field ϕ do not possess initial singularities. These singularities vanish as t increases. The special volume increases with cosmic time t and it will become infinite for large values of t, which shows the anisotropic expansion of the universe. For the model (28) expansion (θ), shear scalar (σ), matter density (ρ), isotropic pressure (p) are monotonically decreasing functions of cosmic time t.Ultimately $\theta \to 0, \sigma \to 0, \rho \to 0, p \to 0$ as $t \to \infty$, which shows that the universe is expanding as time increases. Since in general $\sigma \neq 0$ so, the anisotropy in the model is maintained over time, and the model does not lead to the FRW model in general. But for a specific value of, at $k = 1$ $\sigma \to 0$, this shows that the model isotropizes at a late time for $k = 1$, which leads to the FRW model because shear is zero in the FRW model, which is isotropic and homogeneous. The deceleration parameter $q < 0$ indicates an accelerating universe, and $q > 0$ indicates a decelerating universe, here $q < 0$. Thus, the model (28) represents an accelerating universe. Since spatial volume V increases as time increases, the universe expands. Sáez–Ballester scalar field ϕ monotonically increases with cosmic time t. Thus, the model is an anisotropic, continuously expanding, and rotating universe.

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