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Insights into Soft Semigraphs: AND & OR Operations and Measurement Perspectives

B. George[1], R. K. Thumbakara[2], J. Jose[3], S. P. George[1*]

[1]Department of Mathematics, Pavanatma College, Murickassery, 685604, India

[2]Department of Mathematics, Mar Athanasius College, Kothamangalam, 686666, India

[3]Department of Science and Humanities, Viswajyothi College of Engineering and Technology Vazhakulam, 686670, India

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Abstract

Molodtsov pioneered the concept of soft sets, offering a method to classify elements of a universe based on a specified set of parameters. This approach serves to model vagueness and uncertainty. Semigraphs are a generalized form of graphs introduced by Sampathkumar. The integration of soft set theory into semigraphs led to the creation of soft semigraphs. Due to its adeptness in handling parameterization, the field of soft semi-graph theory is rapidly evolving. This study introduces AND and OR operations within soft semi-graphs and explores some of their characteristics. Furthermore, we present various soft semi-graph measurements, such as distance, radius, diameter, and center.

Keywords: Semigraph; Soft set; Soft semigraph.

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1. Introduction

Conventional formal modeling, reasoning, and computation methods typically exhibit determinism, clarity, and precision. However, the complexities encountered in diverse fields like engineering, medicine, economics, and social sciences often involve data that lacks a clear definition. Various uncertainties present in these problem areas pose challenges for traditional methods. The fuzzy set theory addresses one form of uncertainty, "Fuzziness," arising from elements partially belonging to a set. While it effectively handles uncertainties related to vague or partially belonging elements, it doesn't encompass all uncertainties found in real-world problems. The emergence of soft set theory in 1999 by mathematician Molodtsov [1] offers a more practical approach compared to established theories like probability or fuzzy set theory, owing to its versatility. For example, fuzzy set theory lacks sufficient parameterization tools. Authors such as Maji, Biswas, and Roy [2,3] have expanded on soft set theory, employing it to resolve decision-making problems.

* *Corresponding author*: sijopgorg@pavanatmacollege.org

The notion of soft graphs was introduced by Thumbakara and George [4]. In 2015, Akram and Nawas [5,6] modified the definition of soft graphs. Further advancements in the field were made by Akram and Nawas [7,8], who introduced fuzzy soft graphs, strong fuzzy soft graphs, complete fuzzy soft graphs, and regular fuzzy soft graphs, exploring their properties and potential applications. Akram and Zafar [9,10] pioneered the concepts of soft trees and fuzzy soft trees. Fuzzy soft theory enables handling problems containing uncertain data by combining the characteristics of fuzzy sets and soft sets. Nawaz and Akram [11] explored the applications of fuzzy soft graphs, such as analyzing oligopolistic competition among wireless internet service providers in Malaysia. Akram and Shahzadi [12] also proposed a decision-making approach utilizing Pythagorean Dombi fuzzy soft graphs.

Contributions to the study of soft graphs have been made by Thenge, Jain, and Reddy [13-15]. Due to their utility in handling parameterization, soft graphs represent a growing domain within graph theory. George, Thumbakara, and Jose studied various concepts in soft graphs and introduced soft hypergraphs [16], soft directed graphs [17,18], soft directed hypergraphs [19], and soft Disemigraphs [20], studying their properties. George, Thumbakara, and Jose [21-24] explored various properties of soft graphs. The operation of graph products, a method of combining two graphs, can be extended to soft graphs. They also explored various product operations in soft graphs [25,26] and soft-directed graphs [27-31] and investigated their properties. The concept of semigraphs, a broader version of graphs, was first introduced by Sampathkumar [32,33]. Unlike hypergraphs, semigraphs maintain a specific order of vertices within their edges. When represented on a plane, semigraphs resemble conventional graphs. In 2022, George, Thumbakara, and Jose introduced soft semi-graphs [34,35] by applying soft set principles to semi-graphs and defined some soft semi-graph operations. Moreover, they [36-38] introduced some product operations, connectedness, isomorphisms, and various degrees, graphs, and matrices associated with soft semigraphs. This paper introduces AND and OR operations in soft semi-graphs and investigates some of their properties. We also introduce some measurements in soft semi-graphs, such as distance, radius, diameter, and center.

2. Preliminaries

2.1. *Semigraph*

Sampathkumar introduced the notion of semi-graphs as follows: "A *semi-graph* G is a pair (V, X) where V is a nonempty set whose elements are called vertices of G, and X is a set of n-tuples, called edges of G, of distinct vertices, for various $n \geq 2$, satisfying the following conditions.

1. Any two edges have at most one vertex in common
2. Two edges $(u_1, u_2, \dots, u_n)$ and $(v_1, v_2, \dots, v_m)$ are considered to be equal if and only if
   a. $m = n$ and
   b. either $u_i = v_i$ for $1 \leq i \leq n$, or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

Let $G = (V, X)$ be a semi-graph and $E = (v_1, v_2, \dots, v_n)$ be an edge of G. Then v_1 and v_n are the *end vertices* of E and $v_i, 2 \leq i \leq n-1$ are the *middle vertices (or m-vertices)* of E.

If a vertex v of a semi-graph G appears only as an end vertex, then it is called an *end vertex*. If a vertex v is only a middle vertex, then it is a *middle vertex* or *m-vertex*, while a vertex v is called *middle-cum-end vertex or* (m, e)*-vertex* if it is a middle vertex of some edge and an end vertex of some other edge. A *subedge* of an edge $E = (v_1, v_2, \dots, v_n)$ is a k-tuple $E' = \left(v_{i_1}, v_{i_2}, \dots, v_{i_k}\right)$, where $1 \leq i_1 \leq i_2 \dots \leq i_k \leq n$ or $1 \leq i_k \leq i_{k-1} \leq \dots \leq i_1 \leq n$. We say that the subedge E' is *induced* by the set of vertices $\{v_{i_1}, v_{i_2}, \dots v_{i_k}\}$. A *partial edge* of $E = (v_1, v_2, \dots, v_n)$ is a $(j - i + 1)$-tuple $E\left(v_i, v_j\right) = \left(v_i, v_{i+1}, \dots, v_j\right)$, where $1 \leq i, j \leq n$. $G' = (V', X')$ is a *partial semi-graph* of a semi-graph G if the edges of G' are partial edges of G. Two vertices u and v in a semi-graph G are said to be *adjacent* if they belong to the same edge. If u and v are adjacent and consecutive in order, then they are said to be *consecutively adjacent*. u and v are said to be *e-adjacent* if they are the end vertices of an edge and *1e-adjacent* if both the vertices u and v belong to the same edge and at least one of them is an end vertex of that edge".

2.2. *Soft set*

In 1999, Molodtsov initiated the concept of soft sets. "Let U be an initial universe set and let A be a set of parameters. A pair (F, A) is called a soft set (over U) if and only if F is a mapping of A into the set of all subsets of the set U. That is, $F: A \to \mathcal{P}(U)$".

2.3. *Soft semigraph*

George, Thumbakara, and Jose introduced soft semi-graph by applying the concept of soft set in semi-graph as follows: "Let $G^* = (V, X)$ be a semi-graph having a vertex set V and edge set X. Consider a subset V_1 of V. Then, a partial edge is formed by some or all vertices of V_1 is said to be a *maximum partial edge* or *mp edge* if it is not a partial edge of any other partial edge formed by some or all vertices of V_1. Let X_p be the collection of all partial edges of the semi-graph G and A be a nonempty set. Let a subset R of $A \times V$ be an arbitrary relation from A to V. We define a mapping Q from A to $\mathcal{P}(V)$ by $Q(x) = \{y \in V | xRy\}, \forall x \in A$, where $\mathcal{P}(V)$ denotes the power set of V. Then the pair (Q, A) is a soft set over V. Also, define a mapping W from A to $\mathcal{P}\left(X_p\right)$ by $W(x) = \{$mp edges$< Q(x) >\}$, where $\{$mp edges$< Q(x) >\}$ denotes the set of all mp edges that can be formed by some or all vertices of $Q(x)$ and $\mathcal{P}\left(X_p\right)$ denotes the power set of X_p. The pair (W, A) is a soft set over X_p. Then, we can define a soft semi-graph as follows: The 4-tuple $G = (G^*, Q, W, A)$ is called a *soft semi-graph* of G^* if the following conditions are satisfied:

1. $G^* = (V, X)$ is a semi-graph having a vertex set V and edge set X,
2. A is the nonempty set of parameters,
3. (Q, A) is a soft set over V,
4. (W, A) is a soft set over X_p,
5. $H(a) = \left(Q(a), W(a)\right)$ is a partial semi-graph of $G^*, \forall a \in A$.

Let $G^* = (V, X)$ be a semi-graph and $G = (G^*, Q, W, A)$ be a soft semi-graph of G^* which is also given by $\{H(x): x \in A\}$. Then the partial semi-graph $H(x)$ corresponding to any parameter x in A is called a *p-part* of the soft semi-graph G. An edge present in a soft semi-

graph G of G^* is called an f-*edge*. It may be a partial edge of some edge in G^* or an edge in G^*. A partial edge of any f-edge of a soft semi-graph G is called a p-*edge* of G. An f-edge is a p-edge of itself. An f-edge or a p-edge of a soft semi-graph G is called an fp-*edge* of G." An example of a soft semi-graph is given below.
Let $G^* = (V, X)$ be a semi-graph given in Fig. 1.

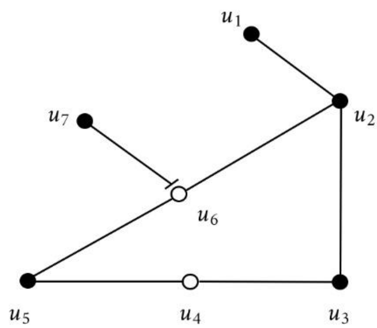

Fig. 1. Semi-graph $G^* = (V, X)$.

Here $V = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ and X= $\{(u_1, u_2), (u_2, u_3), (u_3, u_4, u_5), (u_2, u_6, u_5), (u_6, u_7)\}$.

Let $A = \{u_2, u_5\} \subseteq V$ be a parameter set. Define Q from A to $\mathcal{P}(V)$ by $Q(x) = \{y \in V | xRy \Leftrightarrow x = y$ or x and y are adjacent$\}$, $\forall x \in A$ and W from A to $\mathcal{P}(X_p)$ by $W(x) = \{mp\text{-edges}\langle Q(x)\rangle\}, \forall x \in A$. That is, $Q(u_2) = \{u_1, u_2, u_3, u_5, u_6\}$ and $Q(u_5) = \{u_2, u_3, u_4, u_5, u_6\}$. Also $W(u_2) = \{(u_1, u_2), (u_2, u_6, u_5), (u_2, u_3)\}$ and $W(u_5) = \{(u_2, u_6, u_5), (u_3, u_4, u_5), (u_2, u_3)\}$. Here $H(u_2) = (Q(u_2), W(u_2))$ and $H(u_8) = (Q(u_8), W(u_8))$ are partial semi-graphs of G^* as shown below in Fig. 2. Hence $G = \{H(u_2), H(u_5)\}$ is a soft semi-graph of G^*.

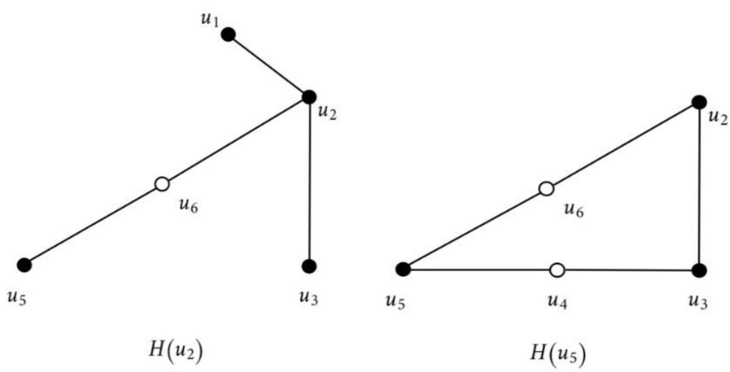

Fig. 2. Soft Semi-graph $G = \{H(u_2), H(u_5)\}$

Let $G^* = (V, X)$ be a semi-graph having a vertex set V and edge set X. Also, let $G_1 = (G^*, Q_1, W_1, A_1)$ and $G_2 = (G^*, Q_2, W_2, A_2)$ be two soft semi-graphs of G^*. Then G_2 is a *soft partial semi-graph* of G_1 if

1. $A_2 \subseteq A_1$,
2. $H_2(x) = \big(Q_2(x), W_2(x)\big)$ is a partial semi-graph of $H_1(x) = \big(Q_1(x), W_1(x)\big)$, for all $x \in A_2$.

Let $G^* = (V, X)$ be a semi-graph given in Fig. 3 having a vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$ and the edge set X = {(v_1, v_2, v_3, v_4), (v_1, v_5, v_6, v_7), (v_3, v_6)}.

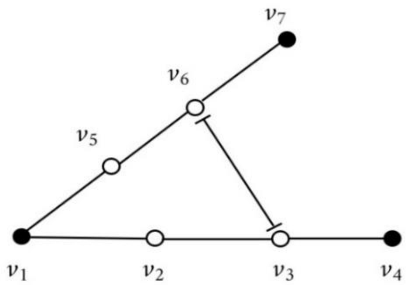

Fig. 3. Semi-graph $G^* = (V, X)$.

Let the parameter set be $A_1 = \{v_3, v_5\} \subseteq V$. Define $Q_1: A_1 \to \mathcal{P}(V)$ by $Q_1(x) = \{y \in V | xRy \Leftrightarrow x = y$ or x and y are adjacent$\}$, $\forall x \in A_1$ and $W_1: A_1 \to \mathcal{P}(X_p)$ by $W_1(x) = \{mp\text{-edges}\langle Q_1(x)\rangle\}, \forall x \in A_1$. That is, $Q_1(v_3) = \{v_1, v_2, v_3, v_4, v_6\}$ and $Q_1(v_5) = \{v_1, v_5, v_6, v_7\}$. Also $W_1(v_3) = \{(v_3, v_6), (v_1, v_2, v_3, v_4)\}$ and $W_1(v_5) = \{(v_1, v_5, v_6, v_7)\}$. Then (Q_1, A_1) is a soft set over V and (W_1, A_1) is a soft set over X_p. Here $H_1(v_3) = \big(Q_1(v_3), W_1(v_3)\big)$ and $H_1(v_5) = \big(Q_1(v_5), W_1(v_5)\big)$ are partial semi-graphs of G^* as shown in Fig. 4. Hence $G_1 = \{H_1(v_3), H_1(v_5)\}$ is a soft semi-graph of G^*.

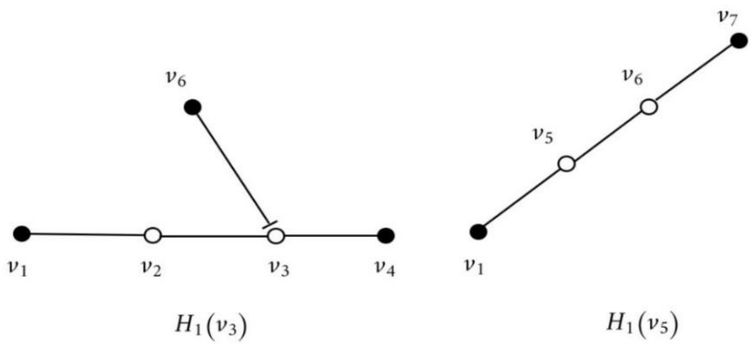

Fig. 4. Soft Semi-graph $G_1 = \{H_1(v_3), H_1(v_5)\}$.

Let $A_2 = \{v_3\} \subseteq V$ be another parameter set. Define $Q_2: A_2 \to \mathcal{P}(V)$ by Q_2(x) ={y ∈V | xRy ⇔x=y or x and y are consecutively adjacent}, $\forall x \in A_2$ and $W_2: A_2 \to \mathcal{P}(X_p)$ by $W_2(x) =$

$\{mp\text{-edges}\langle Q_2(x)\rangle\}, \forall x \in A_2$. That is, $Q_2(v_3) = \{v_2, v_3, v_4, v_6\}$ and $W_2(v_3) = \{(v_3, v_6), (v_2, v_3, v_4)\}$. Then (Q_2, A_2) is a soft set over V and (W_2, A_2) is a soft set over X_p. Here $H_2(v_3) = \big(Q_2(v_3), W_2(v_3)\big)$ is a partial semi-graph of G^* as shown in Fig. 5. Hence $G_2 = \{H_2(v_3)\}$ is a soft semi-graph of G^*.

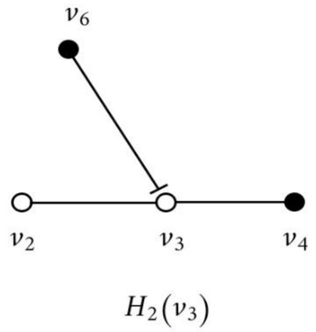

Fig. 5. Soft Semi-graph $G_2 = \{H_2(v_3)\}$.

Here $G_2 = (G^*, Q_2, W_2, A_2) = \{H_2(v_3)\}$ is a soft partial semi-graph of $G_1 = (G^*, Q_1, W_1, A_1) = \{H_1(v_3), H_1(v_5)\}$ since

1. $A_2 \subseteq A_1$,
2. $H_2(x) = \big(Q_2(x), W_2(x)\big)$ is a partial semi-graph of $H_1(x) = \big(Q_1(x), W_1(x)\big)$ for all $x \in A_2$.

3. AND Operation in Soft Semi-Graphs

Let $G^* = (V, X)$ be a semi-graph having a vertex set V and edge set X. Also, let $G_1 = (G^*, Q_1, W_1, A_1)$ and $G_2 = (G^*, Q_2, W_2, A_2)$ be two soft semi-graphs of G^* such that $Q_1(u) \cap Q_2(v) \neq \phi$ for all $(u, v) \in A_1 \times A_2$. Then *AND operation* on G_1 and G_2 denoted by $G_1 \wedge G_2$ is defined as $G_1 \wedge G_2 = G = (G^*, Q, W, A)$, where $A = A_1 \times A_2$ and for all $(u, v) \in A = A_1 \times A_2$, $Q(u, v) = Q_1(u) \cap Q_2(v)$ and $W(u, v) = \{mp\text{-edges } \langle Q_1(u) \cap Q_2(v)\rangle\} = \{mp\text{-edges } \langle Q(u, v)\rangle\}$. If $H(u, v) = \big(Q(u, v), W(u, v)\big), \forall (u, v) \in A$, then $G_1 \wedge G_2 = \{H(u, v): (u, v) \in A\}$.

Example 3.1: Consider a semi-graph $G^* = (V, X)$ given in Fig. 6 having vertex set $V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}\}$ and edge set X= {(v_1, v_2, v_3), (v_1, v_4), (v_3, v_4, v_8), (v_3, v_5, v_6), (v_4, v_6), (v_8, v_7, v_6), (v_8, v_9), (v_7, v_9), (v_9, v_{10}, v_{11}), (v_9, v_{13}, v_{12}), (v_{11}, v_{12})}.

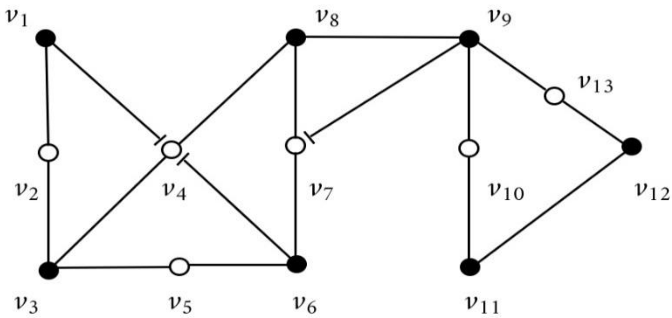

Fig. 6. Semi-graph $G^* = (V, X)$.

Let $A_1 = \{v_6, v_9\} \subseteq V$ be a parameter set. Define a function $Q_1: A_1 \to \mathcal{P}(V)$ by $Q_1(u) = \{v \in V: uRv \Leftrightarrow u = v$ or u and v are adjacent$\}$ for all $u \in A_1$. That is, $Q_1(v_6) = \{v_3, v_4, v_5, v_6, v_7, v_8\}$ and $Q_1(v_9) = \{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}\}$. Then (Q_1, A_1) is a soft set over V. Define another function $W_1: A_1 \to \mathcal{P}(X_p)$ by $W_1(u)$ = $\{mp$-edges $\langle Q_1(u)\rangle\}$ for all $u \in A_1$. That is, $W_1(v_6) = \{(v_4, v_6), (v_3, v_5, v_6), (v_6, v_7, v_8), (v_3, v_4, v_8)\}$ and $W_1(v_9) = \{(v_8, v_9), (v_7, v_9), (v_8, v_7), (v_9, v_{10}, v_{11}), (v_{11}, v_{12}), (v_9, v_{13}, v_{12})\}$. Then (W_1, A_1) is a soft set over X_p. Also $H_1(v_6) = \left(Q_1(v_6), W_1(v_6)\right)$ and $H_1(v_9) = \left(Q_1(v_9), W_1(v_9)\right)$ are partial semi-graphs of G^* as shown in Fig. 7. Hence $G_1 = \{H_1(v_6), H_1(v_9)\}$ is a soft semi-graph of G^*.

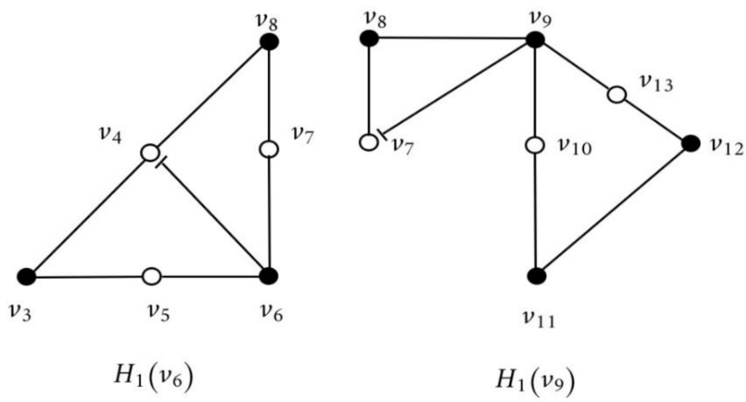

Fig. 7. Soft Semi-graph $G_1 = \{H_1(v_6), H_1(v_9)\}$.

Also, let $A_2 = \{v_8\} \subseteq V$ be another parameter set. Define a function $Q_2: A_2 \to \mathcal{P}(V)$ by $Q_2(u) = \{v \in V: uRv \Leftrightarrow u = v$ or u and v are adjacent$\}$ for all $u \in A_2$. That is, $Q_2(v_8) = \{v_3, v_4, v_6, v_7, v_8, v_9\}$. Then (Q_2, A_2) is a soft set over V. Define another function $W_2: A_2 \to \mathcal{P}(X_p)$ by $W_2(u)$ = $\{$mp-edges $\langle Q_2(u)\rangle\}$ for all $u \in A_2$. That is, $W_2(v_8) = \{(v_3, v_4, v_8), (v_6, v_7, v_8), (v_4, v_6), (v_7, v_9), (v_8, v_9)\}$. Then (W_2, A_2) is a soft set over X_p. Also $H_2(v_8) = \left(Q_2(v_8), W_2(v_8)\right)$ is a partial semi-graph of G^* as shown in Fig. 8. Hence $G_2 = \{H_2(v_8)\}$ is a soft semi-graph of G^*.

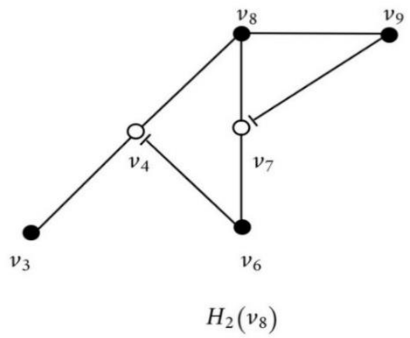

Fig. 8. Soft Semi-graph $G_2 = \{H_2(v_8)\}$.

Thus, we get two soft semi-graphs G_1 and G_2 of the semi-graph G^*.
Then $G_1 \wedge G_2 = (G^*, Q, W, A)$, where $A = A_1 \times A_2 = \{(v_6, v_8), (v_9, v_8)\}$, $Q(v_6, v_8) = Q_1(v_6) \cap Q_2(v_8) = \{v_3, v_4, v_6, v_7, v_8\}$, $W(v_6, v_8) =$ {mp-edges $\langle Q(v_6, v_8)\rangle\} = \{(v_3, v_4, v_8), (v_4, v_6), (v_6, v_7, v_8)\}$, $Q(v_9, v_8) = Q_1(v_9) \cap Q_2(v_8) = \{v_7, v_8, v_9\}$ and $W(v_9, v_8) =$ {mp-edges $\langle Q(v_9, v_8)\rangle\} = \{(v_8, v_7), (v_8, v_9), (v_7, v_9)\}$. Here A is the parameter set, (Q, A) is a soft set over V and (W, A) is a soft set over X_p. Also $H(v_6, v_8) = \left(Q(v_6, v_8), W(v_6, v_8)\right)$ and $H(v_9, v_8) = \left(Q(v_9, v_8), W(v_9, v_8)\right)$ are partial semi-graphs of G^*. Hence $G_1 \wedge G_2 = \{H(v_6, v_8), H(v_9, v_8)\}$ is a soft semi-graph of G^* and is given in Fig. 9.

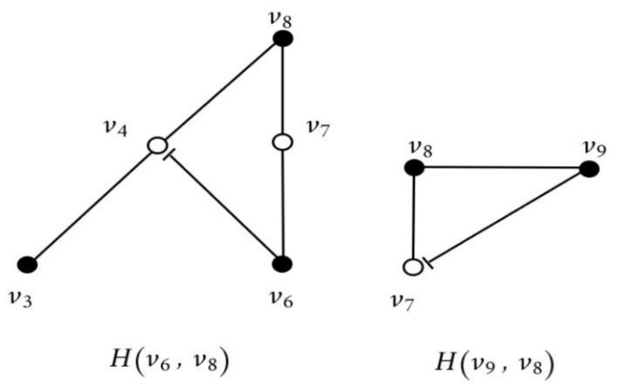

Fig. 9. $G_1 \wedge G_2 = \{H(v_6, v_8), H(v_9, v_8)\}$.

Theorem 3.1: Let $G^* = (V, X)$ be a semi-graph and $G_1 = (G^*, Q_1, W_1, A_1)$ and $G_2 = (G^*, Q_2, W_2, A_2)$ be two soft semi-graphs of G^* such that $Q_1(u) \cap Q_2(v) \neq \phi$ for $(u, v) \in A_1 \times A_2$. Then $G_1 \wedge G_2$ is also a soft semi-graph of G^*.

Proof. AND operation on G_1 and G_2 is defined as $G_1 \wedge G_2 = G = (G^*, Q, W, A)$, where the parameter set $A = A_1 \times A_2$ and for all $(u, v) \in A = A_1 \times A_2$, $Q(u, v) = Q_1(u) \cap Q_2(v)$ and $W(u, v) = \{mp$-edges $\langle Q_1(u) \cap Q_2(v)\rangle\} = \{mp$-edges $\langle Q(u, v)\rangle\}$. Clearly, Q is a mapping from A to $\mathcal{P}(V)$ and W is a mapping from A to $\mathcal{P}(X_p)$. So (Q, A) is a soft set over V and (W, A) is a soft set over X_p. When $(u, v) \in A = A_1 \times A_2$, the corresponding p-part of $G_1 \wedge G_2$ is $H(u, v) = (Q(u, v), W(u, v))$, where $Q(u, v) = Q_1(u) \cap Q_2(v)$ and $W(u, v) = \{mp\text{-edges } \langle Q_1(u) \cap Q_2(v) \rangle\} = \{mp\text{-edges } \langle Q(u, v) \rangle\}$. Here $Q_1(u) \cap Q_2(v) \subseteq V$ and each f-edge in $W(u, v)$ is a partial edge of an edge in G^*. So $H(u, v)$ is a partial semi-graph of G^* for every $(u, v) \in A$. That is, $G = G_1 \wedge G_2$ is a soft semi-graph of G^* since the following conditions are satisfied:

1. $G^* = (V, X)$ is a semi-graph,
2. $A = A_1 \times A_2$ is a nonempty set of parameters,
3. (Q, A) is a soft set over V,
4. (W, A) is a soft set over X_p,
5. $H(u, v) = (Q(u, v), W(u, v))$ is a partial semi-graph of G^* for all $(u, v) \in A = A_1 \times A_2$.

4. OR Operation in Soft Semi-Graphs

Let $G^* = (V, X)$ be a semi-graph having a vertex set V and edge set X. Also, let $G_1 = (G^*, Q_1, W_1, A_1)$ and $G_2 = (G^*, Q_2, W_2, A_2)$ be two soft semi-graphs of G^* . The *OR operation* on G_1 and G_2 denoted by $G_1 \vee G_2$ is defined as $G_1 \vee G_2 = G = (G^*, Q, W, A)$, where $A = A_1 \times A_2$ and for all $(u, v) \in A = A_1 \times A_2$, $Q(u, v) = Q_1(u) \cup Q_2(v)$ and $W(u, v) = \{mp\text{-edges } \langle Q_1(u) \cup Q_2(v) \rangle\} = \{mp\text{-edges } \langle Q(u, v) \rangle\}$.
If $H(u, v) = (Q(u, v), W(u, v)), \forall (u, v) \in A$, then $G_1 \vee G_2 = \{H(u, v): (u, v) \in A\}$.

Example 4.1: Consider the semi-graph G^* given in Figure 6 and its soft semi-graphs G_1 and G_2 given in Figs. 7 and 8 respectively.

Then $G_1 \vee G_2 = (G^*, Q, W, A)$, where $A = A_1 \times A_2 = \{(v_6, v_8), (v_9, v_8)\}$,$Q(v_6, v_8) = Q_1(v_6) \cup Q_2(v_8) = \{v_3, v_4, v_5, v_6, v_7, v_8, v_9\}$, $W(v_6, v_8) =$ {mp-edges$\langle Q(v_6, v_8) \rangle$} $= \{(v_3, v_4, v_8), (v_3, v_5, v_6), (v_4, v_6), (v_6, v_7, v_8), (v_8, v_9), (v_7, v_9)\}$, $Q(v_9, v_8) = Q_1(v_9) \cup Q_2(v_8) = \{v_3, v_4, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}, v_{13}\}$ and $W(v_9, v_8) =$ {mp-edges⟨Q(v_9, v_8) ⟩}= {(v_3, v_4, v_8), (v_6, v_7, v_8), (v_4, v_6),(v_8, v_9),(v_7, v_9),(v_9, v_{10}, v_{11}),(v_{11}, v_{12}}),(v_9, v_{13}, v_{12})}. Here A is the parameter set, (Q, A) is a soft set over V and (W, A) is a soft set over X_p. Also $H(v_6, v_8) = (Q(v_6, v_8), W(v_6, v_8))$ and $H(v_9, v_8) = (Q(v_9, v_8), W(v_9, v_8))$ are partial semi-graphs of G^*. Hence $G_1 \vee G_2 = \{H(v_6, v_8), H(v_9, v_8)\}$ is a soft semi-graph of G^* and is given in Fig. 10.

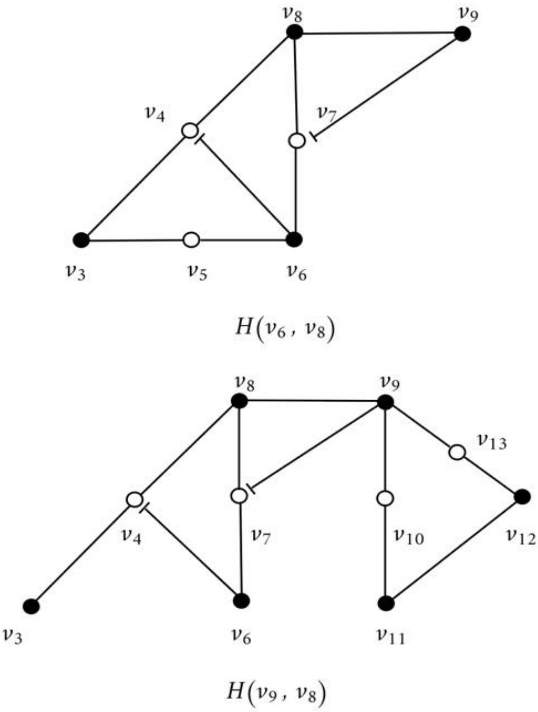

Fig. 10. $G_1 \vee G_2 = \{H(v_6, v_8), H(v_9, v_8)\}$.

Theorem 4.1: Let $G^* = (V, X)$ be a semi-graph and $G_1 = (G^*, Q_1, W_1, A_1)$ and $G_2 = (G^*, Q_2, W_2, A_2)$ be two soft semi-graphs of G^*. Then $G_1 \vee G_2$ is also a soft semi-graph of G^*.

Proof. OR operation on G_1 and G_2 is defined as $G_1 \vee G_2 = G = (G^*, Q, W, A)$, where the parameter set $A = A_1 \times A_2$ and for all $(u, v) \in A = A_1 \times A_2$, $Q(u, v) = Q_1(u) \cup Q_2(v)$ and $W(u, v) = \{mp\text{-edges } \langle Q_1(u) \cup Q_2(v) \rangle\} = \{mp\text{-edges } \langle Q(u, v) \rangle\}$. Clearly, Q is a mapping from A to $\mathcal{P}(V)$ and W is a mapping from A to $\mathcal{P}(X_p)$. So (Q, A) is a soft set over V and (W, A) is a soft set over X_p. When $(u, v) \in A = A_1 \times A_2$, the corresponding p-part of $G_1 \vee G_2$ is $H(u, v) = (Q(u, v), W(u, v))$, where $Q(u, v) = Q_1(u) \cup Q_2(v)$ and $W(u, v) = \{mp\text{-edges } \langle Q_1(u) \cup Q_2(v) \rangle\} = \{mp\text{-edges } \langle Q(u, v) \rangle\}$. Here $Q_1(u) \cup Q_2(v) \subseteq V$ and each f-edge in $W(u, v)$ is a partial edge of an edge in G^*. So $H(u, v)$ is a partial semi-graph of G^* for every $(u, v) \in A$. That is, $G = G_1 \vee G_2$ is a soft semi-graph of G^* since the following conditions are satisfied:

1. $G^* = (V, X)$ is a semi-graph,
2. $A = A_1 \times A_2$ is a nonempty set of parameters,
3. (Q, A) is a soft set over V,
4. (W, A) is a soft set over X_p,

5. $H(u,v) = (Q(u,v), W(u,v))$ is a partial semi-graph of G^* for all $(u,v) \in A = A_1 \times A_2$.

Theorem 4.2: Let $G^* = (V, X)$ be a semi-graph and $G_1 = (G^*, Q_1, W_1, A_1)$ and $G_2 = (G^*, Q_2, W_2, A_2)$ be two soft semi-graphs of G^* such that $Q_1(u) \cap Q_2(v) \neq \phi$ for $(u,v) \in A_1 \times A_2$. Then $G_1 \wedge G_2$ is a soft partial semi-graph of $G_1 \vee G_2$.

Proof. By Theorems 3.1 and 4.1, we have $G_1 \wedge G_2$ and $G_1 \vee G_2$ are soft semi-graphs of G^*. Assume that $G_1 \wedge G_2 = G_\wedge = (G^*, Q_\wedge, W_\wedge, A_\wedge)$ and $G_1 \vee G_2 = G_\vee = (G^*, Q_\vee, W_\vee, A_\vee)$. By the definitions of AND and OR operations, the parameter sets of $G_\wedge$ and $G_\vee$ are respectively $A_\wedge = A_1 \times A_2$ and $A_\vee = A_1 \times A_2$. Clearly $A_\wedge \subseteq A_\vee$. For $(u,v) \in A_\wedge = A_1 \times A_2$, the corresponding p-part $H_\wedge$ of $G_\wedge$ is $(Q_\wedge, W_\wedge)$, where $Q_\wedge(u,v) = Q_1(u) \cap Q_2(v)$ and $W_\wedge(u,v) = \{mp\text{-edges } \langle Q_1(u) \cap Q_2(v) \rangle\} = \{mp\text{-edges } \langle Q_\wedge(u,v) \rangle\}$. Also for $(u,v) \in A_\vee = A_1 \times A_2$, the corresponding p-part $H_\vee$ of $G_\vee$ is $(Q_\vee, W_\vee)$, where $Q_\vee(u,v) = Q_1(u) \cup Q_2(v)$ and $W_\vee(u,v) = \{mp\text{-edges } \langle Q_1(u) \cup Q_2(v) \rangle\} = \{mp\text{-edges } \langle Q_\vee(u,v) \rangle\}$. Clearly $Q_\wedge(u,v) \subseteq Q_\vee(u,v)$ since $Q_1(u) \cap Q_2(v) \subseteq Q_1(u) \cup Q_2(v)$. Also, each partial edge in $W_\wedge(u,v)$ is a partial edge of an edge in $W_\vee(u,v)$. So $H_\wedge(u,v)$ is a partial semi-graph of $1.G_2$ since the following conditions are satisfied:

1. $A_\wedge \subseteq A_\vee$,
2. $H_\wedge(u,v) = (Q_\wedge(u,v), W_\wedge(u,v))$ is a partial semi-graph of $H_\vee(u,v) = (A_\vee(u,v), B_\vee(u,v))$ for all $(u,v) \in A_\wedge = A_1 \times A_2$.

5. Some Measurements in Soft Semi-Graphs

Definition 5.1: A *soft walk* or *s-walk* in a soft semi-graph G is an alternating sequence $v_0E_1v_2E_2 \dots v_{n-1}E_nv_n$ of vertices and fp-edges, beginning with the vertex v_0 and ending with the vertex v_n such that v_{i-1} and v_i are the end vertices or partial end vertices of the fp-edge $E_i, 1 \leq i \leq n$. This s-walk is called a $v_0 - v_n$ s-walk. Here v_0 is called the *origin* and v_n is called the *terminus* of the s-walk. A $v_0 - v_n$ s-walk is *closed* if $v_0 = v_n$. Otherwise, it is called *open*. Also, we can denote a $v_0 - v_n$ s-walk by writing the vertices of the fp-edge E_i instead of E_i. In other words, an s-walk can be represented by a sequence of vertices like $v_0v_1v_2v_3 \dots v_{n-1}v_n$ in which the vertices v_i and v_{i-1} are consecutively adjacent. An s-walk is called *trivial* if it has no fp-edges.

Definition 5.2: An s-walk $v_0E_1v_2E_2 \dots v_{n-1}E_nv_n$ is called a *soft trail* or an *s-trail*, if the fp-edges $E_1, E_2, \dots, E_n$ are such that $E_i \neq E_j$ or E_i is not a partial edge of $E_j, \forall i, j = 1,2, \dots, n$. In an s-trail, vertices may be repeated. Also, note that the fp edges in the form $(v_1, v_2, \dots, v_{n-1}, v_n)$ and $(v_n, v_{n-1}, \dots, v_2, v_1)$ are the same.

Remark 5.1: Suppose $E = (v_1, v_2, \dots, v_i, v_{i+1}, \dots v_r, v_{r+1}, v_{n-1}, v_n)$ is an f-edge of the soft semi-graph G. Then, we treat $(v_i, v_{i+1}, \dots, v_r, v_{r+1})$ and $(v_{r+1}, v_r, \dots, v_{i+1}, v_i)$ as the same partial edge of E. Keep this in mind, while verifying the conditions for an s-trail. For example, if $E = (v_1, v_3, v_4, v_5)$ is an edge of G, then $E_1 = (v_3, v_4, v_5)$ and $E_2 = (v_4, v_3)$ are partial edges of E. Also, E_2 is a partial edge of E_1.

Definition 5.3: A $v_0 - v_n$ *soft path* or a $v_0 - v_n$ *s-path* is a $v_0 - v_n$ s-trail, in which all the vertices are distinct. An s-path will also be an s-trail.

Definition 5.4: The *length* of an s-walk is defined as the number of fp-edges present in that s-walk. Similarly, we define the length of an s-trail or an s-path.

Definition 5.5: Let $G = (G^*, Q, W, A)$ be a soft semi-graph represented by $\{H(x): x \in A\}$ and let u and v be any two vertices of G. Then we say that u and v are *connected* in G, if there is an s-path between u and v in at least one p-part $H(x)$ of G. A p-part $H(x)$ of G for some $x \in A$, is said to be *connected* if every two of its vertices are connected. That is, the p-part $H(x)$ is connected if for every two vertices in $Q(x)$, there is an s-path between them in that p-part.

Definition 5.6: A soft semi-graph G is said to be *connected* if every two of its vertices are connected. i.e., for every two vertices in $\cup_{x\in A} Q(x)$, there is an s-path between them in at least one p-part $H(x)$ of G.

Definition 5.7: Let $G^* = (V, X)$ be a semi-graph and $G = (G^*, Q, W, A)$ be a soft semi-graph of G^* represented by $\{H(x): x \in A\}$. Let u and v be any two vertices in a connected p-part $H(x)$ of G. Then, the *p-part distance* between u and v in $H(x)$, denoted by $d(u, v)[H(x)]$, is defined as the length of the shortest $u - v$ s-path in $H(x)$. That is, $d(u, v)[H(x)]$ is the number of fp-edges included in the shortest $u - v$ s-path in $H(x)$.

Definition 5.8: Let v be a vertex in a connected p-part $H(x)$ of G. That is, $v \in Q(x)$. Then, the *p-part eccentricity* of v in $H(x)$, denoted by $e(v)[H(x)]$, is defined as $e(v)[H(x)] = \max\{d(u, v)[H(x)]: u \in Q(x), u \neq v\}$.

Definition 5.9: The *p-part radius* of a connected p-part $H(x)$ of G, denoted by $r_p[H(x)]$, can be defined as $r_p[H(x)] = \min\{e(v)[H(x)]: v \in Q(x)\}$.

Definition 5.10: The *p-part diameter* of a connected p-part $H(x)$ of G, denoted by $d_p[H(x)]$, can be defined as $d_p[H(x)] = \max\{e(v)[H(x)]: v \in Q(x)\}$.

Definition 5.11: The *soft radius* or *s-radius* $r_s(G)$ of a soft semi-graph G, whose p-part $H(x)$ is connected, $\forall x \in A$, is defined as $r_s(G) = \min\{r_p[H(x)]: x \in A\}$.

Definition 5.12: The *soft diameter* or *s-diameter* $d_s(G)$ of a soft semi-graph G whose p-part $H(x)$ is connected, $\forall x \in A$, is defined as $d_s(G) = \max\{d_p[H(x)]: x \in A\}$.

Definition 5.13: The *p-part center* of a connected p-part $H(x)$ of G, denoted by $C_p[H(x)]$, is defined as the set of all vertices in $Q(x)$ such that their p-part eccentricity equal to the corresponding p-part radius. That is, $C_p[H(x)] = \{v \in Q(x): e(v)[H(x)] = r_p[H(x)]\}$.

Definition 5.14: The *soft center* or the *s-center* $C_s(G)$ of a soft semi-graph G, whose p-part $H(x)$ is connected, $\forall x \in A$, is defined as the set of all p-part centers of G. That is, $C_s(G) = \{v \in C_p[H(x)]: x \in A\}$.

Example 5.1: Let $G^* = (V, X)$ be a semi-graph given in Fig. 11,

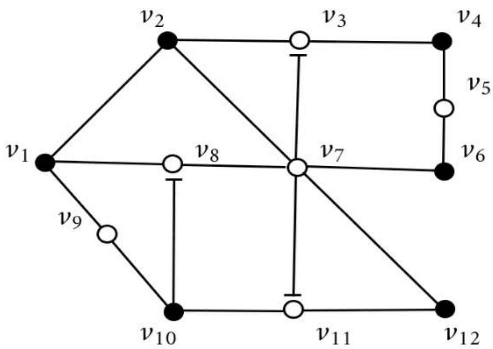

Fig. 11. Semi-graph $G^* = (V, X)$.

Here V= $\{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}, v_{12}\}$ and X= $\{(v_1, v_2), (v_2, v_3, v_4), (v_1, v_8, v_7, v_6), (v_1, v_9, v_{10}), (v_{10}, v_{11}, v_{12}), (v_8, v_{10}), (v_2, v_7, v_{12}), (v_3, v_7, v_{11}), (v_4, v_5, v_6)\}$.

Let $A = \{v_1, v_3\} \subseteq V$ be a parameter set. Define Q from A to $\mathcal{P}(V)$ by $Q(x) = \{y \in V | xRy \Leftrightarrow x = y$ or x and y are adjacent in $G^*\}, \forall x \in A$ and W from A to $\mathcal{P}(X_p)$ by $W(x) = \{mp\text{-edges}\langle Q(x)\rangle\}, \forall x \in A$.

That is, $Q(v_1) = \{v_1, v_2, v_9, v_{10}, v_8, v_7, v_6\}$ and $Q(v_3) = \{v_2, v_3, v_4, v_7, v_{11}\}$.

Also $W(v_1) = \{(v_1, v_2), (v_1, v_8, v_7, v_6), (v_8, v_{10}), (v_1, v_9, v_{10}\}$ and $W(v_3) = \{(v_2, v_3, v_4), (v_3, v_7, v_{11}), (v_2, v_7)\}$. Here $H(v_1) = \big(Q(v_1), W(v_1)\big)$ and $H(v_3) = \big(Q(v_3), W(v_3)\big)$ are partial semi-graphs of G^* as shown below in Fig. 12.

Hence $G = \{H(v_1), H(v_3)\}$ is a soft semi-graph of G^*. Here, the p-parts $H(v_1)$ and $H(v_3)$ are connected since every two vertices of $H(v_1)$ and $H(v_3)$ are connected by an s-path in the corresponding p-part. For every vertex v in $Q(v_1)$, we define the p-part eccentricity, $e(v)[H(v_1)]$ as follows: $e(v)[H(v_1)]$ =max$\{d(u,v)[H(v_1)]: u \in Q(v_1), u \neq v\}$. That is, $e(v_1)[H(v_1)] = 1, e(v_2)[H(v_1)] = 2, e(v_8)[H(v_1)] = 2, e(v_7)[H(v_1)] = 2, e(v_6)[H(v_1)] = 2, e(v_9)[H(v_1)] = 2, e(v_{10})[H(v_1)] = 2$. Therefore, the p-part radius, $r_p[H(v_1)]$ =min$\{e(v)[H(v_1)]: v \in Q(v_1)\} = 1$ and the p-part diameter, $d_p[H(v_1)]$ =max$\{e(v)[H(v_1)]: v \in Q(v_1)\} = 2$. For every vertex v in $Q(v_3)$, we define the p-part eccentricity, $e(v)[H(v_3)]$ as follows: $e(v)[H(v_3)]$ =max$\{d(u,v)[H(v_3)]: u \in Q(v_3), u \neq v\}$. That is, $e(v_2)[H(v_3)] = 2, e(v_3)[H(v_3)] = 1, e(v_4)[H(v_3)] = 2, e(v_7)[H(v_3)] = 2, e(v_{11})[H(v_3)] = 2$. Therefore, the p-part radius, $r_p[H(v_3)]$ =min$\{e(v)[H(v_3)]: v \in Q(v_3)\} = 1$ and the p-part diameter, $d_p[H(v_3)]$ =max$\{e(v)[H(v_3)]: v \in Q(v_3)\} = 2$.

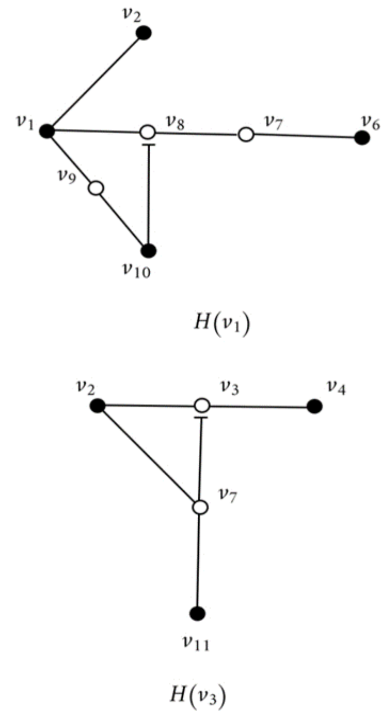

Fig. 12. Soft Semi-graph $G = \{H(v_1), H(v_3)\}$.

Therefore, the s-radius, $r_s(G)$ =min$\{r_p[H(x)]: x \in A\}$ =min$\{r_p[H(v_1)], r_p[H(v_3)]\}$ =min$\{1,1\} = 1$ and the s-diameter, $d_s(G)$ =max$\{d_p[H(x)]: x \in A\}$ =max$\{d_p[H(v_1)], d_p[H(v_3)]\}$ =max$\{2,2\} = 2$. Also, the p-part centers $C_p[H(v_1)] = \{v \in Q(v_1): e(v)[H(v_1)] = r_p[H(v_1)]\} = \{v_1\}$ and $C_p[H(v_3)] = \{v \in Q(v_3): e(v)[H(v_3)] = r_p[H(v_3)]\} = \{v_3\}$. Therefore, the s-center, $C_s(G) = \{v \in C_p[H(x)]: x \in A\} = \{v_1, v_3\}$.

Remark 5.2: We define p-part eccentricity, p-part radius, and p-part diameter in a p-part $H(x)$ of G, if $H(x)$ is connected. Also, we define s-radius and S diameter of a soft semi-graph G if all p-parts of G are connected. If two vertices u and v are not connected by an s-path in a p-part $H(x)$, we define $d(u,v)[H(x)]$ to be infinite.

Theorem 5.1: For any three vertices u, v, w in a p-part $H(x)$ of G, we have $d(u,w)[H(x)] \leq d(u,v)[H(x)] + d(v,w)[H(x)]$.

Proof. If there is no s-path connecting u and v or v and w, then $d(u,v)[H(x)] = \infty$ or $d(v,w)[H(x)] = \infty$ and we have nothing to prove. Suppose that this is not the case in $H(x)$. Then, there is a $u - v$ s-path of length $d(u,v)[H(x)]$ and a $v - w$ s-path of length $d(v,w)[H(x)]$. Joining these two s-paths together, we get an s-walk between u and w of length $d(u,v)[H(x)] + d(v,w)[H(x)]$. This s-walk must contain a $u - v$ s-path having a length not greater than $d(u,v)[H(x)] + d(v,w)[H(x)]$. The length of the shortest $u - w$

s-path is denoted by $d(u,w)[H(x)]$. So, definitely $d(u,w)[H(x)] \leq d(u,v)[H(x)] + d(v,w)[H(x)]$.

Theorem 5.2: If $H(x)$ is a connected p-part of a soft semi-graph G having at least three vertices, then $r_p[H(x)] \leq d_p[H(x)] \leq 2r_p[H(x)]$.

Proof. By the definitions of p-part radius and p-part diameter itself, $r_p[H(x)] \leq d_p[H(x)]$. So, we have to prove that. $d_p[H(x)] \leq 2r_p[H(x)]$. Let u and w be two vertices in $H(x)$ such that $d(u,w)[H(x)] = d_p[H(x)]$. Let v be any vertex in the p-part center of $H(x)$. Then, we have $d_p[H(x)] = d(u,w)[H(x)] \leq d(u,v)[H(x)] + d(v,w)[H(x)] \leq 2e(v)[H(x)] = 2r_p[H(x)]$. That is, $r_p[H(x)] \leq d_p[H(x)] \leq 2r_p[H(x)]$.

Theorem 5.3: Let u and w be any two vertices in a connected p-part $H(x)$ of a soft semi-graph G. If $d(u,w)[H(x)] \geq 2$, then there is a vertex v in $H(x)$ such that $d(u,w)[H(x)] = d(u,v)[H(x)] + d(v,w)[H(x)]$.

Proof. Assume that $d(u,w)[H(x)] \geq 2$. That is, there is a shortest $u - w$ s-path of length at least two. So, there must be at least one vertex v in this path between u and w. Then, the portion of the $u - w$ s-path from u to v must be the shortest $u - v$ s-path. That is, any $u - v$ s-path P present in $H(x)$ has a length greater than or equal to the length of this $u - v$ s-path. Otherwise, we will get a $u - w$ s-path having a length less than $d(u,w)[H(x)]$, by replacing the portion from u to v in the shortest $u - w$ s-path by P, which is not possible. Similarly, we get the portion of the shortest $u - w$ s-path from v to w as the shortest $v - w$ s-path. Therefore, $d(u,w)[H(x)] = d(u,v)[H(x)] + d(v,w)[H(x)]$.

6. Conclusion

Soft semi-graphs emerged by integrating the principles of soft set theory into semigraphs. Parameterization furnishes a range of descriptions of intricate relations represented by semigraphs. This theory of soft semi-graph holds significance for its adeptness in employing parameterization. Our study introduces AND and OR operations within soft semi-graphs and explores their characteristics. Additionally, we present various measurements such as distance, radius, diameter, and center within the realm of soft semi-graphs.

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